Lifetime difference in B_s mixing:Standard Model and Beyond

Andriy Badin

Wayne State University

Based on: A.B., Fabrizzio Gabbiani and Alexey A. Petrov Phys.Lett.B653:230-240,2007

B_s mixing: Standard Model and Beyond

- General formalism for mixing phenomena
- B_s mixing in the framework of Standard Mode
- Generic description of mixing phenomena in theories beyond the Standard Model
- Results for particular models

[A.Badin, F. Gabbiani, A.Petrov, Physics Letters B 653 (2007) 230–240]

$$\left|\Psi(t)\right\rangle = \begin{pmatrix} a(t) \\ b(t) \end{pmatrix} \equiv a(t)\left|B_s\right\rangle + b(t)\left|\overline{B_s}\right\rangle \qquad \mathrm{M} - \frac{i}{2}\mathit{\Gamma} = \begin{pmatrix} M_{11} - \frac{i}{2}\Gamma_{11} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{21} - \frac{i}{2}\Gamma_{21} & M_{22} - \frac{i}{2}\Gamma_{22} \end{pmatrix}$$

$$i\frac{d}{dt}\Psi = \left(M - \frac{i}{2}\Gamma\right)\Psi$$

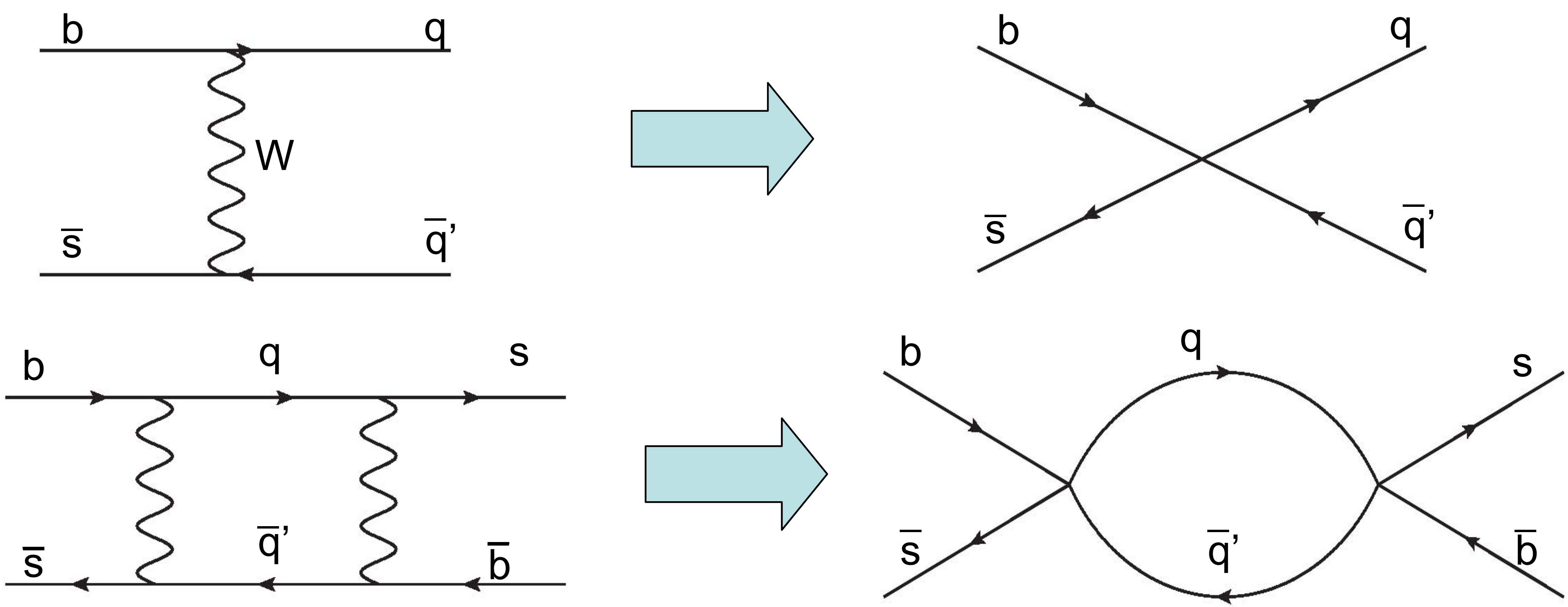
$$M_{11} = M_{22} = M_{B_S}$$

$$\Gamma_{11} = \Gamma_{22} = \Gamma_{B_S}$$

We use optical theorem to relate the off diagonal elements of mixing matrix to the imaginary part of matrix element of the transition operator

$$\Delta\Gamma = -2\Gamma_{12} \quad \text{where} \quad \Gamma_{12} = \frac{1}{2M_{B_s}}\left\langle\overline{B_s}\right|T\left|B_s\right\rangle$$

$$T = \mathrm{Im}\, i\int d^4x\, \mathrm{T}\left(H_{eff}(x)H_{eff}(0)\right)$$



Because of CKM matrix structure, only $q = c$ and $\bar{q} = \bar{c}$ contribute into B_s-$\bar{B}_s$ lifetime difference

B_s Lifetime difference at Leading Order

$$\mathcal{T} = -\frac{G_F^2 m_b^2}{12\pi}(V_{cb}V_{cs}^*)^2 \;\left[F(z)Q(\mu_2) + F_S(z)Q_S(\mu_2)\right]$$

In the heavy-quark limit, the energy release is large, so the correlator is dominated by short-distance physics. An Operator Product Expansion (OPE) can be constructed, which results in an expansion as a series of matrix elements of local operators of increasing dimension suppressed by powers of $1/m_b$

$$\Gamma_{21}(B_s) = -\frac{G_F^2 m_b^2}{12\pi(2M_{B_s})}(V_{cb}V_{cs}^*)^2\sqrt{1-4z}\,\times$$
$$\times\left\{\left[(1-z)\,\left(2\,C_1C_2 + N_cC_2^2\right) + (1-4z)C_1^2/2\right]Q + (1+2z)\left(2\,C_1C_2 + N_cC_2^2 - C_1^2\right)Q_S\right\}$$

$$z = m_c^2/m_b^2 \quad Q = (\bar{b}_i s_i)_{V-A}(\bar{b}_j s_j)_{V-A} \qquad Q_S = (\bar{b}_i s_i)_{S-P}(\bar{b}_j s_j)_{S-P}$$

$$\langle\overline{B}_s|Q|B_s\rangle = f_{B_s}^2 M_{B_s}^2\, 2\left(1+\frac{1}{N_c}\right)B,\ \langle\overline{B}_s|Q_S|B_s\rangle = -f_{B_s}^2 M_{B_s}^2\,\frac{M_{B_s}^2}{(m_b+m_s)^2}\left(2-\frac{1}{N_c}\right)B_S$$

QCD corrections to these Wilson coefficients are also known as well as contribution from penguin diagram

M. Beneke, G. Buchalla, and I. Dunietz, Phys. Rev. D **54**, 4419 (1996).

M. Beneke, G. Buchalla, C. Greub, A. Lenz, and U. Nierste, Phys. Lett. B **459**, 631 (1999) [arXiv:hep-ph/9808385].

Adding $1/m^n$ corrections

$$\Gamma_{21}(B_s) = -\frac{G_F^2 m_b^2}{12\pi(2M_{B_s})}(V_{cb}V_{cs}^*)^2\ \left\{[F(z)+P(z)]\,Q + [F_S(z)+P_S(z)]\,Q_S + \delta_{1/m} + \delta_{1/m^2}\right\}$$

1/m corrections are obtained by expansion of forward scattering amplitude in terms of light quark momentum and by matching result onto operators with derivatives

$$\delta_{1/m} = \sqrt{1-4z}\left\{(1+2z)\left[C_1^2\ (R_2+2R_4) - 2\,(2\,C_1C_2 + N_cC_2^2)\ (R_1+R_2)\right]\right.$$
$$\left. -\ \frac{12z^2}{1-4z}\left[(2\,C_1C_2+N_cC^2)\ (R_2+2R_3) + 2\,C_1^2\,R_3\right]\right\},$$

$$R_1 = \frac{m_s}{m_b}\bar{b}_i\gamma^\mu(1-\gamma_5)s_i\ \bar{b}_j\gamma_\mu(1+\gamma_5)s_j\,,\ \ R_2 = \frac{1}{m_b^2}\bar{b}_i\overleftarrow{D}_\rho\gamma^\mu(1-\gamma_5)\overrightarrow{D}^\rho s_i\ \bar{b}_j\gamma_\mu(1-\gamma_5)s_j\,,$$
$$R_3 = \frac{1}{m_b^2}\bar{b}_i\overleftarrow{D}_\rho(1-\gamma_5)\overrightarrow{D}^\rho s_i\ \bar{b}_j(1-\gamma_5)s_j\,,\ \ R_4 = \frac{1}{m_b}\bar{b}_i(1-\gamma_5)i\overrightarrow{D}_\mu s_i\ \bar{b}_j\gamma^\mu(1-\gamma_5)s_j\,.$$

$$\langle\overline{B}_s|R_1|B_s\rangle = \left(2+\frac{1}{N_c}\right)\frac{m_s}{m_b}\,f_{B_s}^2M_{B_s}^2\,B_1^s,\qquad \langle\overline{B}_s|R_2|B_s\rangle = \left(-1+\frac{1}{N_c}\right)f_{B_s}^2M_{B_s}^2\left(\frac{M_{B_s}^2}{m_b^2}-1\right)B_2^s$$
$$\langle\overline{B}_s|R_3|B_s\rangle = \left(1+\frac{1}{2N_c}\right)f_{B_s}^2M_{B_s}^2\left(\frac{M_{B_s}^2}{m_b^2}-1\right)B_3^s,\quad \langle\overline{B}_s|R_4|B_s\rangle = -f_{B_s}^2M_{B_s}^2\left(\frac{M_{B_s}^2}{m_b^2}-1\right)B_4^s\,.$$

M. Beneke, G. Buchalla, and I. Dunietz, Phys. Rev. D **54**, 4419 (1996).

Adding $1/m^n$ corrections II

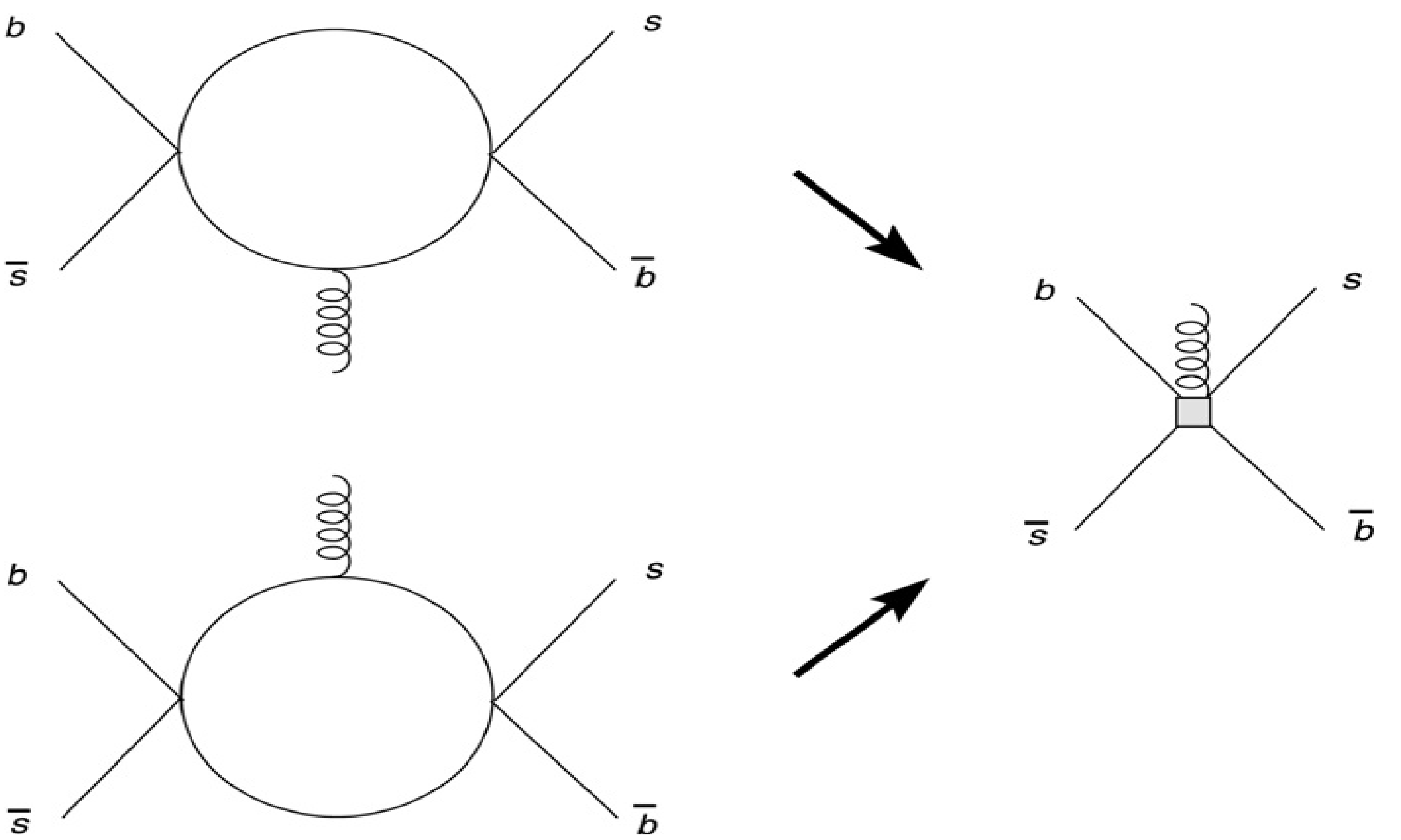
Two classes of corrections arise at this order – kinetic corrections which can be computed in the way analogous to previous case and ones arising from interaction background gluonic field. Kinetic corrections are:

$$
\begin{aligned}
\delta_{1/m^2} &= \frac{24z^2}{(1-4z)^2}(3-10z)\left[C_1^2 W_3 + (2\,C_1C_2 + N_c C_2^2)(W_3 + W_2/2)\right] \\
&- \frac{12z^2}{1-4z}\frac{m_s^2}{m_b^2}\left[C_1^2 Q_S - (2\,C_1C_2 + N_c C_2^2)(Q_S + Q/2)\right] \\
&+ \frac{24z^2}{1-4z}\left[2C_1^2 W_4 - 2\,(2\,C_1C_2 + N_c C_2^2)(W_1 + W_2/2)\right] \\
&- (1-2z)\frac{m_s^2}{m_b^2}(C_1^2 + 2\,C_1C_2 + N_c C_2^2)Q_R.
\end{aligned}
$$

$$
\begin{aligned}
Q_R &= (\bar{b}_i s_i)_{S+P}(\bar{b}_j s_j)_{S+P}\,, \\
W_1 &= \frac{m_s}{m_b}\bar{b}_i \overleftarrow{D}^{\alpha}(1-\gamma_5)\overrightarrow{D}_{\alpha} s_i\; \bar{b}_j(1+\gamma_5)s_j\,, \\
W_2 &= \frac{1}{m_b^4}\bar{b}_i \overleftarrow{D}^{\alpha}\overleftarrow{D}^{\beta}\gamma^{\mu}(1-\gamma_5)\overrightarrow{D}_{\alpha}\overrightarrow{D}_{\beta} s_i\; \bar{b}_j\gamma_{\mu}(1-\gamma_5)s_j\,, \\
W_3 &= \frac{1}{m_b^4}\bar{b}_i \overleftarrow{D}^{\alpha}\overleftarrow{D}^{\beta}(1-\gamma_5)\overrightarrow{D}_{\alpha}\overrightarrow{D}_{\beta} s_i\; \bar{b}_j(1-\gamma_5)s_j\,, \\
W_4 &= \frac{1}{m_b^4}\bar{b}_i \overleftarrow{D}^{\alpha}(1-\gamma_5) i\overrightarrow{D}_{\mu}\overrightarrow{D}_{\alpha} s_i\; \bar{b}_j\gamma^{\mu}(1-\gamma_5)s_j\,,
\end{aligned}
$$

New operators arising at this order

Adding $1/m^n$ corrections III (Gluonic)



$$\mathcal{T}^{G}_{1/\mathrm{m}^2} = - \frac{G_F^2 (V_{cb} V_{cs}^*)^2}{4\pi\sqrt{1-4z}} \left\{ C_1^2 \left[(1-4z) P_1 - (1-4z) P_2 + 4z P_3 - 4z P_4 \right] \right.$$
$$\left. + \; 4\, C_1 C_2 z \left[P_5 + P_6 - P_7 - P_8 \right] \right\} .$$

$$
\begin{aligned}
P_1 &= \bar{b}_i\gamma^\mu(1-\gamma_5)s_i\ \bar{b}_k\gamma^\nu(1-\gamma_5)t^a_{kl}\widetilde{G}^a_{\mu\nu}s_l\,, \\
P_2 &= \bar{b}_k\gamma^\mu(1-\gamma_5)t^a_{kl}\widetilde{G}^a_{\mu\nu}s_l\ \bar{b}_i\gamma^\nu(1-\gamma_5)s_i\,, \\
P_3 &= \frac{1}{m_b^2}\bar{b}_i\overleftarrow{D}^\mu\overleftarrow{D}^\alpha\gamma^\alpha(1-\gamma_5)s_i\ \bar{b}_k\gamma_\nu(1-\gamma_5)t^a_{kl}\widetilde{G}^a_{\mu\nu}s_l\,, \\
P_4 &= \frac{1}{m_b^2}\bar{b}_k\overleftarrow{D}^\nu\overleftarrow{D}^\alpha\gamma^\mu(1-\gamma_5)t^a_{kl}\widetilde{G}^a_{\mu\nu}s_l\ \bar{b}_i\gamma_\alpha(1-\gamma_5)s_i\,, \\
P_5 &= \frac{1}{m_b^2}\bar{b}_k\overleftarrow{D}^\nu\overleftarrow{D}^\alpha\gamma^\mu(1-\gamma_5)s_i\ t^a_{kl}\widetilde{G}^a_{\mu\nu}\ \bar{b}_i\gamma_\alpha(1-\gamma_5)s_l\,, \\
P_6 &= \frac{1}{m_b^2}\bar{b}_i\overleftarrow{D}^\nu\overleftarrow{D}^\alpha\gamma^\mu(1-\gamma_5)s_k\ t^a_{kl}\widetilde{G}^a_{\mu\nu}\ \bar{b}_l\gamma_\alpha(1-\gamma_5)s_i\,, \\
P_7 &= \frac{1}{m_b^2}\bar{b}_k\overleftarrow{D}^\mu\overleftarrow{D}^\alpha\gamma^\alpha(1-\gamma_5)s_i\ t^a_{kl}\widetilde{G}^a_{\mu\nu}\ \bar{b}_i\gamma_\nu(1-\gamma_5)s_l\,, \\
P_8 &= \frac{1}{m_b^2}\bar{b}_i\overleftarrow{D}^\mu\overleftarrow{D}^\alpha\gamma^\alpha(1-\gamma_5)s_k\ t^a_{kl}\widetilde{G}^a_{\mu\nu}\ \bar{b}_l\gamma_\nu(1-\gamma_5)s_i.
\end{aligned}
$$

Parameterization of operators contributing at $1/m^2$

$$\langle \overline{B}_s | Q_R | B_s \rangle = -(2 - \frac{1}{N_c}) f_{B_s}^2 M_{B_s}^2 \frac{M_{B_s}^2}{(m_b + m_s)^2} \alpha_1$$

$$\langle \overline{B}_s | W_1 | B_s \rangle = \frac{m_s}{m_b} \left(1 + \frac{1}{2N_c}\right) f_{B_s}^2 M_{B_s}^2 \left(\frac{M_{B_s}^2}{m_b^2} - 1\right) \alpha_1$$

$$\langle \overline{B}_s | W_2 | B_s \rangle = \frac{1}{2} \left(-1 + \frac{1}{N_c}\right) f_{B_s}^2 M_{B_s}^2 \left(\frac{M_{B_s}^2}{m_b^2} - 1\right)^2 \alpha_3$$

$$\langle \overline{B}_s | W_3 | B_s \rangle = \frac{1}{2} \left(1 + \frac{1}{2N_c}\right) f_{B_s}^2 M_{B_s}^2 \left(\frac{M_{B_s}^2}{m_b^2} - 1\right)^2 \alpha_4$$

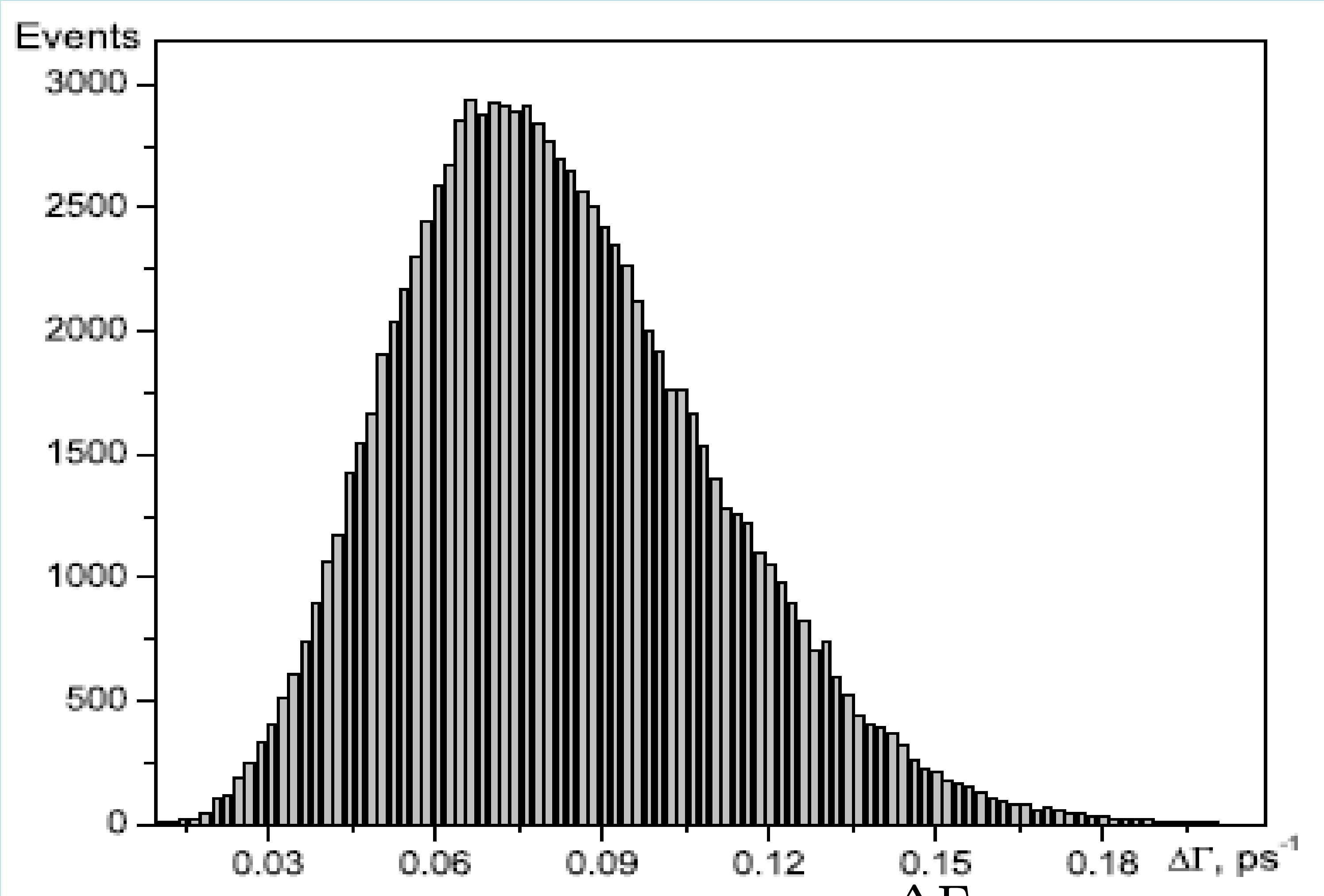
$$\langle \overline{B}_s | W_4 | B_s \rangle = -\frac{1}{2} f_{B_s}^2 M_{B_s}^2 \left(\frac{M_{B_s}^2}{m_b^2} - 1\right)^2 \alpha_5.$$

} Kinetic corrections

$$\langle B_s | P_i | B_s \rangle = \frac{1}{4} f_{B_s}^2 M_{B_s}^2 \left(\frac{M_{B_s}^2}{m_b^2} - 1\right)^2 \beta_i.$$

} Gluonic corrections

$$\begin{aligned}\Delta\Gamma_{B_s} = & [0.0005B + 0.1732B_s + 0.0024B_1 - 0.0237B_2 - 0.0024B_3 - 0.0436B_4 \\ & +2\times10^{-5}\alpha_1 + 4\times10^{-5}\alpha_2 + 4\times10^{-5}\alpha_3 + 0.0009\alpha_4 - 0.0007\alpha 5 \\ & +0.0002\beta_1 - 0.0002\beta_2 + 6\times10^{-5}\beta_3 - 6\times10^{-5}\beta_4 - 1\times10^{-5}\beta_5 \\ & -1\times10^{-5}\beta_6 + 1\times10^{-5}\beta_7 + 1\times10^{-5}\beta_8]\,(ps^{-1})\end{aligned}$$



$$\Delta\Gamma_{B_s} = 0.072 + 0.034 - 0.030 ps^{-1} \qquad \frac{\Delta\Gamma_{B_s}}{\Gamma_{B_s}} = 0.104 \pm 0.049$$

Possibility of New Physics

$$\frac{\Delta\Gamma}{2\Gamma} = -\frac{4\sqrt{2}G_f}{M_B\Gamma_B}\sum_{q,q'} V^*_{bq}V_{sq'}D_{qq'}\left(K_1\delta_{ik}\delta_{jl} + K_2\delta_{il}\delta_{jk}\right)\sum_{n=1}^{5} I_n\left\langle O_n^{ijkl}\right\rangle$$

$$I_1(y,z) = -\frac{km_b}{48\pi}\left(1-2(y+z)+(y-z)^2\right)$$

$$\left\langle O_1^{ijkl}\right\rangle = \left\langle B_s\left|\bar{b}_k\Gamma_\mu\gamma_\nu\Gamma_2 s_j\bar{b}_l\Gamma_1\gamma^\nu\Gamma^\mu s_i\right|\overline{B_s}\right\rangle; \quad I_2(y,z) = -\frac{k}{24\pi m_b}\left(1+(y+z)-2(y-z)^2\right)$$

$$\left\langle O_2^{ijkl}\right\rangle = \left\langle B_s\left|\bar{b}_k\Gamma_\mu\hat{p}\Gamma_2 s_j\bar{b}_l\Gamma_1\hat{p}\Gamma^\mu s_i\right|\overline{B_s}\right\rangle; \quad I_3(y,z) = \frac{k}{8\pi}\sqrt{z}\left(1+y-z\right)$$

$$\left\langle O_3^{ijkl}\right\rangle = \left\langle B_s\left|\bar{b}_k\Gamma_\mu\Gamma_2 s_j\bar{b}_l\Gamma_1\hat{p}\Gamma^\mu s_i\right|\overline{B_s}\right\rangle; \quad I_4(y,z) = -\frac{k}{8\pi}\sqrt{y}\left(1-y+z\right)$$

$$\left\langle O_4^{ijkl}\right\rangle = \left\langle B_s\left|\bar{b}_k\Gamma_\mu\hat{p}\Gamma_2 s_j\bar{b}_l\Gamma_1\Gamma^\mu s_i\right|\overline{B_s}\right\rangle; \quad I_5(y,z) = \frac{km_b}{4\pi}\sqrt{zy}$$

$$\left\langle O_5^{ijkl}\right\rangle = \left\langle B_s\left|\bar{b}_k\Gamma_\mu\Gamma_2 s_j\bar{b}_l\Gamma_1\Gamma^\mu s_i\right|\overline{B_s}\right\rangle; \quad k \equiv \frac{m_b}{2}\left(1-2(y+z)+(y-z)^2\right)^{1/2};$$

$$y \equiv \left(\frac{m_{q'}}{m_b}\right)^2;\ z \equiv \left(\frac{m_q}{m_b}\right)^2$$

$\Gamma_{1,2}$ $D_{qq'}$ NP parameters

[1] Eugene Golowich, Sandip Pakvasa, Alexey A. Petrov. Phys. Rev. Lett. 98, 181801 (2007)

Models of NP considered

- Multi Higgs Model

This model contains charged Higgs bosons as parts of the extended Higgs sector and provides new flavor-changing interactions mediated by charged Higgs bosons

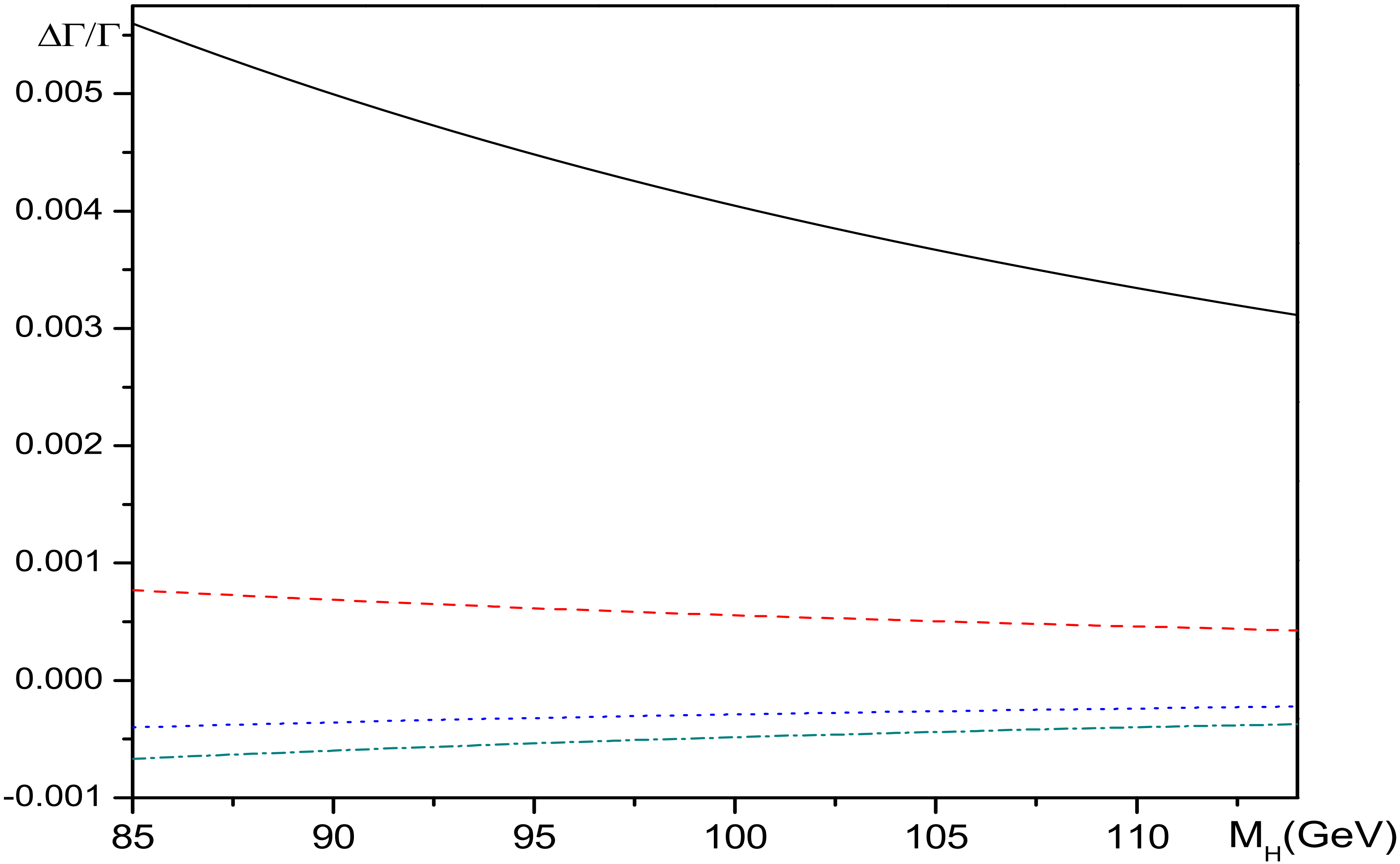
$$\mathcal{H}_{ChH}^{\Delta B=1} = -\frac{\sqrt{2}G_F}{M_H^2}\ \overline{b}_i\overline{\Gamma}_1 q'_i\ \overline{q}_j\overline{\Gamma}_2 s_j$$

where $\overline{\Gamma}_i,\ i = 1, 2$ are

$$\begin{aligned}\overline{\Gamma}_1 &= m_b V_{cb}^* \cot\beta P_L - m_c V_{cb}^* \tan\beta P_R \\ \overline{\Gamma}_2 &= m_s V_{cs} \cot\beta P_R - m_c V_{cs} \tan\beta P_L\end{aligned}$$

For values of $M_H = 85 GeV$ and $\cot\beta = 0.05$ it gives $y_{ChH} \approx 0.006$

Dependence of y_{ChH} on mass of charged Higgs boson: solid line - $\tan\beta = 20$,dashed line - $\tan\beta = 10$, dotted line - $\tan\beta = 5$, dash-dotted line - $\tan\beta = 3$



- Left-Right Symmetric Model

Left Right Symmetric Model (LRSM) assumes the extended $SU(2)_L(2)_R$ symmetry of the theory, which restores parity at high energies

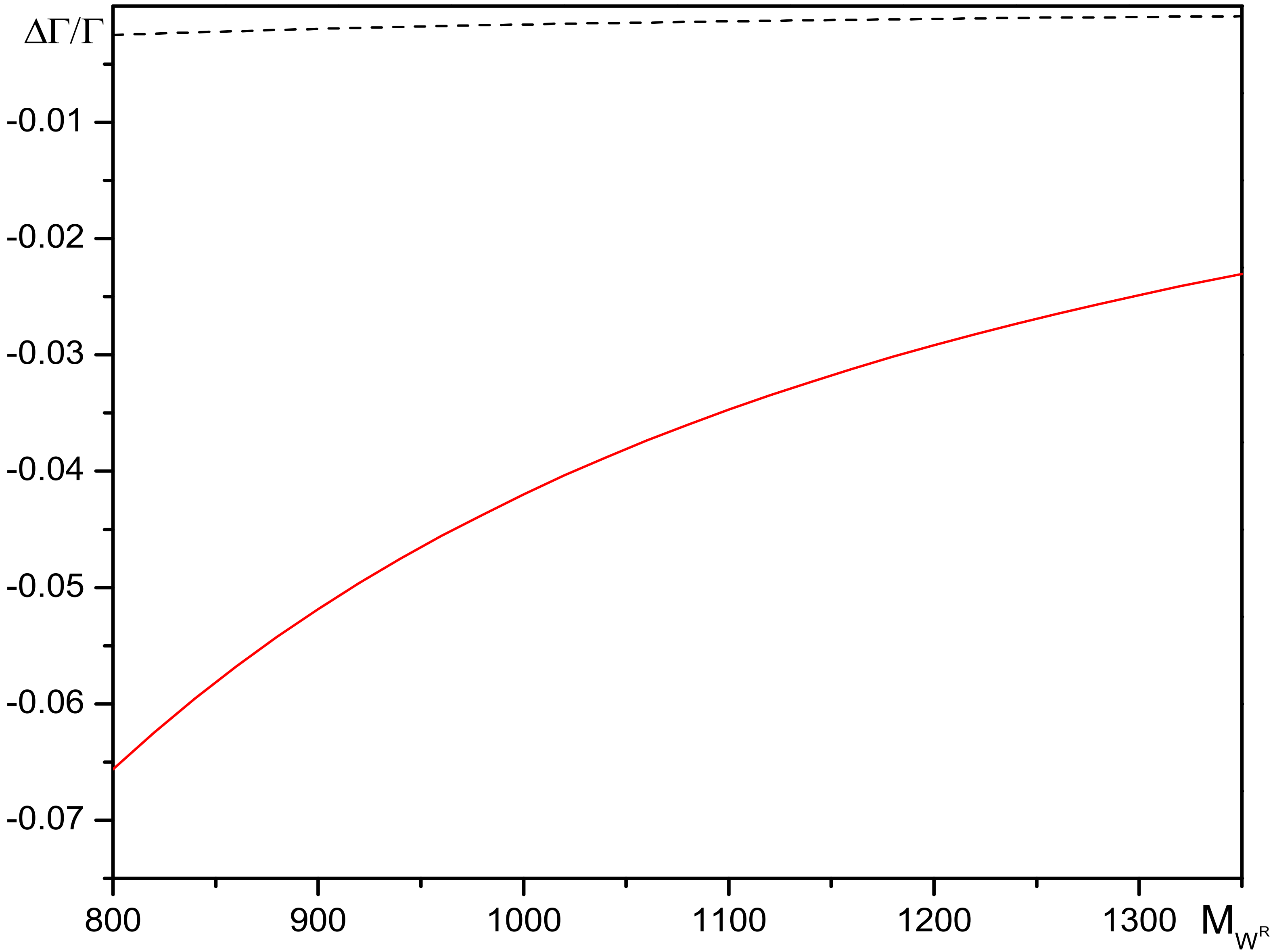
$$\overline{\Gamma}_{1,2} = \gamma^\mu P_R$$

$$D_{qq'} = V_{cb}^{*(R)} V_{cs}^{(R)} \frac{G_F^{(R)}}{\sqrt{2}}$$

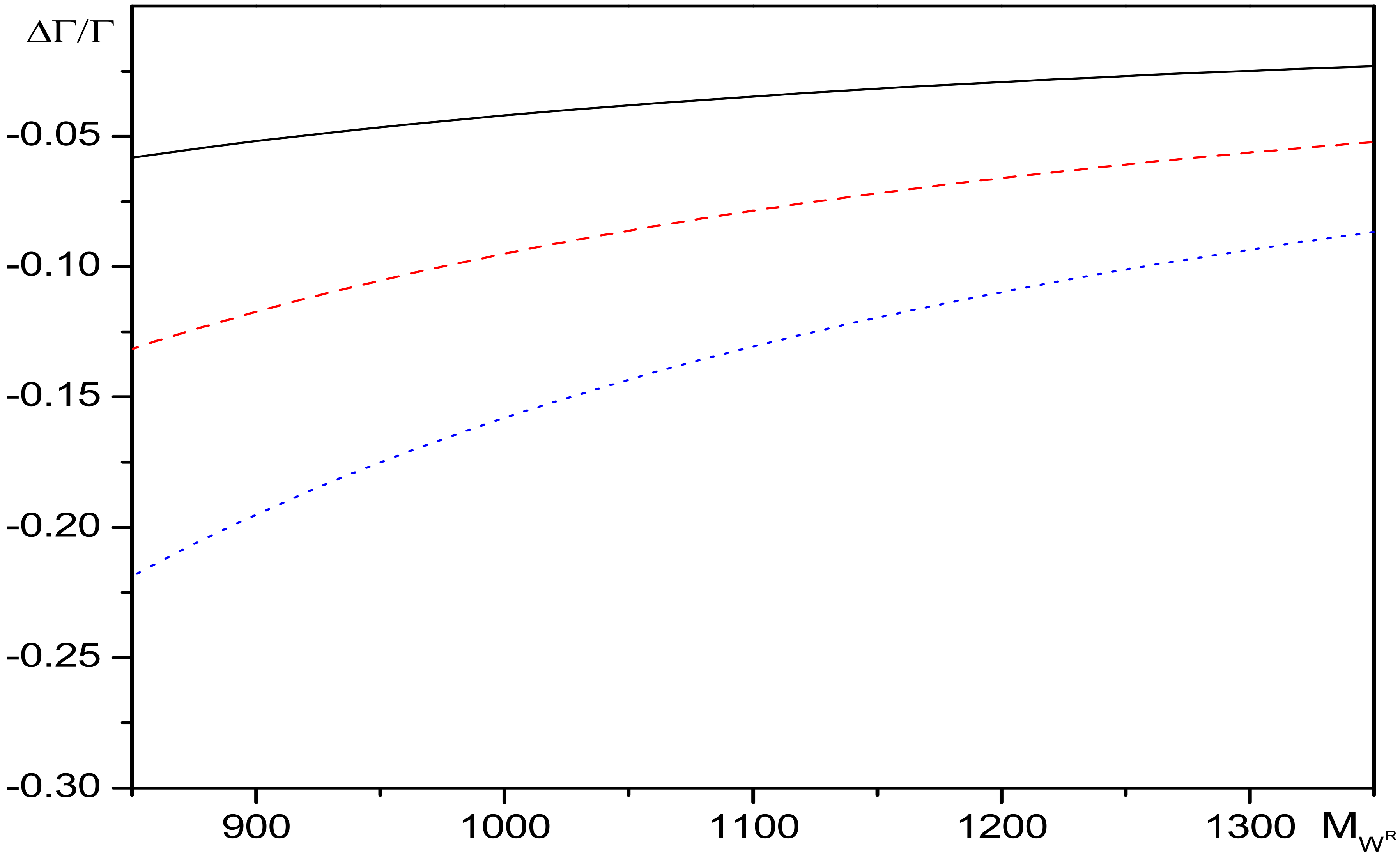
here $\frac{G_F^{(R)}}{\sqrt{2}} = g_R^2/8M_{W^(R)}^2$ and for future calculations we take $g_L = k g_R$

One of possible realizations of such scenario which gives the biggest numerical value of y_{LR} is a "Non-manifest LR" ($V_{ij}^{(R)} \approx 1$) with $M_{W^(R)} = 1\ TeV$ value of $y_{LR} \approx -0.04$ was obtained. In case of "manifest LR" ($(V_{ij}^{(R)} = V_{ij})$) contribution from this model is less.

Dependence of y_{LR} on mass of $W^{(R)}$ boson. Solid line "Non manifest" case, dashed - "manifest"



Dependence of $\Delta\Gamma/\Gamma$ on $M_W^{(R)}$ in non-manifest LRSM. Solid line: $\kappa = 1$ dashed line: $\kappa = 1.5$, dotted line: $\kappa = 2$.



Conclusions:

- $1/m^2$ corrections to difference oi lifetimes of B_s mesons computed
- They depend on 13 non-perturbative parameters
- Adopted statistical approach to obtain numerical value of lifetime difference. Corrections appeared to be small
- Possibility of NP contribution to this observable considered. Several models of NP considered as an example