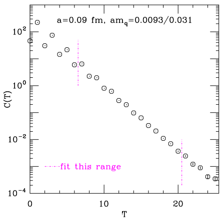
Strange quark content of the nucleon
DPF 2009



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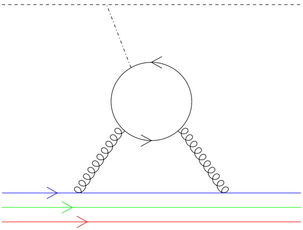
Introduction

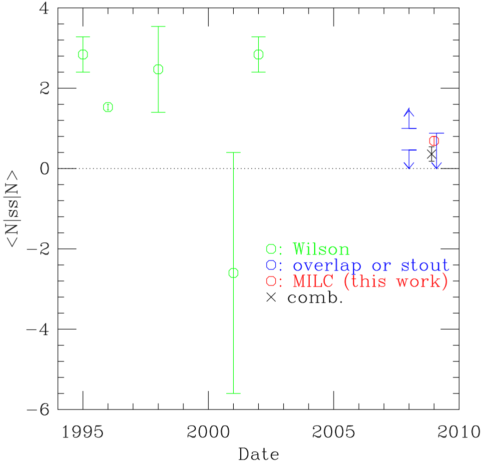
- $\langle N|\bar{q}q|N\rangle$
- Interesting for interaction of dark matter candidates (SUSY, *e.g.*) with matter (like detectors)
- For $\bar{c}c$, $\bar{b}b$ and $\bar{t}t$, do it perturbatively
- $\bar{u}u + \bar{d}d$ is harder, we defer it
- For $\bar{s}s$, previous lattice calculations all over the map

Dark matter scattering?



- Scatter off dark matter particle
- Heavy quarks important
- higgs coupling $\approx m_q$, but in P.T. $\langle N|\bar{q}q|N\rangle \approx 1/m_q$.
- P.T. no good for strange content
  Is it enhanced? suppressed?

Lattice computations



- Previous lattice calculations
- M. Fukugita, Y. Kuramashi, M. Okawa, A. Ukawa, arXiv:hep-lat/9408002, Phys. Rev. D **51**, 5319 (1995); S.J. Dong, J.-F. Lagaë, K.F. Liu, arXiv:hep-ph/960225, Phys. Rev. D **54**, 5496 (1996); SESAM-Collaboration: S. Güsken *et al.*, arXiv:hep-lat/9809066, Phys. Rev. D **59** (1999) 054504; C. Michael, C. McNeile and D. Hepburn (UKQCD collaboration), hep-lat/010902, Nucl. Phys Proc. Suppl. **106**, 293 (2002); H. Ohki *et al.*, arXiv:0810.4223, PoS(LATTICE 2008) 126; arXiv:0806.4744, Phys. Rev. D **78** (2008) 054502; G Bali, S. Collins and A. Schafer, arxiv 0811.0807, PoS(LATTICE2008) 161, 2008. R.D. Young and A.W. Thomas , arXiv:0901.3310.

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Good things about this calculation

- MILC lattices with dynamical u, d and s quarks
- Action with chiral symmetry $\rightarrow$ no additive quark mass renormalization
- Three different lattice spacings
- Range of light quark masses $0.1 m_{strange} < m_{light} < m_{strange}$
- High statistics: up to 2000 lattices/ensemble, 22000 lattices total
- Systematic error estimates

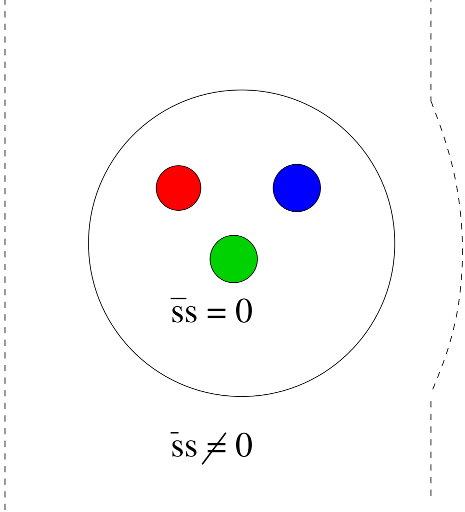
Results

- Isn't this supposed to be at the end?
  But sometimes I don't get to the end.
- $\langle N|\bar{s}s|N\rangle = \frac{\partial M_N}{\partial m_s} = 0.69 \pm 0.07_{\mathrm{statistical}} \pm 0.09_{\mathrm{systematic}}$,
  with m_s in the $\overline{ms}$ regularization at 2 GeV. (but Z_m isn't known very well)
- $m_s \frac{\partial M_N}{\partial m_s} = 59(6)(8)$ MeV
  This is RNG invariant (but m_s isn't known very well)
- $F_{T_s} = \frac{m_s}{M_N} \frac{\partial M_N}{\partial m_s} = 0.063(6)(9)$
- see `arxiv:0905.2432`

Why should there be strange quarks in the nucleon?

- Virtual pairs, like on “higgs scattering” slide?
  Seems like that should be small
- The strange quarks are not in the nucleon
  — they are in the vacuum

Why should there be strange quarks in N?



- The nucleon is like an air bubble in glass
- A “bag model” intuitive picture
- $-\bar{\psi}\psi$ is large in vacuum, small in nucleon
- dashed lines are wavefronts for incident/scattered particle

Algebra, quickly

- What we really mean is
  $``\langle N|\bar{s}s|N\rangle" = \left\langle N \middle| \int d^3x\, \bar{s}s \middle| N \right\rangle - \left\langle 0 \middle| \int d^3x\, \bar{s}s \middle| 0 \right\rangle$
- From differentiating path integral expression for M_N,
  $\langle N|\bar{s}s|N\rangle = \left.\frac{\partial M_N}{\partial m_s}\right|_{\alpha_s, m_l, \ldots}$
- In lattice calculation, M_N is function of correlator $C(t)$ over some distance range: $M_N = f(C(a), C(2a), \ldots)$
- Chain rule: $\frac{\partial M_N}{\partial m_s} = \sum_t \frac{\partial M_N}{\partial C(t)} \frac{\partial C(t)}{\partial m_s}$
- But $\frac{\partial C(t}{\partial m_s} = \langle C(t)\bar{s}s\rangle - \langle C(t)\rangle \langle \bar{s}s\rangle$
- If the lattice operator created nothing but nucleon, this would just reverse the Feynman-Hellman theorem steps above
- A (quark-line) disconnected correlation function → Hard

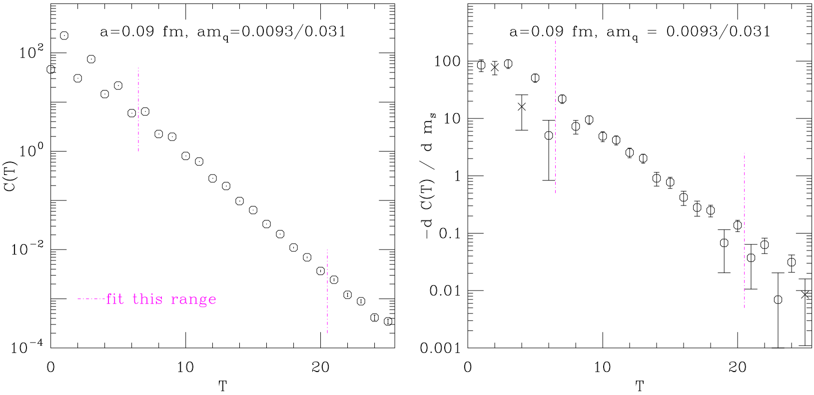
But why should nucleon mass depend on m_s?

- Does not mean that physical M_N depends greatly on m_s
- Changing m_s instead changes **all** lattice quantities; we usually would interpret this as a change in a
- Again, crucial point is holding action at the UV cutoff fixed
- Note contrast to customary definitions of scale with unphysical quark masses, such as "keep r_0 fixed at 0.47 fm"
- Extreme counterexample: you could use the nucleon – set $M_N = 940$ MeV. Then $\frac{\partial M_N}{\partial m_s} = 0$.

How to do this: “data mining”

- MILC lattices, stored — dynamical u, d and s quarks
- Range of dynamical quark masses —
  $0.1 m_s < m_{light} < 1.2 m_s$
- Several lattice spacings — $a = 0.12$, 0.09, 0.06 fm used here
- High statistics (cheap staggered quarks)
- Nucleon correlators already computed on most of them
- Have several stochastic estimations of $\bar{s}s$ on each lattice
- (We did need to complete spectrum computations on some lattice sets.)

Nucleon correlator and its derivative



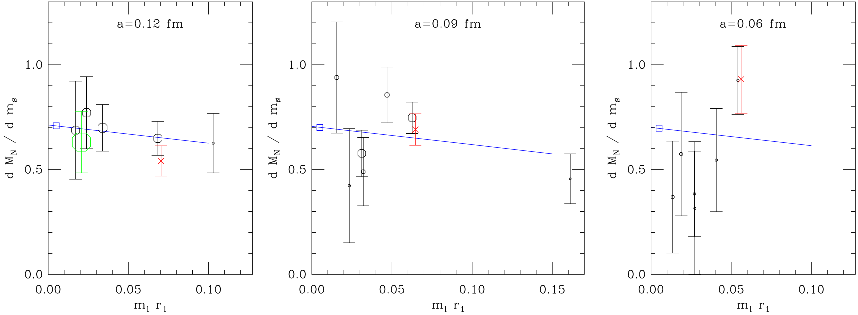
Issues

- Choosing a range of distance to fit (next slide)
- statistical errors - jackknife method
- Adjust to physical strange quark mass (small adjustment)
- Extrapolate to physical light quark mass
- Extrapolate to zero lattice spacing (continuum)
- $\frac{\partial M_N}{\partial m_s} = A + B m_l r_1 + C(a/r_1)^2$
  (no $m_l^{1/2}$ or $m_l \log(m_l)$)
- Make a list of systematic errors

Choosing a fit range

- Choose too low a minimum distance and excited state pollution spoils results
- Choose too high a minimum distance and statistics are too weak
- Can choose a lower minimum distance than in precision M_N calculations.
  Accuracy in the 10% range is okay, so a little pollution is tolerable
- Look for plateaus; choose similar d_{min} in physical units for all ensembles
- $T_{min} \approx 0.6fm =$ best compromise between statistical and systematic errors
- Estimate effect by seeing what $T_{min} \approx 0.6$ fm. does to M_N etc.

$\frac{\partial M_N}{\partial m_s}$ on all the ensembles



The derivative $\frac{\partial M_N}{\partial m_s}$ on the various ensembles.

Adjusted to the correct strange quark mass

Converted to the $\overline{ms}(2\ \mathrm{GeV})$ regularization.

r_1 from heavy quark potential, $r_1 \approx 0.31$ fm.

Symbol size $\propto$ to the number of lattices in the ensemble; largest symbols $\approx$ about 2000 lattices.

The points at $m_l r_1 \approx 0.05$ ($m_l \approx 0.4 m_s$) show (crosses) the value before adjusting to the correct strange quark mass.

The lines are the continuum and chiral fit, evaluated at the corresponding lattice spacing.

Error budget

Source	Estimate
statistical	0.070
Excited states	0.069 (10%)
Finite volume	0.021 (3%)
Higher order χPT	0.049 (7%)
Error in Z_m	0.028 (4%)
Combined systematic	0.09

Error budget.

Strange quark mass adjustment

- Don't have a lot of variety in values of m_s in the MILC lattices
- But we *do have a lot of variety in m_l, including some with* $m_l = .6 m_s$
- Treat these heavy light quarks as lighter strange quarks
- Compare $\frac{\partial M_N}{\partial m_s}$ and $\frac{\partial M_N}{\partial m_l}$ to estimate $\frac{\partial^2 M_N}{\partial {m_s}^2}$
- From five ensembles with the heaviest light quarks: $\frac{\partial^2 M_N}{\partial {m_s}^2} = -2.2(5)$

Light quark mass adjustment

- Here we have enough different m_l's to do a fit
- Not complicated by chiral behavior: no "dangerous terms" in $\frac{\partial M_N}{\partial m_s}$ as m_l goes to 0
  - This is *not true for* $\frac{\partial M_N}{\partial m_l}$ *– needs a real chiral fit*
- Fit to linear in m_l plus constant

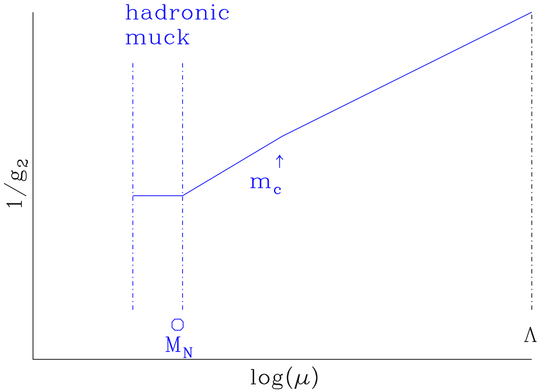
Continuum extrapolation

- Statistics at finer lattice spacings too poor and lattice-spacing dependence too small
- Will have large statistical error on lattice-spacing dependence from a naive fit
- Other hadronic quantities have around 10% error between $a = 0.12fm$ and the continuum
- Use a Bayesian prior (Gaussian) to constrain the lattice-spacing dependence to $0 \pm 10\%$
- Lowest error term goes as a^2 (Asqtad)
- Combine this with the m_l dependence in one fit: $\frac{\partial M_N}{\partial m_s} = A + B(m_l r_1) + C(a/r_1)^2$

Extra slides

How QCD works

Theory fixed at cutoff, hadronic mess happens when coupling gets big. ($1/g^2$ gets small)
Cusp (hard to see) at m_c.



If m_c changes

Change the quark mass, and where $1/g^2$ gets large shifts.

$F_{T_c} = \frac{\partial log(M_N)}{\partial log(m_c)}$ (Shifman, Vainstein, Zakharov; Kryjewski)