Recent Progress in Field and String Theory

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AdS/CFT

AdS/CFT continues to have a huge influence in the string and field theory community.

paper	citations	this year
J. Maldacena (1997)	6200	370
S. Gubser, I. Klebanov, A. Polyakov (1998)	3600	240
E. Witten (1998)	4100	260

12 years and still going strong!

S. Weinberg (1967)	7000	60

What is AdS/CFT?

maximally supersymmetric SU(N) Yang-Mills in 3+1 dimensions
is equivalent to
type IIB string theory in a $AdS_5 \times S^5$ background.

This statement is interesting because

- The low energy limit of string theory is gravity. We can use the correspondence to define a theory of quantum gravity.
- The conjectured correspondence is basically a strong-weak coupling duality. We can use classical gravity to calculate quantities in a strongly interacting field theory.
- There are *many* generalizations.

Basic facts about string theory

- There are open strings (strings with end points) and closed strings (loops).
- Strings may split with a likelihood g_s.
- Strings have a tension $T_0 = 1/\ell_s^2$ where ℓ_s is the string scale.
- Open strings end on massive objects called D-branes. These D-branes have a $m \sim 1/g_s$.

Thinking of time as an extra dimension, strings become surfaces — worldsheets.

What is AdS/CFT?

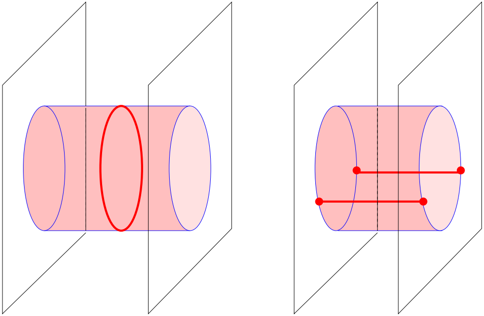


Figure: Open/closed duality (artwork by R. Peschanski)

- D-branes are surfaces strings end on
- the lowest closed string mode is the graviton
- the lowest open string mode is a gauge boson

What happened this year in field and string theory

Three developments this year in string and field theory, all connected to the AdS/CFT correspondence.

- Scattering amplitudes of $\mathcal{N} = 4$ Super Yang-Mills.
- AdS/CFT for the M2-brane.
- Applications of AdS/CFT to nuclear and condensed matter physics.

AdS/CFT and the S-matrix for $\mathcal{N} = 4$ SYM

Figure: S Matrix '09, with apologies to local car manufacturers.

What happened this year for scattering

- Continuing improvements in methodology (Arkani-Hamed, Cachazo, Cheung, Kaplan).
- A new symmetry: dual conformal invariance (Alday, Maldacena).
- Is $\mathcal{N} = 8$ supergravity finite? (Bern, Carrasco, Dixon, Johansson, Roiban)

Some notation

As noticed originally by Parke and Taylor (1986), gluon scattering amplitudes are more compactly expressed in a spinorial, color stripped, on-shell form.

$$\mathcal{A}(p_i, \epsilon_i) \to \mathcal{A}(\lambda_i, \tilde{\lambda}_i, h_i) \quad , \quad h_i = \pm \ .$$

Positive and negative chirality spinors: $\lambda^a, \tilde{\lambda}^{\dot{a}}$ where $a, \dot{a} = 1, 2$.
Momentum:

$$\lambda_a \tilde{\lambda}_{\dot{a}} = p_{a\dot{a}} = \sigma^{\mu}_{a\dot{a}} p_\mu \qquad \text{where} \qquad p_\mu p^\mu = \det(p_{a\dot{a}}) \ .$$

Positive and negative helicity polarization tensors:

$$\epsilon_{a\dot{a}} = \frac{\lambda_a \tilde{\mu}_{\dot{a}}}{[\tilde{\lambda}, \tilde{\mu}]} \qquad \text{and} \qquad \tilde{\epsilon}_{a\dot{a}} = \frac{\mu_a \tilde{\lambda}_{\dot{a}}}{\langle \mu, \lambda \rangle} \ .$$

$$\langle 12 \rangle = \langle \lambda_1, \lambda_2 \rangle = \epsilon_{ab} \lambda_1^a \lambda_2^b \ ,$$

$$[12] = [\tilde{\lambda}_1, \tilde{\lambda}_2] = \epsilon_{\dot{a}\dot{b}} \tilde{\lambda}_1^{\dot{a}} \tilde{\lambda}_2^{\dot{b}} \ .$$

The simplifications continue

- Before the spinor notation, tree level gluon scattering amplitudes took pages and pages to write down.
- With spinor notation, things got much simpler

$$\mathcal{A}(+-+-) = \frac{\langle 24 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle} \ .$$

- This year, Arkani-Hamed, Cachazo, Cheung, Kaplan introduced an "ambidextrous" notation where

$$\mathcal{A}(+-+-) = \mathrm{sgn}(W_1 \cdot Z_2)\mathrm{sgn}(Z_2 \cdot W_3)\mathrm{sgn}(W_3 \cdot Z_4)\mathrm{sgn}(Z_4 \cdot W_1) \ .$$

The ambidextrous notation

- W:
  - Fourier transform with respect to λ, introducing a new conjugate variable $\tilde{\mu}$.
  - Call $W = (\tilde{\mu}, \tilde{\lambda})$.
- Z:
  - Fourier transform with respect to $\tilde{\lambda}$, introducing a new conjugate variable μ.
  - Call $Z = (\lambda, \mu)$.
- Define $W \cdot Z = \tilde{\mu}\lambda - \mu\tilde{\lambda}$.

Why we care about trees, Part I

With the unitarity cut construction, we can sew together tree amplitudes to get on-shell loop amplitudes. With higher loop SYM amplitudes in hand, we can put AdS/CFT to a stringent test.

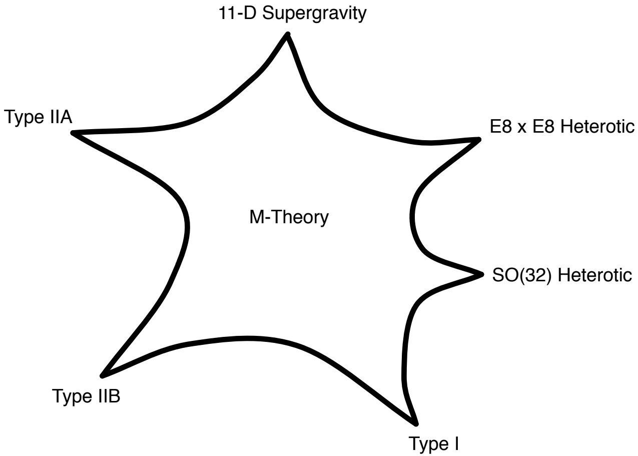
- Using AdS/CFT and T-duality, Alday and Maldacena saw that SYM amplitudes have a hidden dual conformal symmetry.
- Gluon scattering amplitudes map to polygonal Wilson loops with null sides in a T-dual space-time. The length of the side is the momentum of the original gluon.
- This symmetry has been very useful in constraining conjectures for all loop gluon scattering amplitudes in $\mathcal{N} = 4$ SYM.

Why we care about trees, Part II

By a specialization of the Kawai, Llewellyn, and Tye (1986) results for open and closed string scattering amplitudes, supergravity amplitudes are essentially squares of SYM amplitudes. Better methodology has led to recent results suggesting $\mathcal{N} = 8$ supergravity is better behaved (perturbatively) than expected.

- There are old arguments in the literature, based on supersymmetry, that an n-loop amplitude in $\mathcal{N} = 8$ supergravity cannot be divergent for $n \leq 9$.
- For those who prefer a more constructive approach, there is a small community of people who are using these spinorial and unitarity cut techniques to actually calculate the loop amplitudes. See for example Bern, Carrasco, Dixon, Johansson, Roiban (4-loop, 4-point).

The M2-brane mini-revolution



What is M-theory?

AdS/CFT helps us figure out the answer.

- The fundamental degrees of freedom of M-theory are 2+1 and 5+1 dimensional objects: M2-branes and M5-branes.
- A generalization of the AdS/CFT correspondence states that the superconformal fixed point of maximally supersymmetric SU(N) Yang-Mills theory in 2+1 dimensions is dual to M-theory in a $AdS_4 \times S^7$ background.
- Based on analogy to the AdS_5/CFT_4 correspondence, this fixed point theory should be the low energy degrees of freedom on a stack of N M2-branes.

Epiphenomena

paper	date	citations
Gustavsson	9/07	259
Bagger, Lambert	11/07	281
Aharony, Bergman, Jafferis, Maldacena	6/08	305

this year people are continuing to work out the details:
Benna, Klebanov, Klose — mysteries of monopole operators
Aganagic, Hanany — generalizing through geometry
Imamura — exploring the space of gauge invariant operators
...
and my apologies to the many other authors that I don't have room to list ...

A Double Chern-Simons Theory

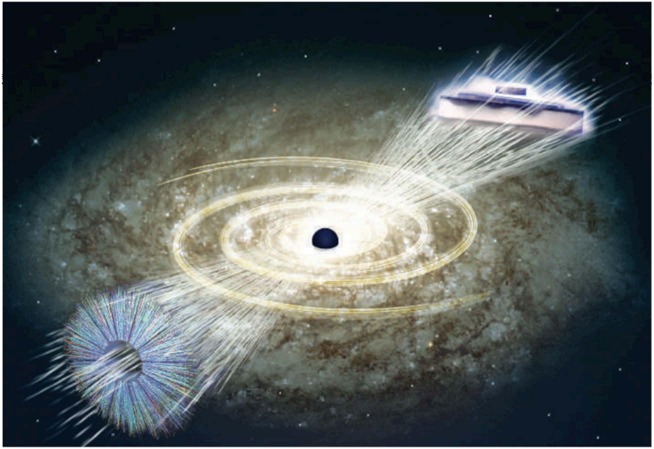
Aharony, Bergman, Jafferis, Maldacena (2008) wrote down a Lagrangian for the superconformal fixed point theory describing the low energy degrees of freedom on a stack of N M2-branes at a $\mathbb{C}^4/\mathbb{Z}_k$ singularity.

- $U(N) \times U(N)$ Chern-Simons theory with level $(k, -k)$.
- Bifundamental chiral superfields, A_1, A_2 and B_1, B_2.
- Superpotential

$$W = \frac{4\pi}{k} \mathrm{Tr}(A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1) \ .$$

- More or less explicit $\mathcal{N} = 6$ supersymmetry enhanced to $\mathcal{N} = 8$ for $k = 1, 2$.

AdS/CFT and the Real World



D. PARKER/IMI/UNIV. BIRMINGHAM HIGH TC CONS./SPL

Some Dictionary Entries

- Hawking temperature of the black hole is the temperature of the field theory.
- Electric charge of the black hole is the charge density of the field theory.
- Magnetic charge of the black hole is the magnetic field of the field theory.
- A black hole with scalar hair is dual to a superfluid (or superconducting) phase of the field theory.

The Progress this Year

AdS/CFT is a useful tool for studying time dependent and nonzero density processes in strongly interacting field theories:

- Chesler, Yaffe; Bhattacharyya, Minwalla: Plasma relaxation times from black hole formation.
- Herzog, Yarom: Relativistic superfluid from a hairy black hole.
- Denef, Hartnoll; Ammon, Erdmenger, Kaminski, Kerner; Gubser, Herzog, Pufu, Tesileanu: Beyond phenomenology; embedding the holographic superfluid/superconductor in string theory.

again, my apologies . . .

AdS/CFT versus the lattice or perturbation theory

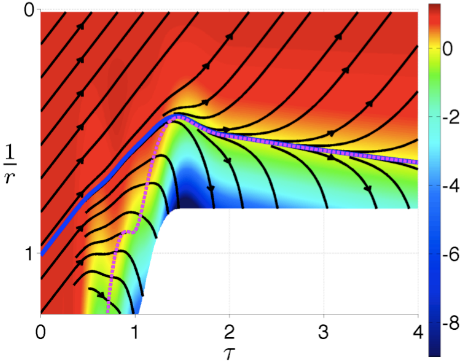


Figure: The congruence of outgoing radial null geodesics. The solid blue line is the event horizon. The dashed magenta line is the apparent horizon. From Chesler, Yaffe.

"We found that the entire process of plasma creation ... can occur in times as short as one to two times $1/T_*$, where T_* is the local temperature at the onset of the hydrodynamic regime." — Chesler and Yaffe

Second Sound of Helium-4 from Khalatnikov's Book

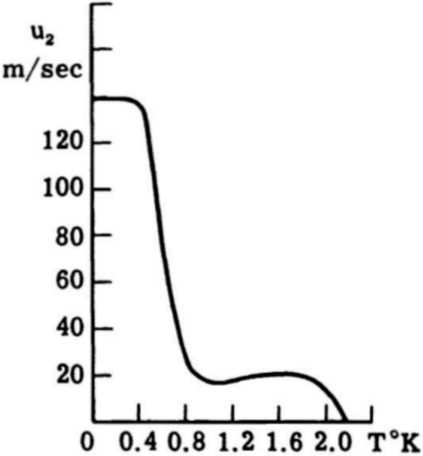
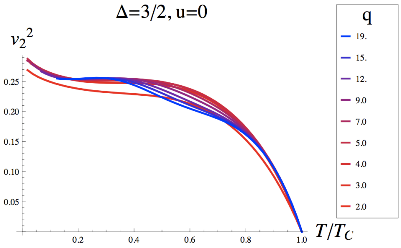


Figure 4. The temperature dependence of the velocity of second sound.

Second Sound

a) 

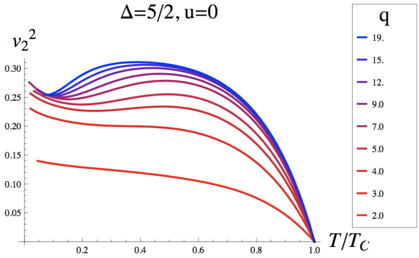
b) 

Figure: The speed of second sound as a function of T/T_c, computed by evaluating thermodynamic derivatives: a) $O_{3/2}$ scalar, b) $O_{5/2}$ scalar for a 3+1 dimensional field theory. The speed of second sound vanishes as $T \to T_c$.

with Amos Yarom

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Things I didn't mention. . .

- Kerr/CFT Correspondence: Hartmann, Strominger, Song, . . .
- Hořava-Lifshitz gravity: Hořava, . . .
- $\mathcal{N} = 2$ gauge theories from wrapped M5-branes: Gaiotto, . . .

Extra Slides

The Viscosity Bound

- Kovtun, Son, and Starinets (2004) conjectured that the viscosity to entropy density ratio is bounded below by

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B} \ .$$

- Kats and Petrov (2007) provided a controlled four-dimensional gauge theory with a gravity dual where the bound is violated by $1/N$ corrections.
- Brigante, Liu, Myers, Shenker, Yaida (2007) provided a class of pheonomenological AdS/CFT models where it seems very plausible the bound is violated.
- Both of these recent papers rely on computing higher curvature corrections to the Einstein gravity result.

- $g^2_{\mathrm{YM}} N \equiv \lambda = (L/\ell_s)^4$ where L is the radius of curvature of AdS_5 and ℓ_s is a length scale that sets the tension of the type IIB strings.
- $4\pi g^2_{\mathrm{YM}} = g_s$ where g_s is the string coupling constant
- Double scaling limit: $N \to \infty$ with λ large and fixed,

Various Limits

- $N/k = \lambda$ is the 't Hooft coupling. When $\lambda \ll 1$, the field theory is weakly coupled.
- When $N \ll k^5$, a type IIA supergravity approximation of M-theory becomes valid.
- When $N \gg k^5$, the eleven dimensional supergravity approximation is valid.