# Heavy Flavor Theory

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# Motivation: Bottom-up approach

Assume existence of New Physics (NP) at short distances, no specific model

NP not directly accessible to experiment (yet): effects appear indirectly as modifications to interactions among SM particles

Supplement SM lagrangian with terms of dimension higher than four ("higher dimension operators") as allowed by Lorentz Invariance and gauge symmetries.

A term of dimension $n > 4$ appears in the lagrangian with coefficient $c/\Lambda_{\rm NP}^{n-4}$ (with $c \sim 1$)
Hence low energy effects are suppressed by powers of $\Lambda_{\rm NP}$
(just a generic form of the effective field theory of the top-down approach)

Advantages of bottom-up approach:
- fairly general, encompasses many (all?) realistic extensions of SM (model independent)
- few parameters

Disadvantages:
- no clear correlation between long ($\text{GeV}^{-1}$) and very short ($\text{TeV}^{-1}$) distances

In this talk I will avoid translation of bounds into explicit models (typically SUSY)

# Flavor problem

The EFT (either approach) generically contains terms that mediate $\Delta F = 2$ or FCNC decays at tree level and suppressed only by $c/\Lambda_{\rm NP}^{n-4}$ $(\text{with } c \sim 1)$

with $n - 4 = 2$ this requires $\Lambda_{\rm NP}$ in excess of $10^4$ TeV from, *e.g.*, K-mixing

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$$\frac{1}{\Lambda_{\rm NP}^2}\left[z_1^K(\overline{d_L}\gamma_\mu s_L)(\overline{d_L}\gamma^\mu s_L) + z_1^D(\overline{u_L}\gamma_\mu c_L)(\overline{u_L}\gamma^\mu c_L) + z_4^D(\overline{u_L}c_R)(\overline{u_R}c_L)\right]$$

$$|z_1^K| \leq z_{\rm exp}^K = 8.8\times 10^{-7}\left(\frac{\Lambda_{\rm NP}}{1\ {\rm TeV}}\right)^2$$

$$|z_1^D| \leq z_{\rm exp}^D = 5.9\times 10^{-7}\left(\frac{\Lambda_{\rm NP}}{1\ {\rm TeV}}\right)^2$$

$$\mathcal{I}m(z_1^K) \leq z_{\rm exp}^{IK} = 3.3\times 10^{-9}\left(\frac{\Lambda_{\rm NP}}{1\ {\rm TeV}}\right)^2$$

$$\mathcal{I}m(z_1^D) \leq z_{\rm exp}^{ID} = 1.0\times 10^{-7}\left(\frac{\Lambda_{\rm NP}}{1\ {\rm TeV}}\right)^2$$

[Nir; Perez; Weiler; @ Planck2009]

Goal of heavy quark physics: constrain models of new physics, verify with precision SM+CKM

# Plan of the Talk

- Introduction with review of CKM theory
- Purely leptonic decays
- $B\overline{B}$ mixing: $|V_{td}|$; $D\overline{D}$ mxing
- Towards a precision determination of $|V_{cb}|$, $|V_{ub}|$
- Progress in Rare $B$ Decays

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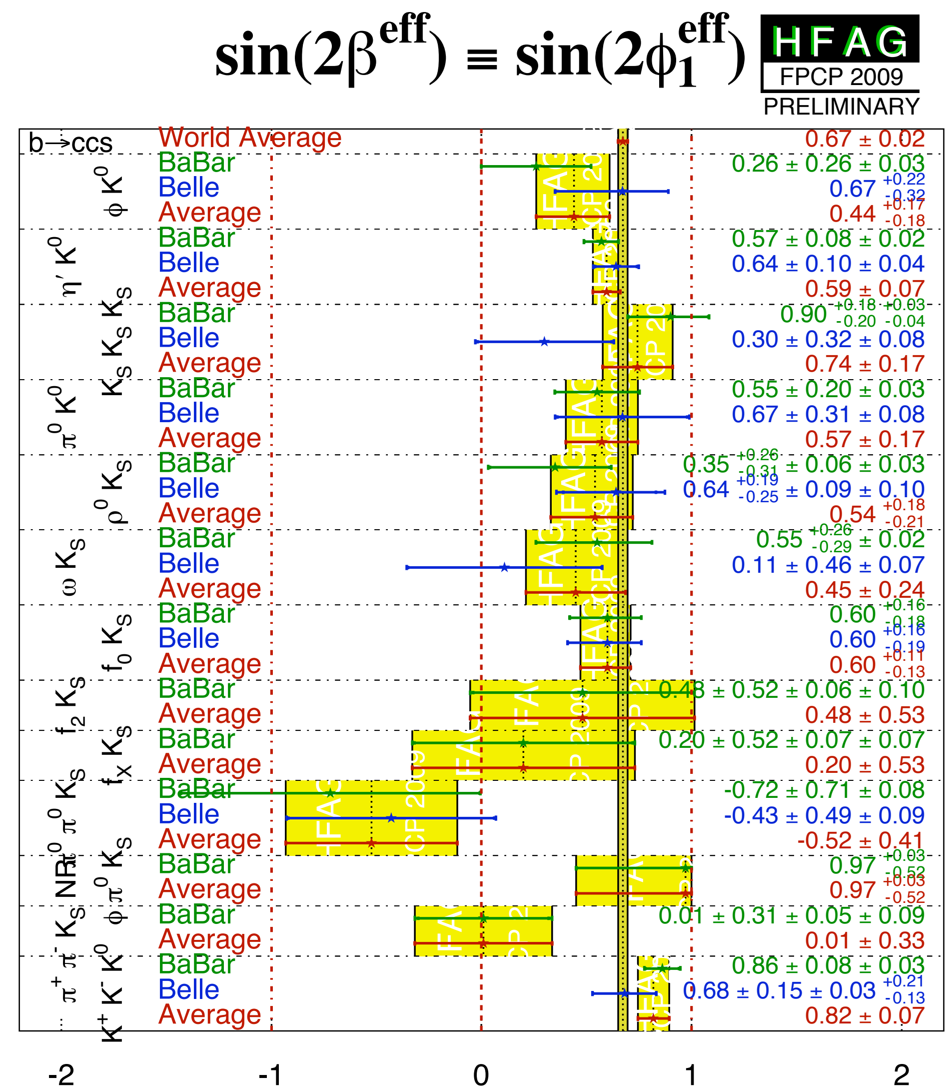
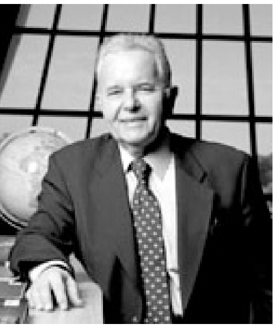


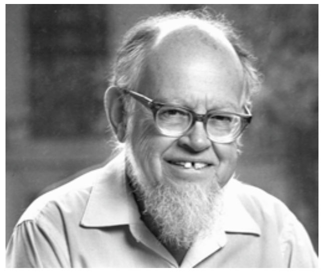

I am sorry I have to leave out many interesting topics (with apologies to speakers in parallel sessions): CPV/angles, two body hadronic decays, sin $2\beta$ determinations from sss penguins (vs ccs trees), heavy flavor at LHC, explicit BSM theories of/with flavor, ...



# The CKM Matrix

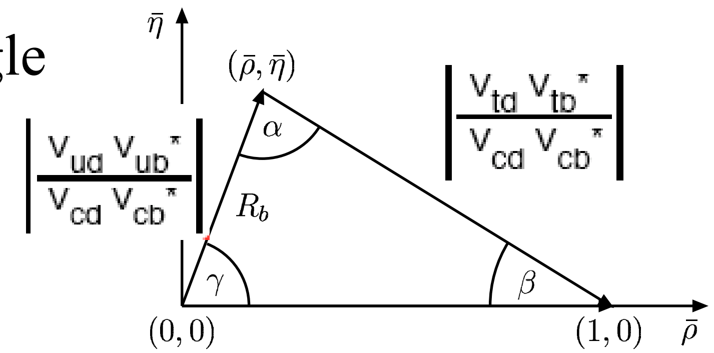
$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- Frequently used Wolfenstein parametrization: four parameters $\lambda,\ A,\ \bar\rho,\ \bar\eta$

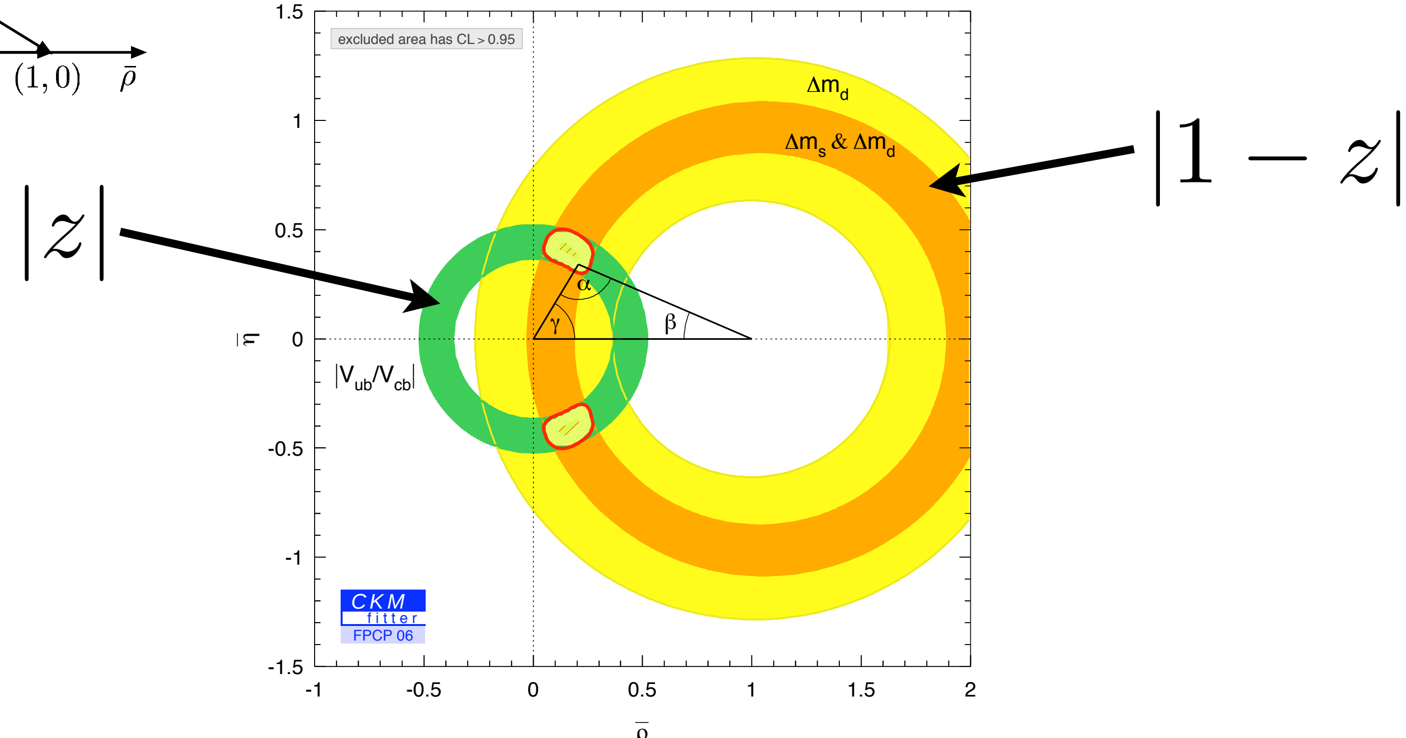


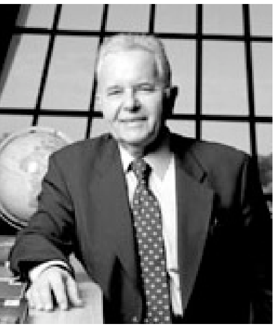
$$V_{\rm CKM} \approx \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\bar\rho - i\bar\eta) \\ -\lambda(1+iA^2\lambda^4\bar\eta) & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\bar\rho-i\bar\eta) & -A\lambda^2(1+i\lambda^2\bar\eta) & 1 \end{pmatrix} + \mathcal{O}(\lambda^6).$$

- CKM Unitarity triangle




- Sides give circles in $z = \bar\rho + i\bar\eta$





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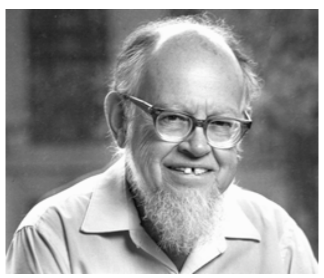

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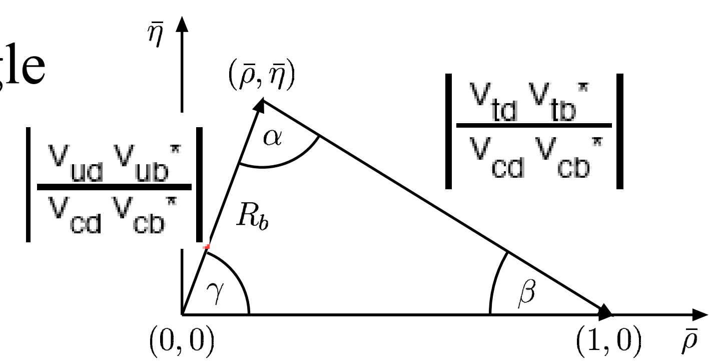


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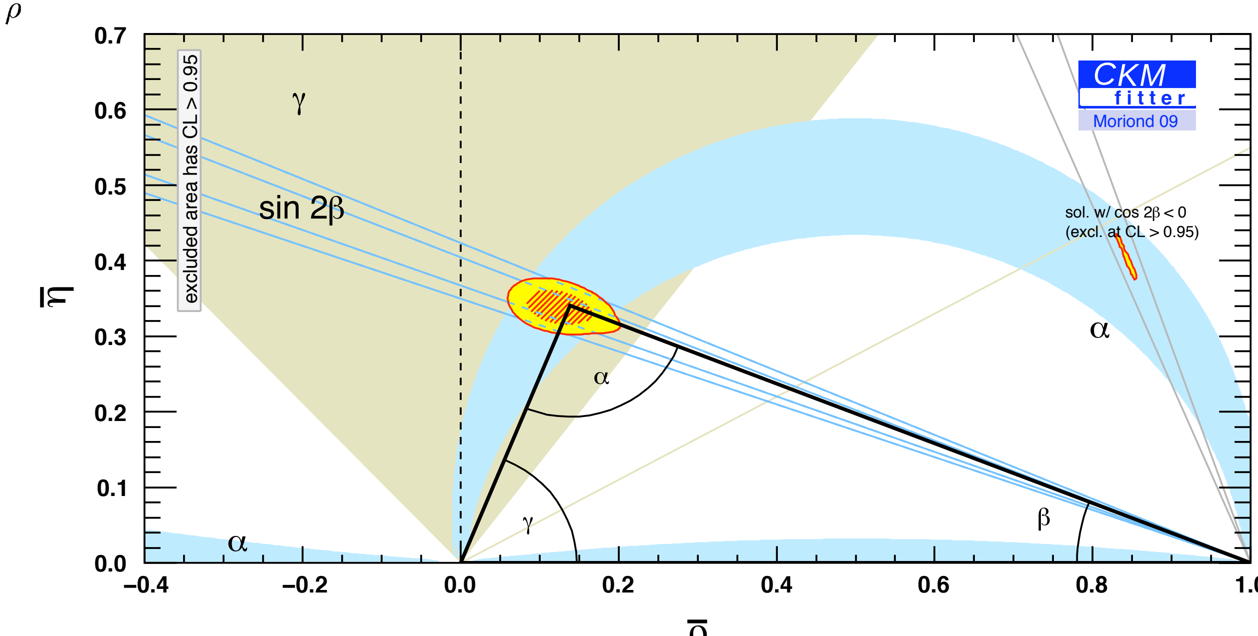
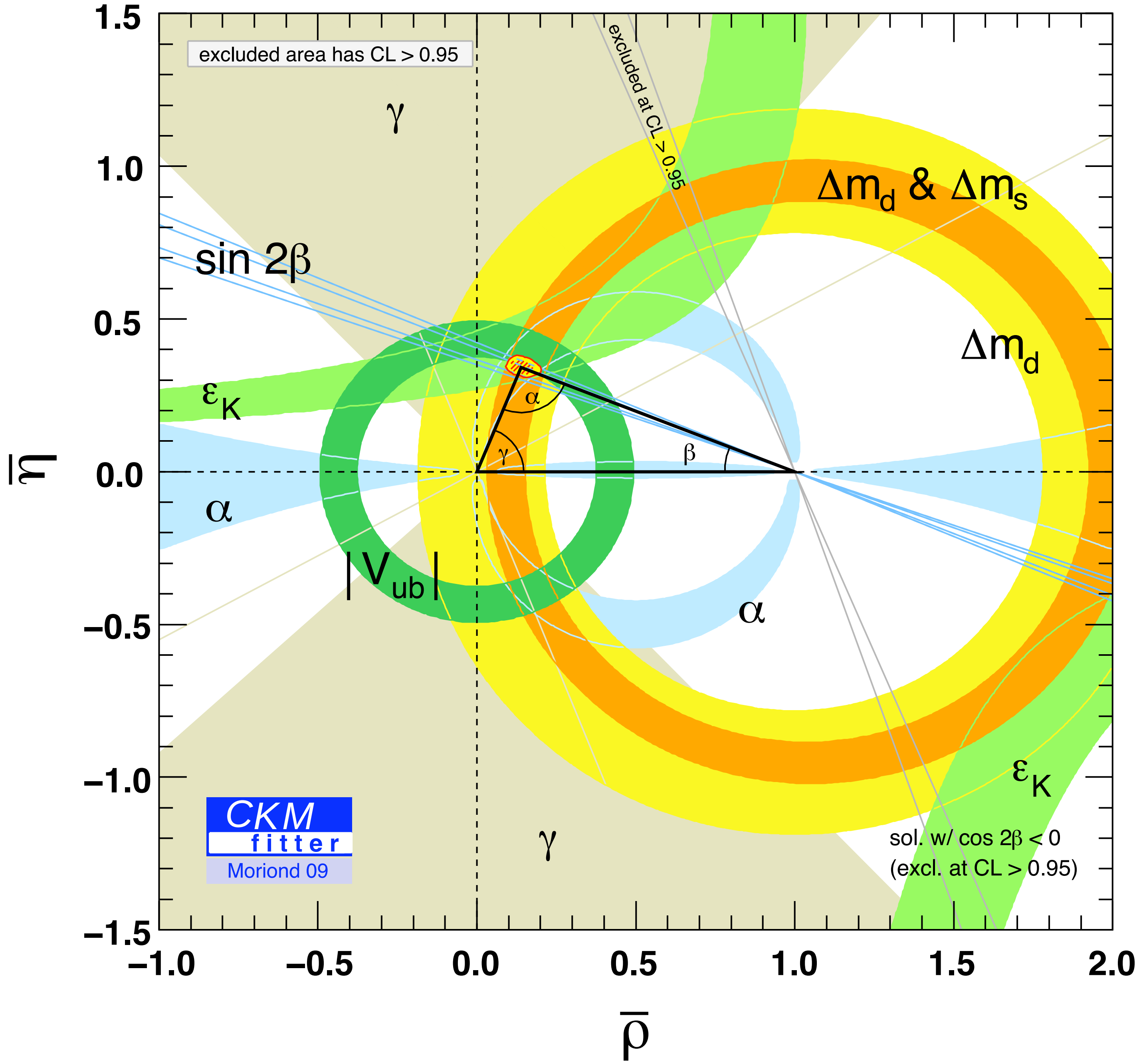
- Angles give ....



Consistency check of CKM theory: global fit



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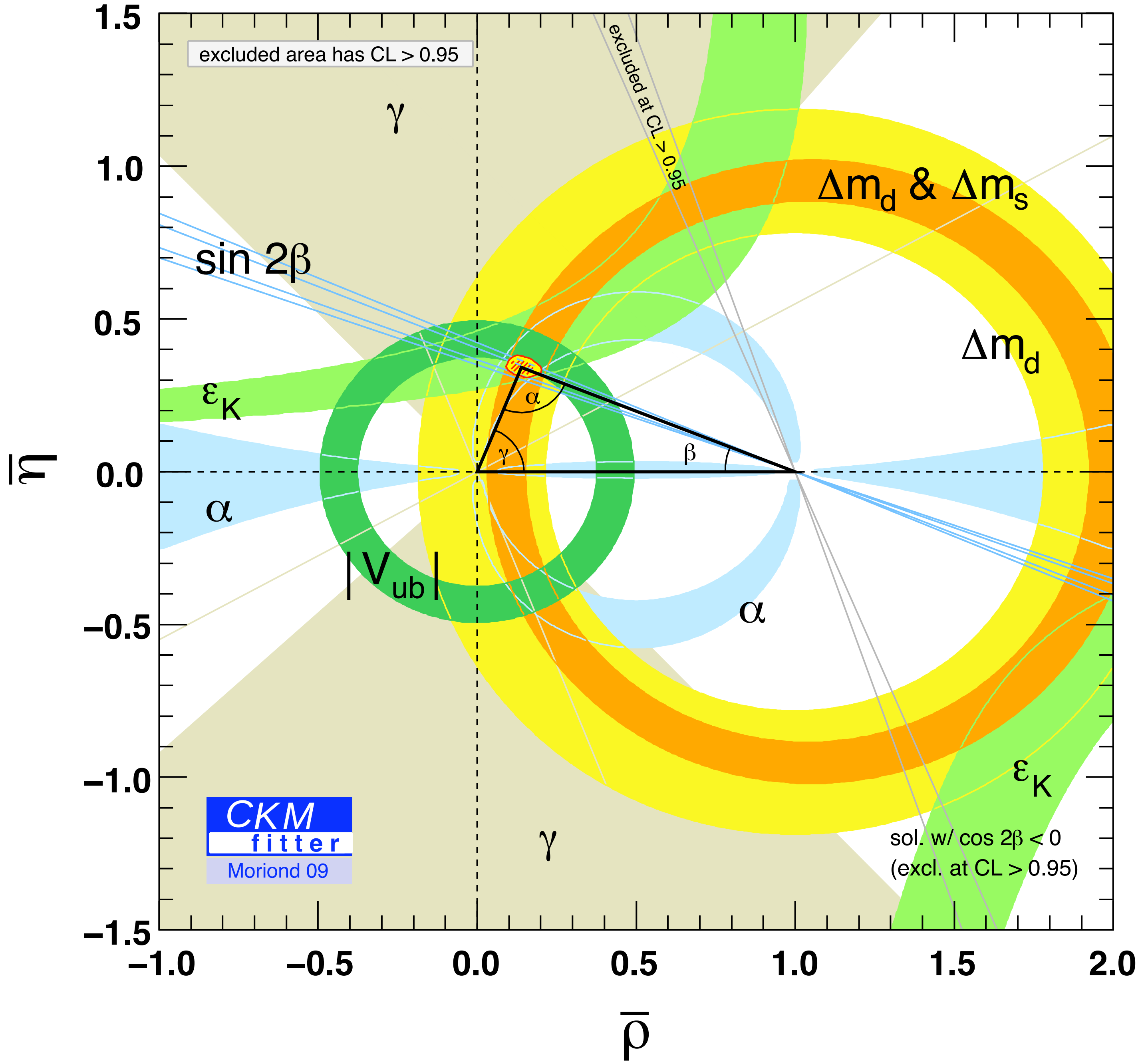


Why care about anything other than sin $2\beta$ and $\Delta m_d/\Delta m_s$?

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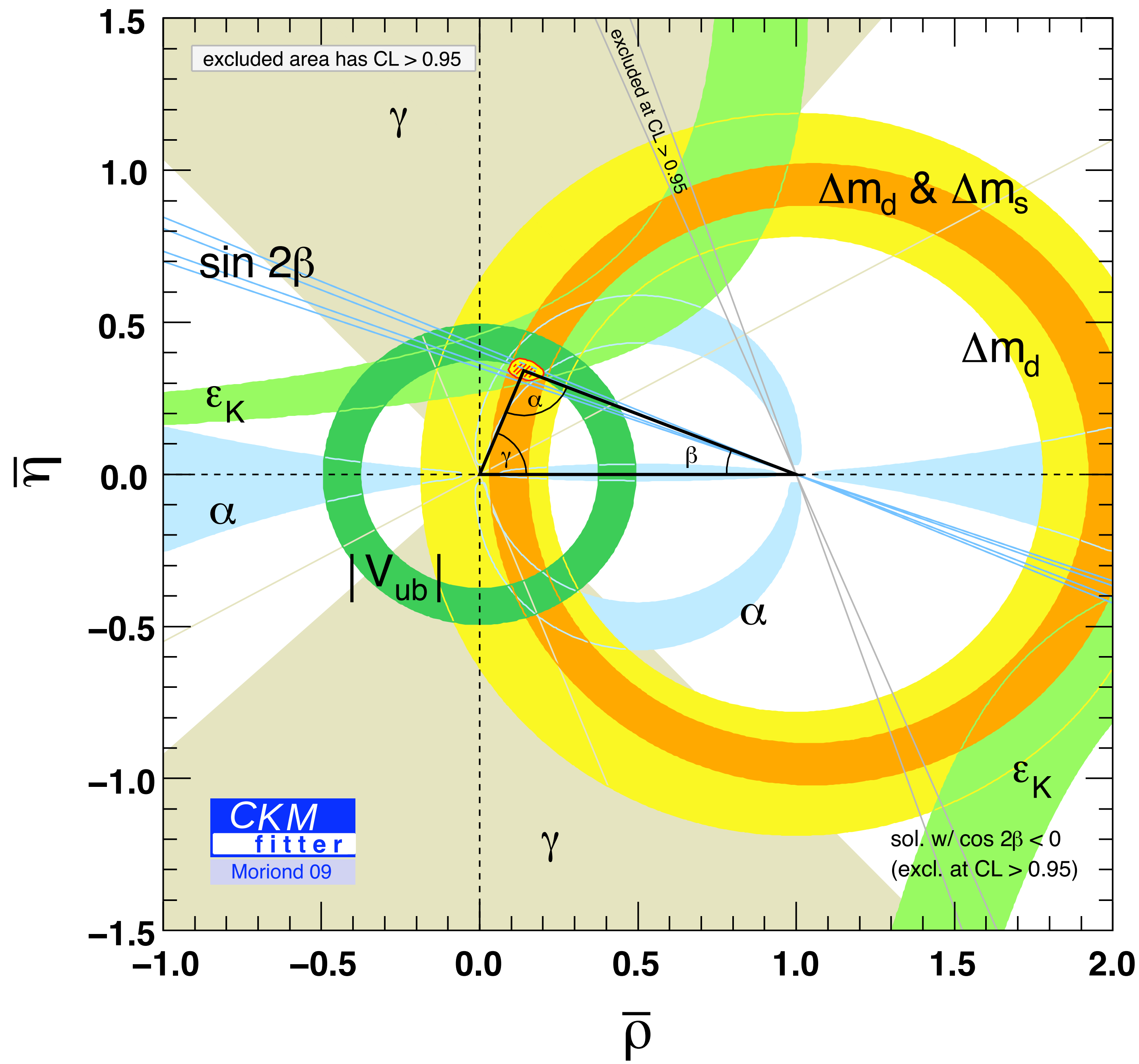
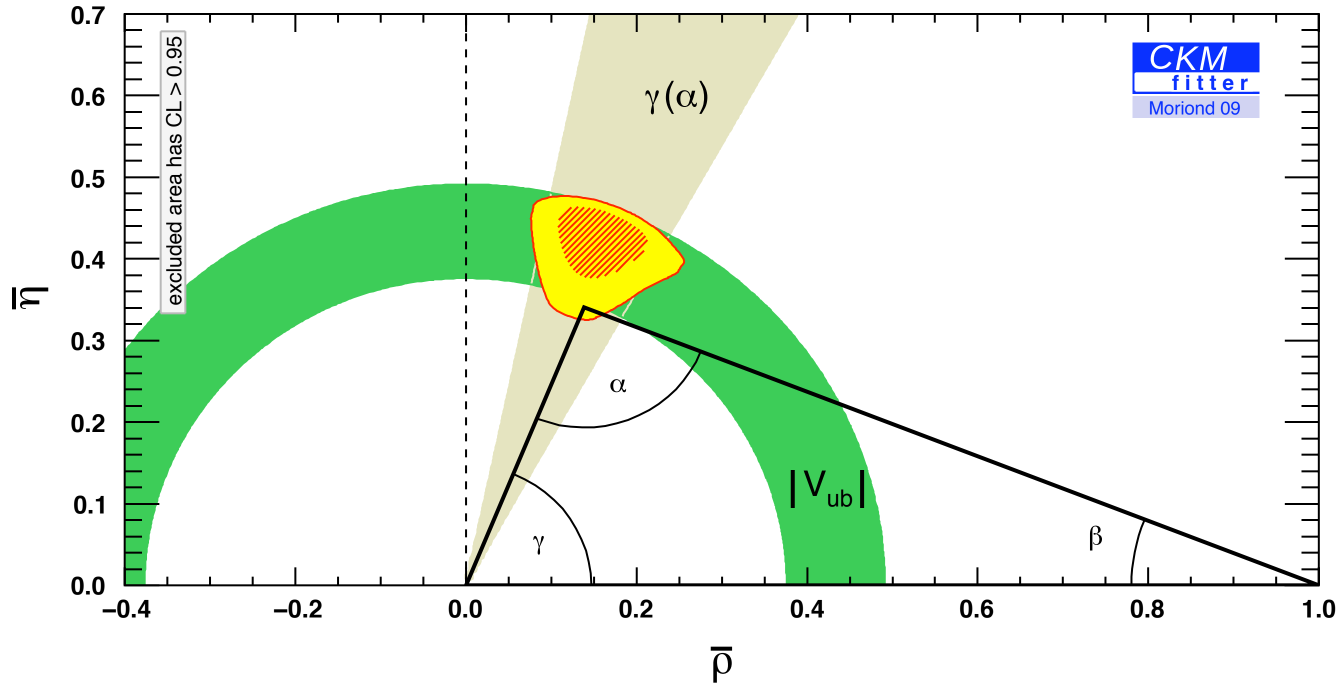
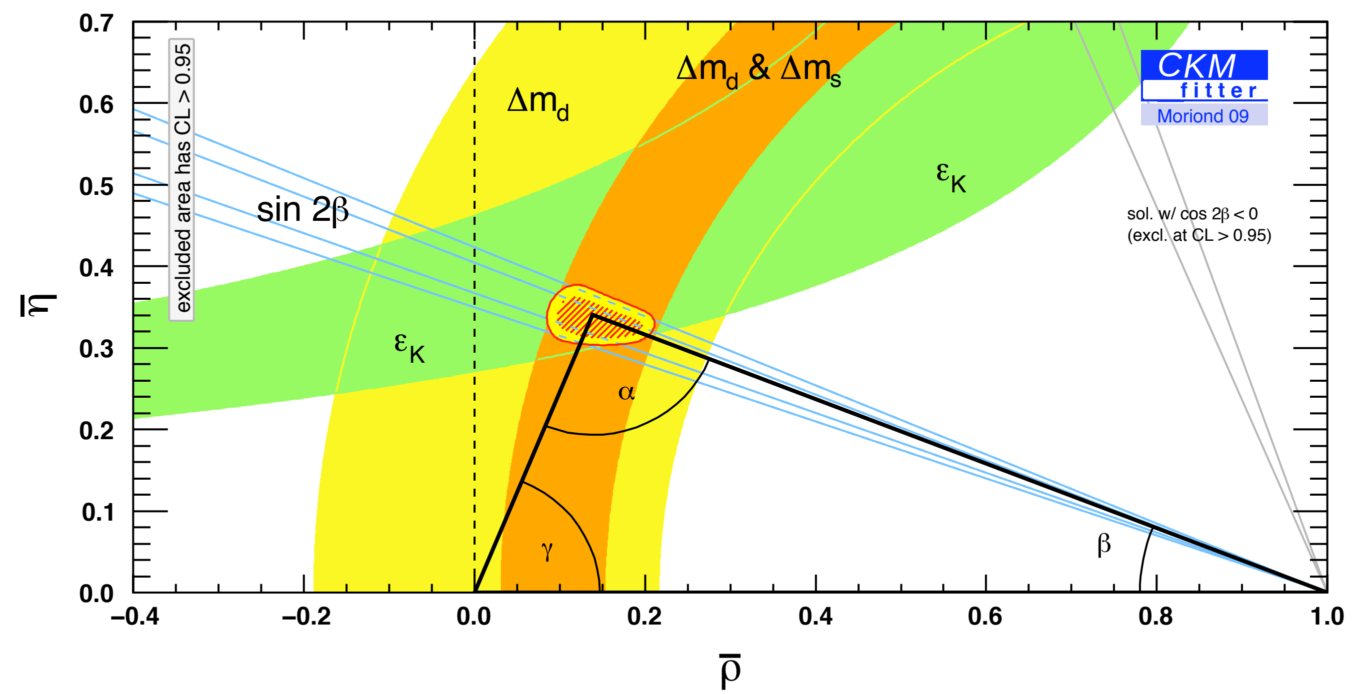



Why care about anything other than $\sin 2\beta$ and $\Delta m_d/\Delta m_s$?

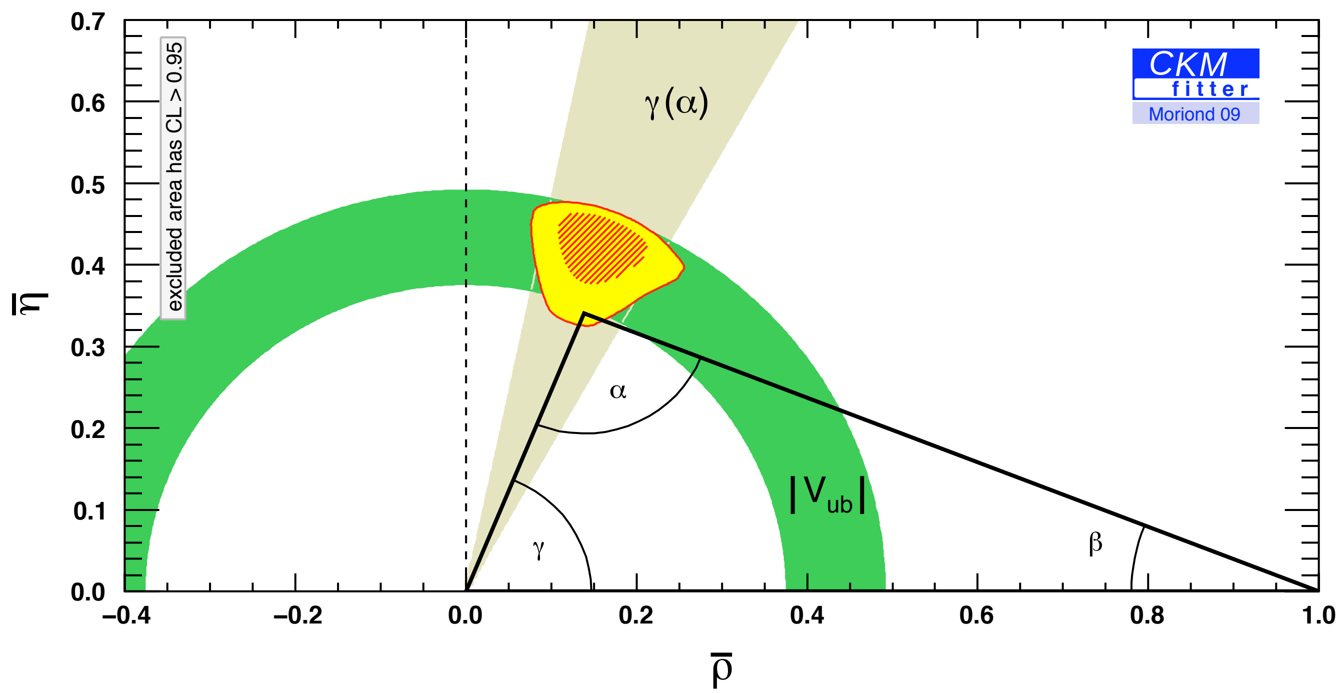
tree vs loop: disentangling new physics form old (orthodoxy: NP enters only at loop level)




Electroweak tree level




EW loop processes

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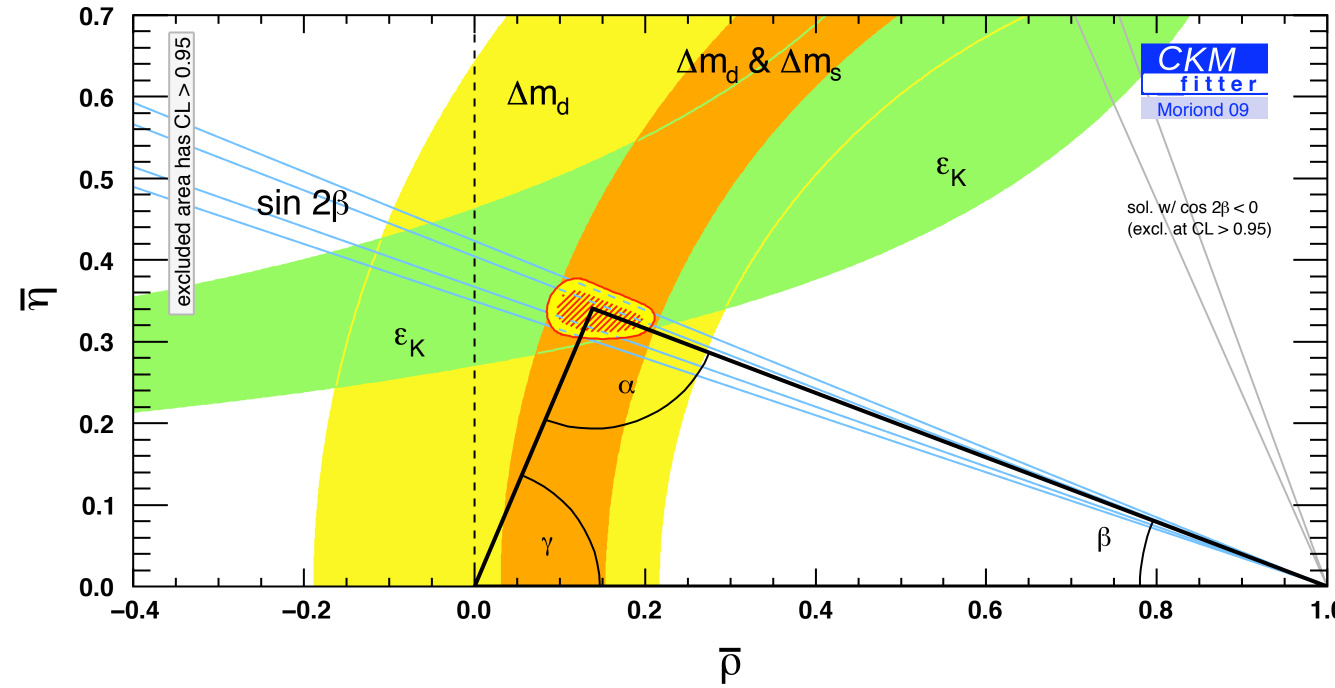
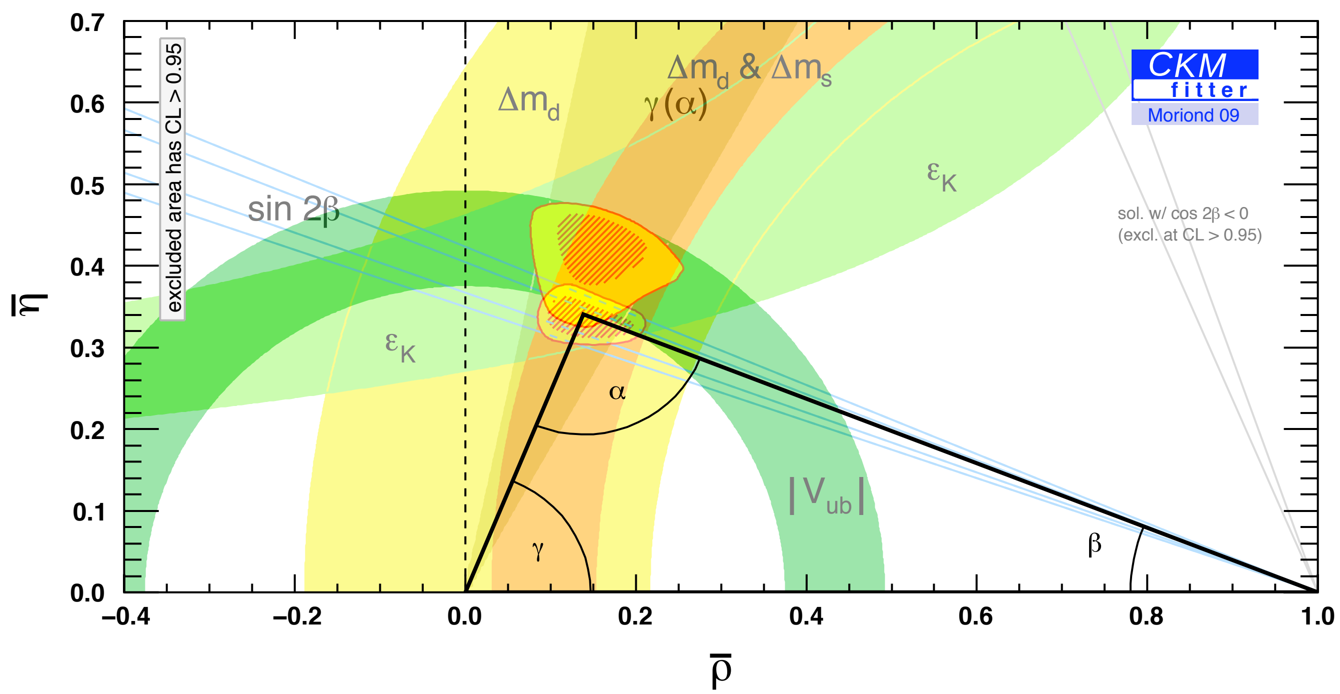
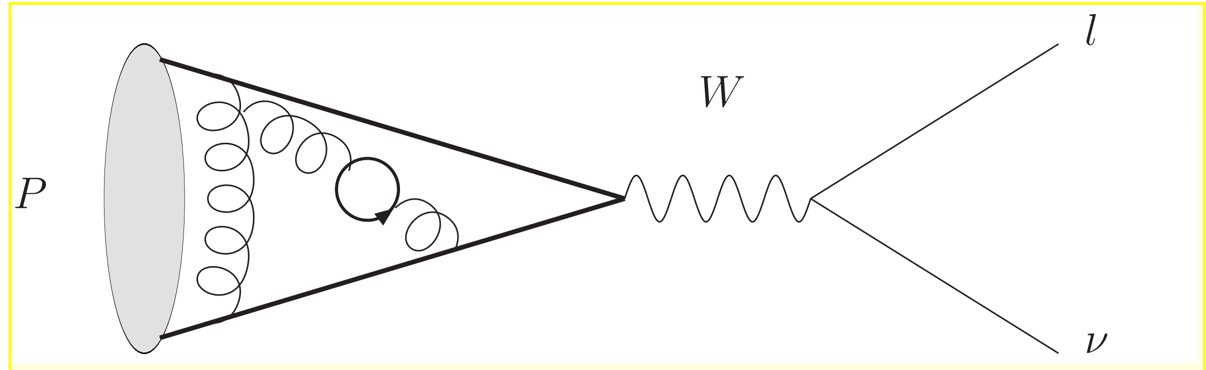


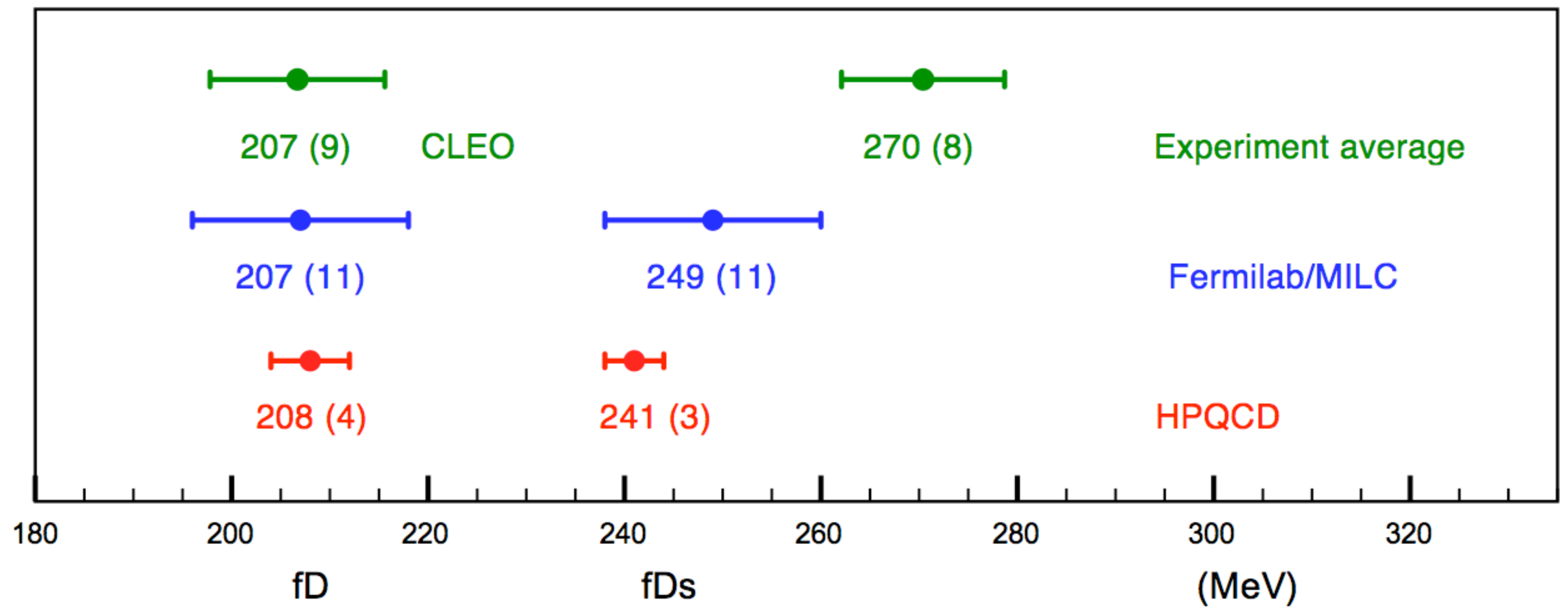

Consistency !?

# Progress and Puzzles in Purely Leptonic *B* and *D* Decays

Theory? B/D decay constants




$$\Gamma(D_s \to \ell\nu_\ell) = \frac{m_{D_s}}{8\pi} f_{D_s}^2 \left|G_F V_{cs}^* m_\ell\right|^2 \left(1 - m_\ell^2/m_{D_s}^2\right)^2,$$




HPQCD/exp discrepancy greater than 3$\sigma$

New Physics?

[Dobrescu, Kronfeld]

- *s*-channel charged higgs exchange, with $y_s \ll y_c$ and $y_c, y_\tau \sim 1$: disfavored by *D* decay data
- *t*-channel charge +2/3 leptoquark exchange; disfavored by bound on $\tau \to \mu ss$
- *u*-channel charge –1/3 leptquark exchange (like *d*-squark)

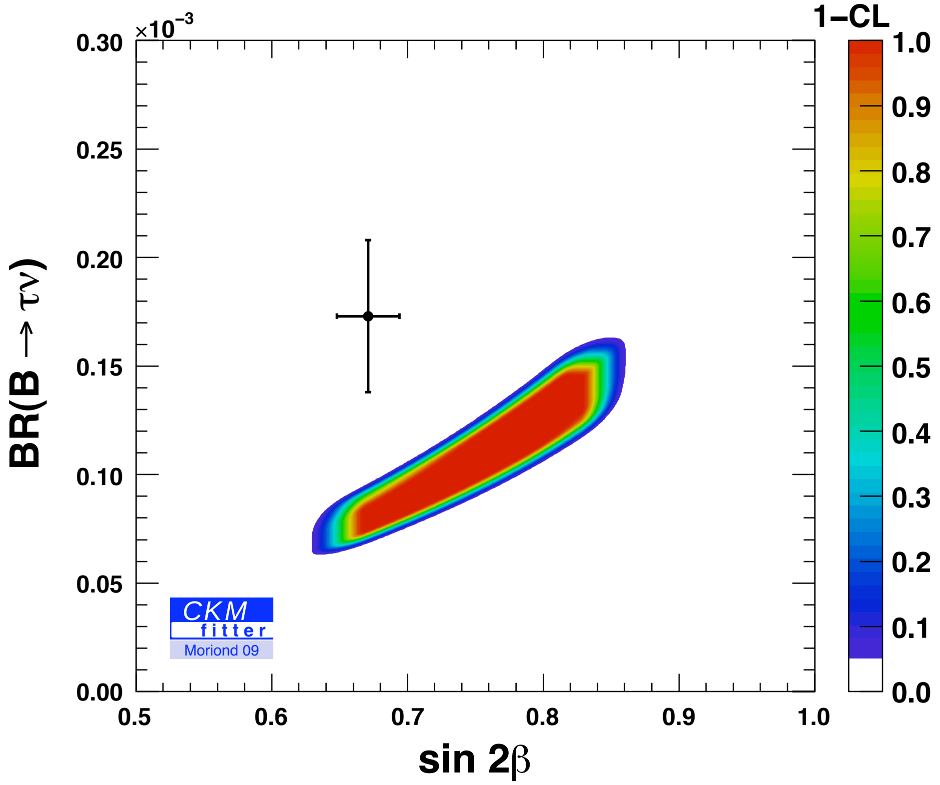
$$\mathscr{L}_{\text{LQ}} = \kappa_{2\ell}\left(\bar{c}_L \ell_L^c - \bar{s}_L \nu_{\ell L}^c\right)\tilde{d} + \kappa'_{2\ell}\,\bar{c}_R \ell_R^c \tilde{d} + \text{H.c.},$$

All tree level!

# $B \to \tau\nu$

Tension between $\sin 2\beta$ and Br ($B \to \tau\nu$):

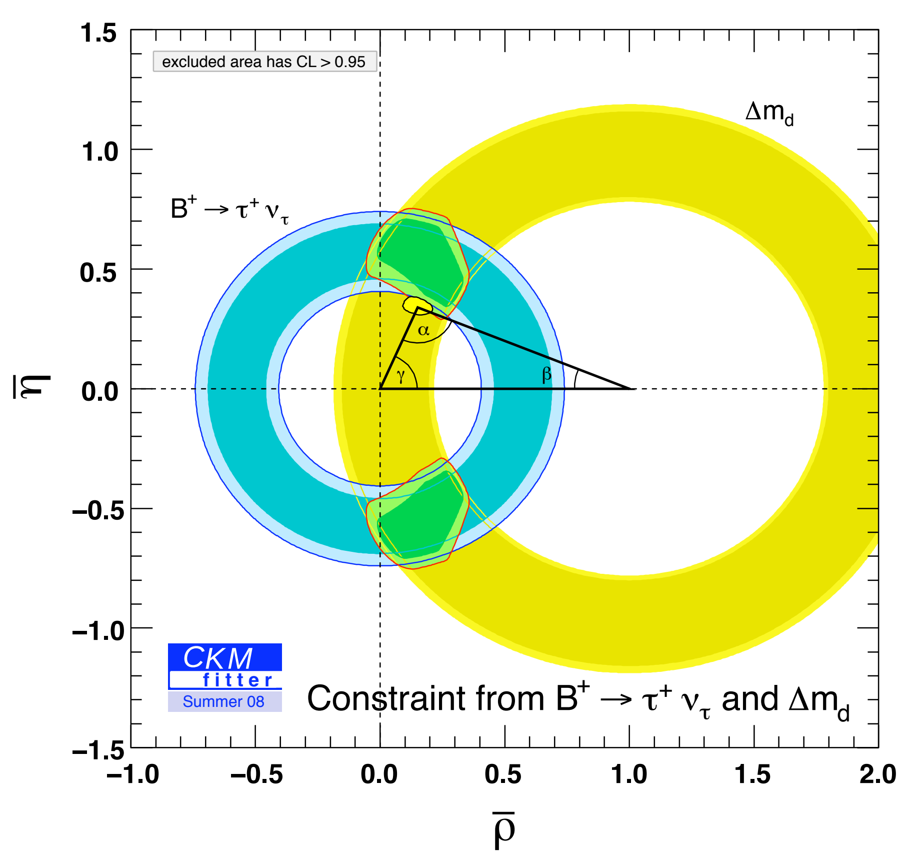
global fit without using these measurements,
cross is from experimental values ($1\sigma$)




Shape of correlation best understood from ratio:

$$\frac{\mathrm{Br}(B \to \tau\nu)}{\Delta m_d} = \frac{3\pi m_\tau^2}{4 m_W^2 S(x_t)} \left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 \tau_{B_+} \frac{1}{|V_{ud}|^2 B_{B_d}} \left(\frac{\sin\beta}{\sin\gamma}\right)^2$$

- Decay constants cancel
- Depends only on bag parameter $B_B$
- Constraint in z plane does not match exactly global fit

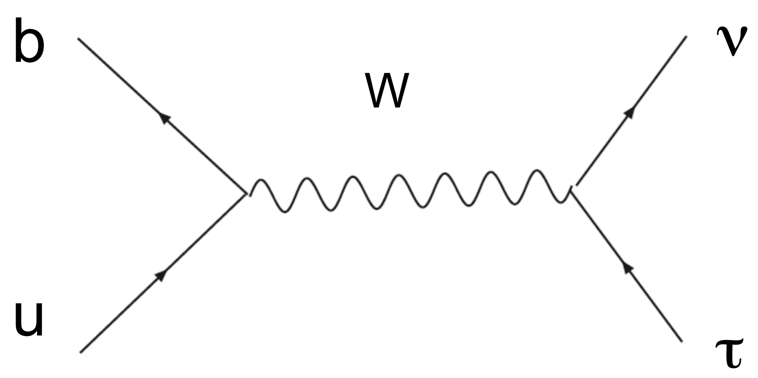


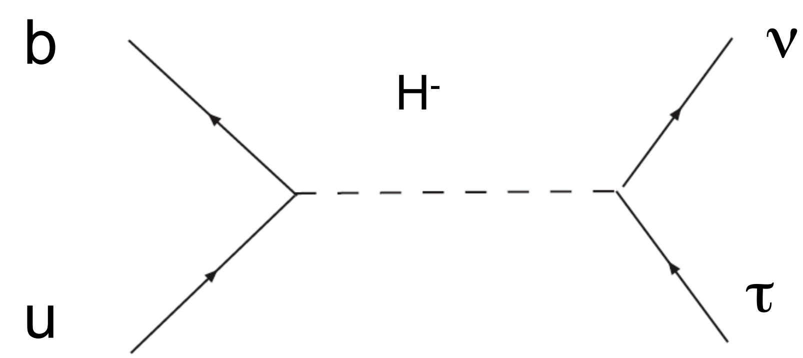
# $B \to \tau\nu$, $B \to X_c\tau\nu$

Sensitive to charged higgs exchange. Normalize to non-tau [Hou; Isidori]

$$\tan\beta = \frac{v_2}{v_1}$$

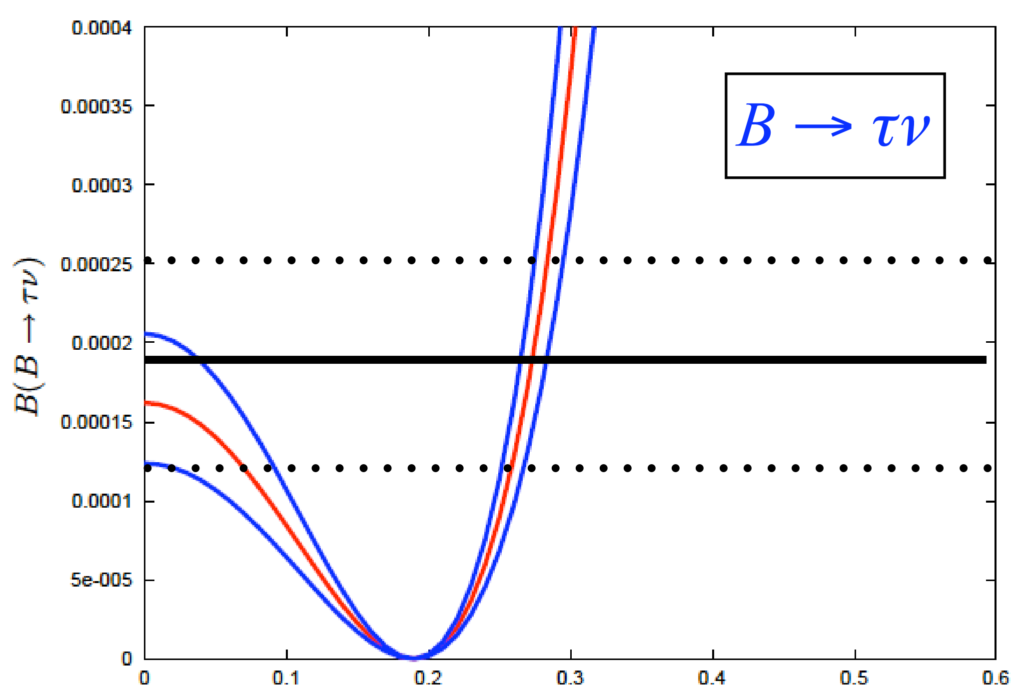
$$r = \frac{\tan\beta}{M_{H^\pm}}$$



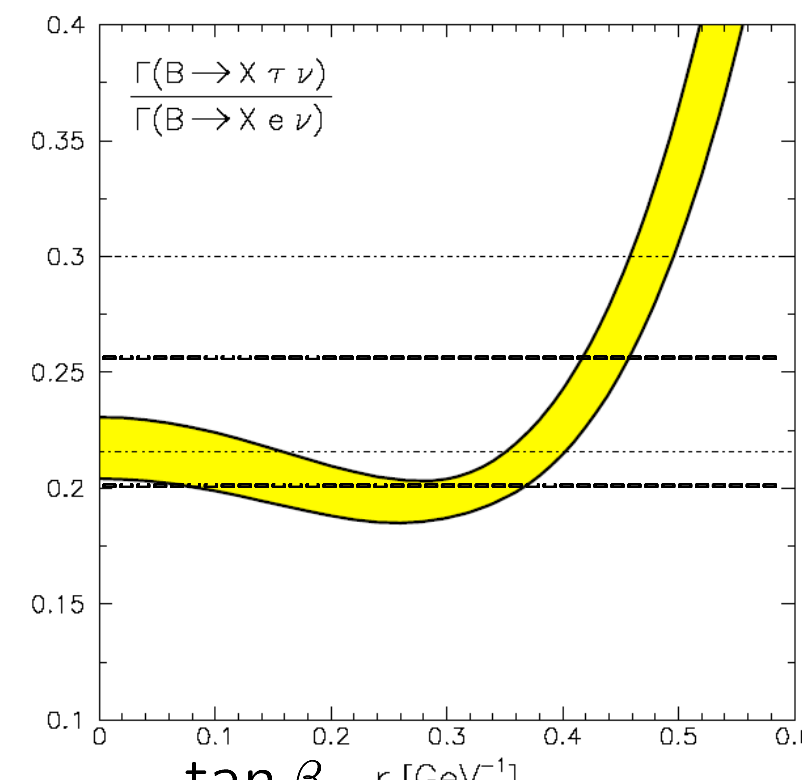




$$\mathrm{Br}(B \to \tau\nu) = (1.79^{+0.56+0.39}_{-0.49-0.46}) \times 10^{-4}$$ (Belle)

$$\mathrm{Br}(B \to \tau\nu X) = (2.48 \pm 0.26) \times 10^{-2}$$ (LEP)




[Ito, Gaur, Okada]




LEP

$$r \equiv \frac{\tan\beta}{m_H}$$

[Grossman, Haber, Nir]

# Neutral Meson Mixing: generalities

time evolution: $$i\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}|P^0(t)\rangle\\ |\overline{P}^0(t)\rangle\end{pmatrix} = \left(M - \frac{i}{2}\Gamma\right)\begin{pmatrix}|P^0(t)\rangle\\ |\overline{P}^0(t)\rangle\end{pmatrix}$$

solution: $$|P_{L,H}\rangle = p\,|P^0\rangle \pm q\,|\overline{P}^0\rangle$$

$$|P_{L,H}(t)\rangle = e^{-(im_{L,H}+\Gamma_{L,H}/2)t}\,|P_{L,H}\rangle$$

with:

$$m = \frac{m_H + m_L}{2}, \qquad \Gamma = \frac{\Gamma_H + \Gamma_L}{2}$$

$$\Delta m = m_H - m_L, \qquad \Delta\Gamma = \Gamma_H - \Gamma_L$$

$$(\Delta m)^2 - \frac{(\Delta\Gamma)^2}{4} = 4\,|M_{12}|^2 - |\Gamma_{12}|^2,$$

$$\Delta m\,\Delta\Gamma = 4\,\mathrm{Re}(M_{12}\Gamma_{12}^*),$$

$$\frac{q^2}{p^2} = \frac{2M_{12}^* - i\,\Gamma_{12}^*}{2M_{12} - i\,\Gamma_{12}}.$$

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Decay amplitudes: $A_f = \langle f|\mathcal{H}|P^0\rangle, \qquad \bar{A}_f = \langle f|\mathcal{H}|\overline{P}^0\rangle$

physical quantities:

- $\left|\frac{\bar{A}_{\bar{f}}}{A_f}\right|$ — ≠ 1 if CPV in decay
- $\left|\frac{q}{p}\right|$ — ≠ 1 if CPV in mixing
- $\lambda_f = \frac{q}{p}\frac{\bar{A}_f}{A_f}$ — Im ≠ 0 if CPV in interference decay-mixing
- $\phi_{12} = \arg\left(-\frac{M_{12}}{\Gamma_{12}}\right)$ — sensitive to NP in $M_{12}$ (≈ 0 in SM in $K$, $B_{d,s}$)
- $\mathrm{Im}\,\frac{\Gamma_{12}}{M_{12}} = \frac{1-|q/p|^4}{1+|q/p|^4}$ — dilepton asymm. ($A_{SL}$ for B's) non-pertubative: incalculable (OPE in B no better than lifetime)

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*Cases*

$\Delta m \gg |\Delta\Gamma|$ (e.g., for $B_{d,s}$)

$$\Delta m = 2|M_{12}|(1 + \mathcal{O}(\Gamma_{12}/M_{12}))$$
$$\Delta\Gamma = -2|\Gamma_{12}|\cos\phi_{12}(1 + \mathcal{O}(\Gamma_{12}/M_{12}))$$

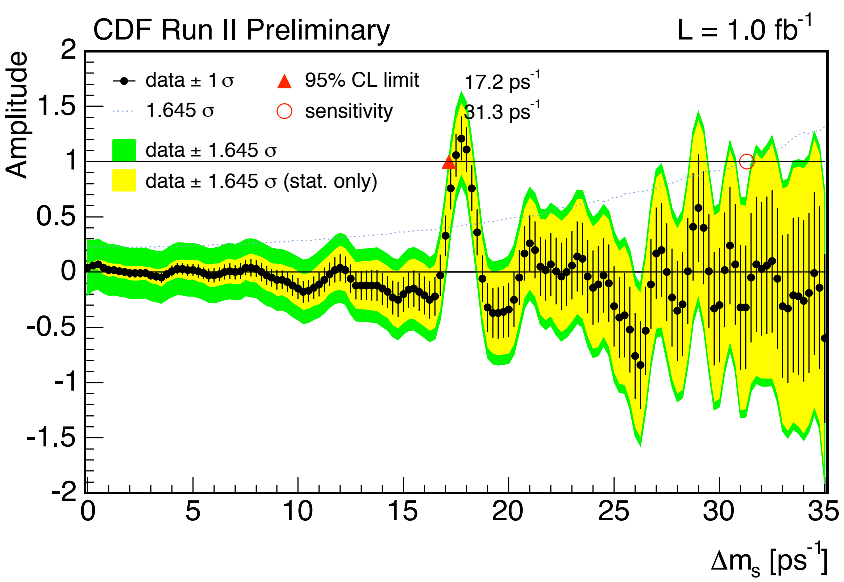
- $\phi_{12}$ suppressed in SM (Bd,s)
- NP can only reduce $|\Delta\Gamma|$
- $q/p = -\arg(M_{12})(1+\ldots)$ so time dependent CP asymmetries sensitive no NP in $M_{12}$

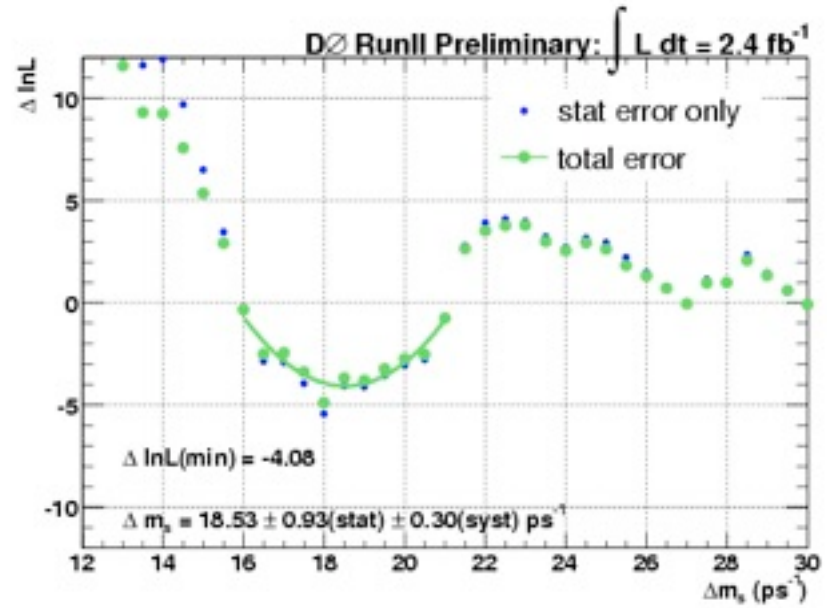
$\Delta m \ll |\Delta\Gamma|$

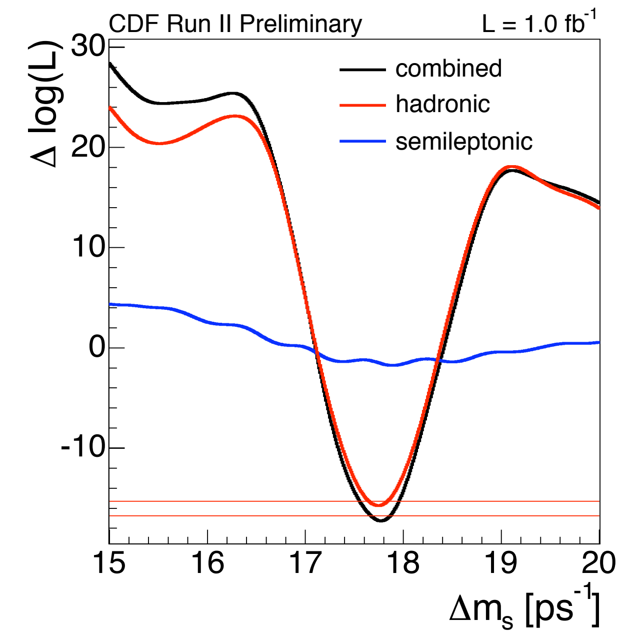
$$\Delta m = 2|M_{12}\cos\phi_{12}|(1 + \mathcal{O}(M_{12}/\Gamma_{12}))$$
$$\Delta\Gamma = \mp 2|\Gamma_{12}|(1 + \mathcal{O}(M_{12}/\Gamma_{12}))$$

- $q/p = -\arg(\Gamma_{12})(1+\ldots)$ depends weakly on $M_{12}$
- if $\Delta m \ll |\Delta\Gamma|$ and no CPV in D decay
$$\arg\lambda_{K^+K^-} \propto 2\left|\frac{M_{12}}{\Gamma_{12}}\right|^2\sin(2\phi_{12}).$$
- reduced sensitivity to NP in $M_{12}$ (even for dominant NP)

# $B\bar{B}$ mixing: $|V_{td}|$




**Theory:**

$$\frac{|V_{td}|}{|V_{ts}|} = \xi \sqrt{\frac{\Delta m_s \ m_{B_s}}{\Delta m_d \ m_{B_d}}} \qquad \xi^2 = \frac{B_{B_s} f_{B_s}^2}{B_{B_d} f_{B_d}^2}$$

**Lattice:** $\xi = 1.211 \pm 0.038 \pm 0.024_{estimate}$

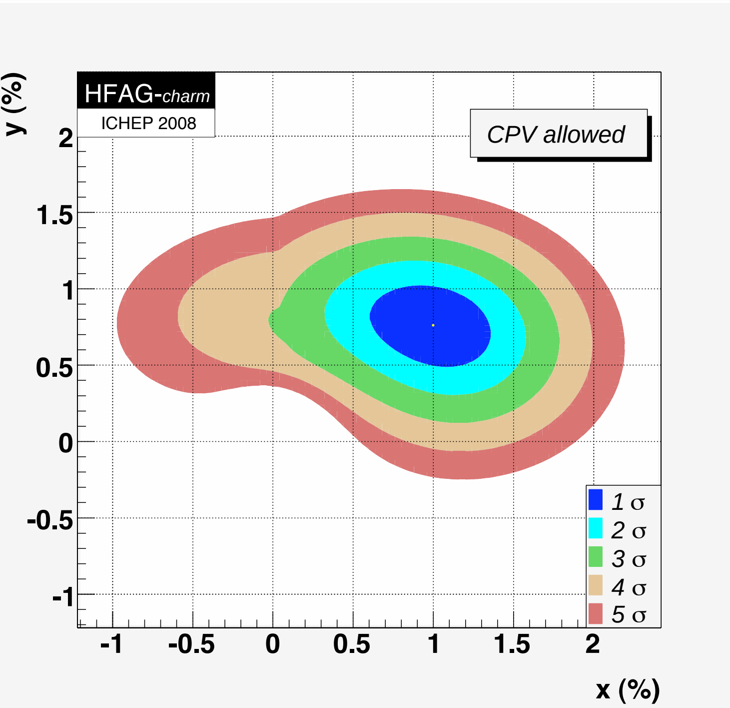
I'll believe a 3% lattice theory error when the lattice has produced one successful prediction and several 3% postdictions
However, here the calculation is really of $\xi^2-1$, and the error is 16%
Chiral lag gives only chiral logs, so error in $\xi^2-1 \approx 0.3$ is ~100%

$$\boxed{\frac{|V_{td}|}{|V_{ts}|} = 0.2060 \pm 0.0012(\mathrm{exp})^{+0.0081}_{-0.0060}(\mathrm{th})}$$

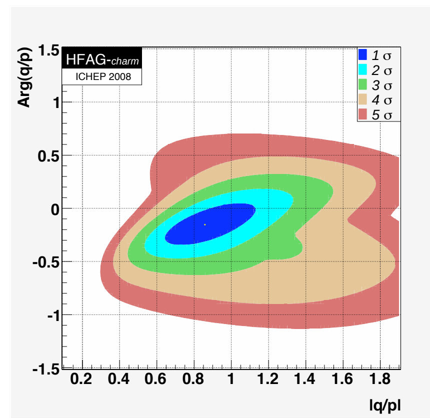
using $\xi = 1.210^{+0.047}_{-0.035}$

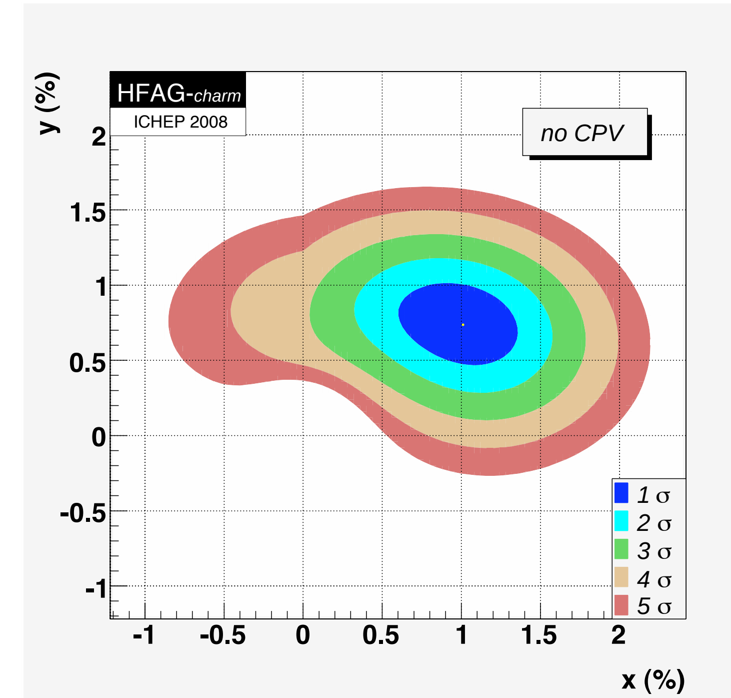
[CDF & D0: 0905.1109]

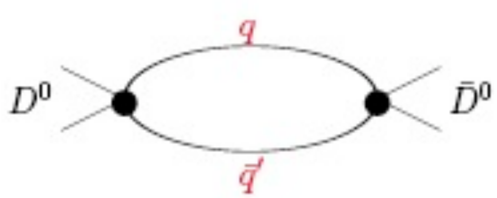
# $D\bar{D}$ mixing



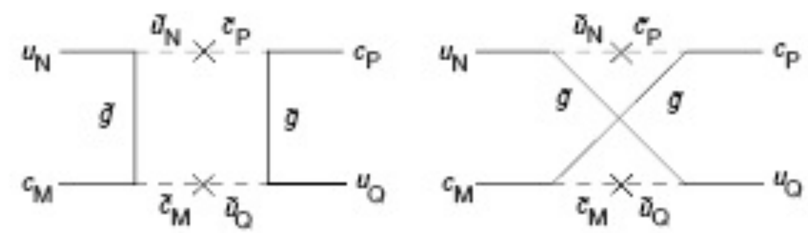

$$x = \frac{\Delta m}{\Gamma}\,, \qquad y = \frac{\Delta\Gamma}{2\Gamma}$$



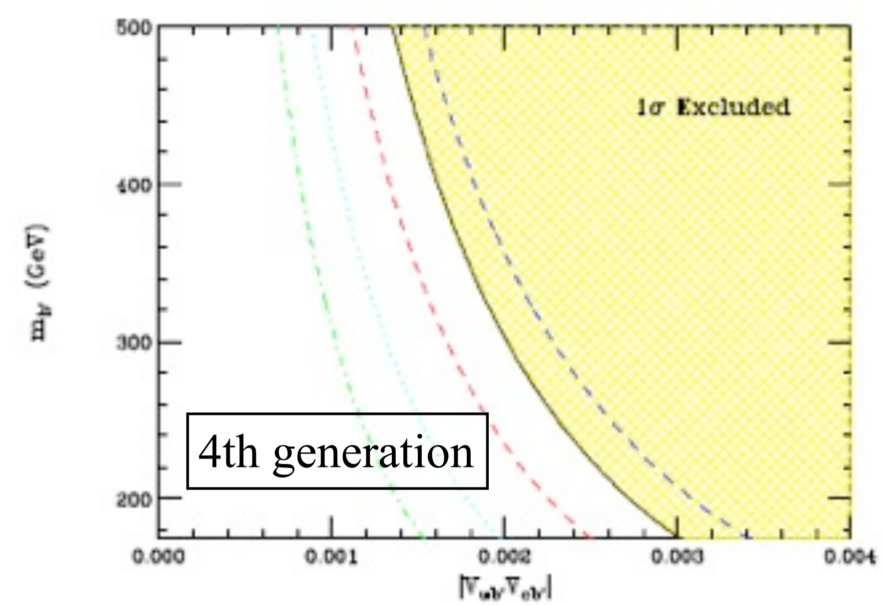

$q, q' = d, s, b$



(a susy box)

- No $t$-quark in box diagram: GIM suppressed $\Delta m$
- $x, y$ likely dominated by long distance physics
- Expect $x \sim y$ and $x, y \sim \sin^2\theta_C \times$ [SU(3) breaking]
- NP in $M_{12}$, rather than in $\Gamma_{12}$:

  $M_{12} \sim$ loop, $\Gamma_{12} \sim$ tree
- May still obtain useful bounds by assuming no perverse cancellation between SM and NP, then demand NP contribution is less than measured




[et al and A. Petrov]
[see also Ciuchini et al]

# $|V_{cb}|$: inclusive decays $B \to X_c \ell \nu$

- <u>Theory</u>:
  Combine Heavy Quark Mass Expansion (HQET) and Operator Product Expansion (OPE)
  [Chay, Georgi, BG; Voloshin, Shifman; Bigi et al; Manohar, Wise, Bloek et al]

- OPE: expand since $m_b$ is larger than any scale in HQET matrix element
- Decay rate for $B \to X_c \ell \nu_\ell$

$$\Gamma_n/\Gamma_0 \sim (\Lambda_{\rm QCD}/m_b)^n$$

$$\Gamma \;=\; \Gamma_0 + \quad \Gamma_1 + \quad \Gamma_2 + \quad \Gamma_3 + \quad \ldots$$

| | $\Gamma_0$ | $\Gamma_1$ | $\Gamma_2$ | $\Gamma_3$ | |
|---|---|---|---|---|---|
| 6 parameters | $m_b$ | | $\lambda_1$ | $\mathcal{T}_{1,2,3}, \rho_1$ | With $1/m_c$ expansion |
| | $m_b, m_c$ | | $\lambda_{1,2}$ | $\rho_{1,2}$ | No $1/m_c$ expansion |

- $\Gamma_i$ given in terms of few non-perturbative parameters + expansion in $\alpha_s(m_b)$
- $\Gamma_0$ is free quark decay rate
- $\Gamma_1 = 0$ (Luke's theorem)
- Local quark-hadron duality is mildly used (to show a correction is small)
- Moments of distribution have differing sensitivity to non-perturbative parameters

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Not the same person

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- Decay rate for $B \to X_c \ell \nu_\ell$

$$\Gamma_n/\Gamma_0 \sim (\Lambda_{\rm QCD}/m_b)^n$$

$$\Gamma = \Gamma_0 + \Gamma_1 + \Gamma_2 + \Gamma_3 + \ldots$$

$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

6 parameters $\begin{cases} m_b & \lambda_1 & \mathcal{T}_{1,2,3}, \rho_1 \\ m_b, m_c & \lambda_{1,2} & \rho_{1,2} \end{cases}$

$\leftarrow$ With $1/m_c$ expansion

$\leftarrow$ No $1/m_c$ expansion

- $\Gamma_i$ given in terms of few non-perturbative parameters + expansion in $\alpha_s(m_b)$
- $\Gamma_0$ is free quark decay rate
- $\Gamma_1 = 0$ (Luke's theorem)
- Local quark-hadron duality is mildly used (to show a correction is small)
- Moments of distribution have differing sensitivity to non-perturbative parameters

# $|V_{cb}|$: inclusive decays $B \to X_c \ell \nu$

- Theory:
  Combine Heavy Quark Mass Expansion (HQET) and Operator Product Expansion (OPE)
  [Chay, Georgi, BG; Voloshin, Shifman; Bigi et al; Manohar, Wise, Bloek et al]

- OPE: expand since $m_b$ is larger than any scale in HQET matrix element
- Decay rate for $B \to X_c \ell \nu_\ell$

$\Gamma_n/\Gamma_0 \sim (\Lambda_{\rm QCD}/m_b)^n$

$$\Gamma = \Gamma_0 + \Gamma_1 + \Gamma_2 + \Gamma_3 + \ldots$$

6 parameters:
- $m_b$, $\lambda_1$, $\mathcal{T}_{1,2,3}, \rho_1$ ← With $1/m_c$ expansion
- $m_b, m_c$, $\lambda_{1,2}$, $\rho_{1,2}$ ← No $1/m_c$ expansion

- $\Gamma_i$ given in terms of few non-perturbative parameters + expansion in $\alpha_s(m_b)$
- $\Gamma_0$ is free quark decay rate
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- Moments of distribution have differing sensitivity to non-perturbative parameters

# Moment Analysis

[Falk et al; Kapustin, Ligeti; Gambino, Uraltsev; D. Benson et al; Bauer et al]

Lepton Energy

$$\langle E_\ell^n \rangle_{E_{\rm cut}} \equiv \frac{R_n(E_{\rm cut},0)}{R_0(E_{\rm cut},0)}$$

where

$$R_n(E_{\rm cut},M) \equiv \int_{E_{\rm cut}} (E_\ell - M)^n \frac{d\Gamma}{dE_\ell}\, dE_\ell$$

Hadronic Mass

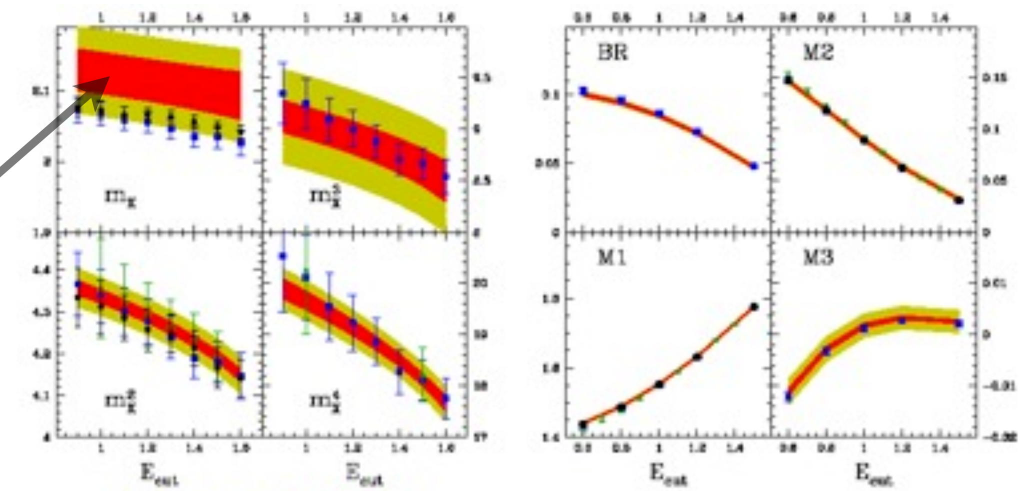
$$\langle m_X^{2n} \rangle_{E_{\rm cut}} \equiv \frac{\int_{E_{\rm cut}} (m_X^2)^n \frac{d\Gamma}{dm_X^2}\, dm_X^2}{\int_{E_{\rm cut}} \frac{d\Gamma}{dm_X^2}\, dm_X^2}$$

Photon Energy in $B \to X_s \gamma$

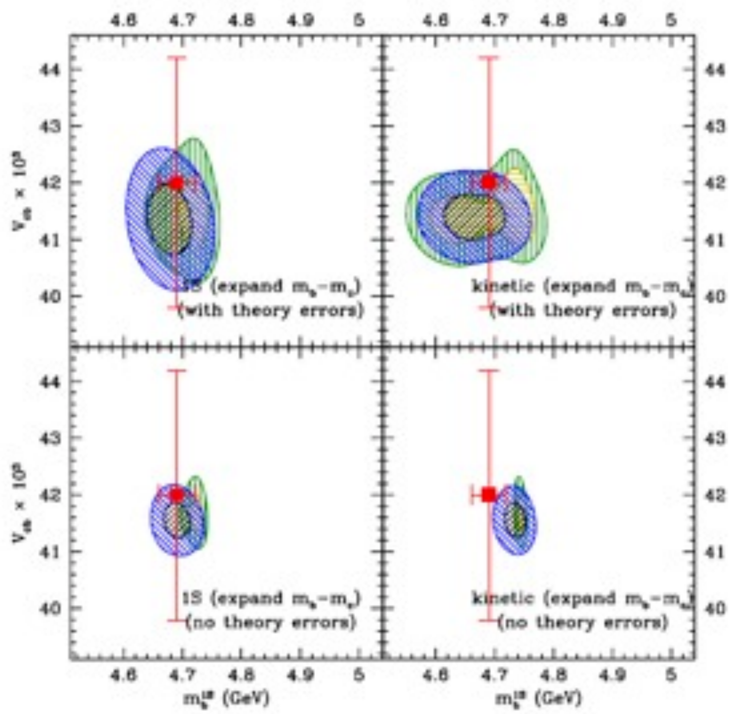
$$\langle E_\gamma^n \rangle \equiv \frac{\int_{E_{\rm cut}} (E_\gamma)^n \frac{d\Gamma}{dE_\gamma}\, dE_\gamma}{\int_{E_{\rm cut}} \frac{d\Gamma}{dE_\gamma}\, dE_\gamma}$$

# Global Analysis

- Data: BaBar, BELLE, CDF, CLEO, DELPHI
- With/without $1/m_c$ expansion
- Compare mass schemes: 1S, PS, …
- Half integer hadronic moments error badly behaved

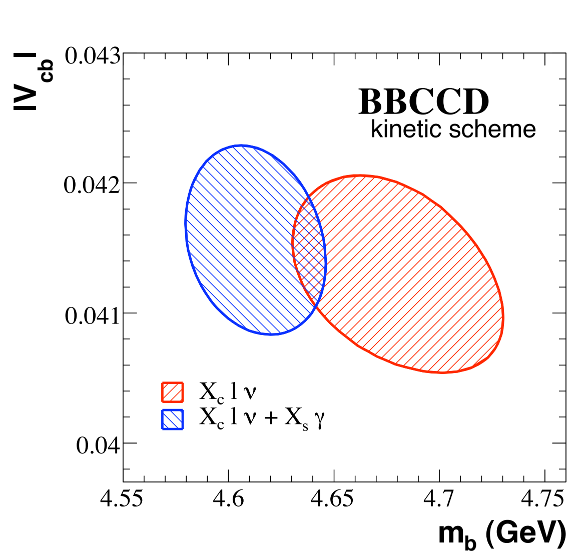


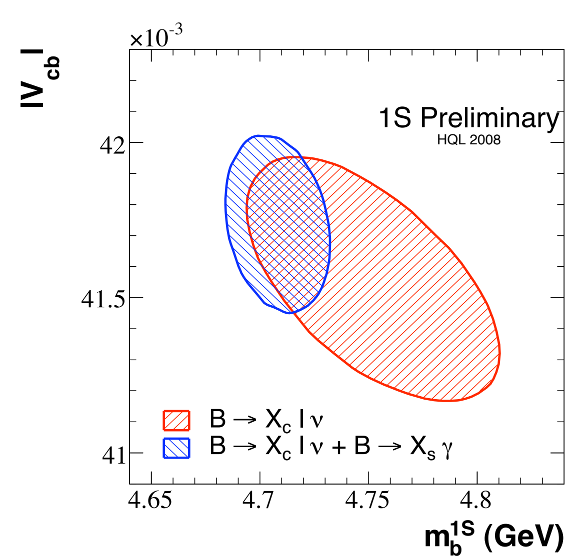

Exp: BABAR (□), CLEO (△), BELLE (•)




- Hoang's $m_b^{1S}$ with PDG's $|V_{cb}|$
- $\Delta\chi^2 = 1$ (black & yellow), $\Delta\chi^2 = 4$ (blue & green)
- yellow/green: Omit restriction on range of $(\Lambda_{QCD})^3$ parameters

+ Recent



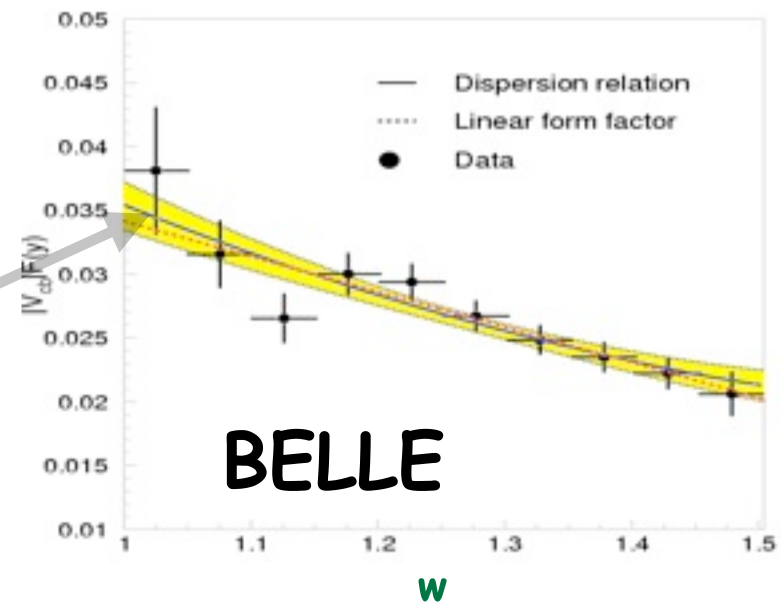

[Schwanda - 0903.3648]

(beware of different vertical scales!)

# $|V_{cb}|$: exclusive decays $B \to (D, D^*)\ell\nu$

$$\frac{d\Gamma(B\to D^*\ell\bar\nu)}{dw} = \frac{G_F^2 m_B^5}{48\pi^3} r_*^3 (1-r_*)^2 \sqrt{w^2-1}\,(w+1)^2 \times \left[1+\frac{4w}{1+w}\frac{1-2wr_*+r_*^2}{(1-r_*)^2}\right] |V_{cb}|^2 \mathcal{F}_*{}^2(w)$$

$$\frac{d\Gamma(B\to D\ell\bar\nu)}{dw} = \frac{G_F^2 m_B^5}{48\pi^3} r^3 (1+r)^2 (w^2-1)^{3/2} |V_{cb}|^2 \mathcal{F}^2(w)$$




- HQET gives $\mathcal{F}(1) = \mathcal{F}_*(1) = 1$
- Luke's theorem

$$\mathcal{F}_*(1) - 1 = \mathcal{O}(\Lambda_{QCD}/m_c)^2$$

$$\mathcal{F}_*(w) = \mathcal{F}_*(1)[1+\rho^2(w-1)+c(w-1)^2+\ldots]$$

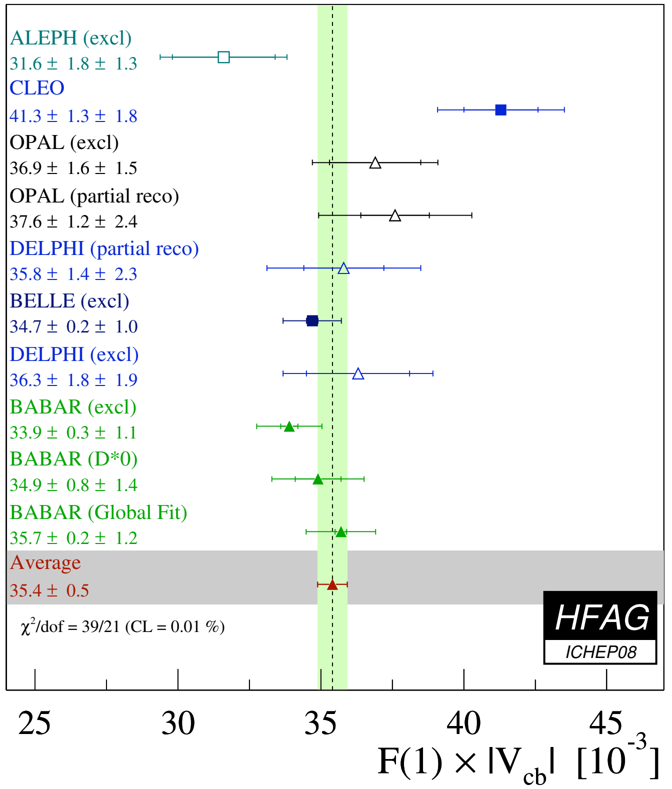
- Extrapolation to $w = 1$ constrained by dispersion relations (unitarity/analiticity)
  [Boyd et al; Caprini et al; Bjorken; Uraltsev; Oliver et al]

- Lattice $\mathcal{F}_*(1) = 0.917 \pm 0.008 \pm 0.005$
  [Dvitiis et al; Laiho et al]

HFAG (ICHEP 2008) $\mathcal{F}_*(1)|V_{cb}| = (35.41 \pm 0.52) \times 10^{-3}$

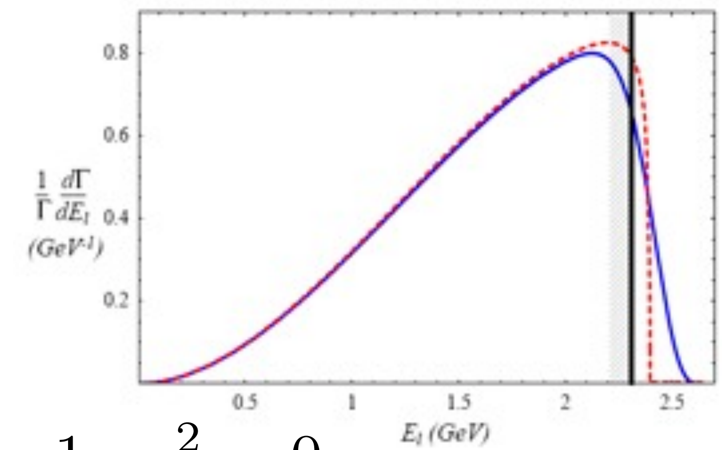
DPF-Detroit (me, added in quad)

$$|V_{cb}| = (38.62 \pm 0.69) \times 10^{-3}$$




Deviates from inclusive by $4.5\sigma$

# End point spectra in $B \to X_u \ell \nu$ and $B \to X_s \gamma$

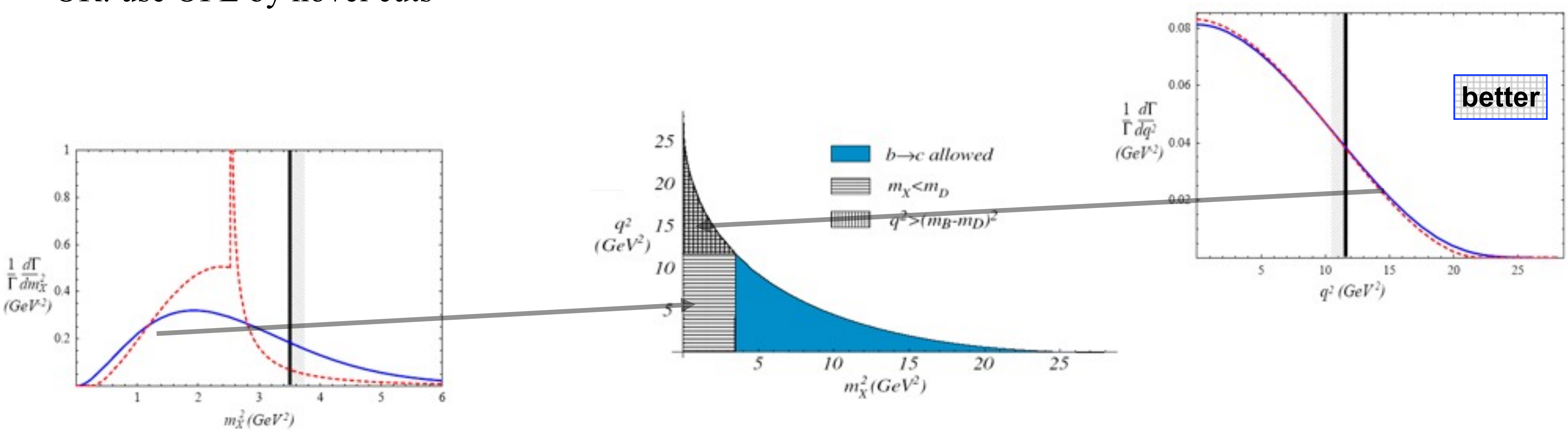



- Need to impose large E$_\ell$-cut to remove background from $B \to X_c \ell \nu$
- OPE breaks down near end of spectrum.
- Maybe re-summed (in restricted range ) into unknown "shape function" *f(x)*:
  [Bigi et al; Neubert]

$$2M_B f(x) = \langle B|\bar{Q}_v \delta(x + in \cdot D) Q_v |B\rangle \qquad n \cdot v = 1,\ n^2 = 0$$

- Universal: measure in $B \to X_s \gamma$ and use in $B \to X_u \ell \nu$
- Moments analysis: re-sum short distance corrections
  [Leibovich et al; Neubert]

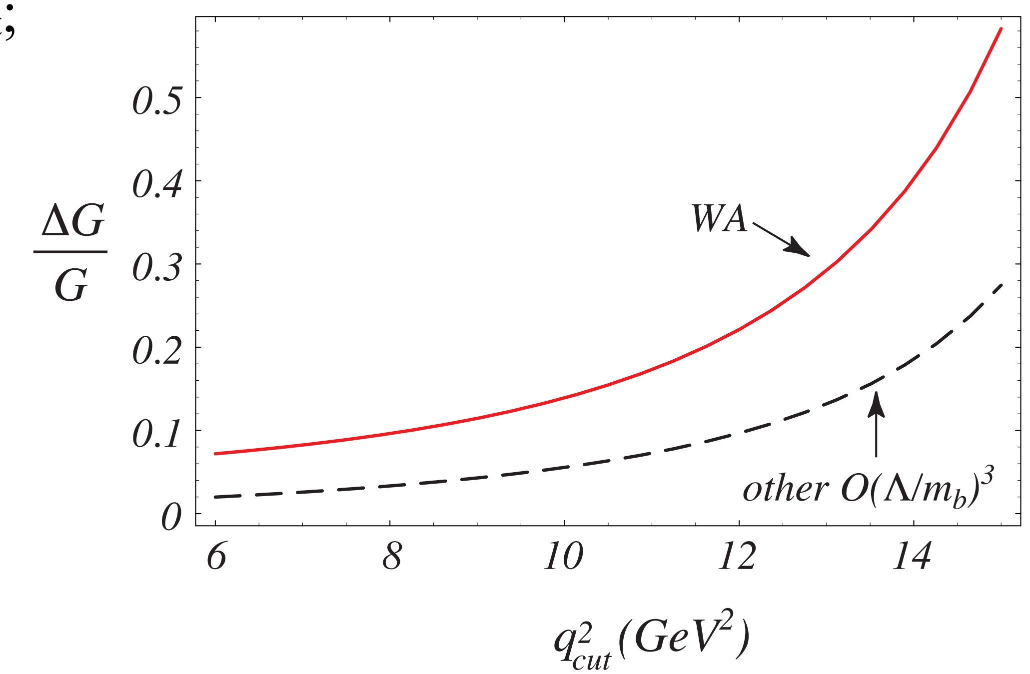
- OR: use OPE by novel cuts



# $|V_{ub}|$: inclusive rate $B \to X_u \ell \nu$

- Theoretical Uncertainties
  [Bauer et al; Leibovich et a al; Neubert]
  - Weak annihilation contribution independent of $q^2_{\rm cut}$ and $m_{\rm cut}$; depends on magnitude of factorization violation

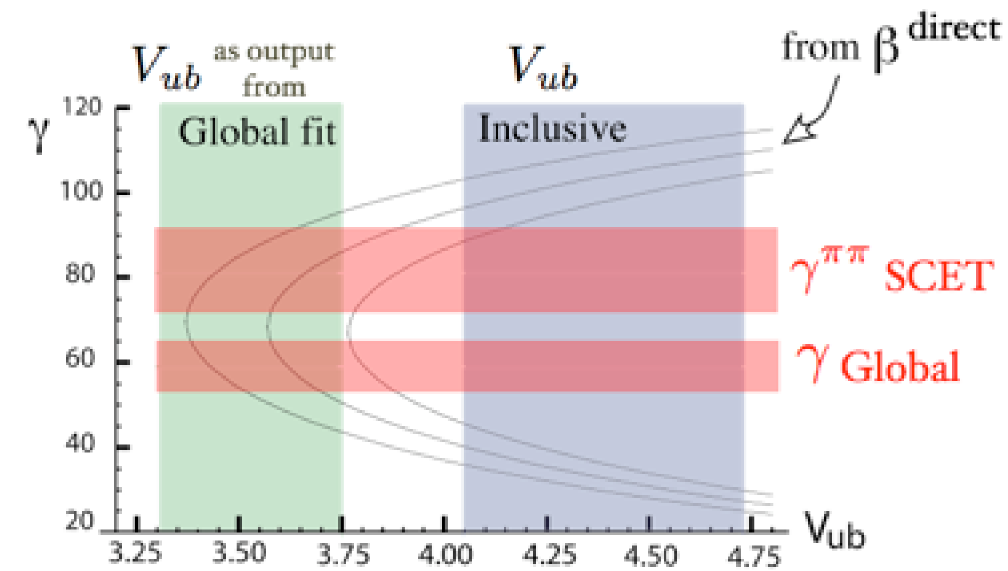
$$\Gamma(q^2_{\rm cut}, m_{\rm cut}) \equiv \frac{G_F^2 |V_{ub}|^2 \,(4.7\,{\rm GeV})^5}{192\pi^3}\, G({q_{\rm cut}}^2, m_{\rm cut})$$




  - Universality violation in shape function
    - sub-leading shape functions

  - $\alpha_s(\sqrt{\Lambda m_b})*\Lambda/m_b$ "brick wall"
    - numerics: $\alpha_s(\sqrt{\Lambda m_b})*\Lambda/m_b$ at least 5%
      but there are ~10 terms so guesstimate $\sqrt{(10)}*5\% = 15\%$

- Inclusive tension



- Results I: novel cuts (not so novel any more)
  - Different analysis (ie choices of cuts, moments, etc) require different calculations:
    - BLNP - B.O. Lange, M. Neubert and G. Paz, Phys. Rev. D72:073006 (2005) [arXiv:hep-ph/0504071v3]
    - DGE - J.R. Andersen and E. Gardi, JHEP 0601:097 (2006) [arXiv:hep-ph/0509360v2]. and [arXiv:0806.4524]
    - GGOU - P. Gambino, P. Giordano, G. Ossola, N. Uraltsev, JHEP 0710:058,2007 [arXiv:0707.2493].
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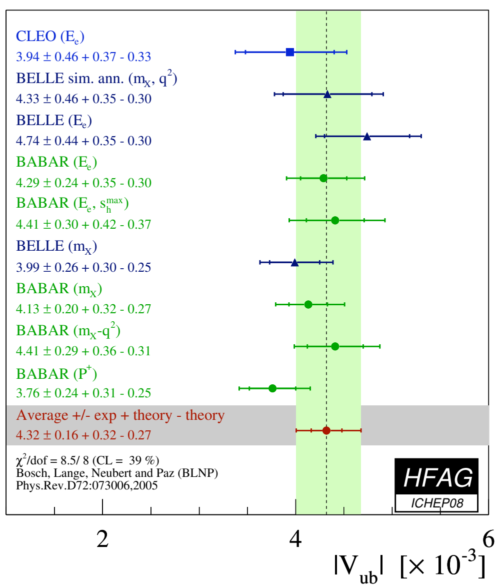
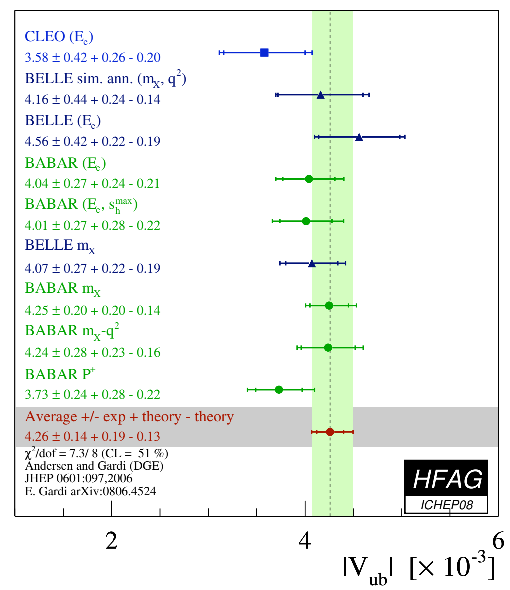
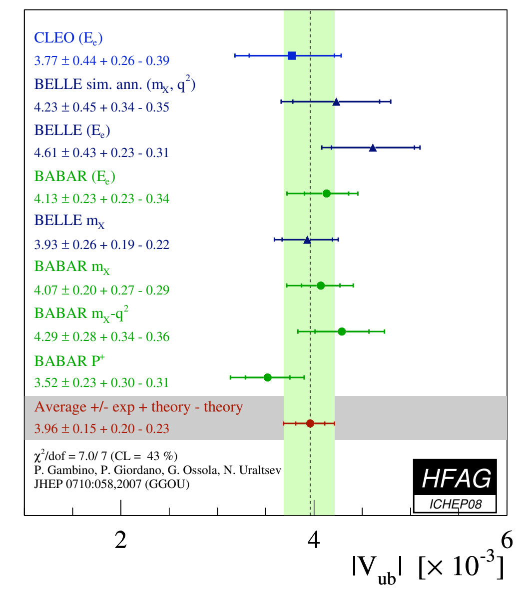
BLNP



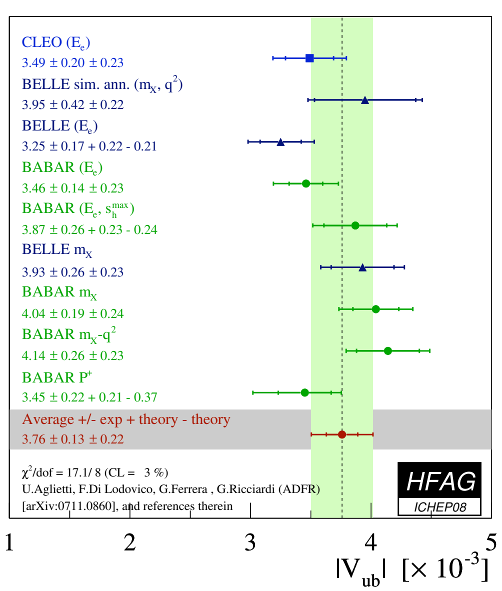

DGE



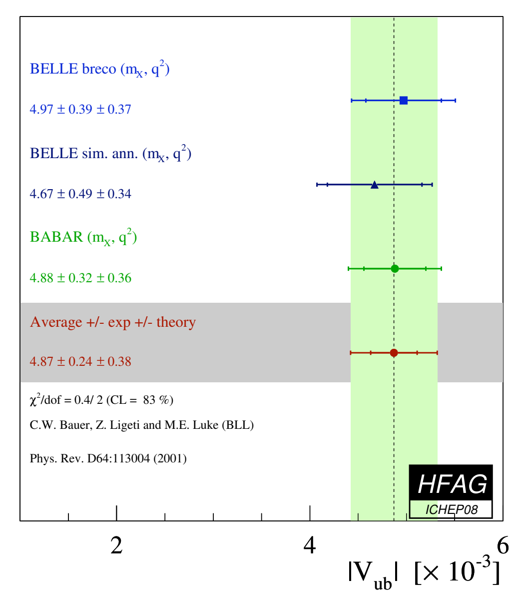

GGOU




DDFR




BLL



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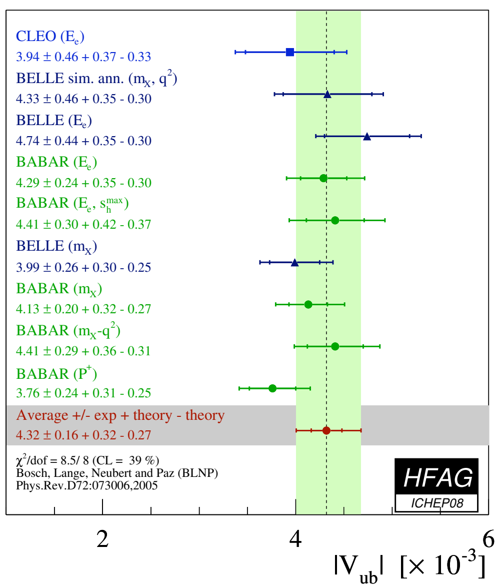
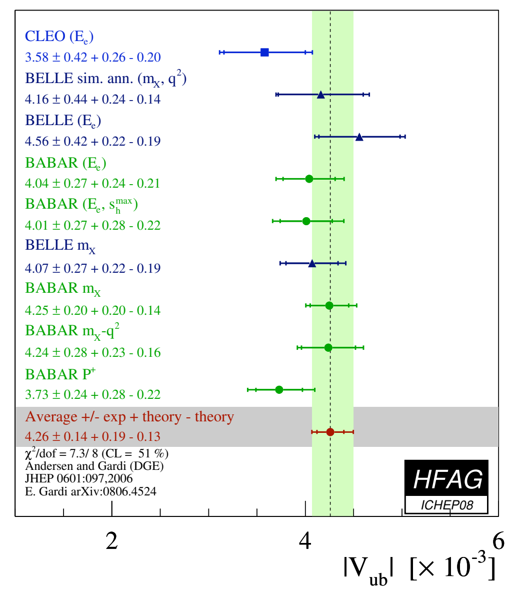
BLNP




DGE

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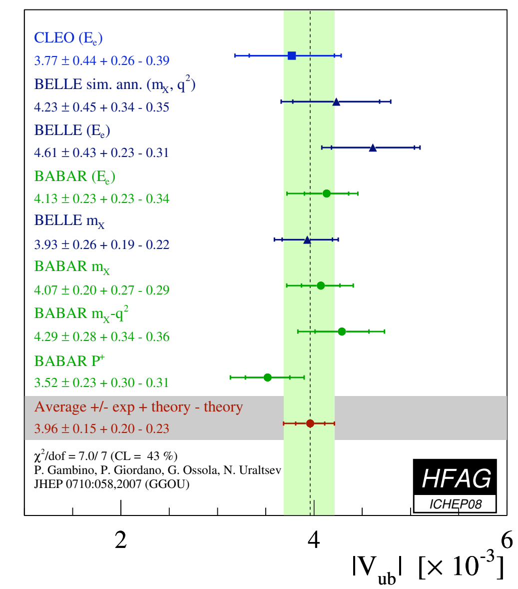


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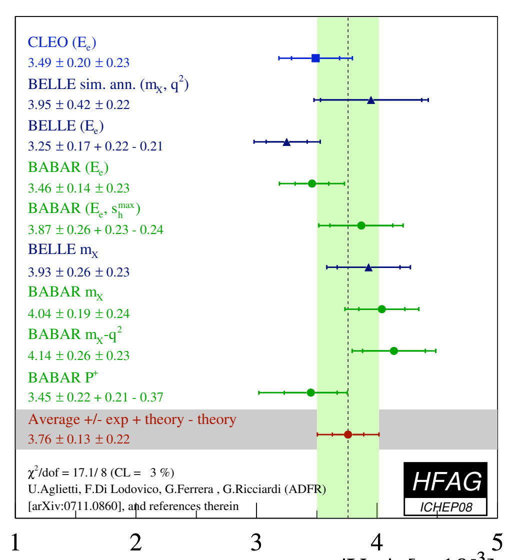


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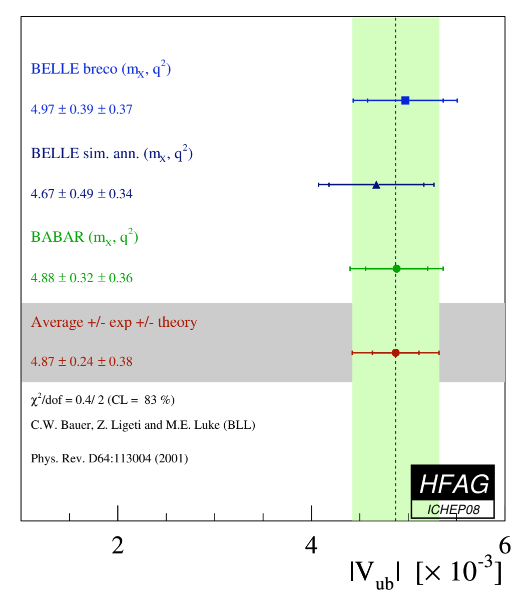
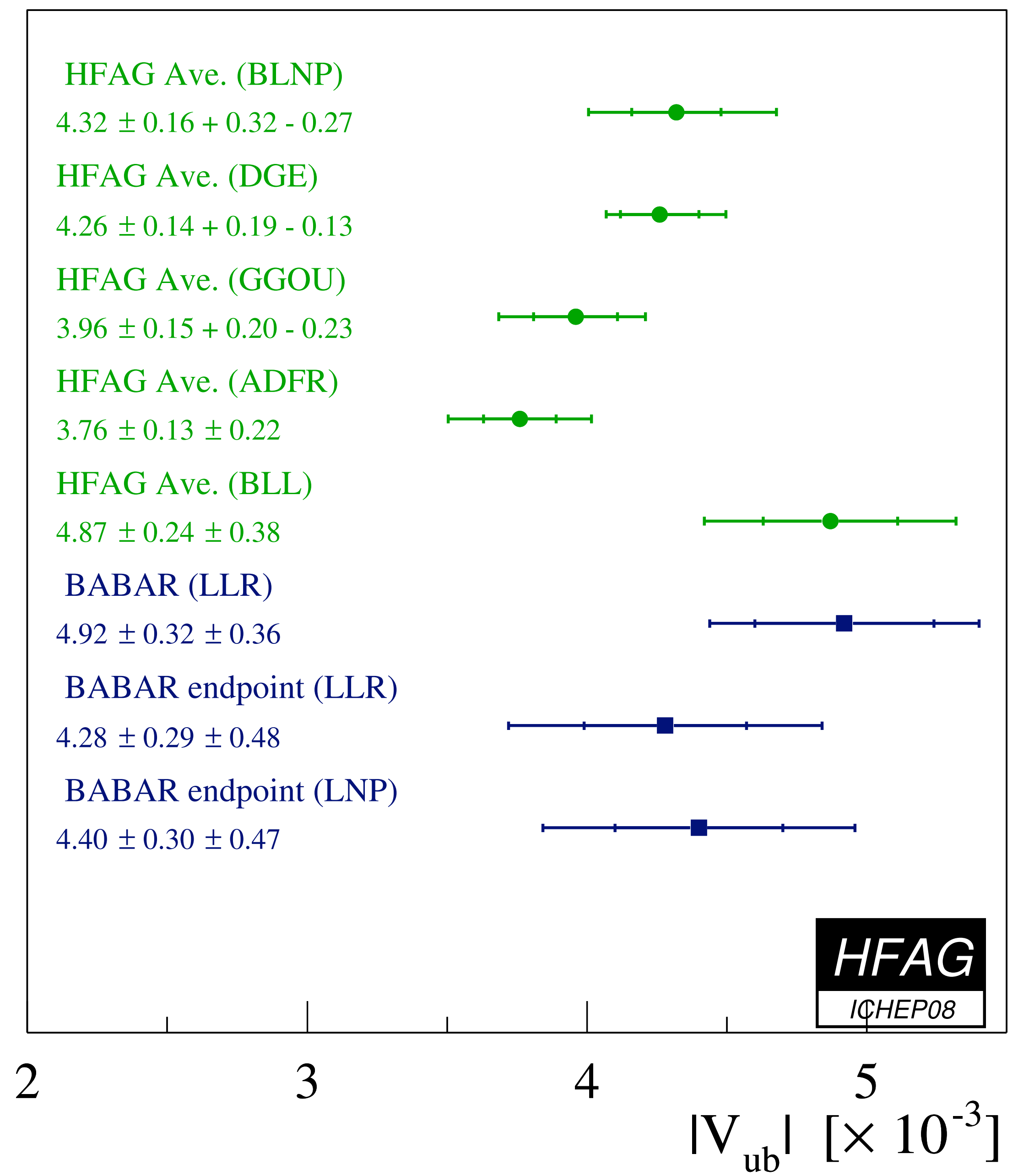



<u>Spread indicates underestimated theoretical uncertainties</u>

(in 2005 Belle and BaBar reported results with 5% theory uncertainty)

- Results II: moments radiative/semileptonic
  - LLR - Leibovich, Low, and Rothstein, Phys.Rev.D62:014010,2000 [arXiv:hep-ph/0001028v2], and Phys.Lett.B486:86-91,2000 [arXiv:hep-ph/0005124v1]
  - LNP, Lange, Neubert and Paz (JHEP 0510 (2005) 084 [arXiv:hep-ph/0508178v2] and JHEP 0601 (2006) 104 [arXiv:hep-ph/0511098v1])



# $|V_{ub}|$: exclusive decays

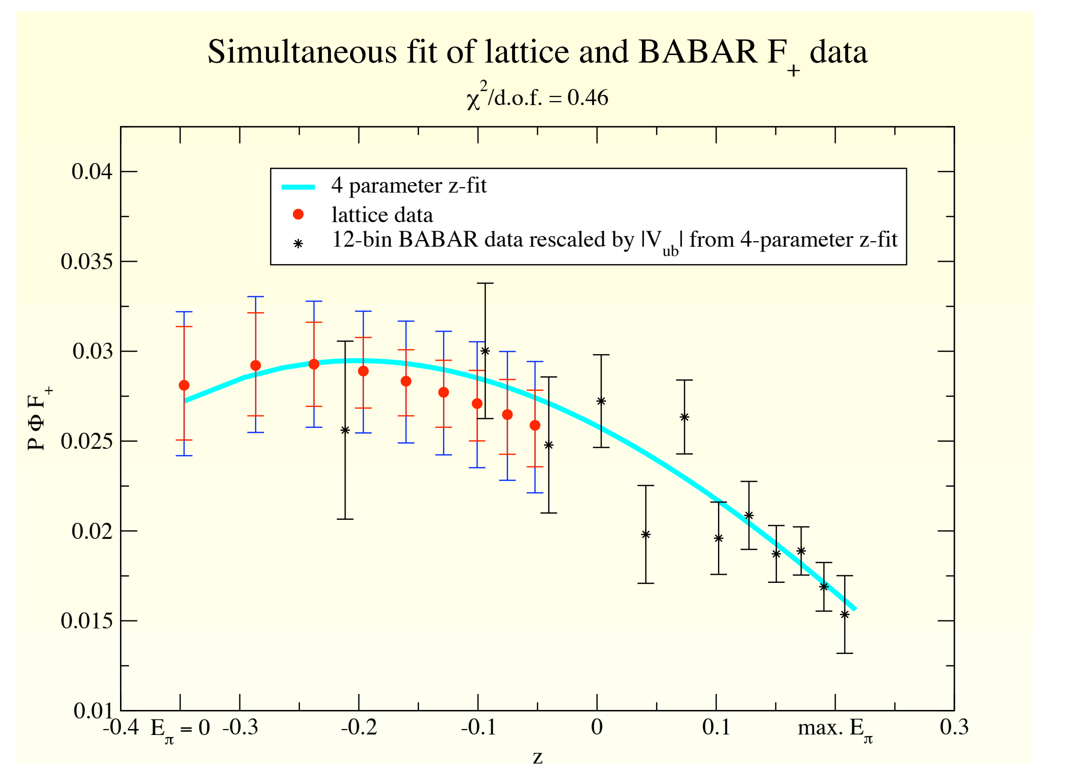
$$\mathrm{Br}(B \to \pi \ell \nu) = |V_{ub}|^2 \int_0^{q^2_{\max}} dq^2 \, f_+^{B\to\pi}(q^2)^2 \times (\text{trivial factors})$$

- Problem:
  - experiment gives low $q^2$ data
  - lattice gives form factor at high $q^2$
  - extrapolation introduces error
- Moving NRQCD: low data from lattice
  [K. Wong]
- Dispersion relations: *combine* lattice and experimental data over full $q^2$ region fitting to **model-independent** expression based on analiticity and unitarity
  [Arnesen et al; Becher an dHil; Ball; Mackenzie and Van de Water]

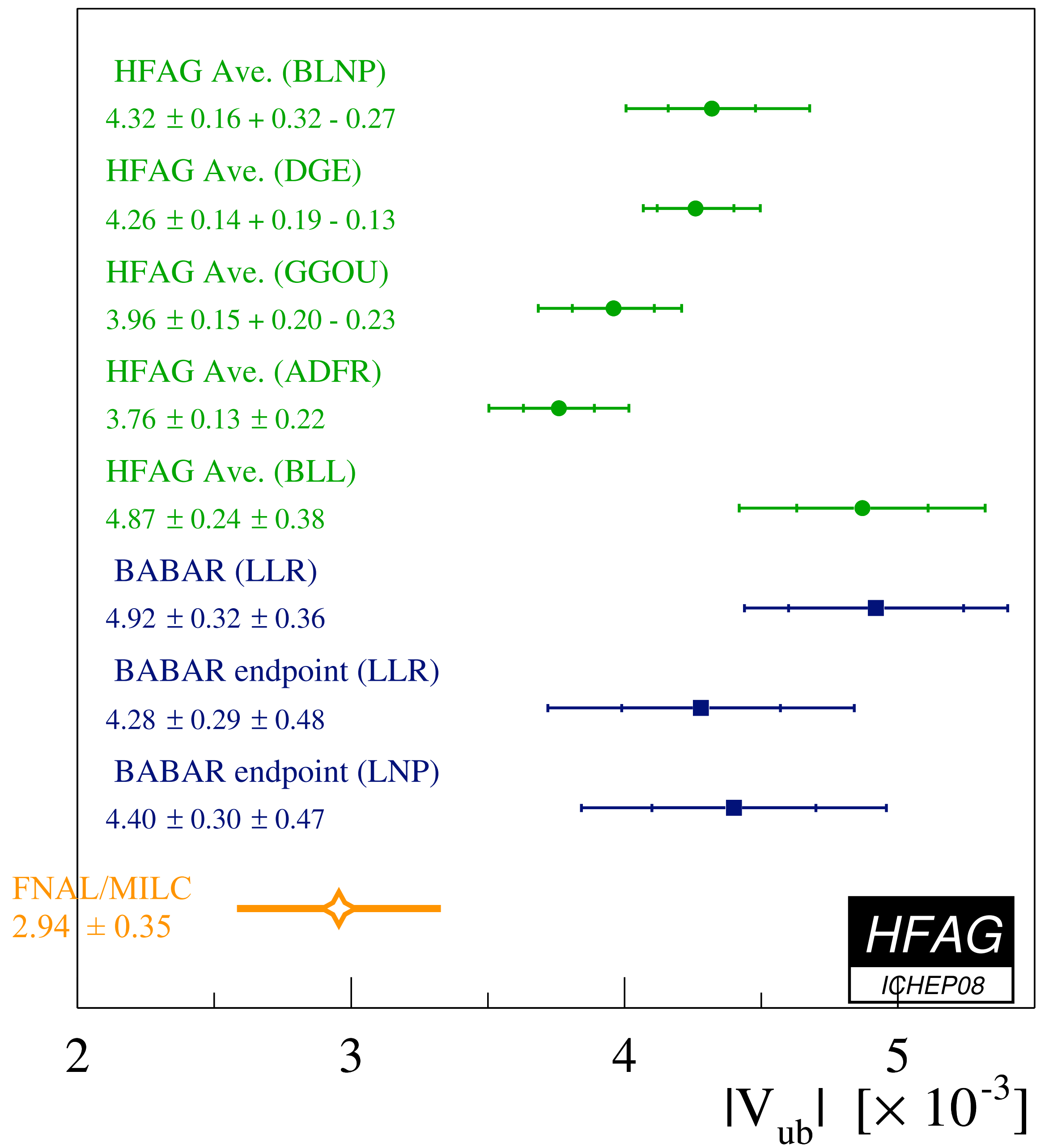
FNAL/MILC $N_f = 2+1$

$|V_{ub}| = (2.94 \pm 0.35) \times 10^{-3}$ (12% error)

(Van de Water, Lattice 2008)



Tension between inclusive and exclusive?




(FNAL/MILC point not from HFAG)

# Other ways to get $|V_{ub}|$

- $\mathcal{B}(B \to \ell\bar\nu)$ measures $f_B \times |V_{ub}|$ — need $f_B$ from lattice
- "Grinstein-type double ratio" inspired ideas (HQS / chiral symmetry suppressions)
  - $\dfrac{f_B}{f_{B_s}} \times \dfrac{f_{D_s}}{f_D}$ — lattice: double ratio $= 1$ within few $\%$ [Grinstein '93]
  - $\dfrac{f^{(B\to\rho\ell\bar\nu)}}{f^{(B\to K^*\ell^+\ell^-)}} \times \dfrac{f^{(D\to K^*\ell\bar\nu)}}{f^{(D\to\rho\ell\bar\nu)}}$ or $q^2$ spectra — accessible soon? [ZL, Wise; Grinstein, Pirjol]

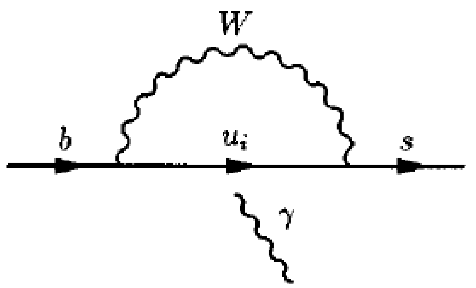
    CLEO-C $D \to \rho\ell\bar\nu$ data still consistent with no $SU(3)$ breaking in form factors [ZL, Stewart, Wise]

    Could lattice do more to pin down the corrections?

    Worth looking at similar ratio with $K$, $\pi$ — role of $B^*$ pole...?
  - $\dfrac{\mathcal{B}(B \to \ell\bar\nu)}{\mathcal{B}(B_s \to \ell^+\ell^-)} \times \dfrac{\mathcal{B}(D_s \to \ell\bar\nu)}{\mathcal{B}(D \to \ell\bar\nu)}$ — very clean... after 2015? [Ringberg workshop, '03]
  - $\dfrac{\mathcal{B}(B_u \to \ell\bar\nu)}{\mathcal{B}(B_d \to \mu^+\mu^-)}$ — even cleaner... ever possible? [Grinstein, CKM'06]

source: Z. Ligeti, Lattice QCD Meets Experiment Workshop,

# Rare $B$ Decays: $b \to s\gamma$




- Sensitive to New Physics
  - Rate
  - CP asymmetry
- Experimental Measurements
  - Precise Rate (~7% HFAG2008)
  - Asymmetry will improve
- Largely Under Control (non-perturbative effects ~5% in rate)

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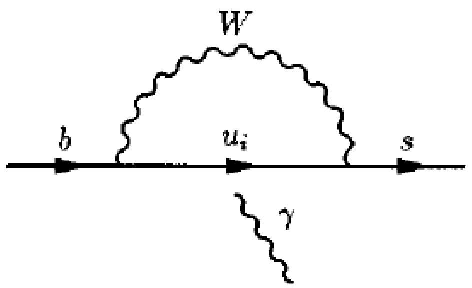


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$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QCD}\times\text{QED}}(u,d,s,c,b) + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{8} C_i(\mu)\, Q_i$$

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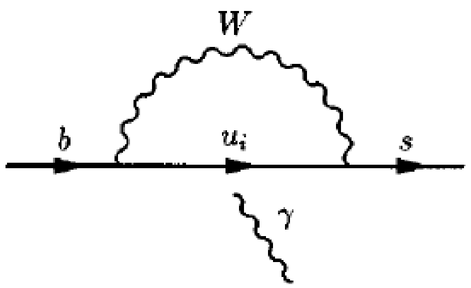


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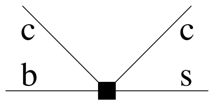
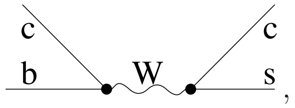
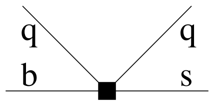
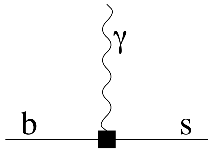
$Q_{1,2} =$ 

$= (\bar{s}\Gamma_i c)(\bar{c}\Gamma_i' b),$ **from** 

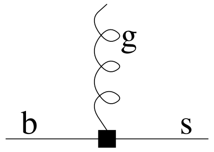
$|C_i(m_b)| \sim 1$

$Q_{3,4,5,6} =$ 

$= (\bar{s}\Gamma_i b)\sum_q (\bar{q}\Gamma_i' q),$ $|C_i(m_b)| < 0.07$

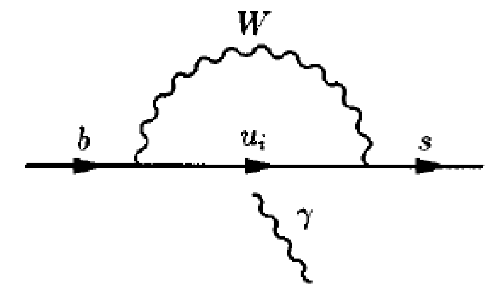
$Q_7 =$ 

$= \frac{e m_b}{16\pi^2}\, \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu},$ $C_7(m_b) \simeq -0.3$

$Q_8 =$ 

$= \frac{g m_b}{16\pi^2}\, \bar{s}_L \sigma^{\mu\nu} T^a b_R G^a_{\mu\nu},$ $C_8(m_b) \simeq -0.15$

# Rare $B$ Decays: $b \to s\gamma$

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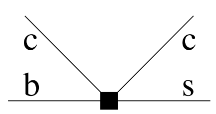
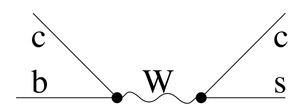
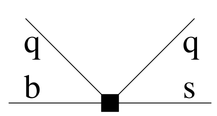
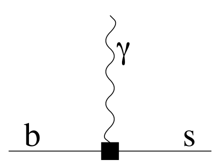
$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm QCD\times QED}(u,d,s,c,b) + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{8} C_i(\mu)\, Q_i$$

$Q_{1,2} =$  $= (\bar{s}\Gamma_i c)(\bar{c}\Gamma'_i b)$, **from** 

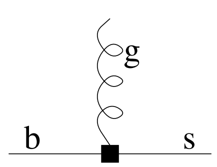


$Q_{3,4,5,6} =$  $= (\bar{s}\Gamma_i b)\sum_q(\bar{q}\Gamma'_i q)$,



$Q_7 =$  $= \frac{e m_b}{16\pi^2}\, \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}$,



$Q_8 =$  $= \frac{g m_b}{16\pi^2}\, \bar{s}_L \sigma^{\mu\nu} T^a b_R G^a_{\mu\nu}$,



$|C_i(m_b)| \sim 1$

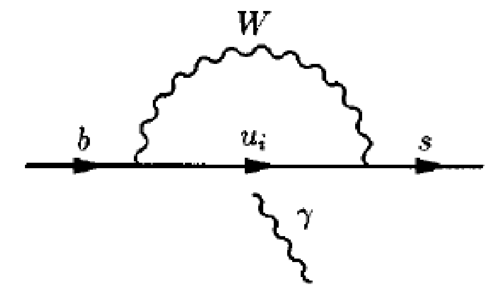
$|C_i(m_b)| < 0.07$

$C_7(m_b) \simeq -0.3$

$C_8(m_b) \simeq -0.15$

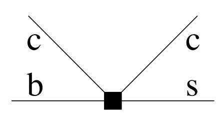
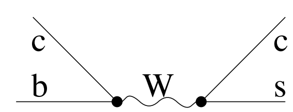
Known to NNLO

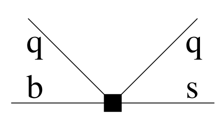
# Rare $B$ Decays: $b \to s\gamma$

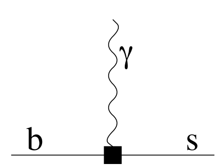


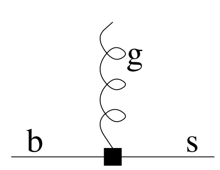

- Sensitive to New Physics
  - Rate
  - CP asymmetry
- Experimental Measurements
  - Precise Rate (~7% HFAG2008)
  - Asymmetry will improve
- Largely Under Control (non-perturbative effects ~5% in rate)

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm QCD\times QED}(u,d,s,c,b) + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{8} C_i(\mu)\, Q_i$$

$Q_{1,2} =$  $= (\bar{s}\Gamma_i c)(\bar{c}\Gamma_i' b)$, **from** 


$Q_{3,4,5,6} =$  $= (\bar{s}\Gamma_i b)\Sigma_q(\bar{q}\Gamma_i' q)$,

$Q_7 =$  $= \frac{e m_b}{16\pi^2}\, \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}$,

$Q_8 =$  $= \frac{g m_b}{16\pi^2}\, \bar{s}_L \sigma^{\mu\nu} T^a b_R G^a_{\mu\nu}$,

$|C_i(m_b)| \sim 1$

$|C_i(m_b)| < 0.07$

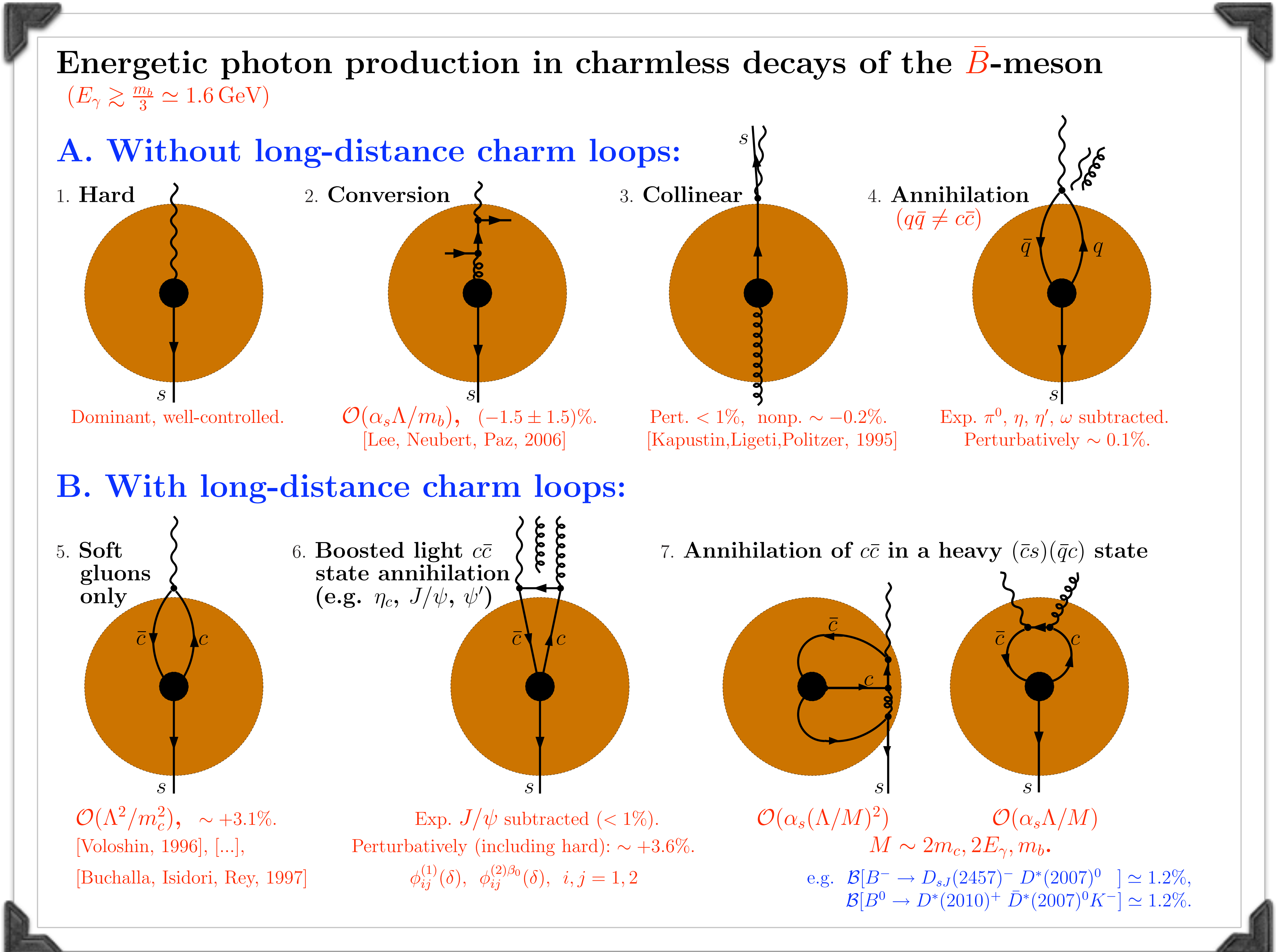
$C_7(m_b) \simeq -0.3$

$C_8(m_b) \simeq -0.15$

Known to NNLO

Constrain/discover new physics by determining $C_7(m_b)$ from inclusive radiative decay

Relative size of various contributions have been studied




lifted from Misiak

Three steps in the determination $C_7(m_b)$:

1. Matching: Choose $C_i(M)$ so that the Fermi effective theory and the SM, renormalized at $M \sim M_W$

2. Running: Compute anomalous dimension matrices and use RG-equation to compute $C_i(m_b)$

3. Matrix Elements: Perturbative calculation of amplitudes in EFT (renormalized at $m_b$)

## *A prodigious effort!*

**Status of the SM calculations for $\bar{B} \to X_s\gamma$ (Courtesy: M. Misiak)**

Matching $(\mu_0 \sim M_W, m_t)$:

$$C_i(\mu_0) = C_i^{(0)}(\mu_0) + \frac{\alpha_s(\mu_0)}{4\pi} C_i^{(1)}(\mu_0) + \left(\frac{\alpha_s(\mu_0)}{4\pi}\right)^2 C_i^{(2)}(\mu_0)$$

$i = 1, ..., 6$: tree, 1-loop, 2-loop [ Bobeth, Misiak, Urban, NPB 574 (2000) 291]

$i = 7, 8$: 1-loop, 2-loop, 3-loop [ Steinhauser, Misiak, hep-ph/0401041]

The 3-loop matching has less than 2% effect on BR($\bar{B} \to X_s\gamma$)

Mixing:

$$\hat{\gamma} = \frac{\alpha_s}{4\pi}\begin{pmatrix} 1L & 2L \\ 0 & 1L \end{pmatrix} + \left(\frac{\alpha_s}{4\pi}\right)^2 \begin{pmatrix} 2L & 3L \\ 0 & 2L \end{pmatrix} + \left(\frac{\alpha_s}{4\pi}\right)^3 \begin{pmatrix} 3L & 4L \\ 0 & 3L \end{pmatrix}$$

Haisch, Gorbahn, Gambino, Schröder, Czakon

Matrix elements $(\mu_b \sim m_b)$:

$$\langle O_i\rangle(\mu_b) = \langle O_i\rangle^{(0)}(\mu_b) + \frac{\alpha_s(\mu_b)}{4\pi}\langle O_i\rangle^{(1)}(\mu_b) + \left(\frac{\alpha_s(\mu_b)}{4\pi}\right)^2 \langle O_i\rangle^{(2)}(\mu_b)$$

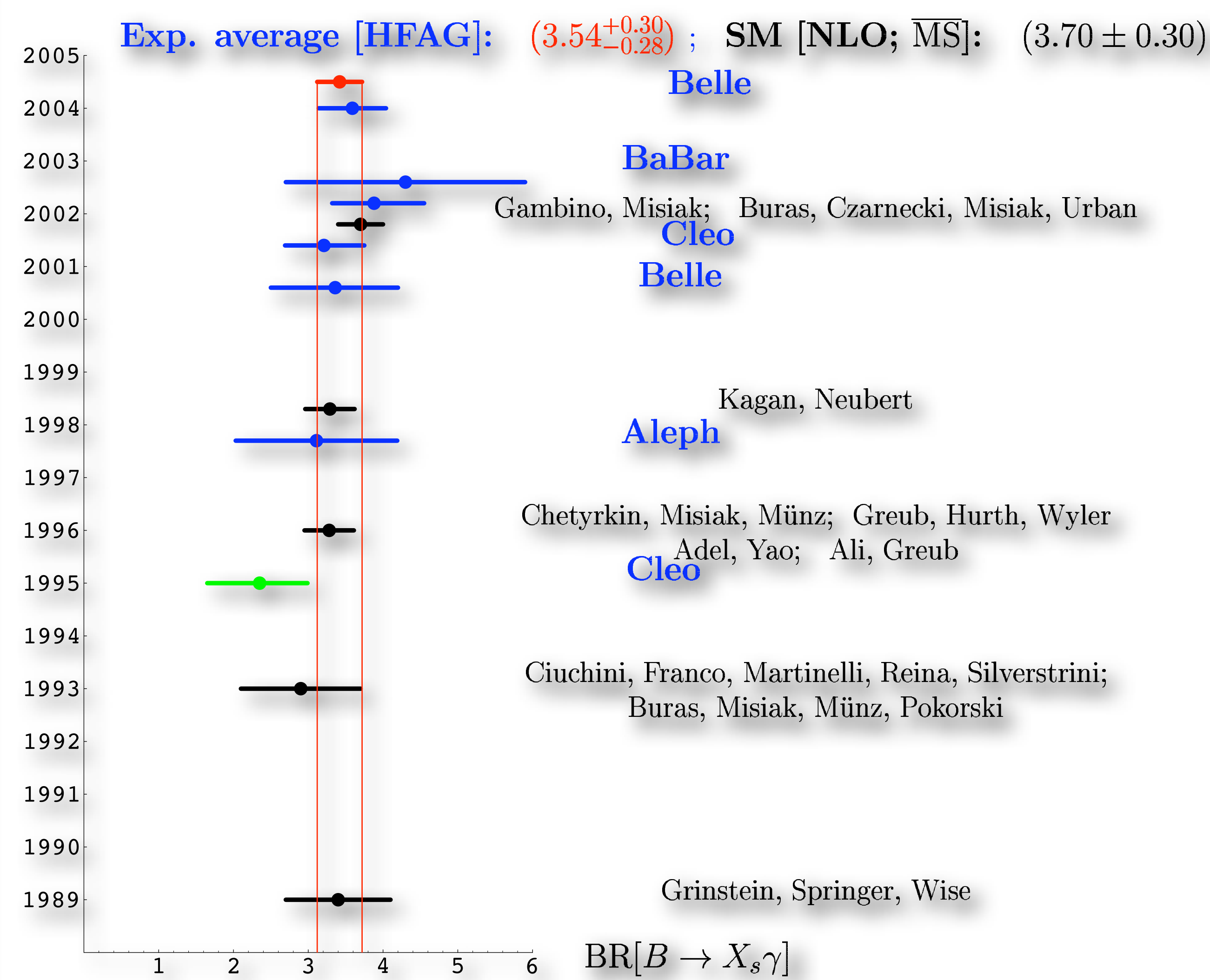
$i = 1, ..., 6$: 1-loop, 2-loop, 3-loop [Bieri, Greub, Steinhauser, hep-ph/0302051 ]

$i = 7, 8$: tree, 1-loop, $\mathcal{O}(\alpha_s^2 n_f)$, Steinhauser, Misiak 2-loop [Greub, Hurth, Asatrian]

August 20, 2004
ichep'04, Beijing

Review of Heavy Quark Physics – Theory (page 17)

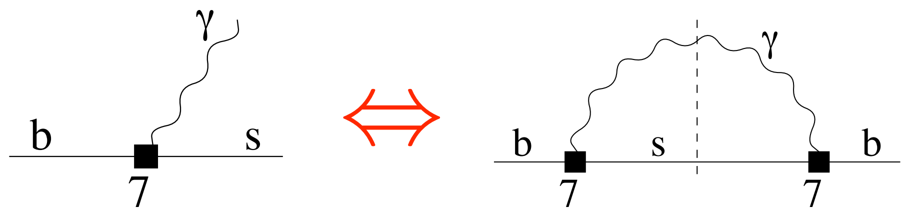
Ahmed Ali
DESY, Hamburg



Status of calculation of matrix elements

$$\Gamma(b \to X_s^{\text{parton}}\gamma)_{E_\gamma > E_0} = \frac{G_F^2 m_b^5 \alpha_{\text{em}}}{32\pi^4} |V_{ts}^* V_{tb}|^2 \sum_{i,j=1}^{8} C_i(\mu_b) C_j(\mu_b) G_{ij}(E_0, \mu_b)$$

LO




$G_{77} = 1$, all others vanish

NLO

$i, j = 1, 2, 7, 8$ dominate [Greub, Hurth, Wyler; Ali, Greub](1996)

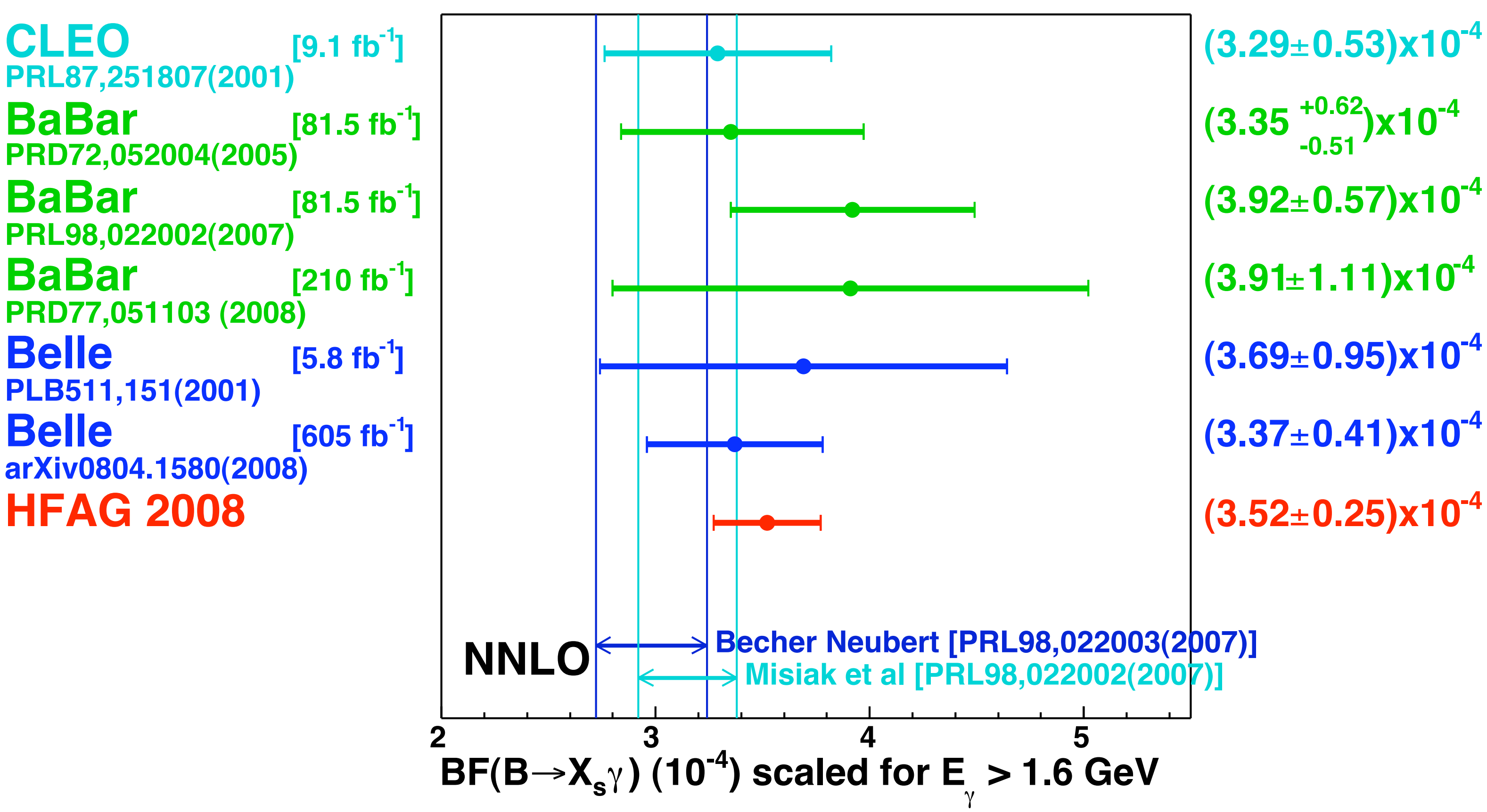
bust rest also known [Buras, Czarnecki, Msisak, Urban; Pott] (2002)

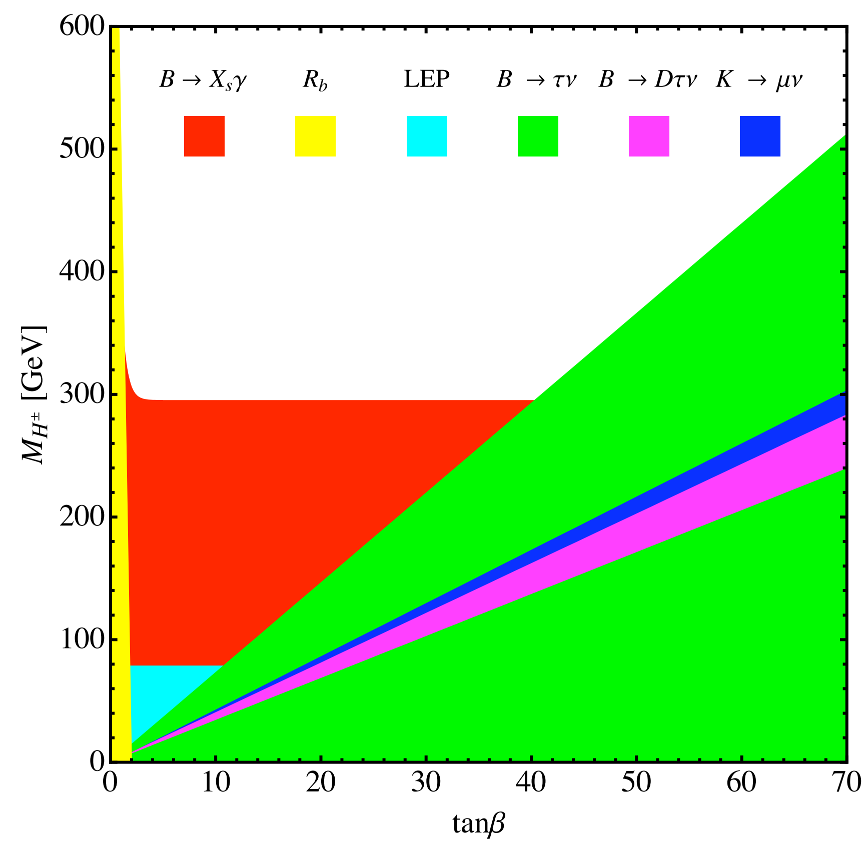
NNLO

only parts of $i, j = 1, 2, 7, 8$ known:

- $G_{77}$ determined [Blokland et al; Melnikov, Mitov; Asatrian et al]
- $G_{11}$, $G_{12}$ and $G_{22}$:
  - 2 particle cuts are $|\text{NLO}|^2$
  - 3,4 particle cuts vanish at endpoint
- Ongoing progress in the rest
  - 2,3,4 particle cuts in $G_{17}$ and $G_{27}$ [Schutzmeier, Czakon, Boughezal]
  - 2,3,4 particle cuts in $G_{78}$ [Asatrian, Ewerth, Ferroglia, Greub, Ossola]

$$\mathcal{B}(\bar{B} \to X_s\gamma)^{\rm NNLO}_{E_\gamma>1.6\ {\rm GeV}} = \begin{cases} (3.15 \pm 0.23) \times 10^{-4}, & \text{hep-ph/0609232, using the 1S scheme,} \\ (3.26 \pm 0.24) \times 10^{-4}, & \text{following the kin scheme analysis of arXiv:0805.0271, but } \overline{m}_c(\overline{m}_c)^{\rm 2loop} \text{ rather than } \overline{m}_c(\overline{m}_c)^{\rm 1loop} \text{ in } P(E_0). \end{cases}$$

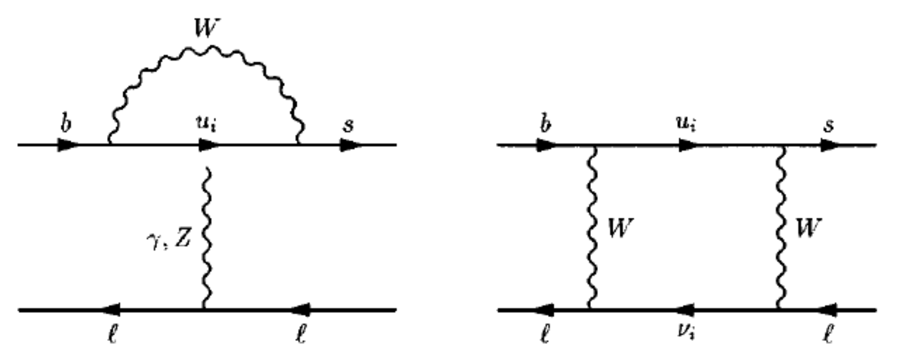


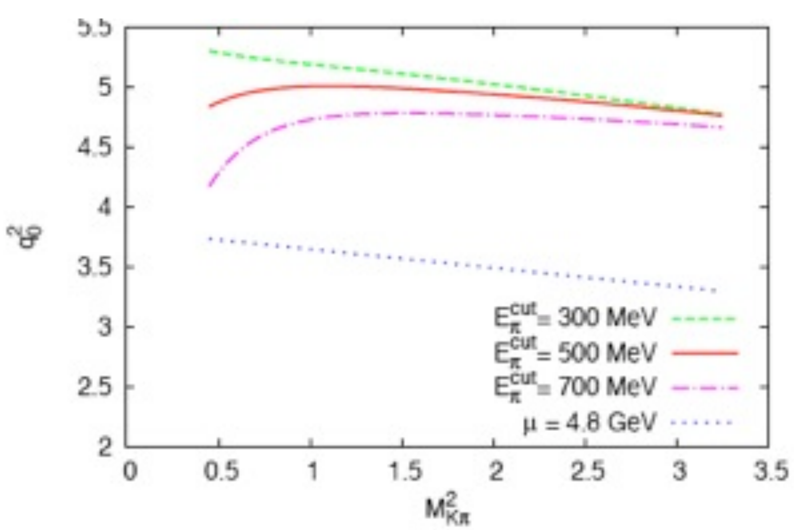



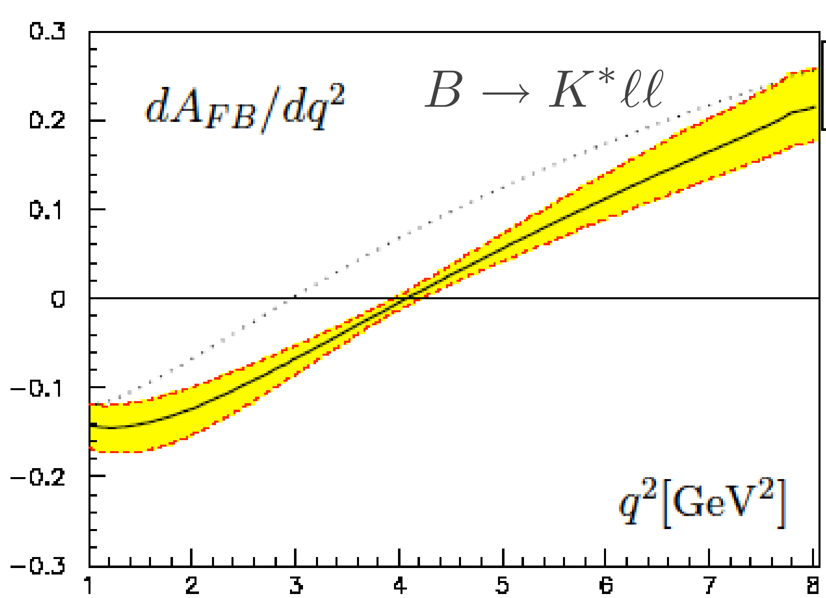

Haisch,arXiv:0805.2141

# Rare $B$ Decays: $b \to s\ell\ell$

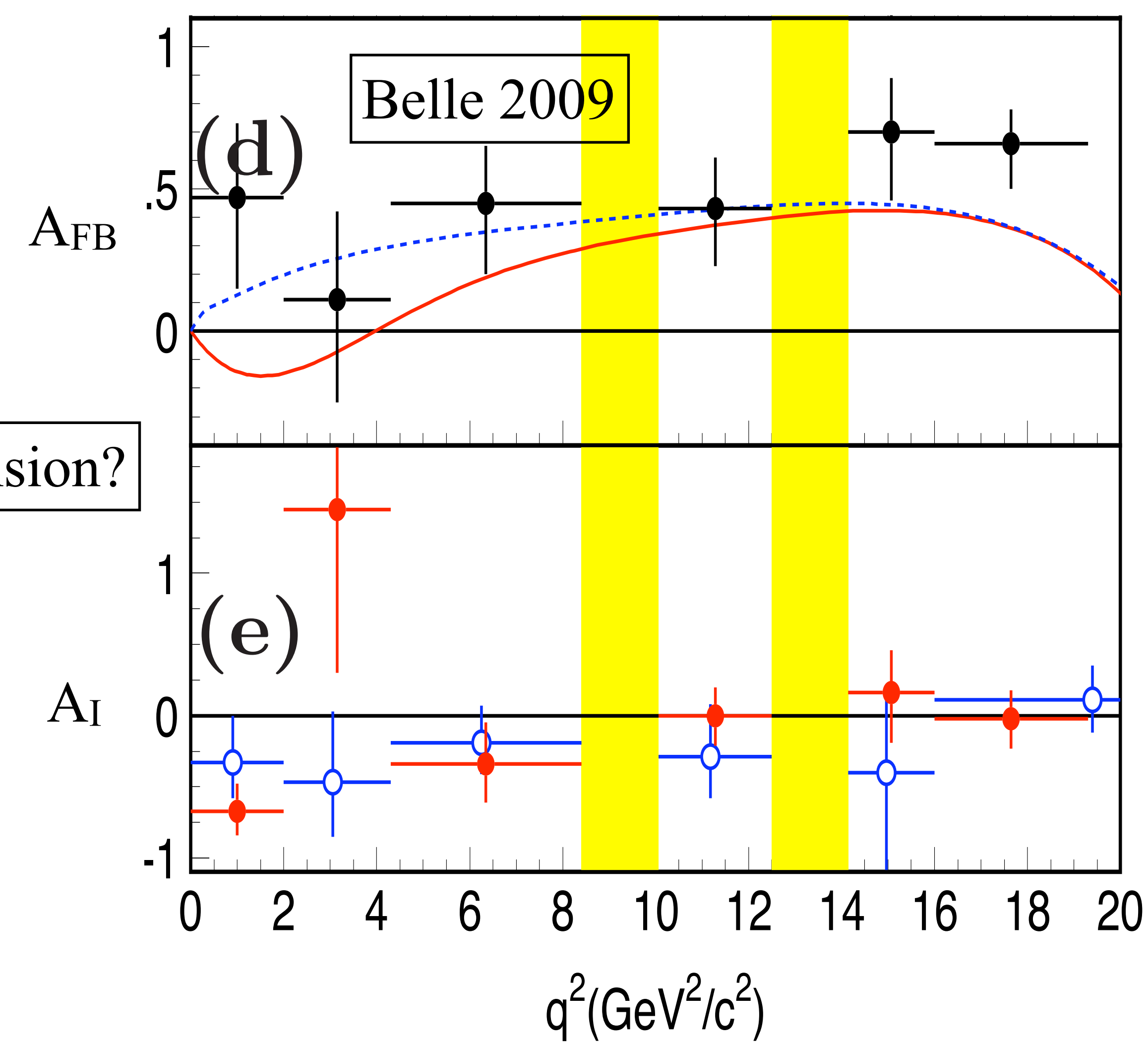
- Requires 1 loop less than radiative
- NNLO complete
- FB asymmetry zero in $B \to K^*\ell\ell$ robust [Burdman] even including non-resonant Kpi [BG, Pirjol]
- Sensitive to new physics




[Ali,Kramer, Zhu]



The end