Heavy Flavor Theory

Benjamin Grinstein
Motivation: Bottom-up approach

Assume existence of New Physics (NP) at short distances, no specific model

NP not directly accessible to experiment (yet): effects appear indirectly as modifications to interactions among SM particles

Supplement SM lagrangian with terms of dimension higher than four (“higher dimension operators”) as allowed by Lorentz Invariance and gauge symmetries.

A term of dimension $n > 4$ appears in the lagrangian with coefficient $c/\Lambda_{\text{NP}}^{n-4}$ (with $c \sim 1$)
Hence low energy effects are suppressed by powers of $\Lambda_{\text{NP}}$
(just a generic form of the effective field theory of the top-down approach)

Advantages of bottom-up approach:
fairly general, encompasses many (all?) realistic extensions of SM (model independent)
few parameters

Disadvantages:
no clear correlation between long (GeV$^{-1}$) and very short (TeV$^{-1}$) distances

In this talk I will avoid translation of bounds into explicit models (typically SUSY)
Flavor problem

The EFT (either approach) generically contains terms that mediate $\Delta F = 2$ or FCNC decays at tree level and suppressed only by $c/\Lambda_{NP}^{n-4}$ (with $c \sim 1$)

with $n - 4 = 2$ this requires $\Lambda_{NP}$ in excess of $10^4$ TeV from, e.g., K-mixing
Flavor problem

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with $n - 4 = 2$ this requires $\Lambda_{NP}$ in excess of $10^4$ TeV from, e.g., K-mixing

$$\frac{1}{\Lambda_{NP}^2} \left[ z^K_1 (d_L \gamma_\mu s_L)(d_L \gamma^\mu s_L) + z^D_1 (u_L \gamma_\mu c_L)(u_L \gamma^\mu c_L) + z^D_4 (u_L c_R)(u_R c_L) \right]$$

$$|z^K_1| \leq z^K_{exp} = 8.8 \times 10^{-7} \left( \frac{\Lambda_{NP}}{1 \text{ TeV}} \right)^2$$

$$|z^D_1| \leq z^D_{exp} = 5.9 \times 10^{-7} \left( \frac{\Lambda_{NP}}{1 \text{ TeV}} \right)^2$$

$$\Im(z^K_1) \leq z^{IK}_{exp} = 3.3 \times 10^{-9} \left( \frac{\Lambda_{NP}}{1 \text{ TeV}} \right)^2$$

$$\Im(z^D_1) \leq z^{ID}_{exp} = 1.0 \times 10^{-7} \left( \frac{\Lambda_{NP}}{1 \text{ TeV}} \right)^2$$

[Nir; Perez; Weiler; @ Planck2009]
Goal of heavy quark physics: constrain models of new physics, verify with precision SM+CKM

Plan of the Talk

• Introduction with review of CKM theory
• Purely leptonic decays
• $B\bar{B}$ mixing: $|V_{ud}|$; $D\bar{D}$ mxing
• Towards a precision determination of $|V_{cb}|$, $|V_{ub}|$
• Progress in Rare $B$ Decays
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Plan of the Talk

- Introduction with review of CKM theory
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- $BB$ mixing: $|V_{td}|$; $DD$ mixing
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I am sorry I have to leave out many interesting topics (with apologies to speakers in parallel sessions): CPV/angles, two body hadronic decays, sin $2\beta$ determinations from sss penguins (vs ccs trees), heavy flavor at LHC, explicit BSM theories of/wiith flavor, ...
The CKM Matrix

\[ V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \]

- Frequently used Wolfenstein parametrization: four parameters \( \lambda, A, \bar{\rho}, \bar{\eta} \)

\[ V_{\text{CKM}} \approx \begin{pmatrix} 1 - \frac{1}{2} \lambda^2 & \lambda & A\lambda^3(\bar{\rho} - i\bar{\eta}) \\ -\lambda(1 + iA^2\lambda^4\bar{\eta}) & 1 - \frac{1}{2} \lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \bar{\rho} - i\bar{\eta}) & -A\lambda^2(1 + i\lambda^2\bar{\eta}) & 1 \end{pmatrix} + O(\lambda^6). \]

- CKM Unitarity triangle

- Sides give circles in \( z = \bar{\rho} + i\bar{\eta} \)

\[ |z| \]

| 1 - z |
The CKM Matrix

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- Angles give ....
Consistency check of CKM theory: global fit
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Why care about anything other than $\sin 2\beta$ and $\Delta m_d/\Delta m_s$?
Consistency check of CKM theory: global fit

Why care about anything other than $\sin 2\beta$ and $\Delta m_d/\Delta m_s$?

...tree vs loop: disentangling new physics form old (orthodoxy: NP enters only at loop level)
Electroweak tree level

EW loop processes
Electroweak tree level

EW loop processes

Consistency ?!
Progress and Puzzles in Purely Leptonic $B$ and $D$ Decays

Theory? $B/D$ decay constants

\[ \Gamma(D_s \rightarrow \ell \nu_\ell) = \frac{m_{D_s}}{8\pi} f^2_{D_s} |G_F V^*_c m_\ell|^2 \left(1 - \frac{m_\ell^2}{m_{D_s}^2}\right)^2, \]

New Physics?

- $s$-channel charged higgs exchange, with $y_s \ll y_c$ and $y_c, y_\tau \sim 1$: disfavored by $D$ decay data
- $t$-channel charge $+2/3$ leptoquark exchange; disfavored by bound on $\tau \rightarrow \mu ss$
- $u$-channel charge $-1/3$ leptoquark exchange (like $d$-squark)

\[ \mathcal{L}_{\text{LQ}} = \kappa_{2\ell} (\bar{c}_L \ell'^L_L - \bar{s}_L \nu^c_{\ell L}) \bar{d} + \kappa'_{2\ell} \bar{c}_R \ell'^c_R \bar{d} + \text{H.c.}, \]

[All tree level!]

HPQCD/exp discrepancy greater than $3\sigma$
$B \to \tau\nu$

Tension between $\sin 2\beta$ and $\text{Br} (B \to \tau\nu)$:

global fit without using these measurements, cross is from experimental values (1$\sigma$)

Shape of correlation best understood from ratio:

$$\frac{\text{Br}(B \to \tau\nu)}{\Delta m_d} = \frac{3\pi m^2_{\tau}}{4m^2_W S(x_t)} \left(1 - \frac{m^2_{\tau}}{m^2_B}\right)^2 \tau_{B^+} \frac{1}{|V_{ud}|^2 B_{B_d}} \left(\frac{\sin\beta}{\sin\gamma}\right)^2$$

- Decay constants cancel
- Depends only on bag parameter $B_B$
- Constraint in $z$ plane does not match exactly global fit
$$B \to \tau \nu, \quad B \to X_c \tau \nu$$

Sensitive to charged higgs exchange. Normalize to non-tau

$$\tan \beta = \frac{v_2}{v_1}$$

$$r = \frac{\tan \beta}{M_{H^\pm}}$$

$$\text{Br}(B \to \tau \nu) = (1.79^{+0.56+0.39}_{-0.49-0.46}) \times 10^{-4}$$  \hspace{1cm} (Belle)

$$\text{Br}(B \to \tau \nu X) = (2.48 \pm 0.26) \times 10^{-2}$$  \hspace{1cm} (LEP)
Neutral Meson Mixing: generalities

time evolution:
\[ i \frac{d}{dt} \left( \frac{\left| P^0(t) \right>}{\left| \bar{P}^0(t) \right>} \right) = \left( M - \frac{i}{2} \Gamma \right) \left( \frac{\left| P^0(t) \right>}{\left| \bar{P}^0(t) \right>} \right) \]

solution:
\[ \left| P_{L,H}(t) \right> = p \left| P^0 \right> \pm q \left| P^0 \right> \]
\[ \left| P_{L,H}(t) \right> = e^{-(i m_{L,H} + \Gamma_{L,H}/2)t} \left| P_{L,H} \right> \]

Decay amplitudes:
\[ A_f = \langle f | H | P^0 \rangle, \quad \bar{A}_f = \langle f | \bar{H} | P^0 \rangle \]

physical quantities:
\[ \frac{|\bar{A}_f|}{A_f} \neq 1 \text{ if CPV in decay} \quad \frac{|q|}{|p|} \neq 1 \text{ if CPV in mixing} \quad \lambda_f = \frac{q}{p} \frac{\bar{A}_f}{A_f} \quad \phi_{12} = \arg \left( -\frac{M_{12}}{\Gamma_{12}} \right) \quad \text{Im} \left( \frac{\Gamma_{12}}{M_{12}} \right) = \frac{1 - |q/p|^4}{1 + |q/p|^4} \]

Cases
\[ \Delta m \gg |\Delta \Gamma| \quad (e.g., \text{ for } B_{d,s}) \]
\[ \Delta m = 2|M_{12}|(1 + \mathcal{O}(\Gamma_{12}/M_{12})) \]
\[ \Delta \Gamma = -2|\Gamma_{12}| \cos \phi_{12} (1 + \mathcal{O}(\Gamma_{12}/M_{12})) \]
\[ \phi_{12} \text{ suppressed in SM (Bd,s)} \]
\[ \text{NP can only reduce } |\Delta \Gamma| \]
\[ q/p = -\arg(M_{12})(1+...) \text{ so time dependent CP asymmetries sensitive no NP in } M_{12} \]

\[ \Delta m \ll |\Delta \Gamma| \]
\[ \Delta m = 2|M_{12}| \cos \phi_{12} (1 + \mathcal{O}(M_{12}/\Gamma_{12})) \]
\[ \Delta \Gamma = \mp 2|\Gamma_{12}|(1 + \mathcal{O}(M_{12}/\Gamma_{12})) \]
\[ q/p = -\arg(\Gamma_{12})(1+...) \text{ depends weakly on } M_{12} \]
\[ \text{if } \Delta m \ll |\Delta \Gamma| \text{ and no CPV in D decay} \]
\[ \arg \lambda_{K+K-} \propto 2 \left| \frac{M_{12}}{\Gamma_{12}} \right|^2 \sin(2\phi_{12}). \]
\[ \text{reduced sensitivity to NP in } M_{12} \text{ (even for dominant NP)} \]
**$B\bar{B}$ mixing: $|V_{td}|$**

**Theory:**

$$\frac{|V_{td}|}{|V_{ts}|} = \xi \sqrt{\frac{\Delta m_s}{\Delta m_d}} \frac{m_{B_s}}{m_{B_d}}$$

$$\xi^2 = \frac{B_{B_s} f_{B_s}^2}{B_{B_d} f_{B_d}^2}$$

I’ll believe a 3% lattice theory error when the lattice has produced one successful prediction and several 3% postdictions. However, here the calculation is really of $\xi^2-1$, and the error is 16%.

Chiral lag gives only chiral logs, so error in $\xi^2-1\approx0.3$ is $\sim100%$

**Lattice:**

$$\xi = 1.211 \pm 0.038 \pm 0.024_{\text{estimate}}$$

$$\frac{|V_{td}|}{|V_{ts}|} = 0.2060 \pm 0.0012(\text{exp}) + 0.0081(\text{th}) - 0.0060(\text{th})$$

using $$\xi = 1.210^{+0.047}_{-0.035}$$

[CDF & D0: 0905.1109]
$D\bar{D}$ mixing

\[ x = \frac{\Delta m}{\Gamma}, \quad y = \frac{\Delta \Gamma}{2\Gamma} \]

- No $t$-quark in box diagram: GIM suppressed $\Delta m$
- $x, y$ likely dominated by long distance physics
- Expect $x \sim y$ and $x, y \sim \sin^2 \theta_C \times [\text{SU(3) breaking}]$
- NP in $M_{12}$, rather than in $\Gamma_{12}$:
  \[ M_{12} \sim \text{loop}, \quad \Gamma_{12} \sim \text{tree} \]
- May still obtain useful bounds by assuming no perverse cancellation between SM and NP, then demand NP contribution is less than measured

$q, q' = d, s, b$

[et al and A. Petrov]
[see also Ciuchini et al]
\[ |V_{cb}|: \text{inclusive decays } B \rightarrow X_c \ell \nu \]

- **Theory:**
  Combine Heavy Quark Mass Expansion (HQET) and Operator Product Expansion (OPE)
  [Chay, Georgi, BG; Voloshin, Shifman; Bigi et al; Manohar, Wise, Bloek et al]

- OPE: expand since \( m_b \) is larger than any scale in HQET matrix element
- Decay rate for \( B \rightarrow X_c \ell \nu \)
  \[ \Gamma_n / \Gamma_0 \sim (\Lambda_{QCD}/m_b)^n \]
  \[ \Gamma = \Gamma_0 + \Gamma_1 + \Gamma_2 + \Gamma_3 + \ldots \]
  \[
  \begin{array}{c}
  \Gamma_0, m_b, m_c, \lambda_1, \lambda_{1,2}, T_{1,2,3}, \rho_1, \rho_{1,2} \\
  \end{array}
  \]
  6 parameters
  With \( 1/m_c \) expansion
  No \( 1/m_c \) expansion

- \( \Gamma_i \) given in terms of few non-perturbative parameters + expansion in \( \alpha_s(m_b) \)
- \( \Gamma_0 \) is free quark decay rate
- \( \Gamma_1 = 0 \) (Luke’s theorem)
- Local quark-hadron duality is mildly used (to show a correction is small)
- Moments of distribution have differing sensitivity to non-perturbative parameters
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  \[ \Gamma = \Gamma_0 + \Gamma_1 + \Gamma_2 + \Gamma_3 + \ldots \]
  \[ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \]
  \[ \begin{align*}
  m_b & \quad \lambda_1 & \quad \mathcal{T}_{1,2,3,} & \quad \rho_1 \\
  m_b, m_c & \quad \lambda_{1,2} & \quad \rho_{1,2}
  \end{align*} \]

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- Not the same person
$|V_{cb}|$: inclusive decays $B \rightarrow X_c \ell \nu$

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$$\Gamma = \Gamma_0 + \Gamma_1 + \Gamma_2 + \Gamma_3 + \ldots$$

$\Gamma_n / \Gamma_0 \sim (\Lambda_{QCD} / m_b)^n$

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Moment Analysis

Lepton Energy

\[ \langle E_{\ell}^n \rangle_{E_{\text{cut}}} \equiv \frac{R_n(E_{\text{cut}}, 0)}{R_0(E_{\text{cut}}, 0)} \]

where

\[ R_n(E_{\text{cut}}, M) \equiv \int_{E_{\text{cut}}} \frac{\Gamma}{dE_{\ell}} \frac{(E_{\ell} - M)^n}{dE_{\ell}} dE_{\ell} \]

Hadronic Mass

\[ \langle m_X^{2n} \rangle_{E_{\text{cut}}} \equiv \frac{\int_{E_{\text{cut}}} \frac{\Gamma}{dm_X^2} \frac{dE_X}{dm_X^2} dE_X^2}{\int_{E_{\text{cut}}} \frac{d\Gamma}{dm_X^2} dE_X^2} \]

Photon Energy in \( B \to X_s \gamma \)

\[ \langle E_{\gamma}^n \rangle \equiv \frac{\int_{E_{\text{cut}}} \frac{\Gamma}{dE_{\gamma}} \frac{dE_{\gamma}}{dE_{\gamma}} dE_{\gamma}}{\int_{E_{\text{cut}}} \frac{d\Gamma}{dE_{\gamma}} dE_{\gamma}} \]
Global Analysis

- Data: BaBar, BELLE, CDF, CLEO, DELPHI
- With/without $1/m_c$ expansion
- Compare mass schemes: $1S$, PS, …
- Half integer hadronic moments error badly behaved

+ Recent

- Hoang's $m_b^{1S}$ with PDG's $|V_{cb}|$
- $\Delta \chi^2 = 1$ (black & yellow), $\Delta \chi^2 = 4$ (blue & green)
- yellow/green: Omit restriction on range of $(\Lambda_{QCD})^3$ parameters

[Schwanda - 0903.3648] (beware of different vertical scales!)
|$V_{cb}$|: exclusive decays $B \rightarrow (D, D^*) \ell \nu$

\[
\frac{d\Gamma(B \rightarrow D^* \ell \bar{\nu})}{dw} = \frac{G_F^2 m_B^5}{48\pi^3} r_*^3 (1-r_*)^2 \sqrt{w^2-1}(w+1)^2 \\
\times \left[ \frac{4w-2wr_* + r_*^2}{1+w} (1-r_*)^2 \right] |V_{cb}|^2 \mathcal{F}_a^2(w)
\]

\[
\frac{d\Gamma(B \rightarrow D \ell \bar{\nu})}{dw} = \frac{G_F^2 m_B^5}{48\pi^3} r_*^3 (1+r)^2 (w^2-1)^{3/2} |V_{cb}|^2 \mathcal{F}_*^2(w)
\]

- HQET gives $\mathcal{F}(1) = \mathcal{F}_*(1) = 1$
- Luke’s theorem
  \[
  \mathcal{F}_*(1) - 1 = \mathcal{O}(\Lambda_{QCD}/m_c)^2
  \]
  \[
  \mathcal{F}_*(w) = \mathcal{F}_*(1) [1 + \rho^2(w-1) + c(w-1)^2 + \ldots]
  \]
- Extrapolation to $w = 1$ constrained by dispersion relations (unitarity/anatonicity)
  [Boyd et al; Caprini et al; Bjorken; Uraltsev; Oliver et al]
- Lattice $\mathcal{F}_*(1) = 0.917 \pm 0.008 \pm 0.005$
  [Dvitiis et al; Laiho et al]

HFAG (ICHEP 2008) $\mathcal{F}_*(1)|V_{cb}| = (35.41 \pm 0.52) \times 10^{-3}$

DPF-Detroit (me, added in quad)

$|V_{cb}| = (38.62 \pm 0.69) \times 10^{-3}$

Deviates from inclusive by $4.5\sigma$
End point spectra in $B \rightarrow X_u \ell \nu$ and $B \rightarrow X_s \gamma$

- Need to impose large $E_\ell$-cut to remove background from $B \rightarrow X_c \ell \nu$.
- OPE breaks down near end of spectrum.
- Maybe re-summed (in restricted range) into unknown “shape function” $f(x)$:
  
  \[ 2 M_B f(x) = \langle B|\bar{Q}_v \delta(x + in \cdot D)Q_v|B \rangle \quad n \cdot v = 1, \quad n^2 = 0 \]

- Universal: measure in $B \rightarrow X_s \gamma$ and use in $B \rightarrow X_u \ell \nu$.
- Moments analysis: re-sum short distance corrections
  
  \[ [\text{Bigi et al; Neubert}] \]

- OR: use OPE by novel cuts

\[ B \rightarrow X^c \ell \nu \quad \text{and} \quad B \rightarrow X^s \gamma \]

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\[ n \cdot v = 1, \quad n^2 = 0 \]
$|V_{ub}|$: inclusive rate $B \rightarrow X_u \ell \nu$

- Theoretical Uncertainties
  [Bauer et al; Leibovich et al; Neubert]
  - Weak annihilation contribution independent of $q^2_{\text{cut}}$ and $m_{\text{cut}}$; depends on magnitude of factorization violation

  $$\Gamma(q^2_{\text{cut}}, m_{\text{cut}}) = \frac{G_F^2 |V_{ub}|^2 (4.7 \text{ GeV})^5}{192\pi^3} G(q^2_{\text{cut}}, m_{\text{cut}})$$

- Universality violation in shape function
  - sub-leading shape functions

- $\alpha_s(\sqrt{\Lambda m_b}) \Lambda/mb$ “brick wall”
  - numerics: $\alpha_s(\sqrt{\Lambda m_b}) \Lambda/mb$ at least 5% but there are $\sim$10 terms so guesstimate $\sqrt{10} \times 5\% = 15\%$

- Inclusive tension
- Results I: novel cuts (not so novel any more)
- Different analysis (ie choices of cuts, moments, etc) require different calculations:

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Spread indicates underestimated theoretical uncertainties
(in 2005 Belle and BaBar reported results with 5% theory uncertainty)
• Results II: moments radiative/semileptonic

\[
\begin{array}{l}
\text{HFAG Ave. (BLNP)} \\
4.32 \pm 0.16 + 0.32 - 0.27 \\
\text{HFAG Ave. (DGE)} \\
4.26 \pm 0.14 + 0.19 - 0.13 \\
\text{HFAG Ave. (GGOU)} \\
3.96 \pm 0.13 + 0.20 - 0.23 \\
\text{HFAG Ave. (ADFR)} \\
3.76 \pm 0.13 \pm 0.22 \\
\text{HFAG Ave. (BLL)} \\
4.87 \pm 0.24 \pm 0.38 \\
\text{BABAR (LLR)} \\
4.92 \pm 0.32 \pm 0.36 \\
\text{BABAR endpoint (LLR)} \\
4.28 \pm 0.29 \pm 0.48 \\
\text{BABAR endpoint (LNP)} \\
4.40 \pm 0.30 \pm 0.47
\end{array}
\]
$|V_{ub}|$: exclusive decays

$$\text{Br}(B \to \pi \ell \nu) = |V_{ub}|^2 \int_0^{q_{\text{max}}^2} dq^2 \frac{f_{+}^{B 	o \pi}(q^2)}{q^2} \times (\text{trivial factors})$$

- Problem:
  - experiment gives low $q^2$ data
  - lattice gives form factor at high $q^2$
  - extrapolation introduces error

- Moving NRQCD: low data from lattice
  [K. Wong]

- Dispersion relations: combine lattice and experimental data over full $q^2$ region fitting to model-independent expression based on analyticity and unitarity
  [Arnesen et al; Becher an dHil; Ball; Mackenzie and Van de Water]

**FNAL/MILC**

$N_f = 2 + 1$

$|V_{ub}| = (2.94 \pm 0.35) \times 10^{-3}$ (12\% error)

(Van de Water, Lattice 2008)
Tension between inclusive and exclusive?

| HFAG Ave. (BLNP) | 4.32 ± 0.16 + 0.32 - 0.27 |
| HFAG Ave. (DGE) | 4.26 ± 0.14 + 0.19 - 0.13 |
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| BABAR endpoint (LNP) | 4.40 ± 0.30 ± 0.47 |

**FNAL/MILC**
2.94 ± 0.35

(FNAL/MILC point not from HFAG)
Other ways to get $|V_{ub}|$

- $\mathcal{B}(B \to \ell \bar{\nu})$ measures $f_B \times |V_{ub}|$ — need $f_B$ from lattice

- "Grinstein-type double ratio" inspired ideas (HQS / chiral symmetry suppressions)
  - $\frac{f_B}{f_{B_s}} \times \frac{f_{D_s}}{f_D}$ — lattice: double ratio = 1 within few %

  [Grinstein '93]

  - $\frac{f(B \to \rho \ell \bar{\nu})}{f(B \to K^* \ell \bar{\nu})} \times \frac{f(D \to K^* \ell \bar{\nu})}{f(D \to \rho \ell \bar{\nu})}$ or $q^2$ spectra — accessible soon?

  [ZL, Wise; Grinstein, Pirjol]

CLEO-C $D \to \rho \ell \bar{\nu}$ data still consistent with no $SU(3)$ breaking in form factors

Could lattice do more to pin down the corrections?

Worth looking at similar ratio with $K$, $\pi$ — role of $B^*$ pole...?

- $\frac{\mathcal{B}(B \to \ell \bar{\nu})}{\mathcal{B}(B_s \to \ell^+ \ell^-)} \times \frac{\mathcal{B}(D_s \to \ell \bar{\nu})}{\mathcal{B}(D \to \ell \bar{\nu})}$ — very clean... after 2015?

  [Ringberg workshop, '03]

- $\frac{\mathcal{B}(B_u \to \ell \bar{\nu})}{\mathcal{B}(B_d \to \mu^+ \mu^-)}$ — even cleaner... ever possible?

  [Grinstein, CKM'06]

source: Z. Ligeti, Lattice QCD Meets Experiment Workshop,
Rare $B$ Decays: $b \rightarrow s \gamma$

- Sensitive to New Physics
  - Rate
  - CP asymmetry
- Experimental Measurements
  - Precise Rate (~7% HFAG2008)
  - Asymmetry will improve
- Largely Under Control (non-perturbative effects ~5% in rate)
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- Largely Under Control (non-perturbative effects ~5% in rate)

\[
\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QCD}\times\text{QED}}(u, d, s, c, b) + \frac{4G_F}{\sqrt{2}} V_{ts} V_{tb} \sum_{i=1}^{8} C_i(\mu) Q_i
\]
Rare $B$ Decays: $b \rightarrow s\gamma$

- Sensitive to New Physics
- Rate
- CP asymmetry
- Experimental Measurements
  - Precise Rate ($\sim 7\%$ HFAG2008)
  - Asymmetry will improve
- Largely Under Control (non-perturbative effects $\sim 5\%$ in rate)

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QCD\times QED}}(u, d, s, c, b) + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{8} C_i(\mu) Q_i$$

- $Q_{1,2} = \begin{array}{c} \bar{c} \\ b \end{array} \begin{array}{c} c \\ s \end{array} = (\bar{s}\Gamma_i c)(\bar{c}\Gamma'_i b)$, from $\begin{array}{c} \bar{c} \\ b \end{array} \begin{array}{c} c \\ W \end{array} \begin{array}{c} c \\ s \end{array}$, $|C_i(m_b)| \sim 1$

- $Q_{3,4,5,6} = \begin{array}{c} \bar{q} \\ b \end{array} \begin{array}{c} q \\ s \end{array} = (\bar{s}\Gamma_i b)\sum_q(\bar{q}\Gamma'_q q)$, $|C_i(m_b)| < 0.07$

- $Q_7 = \begin{array}{c} b \\ s \end{array} = \frac{e m_b}{16\pi^2} \bar{s}L\sigma^{\mu\nu}b_R F_{\mu\nu}$, $C_7(m_b) \sim -0.3$

- $Q_8 = \begin{array}{c} b \\ s \end{array} = \frac{q m_b}{16\pi^2} \bar{s}L\sigma^{\mu\nu}T^a b_R G^a_{\mu\nu}$, $C_8(m_b) \sim -0.15$


**Rare B Decays: \( b \rightarrow s\gamma \)**

- Sensitive to New Physics
  - Rate
  - CP asymmetry
- Experimental Measurements
  - Precise Rate (~7% HFAG2008)
  - Asymmetry will improve
- Largely Under Control (non-perturbative effects ~5% in rate)

\[
\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QCD}\times\text{QED}}(u,d,s,c,b) + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{8} C_i(\mu) Q_i
\]

- \( Q_{1,2} = \) \[\begin{array}{c}
\bar{c} \\
\hspace{1cm} \uparrow \\
\hspace{1cm} \downarrow \\
s \\
\end{array}\] = \((s\Gamma_i c)(\bar{c}\Gamma'_i b)\), from \[\begin{array}{c}
\bar{c} \\
\hspace{1cm} \uparrow \\
W \\
s \\
\end{array}\]

- \( |C_i(m_b)| \sim 1 \)

- \( Q_{3,4,5,6} = \) \[\begin{array}{c}
q \\
\hspace{1cm} \uparrow \\
\hspace{1cm} \downarrow \\
s \\
\end{array}\] = \((s\Gamma_i b)\sum_q (q\Gamma'_q)\),

- \( |C_i(m_b)| < 0.07 \)

- \( Q_7 = \) \[\begin{array}{c}
b \\
\hspace{1cm} \uparrow \\
g \\
s \\
\end{array}\] = \( \frac{em_b}{16\pi^2} \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu} \),

- \( C_7(m_b) \sim -0.3 \)

- \( Q_8 = \) \[\begin{array}{c}
b \\
\hspace{1cm} \uparrow \\
g \\
s \\
\end{array}\] = \( \frac{q m_b}{16\pi^2} \bar{s}_L \sigma^{\mu\nu} T^a b_R G_{\mu\nu}^a \),

- \( C_8(m_b) \sim -0.15 \)

Known to NNLO
Rare $B$ Decays: $b \rightarrow s\gamma$

- Sensitive to New Physics
  - Rate
  - CP asymmetry
- Experimental Measurements
  - Precise Rate (~7% HFAG2008)
  - Asymmetry will improve
- Largely Under Control (non-perturbative effects ~5% in rate)

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QCD\times QED}}(u, d, s, c, b) + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{8} C_i(\mu) Q_i \]

\[
Q_{1,2} = \begin{array}{c}
\bar{c} \\
\bar{b} \\
s
\end{array} = (\bar{s}\Gamma c)(\bar{c}\Gamma' b), \quad \text{from} \quad \begin{array}{c}
\bar{c} \\
\bar{b} \\
s
\end{array}, \quad |C_i(m_b)| \sim 1
\]

\[
Q_{3,4,5,6} = \begin{array}{c}
\bar{q} \\
\bar{b} \\
s
\end{array} = (\bar{s}\Gamma b)\sum_q (\bar{q}\Gamma' q),
\]

\[
Q_7 = \begin{array}{c}
b \\
s
\end{array} = \frac{em_b}{16\pi^2} \bar{s}_L\sigma^{\mu\nu}b_R F_{\mu\nu},
\]

\[
Q_8 = \begin{array}{c}
b \\
s
\end{array} = \frac{gm_b}{16\pi^2} \bar{s}_L\sigma^{\mu\nu}T^a b_R G_{\mu\nu}^a,
\]

Known to NNLO

- $|C_i(m_b)| < 0.07$
- $C_7(m_b) \sim -0.3$
- $C_8(m_b) \sim -0.15$

Constrain/discover new physics by determining $C_7(m_b)$ from inclusive radiative decay
Energetic photon production in charmless decays of the $\bar{B}$-meson

$(E_\gamma \gtrsim \frac{m_b}{3} \approx 1.6\,\text{GeV})$

**A. Without long-distance charm loops:**

1. Hard

   ![Diagram](image1)

   Dominant, well-controlled. \(\mathcal{O}(\alpha_s\Lambda/m_b)\), \((-1.5 \pm 1.5)\%\).

   [Lee, Neubert, Paz, 2006]

2. Conversion

   ![Diagram](image2)

   Pert. < 1%, nonp. \(~\sim\sim\) \(-0.2\%\).

   [Kapustin,Ligeti,Politzer, 1995]

3. Collinear

   ![Diagram](image3)

4. Annihilation \((qq \neq \bar{c}c)\)

   ![Diagram](image4)

   Exp. \(\pi^0, \eta, \eta', \omega\) subtracted. Perturbatively \(~\sim\sim\) \(0.1\%\).

**B. With long-distance charm loops:**

5. Soft gluons only

   ![Diagram](image5)

   \(\mathcal{O}(\Lambda^2/m_b^2)\), \(\sim\sim\) \(+3.1\%\).

   [Voloshin, 1996, [...]]


6. Boosted light $c\bar{c}$ state annihilation \((\text{e.g. } \eta_c, J/\psi, \psi')\)

   ![Diagram](image6)

   Exp. \(J/\psi\) subtracted \(<1\%\).

   Perturbatively (including hard): \(\sim\sim\) \(+3.6\%\).

   \(\phi^{(i)}_{ij}(\delta), \phi^{(2)}_{ij}(\delta), i,j = 1,2\)

7. Annihilation of $c\bar{c}$ in a heavy \((\bar{c}s)(\bar{q}c)\) state

   ![Diagram](image7)

   \(\mathcal{O}(\alpha_s\Lambda/M)^2\)

   \(M \sim 2m_c, 2E_{\gamma}, m_b,\)

   e.g. \(B^0 \rightarrow D_s(2457)^+D^*(2007)\bar{D}^*(2007)^0K^-\) \(\sim\sim\) \(1.2\%\),

   \(B^0 \rightarrow D^*(2010)^+\bar{D}^*(2010)^0K^-\) \(\sim\sim\) \(1.2\%\).

lifted from Misiak
Three steps in the determination $C_7(m_b)$:

1. **Matching**: Choose $C_i(M)$ so that the Fermi effective theory and the SM, renormalized at $M \sim M_W$

2. **Running**: Compute anomalous dimension matrices and use RG-equation to compute $C_i(m_b)$

3. **Matrix Elements**: Perturbative calculation of amplitudes in EFT (renormalized at $m_b$)

**A prodigious effort!**
Status of calculation of matrix elements

\[
\Gamma(b \to X_{\text{parton}}^s \gamma)_{E_\gamma > E_0} = \frac{G^2 F m_b^5 \alpha_{\text{em}}}{32\pi^4} |V_{ts}^* V_{tb}|^2 \sum_{i,j=1}^8 C_i(\mu_b) C_j(\mu_b) G_{ij}(E_0, \mu_b)
\]

\( G_{77} = 1, \text{ all others vanish} \)

\( i, j = 1, 2, 7, 8 \) dominate

bust rest also known

[Greub, Hurth, Wyler; Ali, Greub](1996)

only parts of \( i, j = 1, 2, 7, 8 \) known:

- \( G_{77} \) determined [Bloklan et al; Melnikov, Mitov; Asatran et al]
- \( G_{11}, G_{12} \) and \( G_{22} \):
  - 2 particle cuts are \( |\text{NLO}|^2 \)
  - 3, 4 particle cuts vanish at endpoint
- Ongoing progress in the rest
  - 2, 3, 4 particle cuts in \( G_{17} \) and \( G_{27} \) [Schutzmeier, Czakon, Boughezal]
  - 2, 3, 4 particle cuts in \( G_{78} \) [Asatran, Ewerth, Ferroglia, Greub, Ossola]
$$\mathcal{B}(\bar{B} \to X_s \gamma)_{E_{\gamma} > 1.6 \text{ GeV}}^{\text{NNLO}} = \begin{cases} (3.15 \pm 0.23) \times 10^{-4}, & \text{hep-ph/0609232, using the 1S scheme,} \\ (3.26 \pm 0.24) \times 10^{-4}, & \text{following the kin scheme analysis of arXiv:0805.0271, but } \mathcal{M}_c(\mathcal{M}_c)^{2\text{loop}} \text{ rather than } \mathcal{M}_c(\mathcal{M}_c)^{1\text{loop}} \text{ in } P(E_0).} \end{cases}$$
Rare $B$ Decays: $b \rightarrow s \ell \ell$

- Requires 1 loop less than radiative
- NNLO complete
- FB asymmetry zero in $B \rightarrow K^* \ell \ell$ robust [Burdman]
even including non-resonant Kpi [BG, Pirjol]
- Sensitive to new physics
The end