Lattice QCD

Aida X. El-Khadra

University of Illinois

DPF 2009, Jul. 26-31, 2009

Outline

- Introduction and Glossary ⇒ **Appendix**
- History
  - predictions of Lattice QCD
- Tests of lattice QCD:
  - the light hadron spectrum
    - ⇒ quark masses and α_s
- Flavor physics
  - ♣ leptonic and semileptonic Kaon decays
  - ♣ leptonic *D* and *B* meson decays
  - ♣ semileptonic B meson decays
  - ♣ neutral *K* and *B* meson mixing
  - ♣ impact on UT triangle
- Conclusions and Outlook

Not covered in this talk:

- ☐ lattice QCD results with $n_f = 2$ or $n_f = 0$
- ☐ QCD thermodynamics
- ☐ nucleon properties
- ☐ excited hadrons
- ☐ hadron structure
- ☐ topology and chiral properties of QCD
- ☐ lattice field theory calculations for BSM models

- ☐ ….

Strategy

- Lattice QCD action has the same free parameters as continuum QCD:
  quark masses and α_s
- use experimentally measured hadron masses as input, for example:
  π, K, D_s, B_s mesons for u, d, s, c, b quark masses
- need an experimental input to determine the lattice spacing (a) in GeV:
  $2S\text{-}1S$ splitting in Υ system, f_π, Ξ mass, …
  this also determines α_s
- lattice QCD calculations of all other quantities should agree with experiment …

History

1999: MILC started to generate ensembles with $n_f = 2+1$ sea quarks, using the Asqtad action

made publicly available

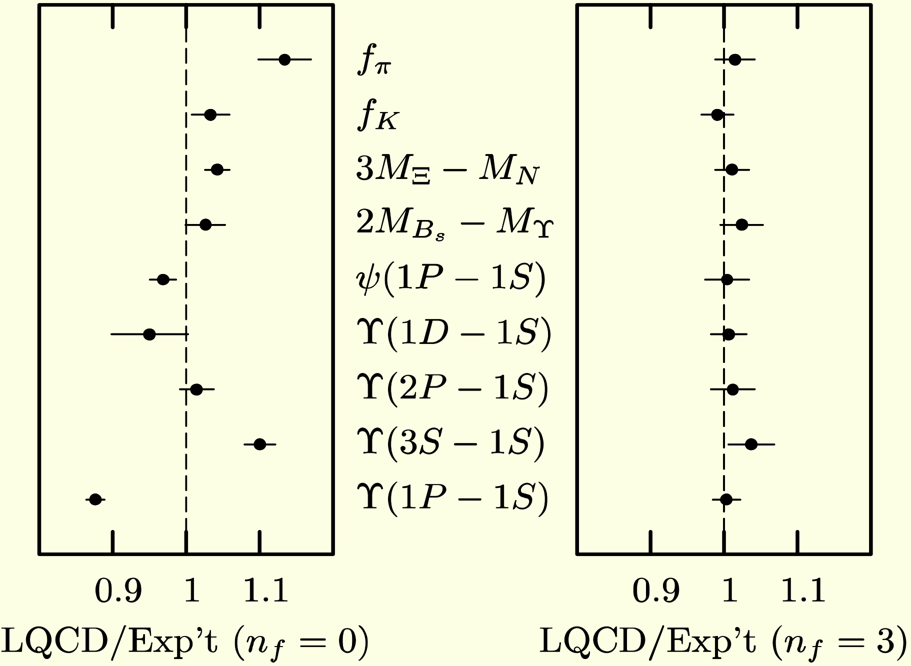
2004: MILC+HPQCD+FNAL tested them against experiment at the 2-3% level

History

MILC+HPQCD+FNAL (Phys. Rev. Lett. 92:022001,2004)

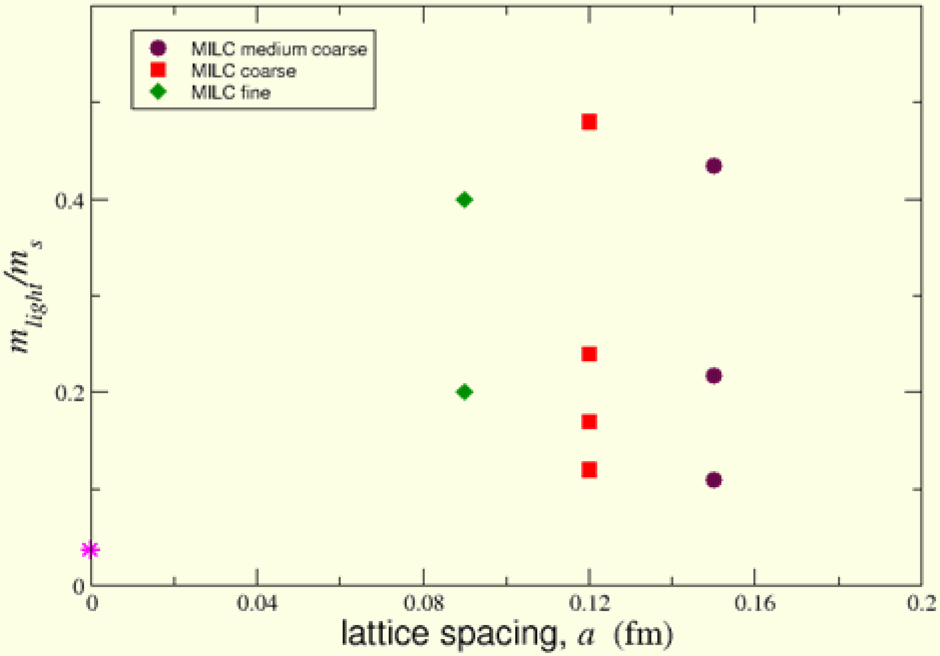
lattice QCD/experiment

before 2004



Postdiction at the 2-3% level

MILC ensembles ca. 2004



First ensembles with $n_f = 2+1$ Asqtad sea

- ~400-500 configurations/ensemble
- additional ensembles at larger a and/or larger m_{light} are not shown here

History

1999: MILC started to generate ensembles with $n_f = 2+1$ sea quarks, using the Asqtad action

made publicly available

2004: MILC+HPQCD+FNAL tested them against experiment at the 2-3% level

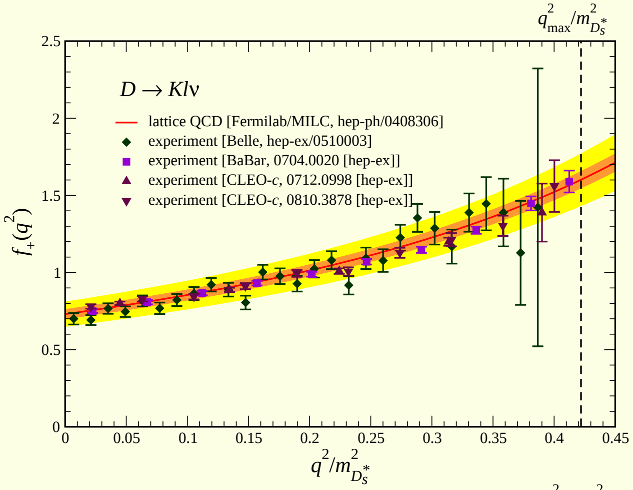
2005: FNAL+MILC: *predictions* for D, D_s meson decay constants and semileptonic form factors (shape) with 7-9 % precision

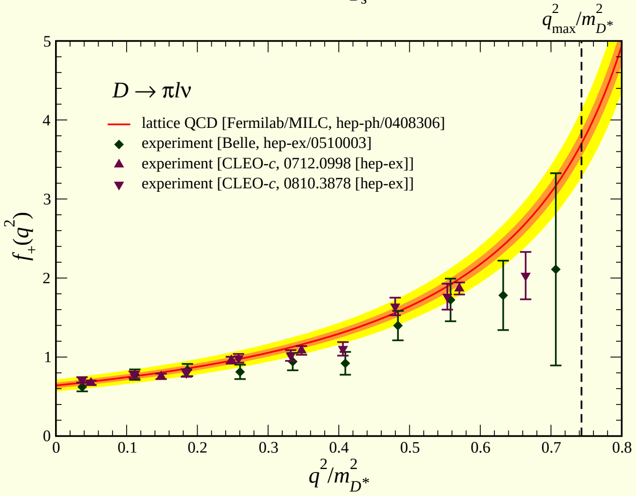
also FNAL+HPQCD: prediction of the B_c mass

History

Form factors for $D \to K l \nu$ and $D \to \pi l \nu$

(Phys. Rev. Lett. 94:011601, 2005)

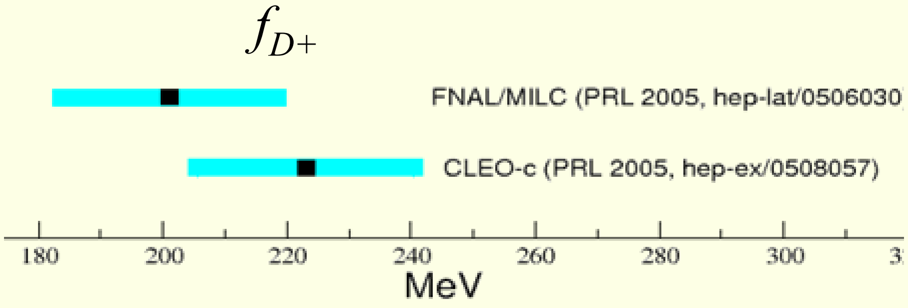


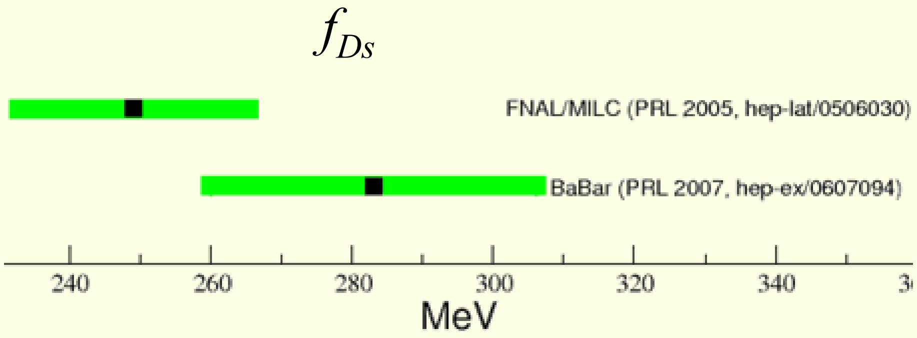


- Normalization agrees with experiment
- *Prediction* of the shape

D^+ and D_s meson decay constants

(Phys. Rev. Lett. 95:1222002, 2005)





- *Predictions* of f_{D+} and f_{Ds} at 7-9% level

History

1999: MILC started to generate ensembles with $n_f = 2+1$ sea quarks, using the Asqtad action made publicly available

2004: MILC+HPQCD+FNAL tested them against experiment at the 2-3% level

2005: FNAL+MILC: *predictions* for D, D_s meson decay constants and semileptonic form factors (shape) with 7-9 % precision
also FNAL+HPQCD: prediction of the B_c mass

2006: RBC/UKQCD started to generate $n_f=2+1$ ensembles with DWF sea quarks.

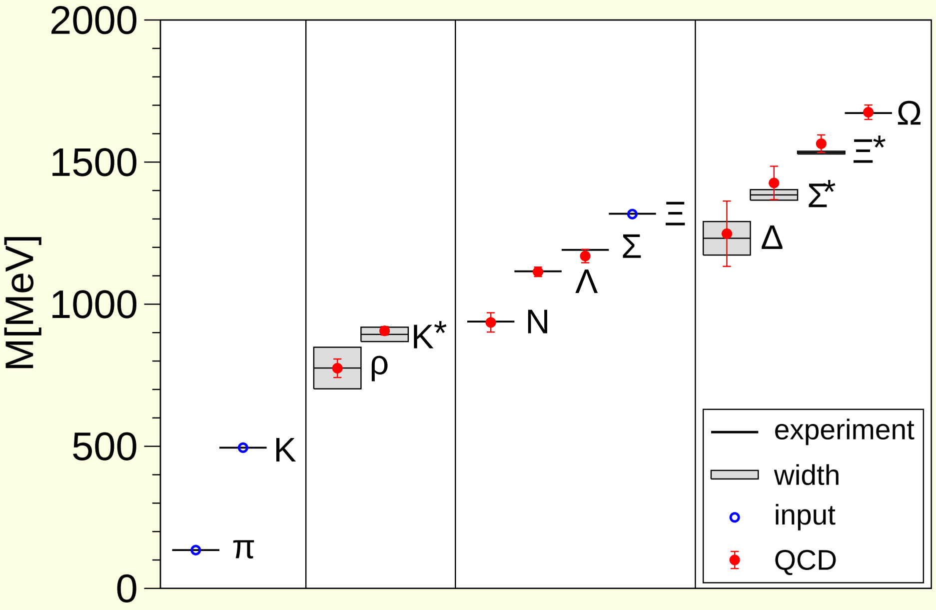
2007: $n_f=2+1$ ensembles are being generated by

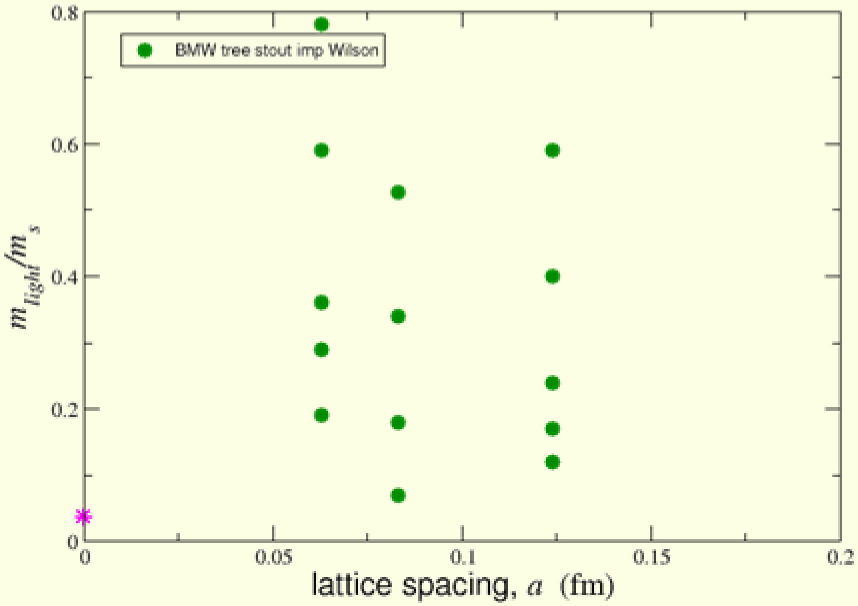
- BMW with tree O(a) improved stout smeared Wilson sea quarks
- PACS-CS with NP O(a) improved Wilson sea quarks
- JLQCD/TWQCD with Overlap sea quarks
- HSC with anisotropic Clover sea quarks

2008: BMW, PACS-CS, MILC: postdictions of the light hadron spectrum (tests of lattice QCD)
also: HSC Phys. Rev. D79, 034502 (2009) and LHPC Phys. Rev. D79 , 054502 (2009)

Tests of Lattice QCD: the hadron spectrum

BMW Science 322, 1224 (2008)



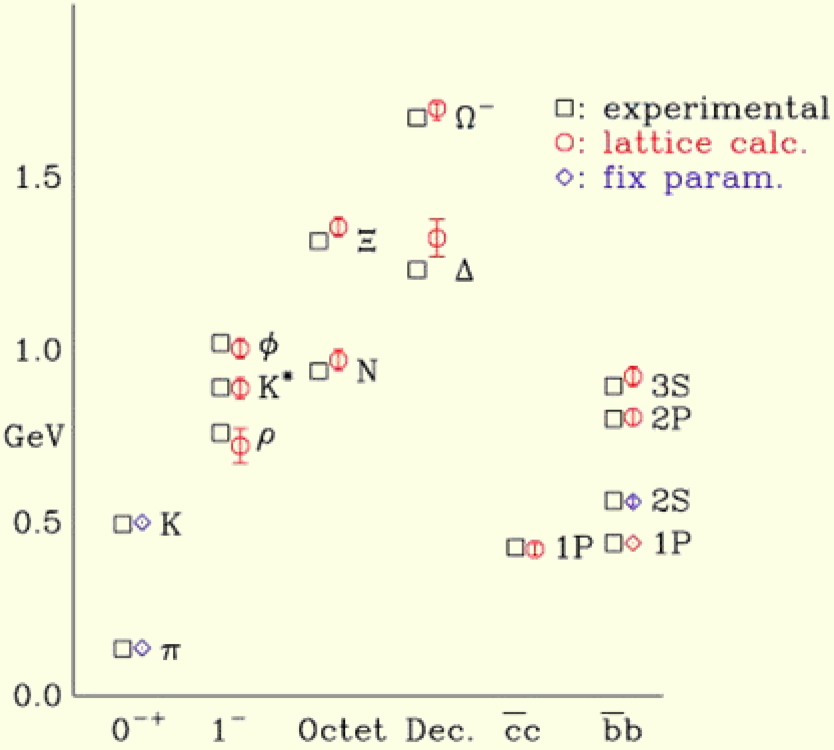


BMW ensembles with tree imp. Wilson action

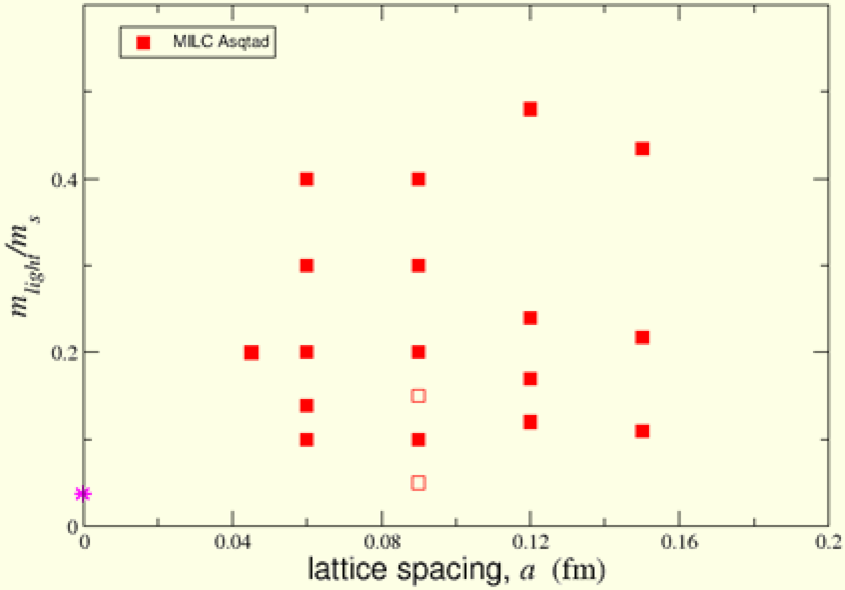
- Stout smeared links
- ~100-200 configurations per ensemble
- $m_\pi \geq 190$ MeV and $m_\pi L > 4$

Tests of Lattice QCD: the hadron spectrum

MILC Phys. Rev. D 70, 094505 (2004) and arXiv:0903.3598



results based on a subset of the MILC ensembles



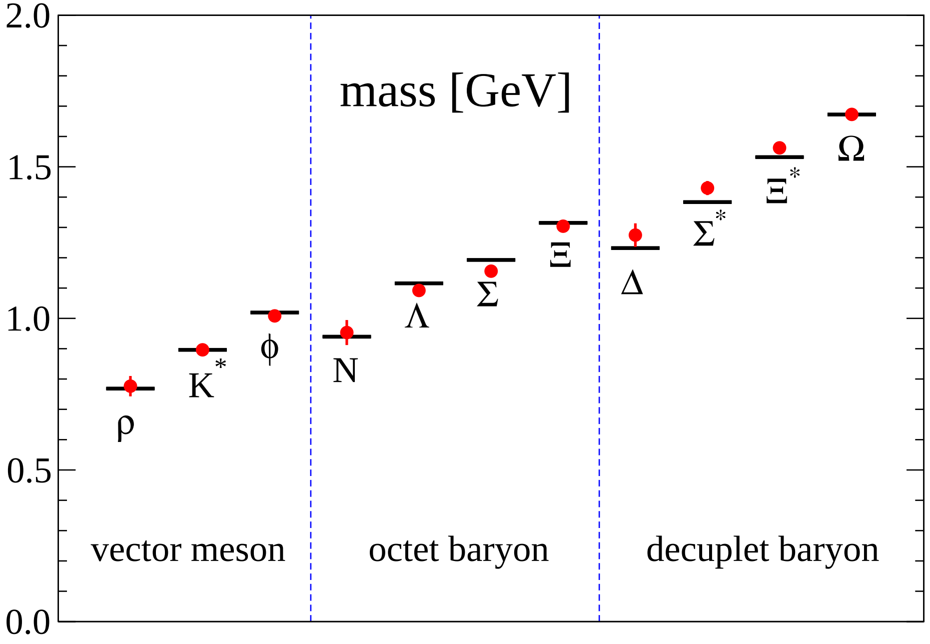
MILC ensembles with the Asqtad action

- ~500-2000 configurations per ensemble
- $m_\pi \geq 240$ MeV and $m_\pi L > 4$
- 5 lattice spacings

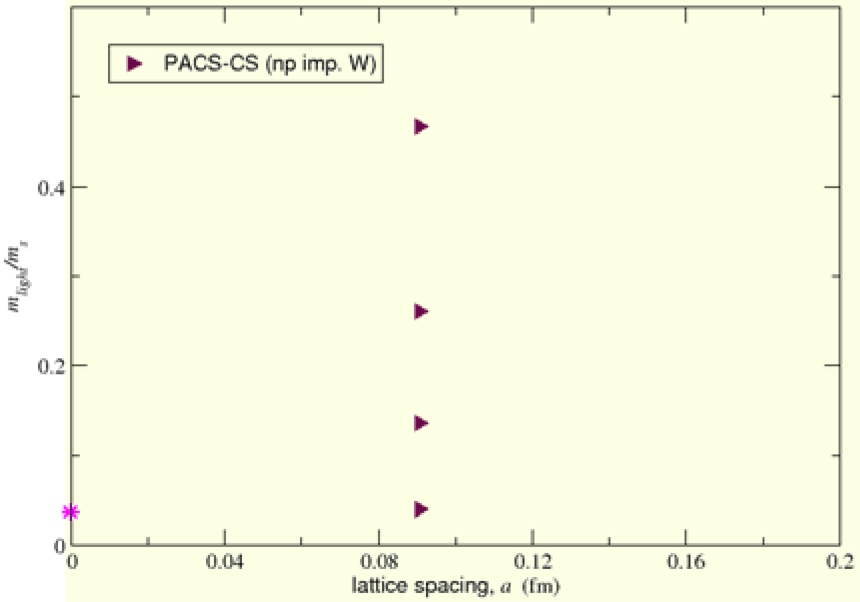
A. El-Khadra, DPF2009, Jul 26-31, 2009

Tests of Lattice QCD: the hadron spectrum

PACS-CS Phys. Rev. D 79, 034503 (2009)



Results based on only one lattice spacing



CS-CS ensembles with a NP imp. Wilson action

- ~100-200 configurations per ensemble
- $m_\pi \geq 156$ MeV and $m_\pi L \geq 2.3$

History

1999: MILC started to generate ensembles with $n_f = 2+1$ sea quarks, using the Asqtad action made publicly available

2004: MILC+HPQCD+FNAL tested them against experiment at the 2-3% level

2005: FNAL+MILC: *predictions* for D, D_s meson decay constants and semileptonic form factors (shape) with 7-9 % precision
also FNAL+HPQCD: prediction of the B_c mass

2006: RBC/UKQCD started to generate $n_f = 2+1$ ensembles with DWF sea quarks.

2007: $n_f = 2+1$ ensembles are being generated by

- BMW with tree O(a) improved stout smeared Wilson sea quarks
- PACS-CS with NP O(a) improved Wilson sea quarks
- JLQCD/TWQCD with Overlap sea quarks
- HSC with anisotropic Clover sea quarks

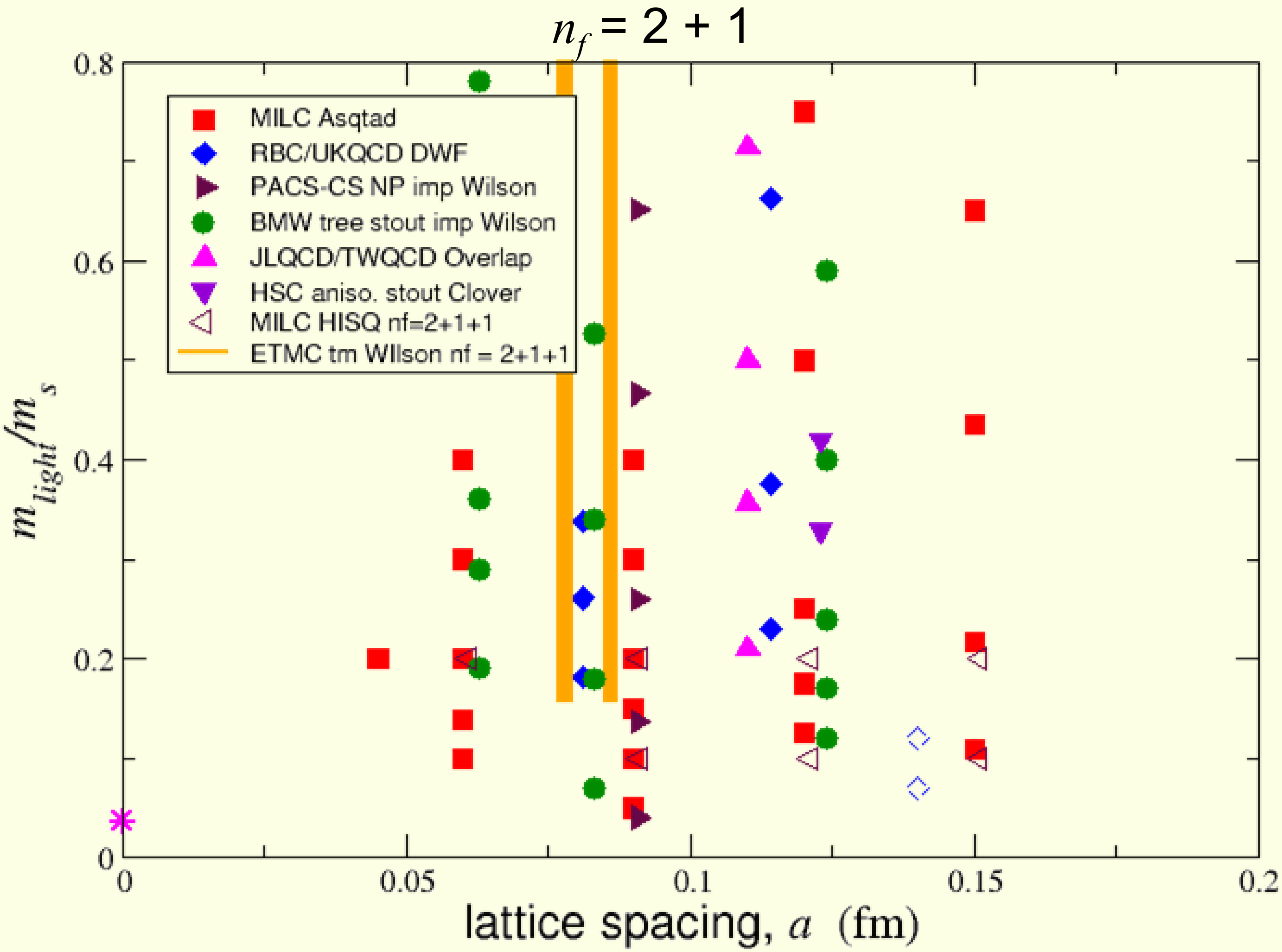
2008: BMW, PACS-CS, MILC: postdictions of the light hadron spectrum (tests of lattice QCD)
also: HSC Phys. Rev. D79, 034502 (2009) and LHPC Phys. Rev. D79 , 054502 (2009)

$n_f = 2+1+1$ ensembles are starting to be generated by

- MILC with HISQ sea quarks
- ETMC with twisted mass Wilson sea quarks

also $n_f = 2+1$ ensembles by QCDSF with SLiNC sea

Overview of simulation parameters today



- ensemble sizes vary between ~100 to ~2000 configurations/ensemble
- volumes vary between $m_\pi L$ ~ 2.5 and $m_\pi L$ ~ 5 (need $m_\pi L \geq 4$ to control finite volume errors at 1%)
- unfilled symbols indicate ensembles currently in production or planned

From the hadron spectrum to quark masses and α_s

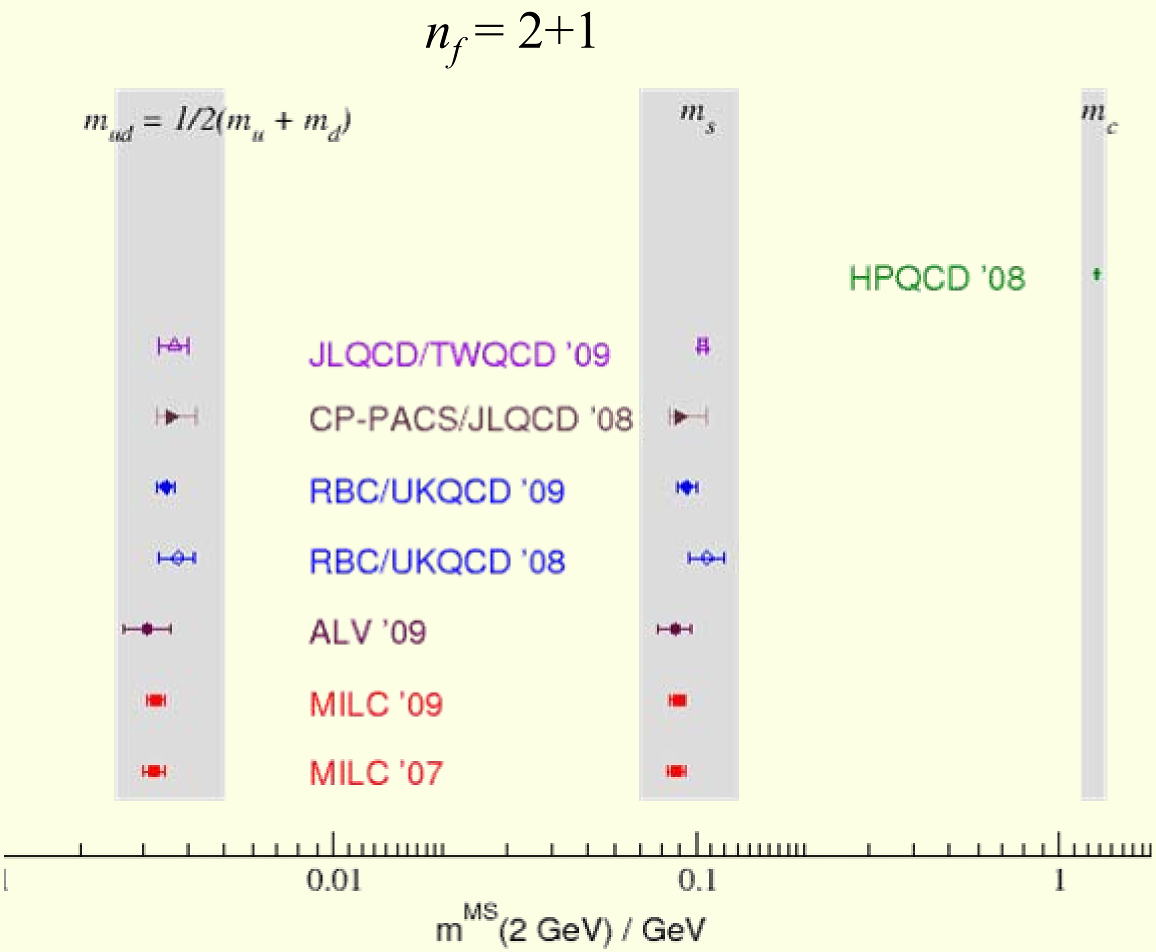
- from experimental inputs (m_π, m_K, etc..) we obtain the bare lattice masses.
- need additional work to determine renormalized quark masses and α_s:

  for α_s:
  calculate additional short distance quantities (Wilson loops, current-current correlators, Schrödinger functional, …)

  for quark masses and α_s:
  » use PT
  » or use nonperturbative renormalization
  » match to $\overline{MS}$ scheme

- new method pioneered by HPQCD (Phys. Rev. D78, 054513 (2008)):
  - calculate moments of current-current correlators with lattice QCD
  - use continuum PT to extract charm quark mass (and α_s)
    (cont. PT done to three or four loop order)
  - agrees well with similar determination from $e+e-$ data

the quark masses



HPQCD '08:
Phys. Rev. D78, 054513 (2008)

JLQCD/TWQCD '09:
J. Noaki, Lattice 2009 **preliminary**

CP-PACS/JLQCD '08:
Phys. Rev. D78, 011502 (2008)

RBC/UKQCD '09:
R. Mawhinney and C. Kelly, Lattice 2009 **preliminary**

RBC/UKQCD '08:
Phys. Rev. D78, 114509 (2008)

ALV '09:
J. Laiho, Chiral Dynamics 2009 **preliminary**

MILC '09:
U. Heller, Lattice 2009 **preliminary**

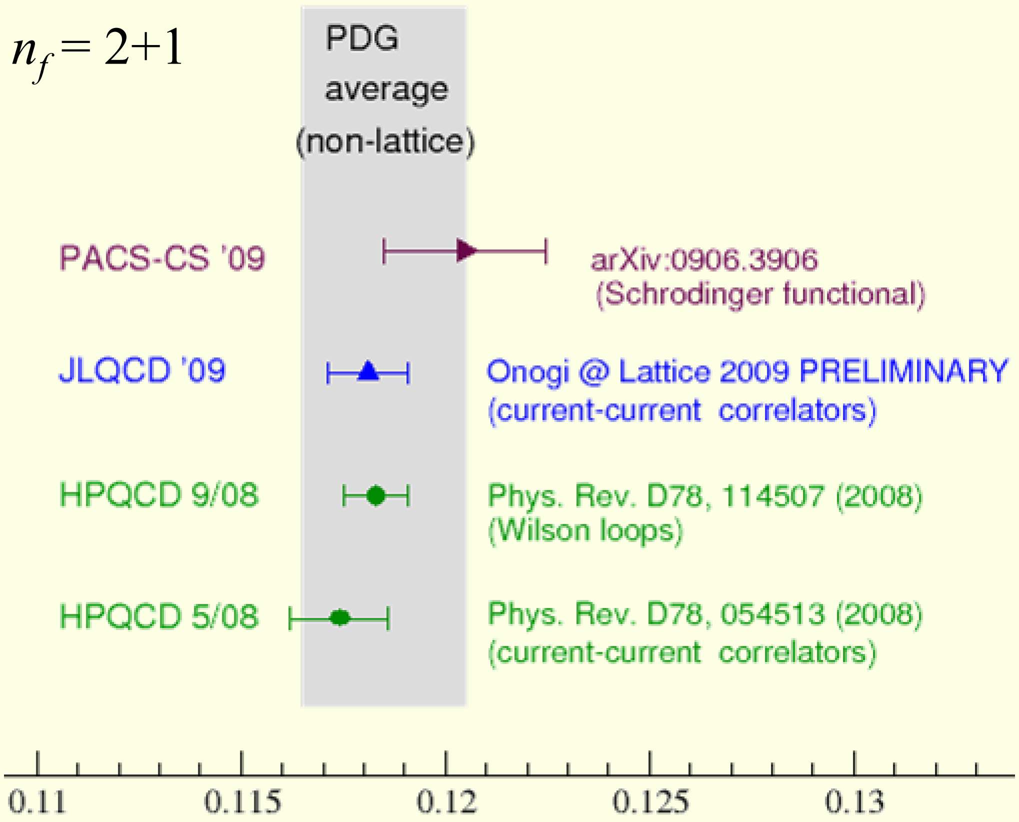
MILC '07:
Lattice '07, arXiv:0710.1118

- grey columns: PDG averages
- open symbols: results based on one lattice spacing

The strong coupling, α_s

$$\alpha_s^{\overline{MS}}(m_z)$$

$n_f = 2+1$



HPQCD 5/08:
- 4 lattice spacings, $a \sim 0.06 - 0.15$ fm
- moments of current-current correlators
- continuum PT to 3 or 4 loop order

HPQCD 9/08:
- 6 lattice spacings, $a \sim 0.045 - 0.18$ fm
- Wilson loops
- lattice PT: calculated to 3-loop order
  + higher order coefficients from fits

JLQCD '09: Preliminary
- one lattice spacing, $a \sim 0.11$ fm
- current-current correlators
- continuum PT to two-loop order

PACS-CS '09:
- three lattice spacings
- Schrödinger functional
- continuum PT to two-loop order

Flavor physics

1. Leptonic and semileptonic Kaon decays
   - $f_K/f_\pi \quad \Rightarrow |V_{us}/V_{ud}|$
   - $K \to \pi l \nu \quad \Rightarrow |V_{us}|$

   $\Big\} \Rightarrow |V_{ud}|$ and $|V_{us}|$
2. Leptonic D and B decays
   - f_D and f_{Ds}
   - f_B and f_{Bs}
3. Semileptonic B decay form factors
   - $B \to D^* l \nu \Rightarrow |V_{cb}|$
   - $B \to \pi l \nu \quad \Rightarrow |V_{ub}|$
4. Neutral K and B meson mixing
   - B_K
   - $\xi \quad \Rightarrow |V_{ts}/V_{td}|$
5. Impact on the UT triangle and on BSM

Lattice QCD program relevant to CKM elements

$$\left(\begin{array}{ccc}
V_{ud} & \boldsymbol{V_{us}} & \boldsymbol{V_{ub}} \\
 & K \to \pi\, l\nu & B \to \pi\, l\nu \\
\multicolumn{2}{c}{\pi \to \mu\nu,\, K \to \mu\nu} & \\
\boldsymbol{V_{cd}} & \boldsymbol{V_{cs}} & \boldsymbol{V_{cb}} \\
D \to \pi\, l\nu & D \to K\, l\nu & B \to D, D^*\, l\nu \\
D \to l\nu & D_s \to l\nu & \\
\boldsymbol{V_{td}} & \boldsymbol{V_{ts}} & V_{tb} \\
B^0 - \overline{B^0}\ \text{mixing} & B_s - \overline{B_s} & \\
K^0 - \overline{K^0} & &
\end{array}\right)$$

K_{l2} and K_{l3} decays

$$\frac{\Gamma(K\to l\nu)}{\Gamma(\pi\to l\nu)} = (known)\times\left|\frac{V_{us}}{V_{ud}}\right|^2\times\left|\frac{f_K}{f_\pi}\right|^2$$

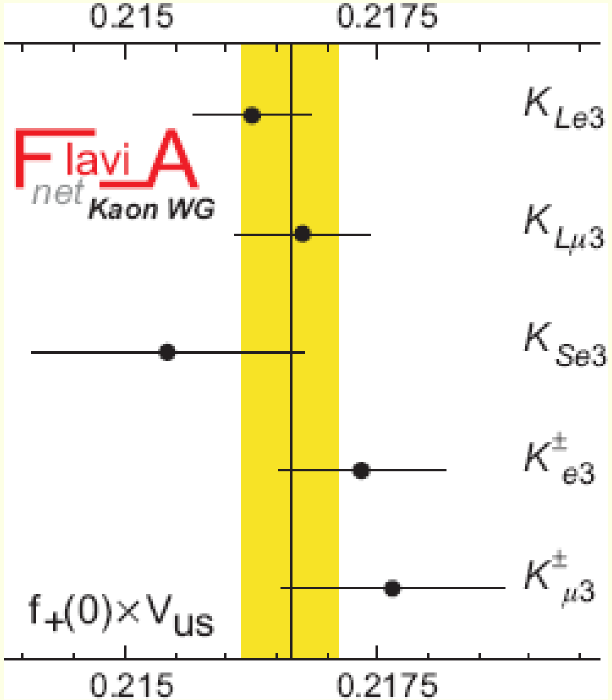
$$\Gamma(K\to\pi\,\ell\,\nu) = (known)\times|V_{us}|^2\times f_+^2(0)\times I_K^\ell$$

I_K^ℓ : phase space integral, exp. measured

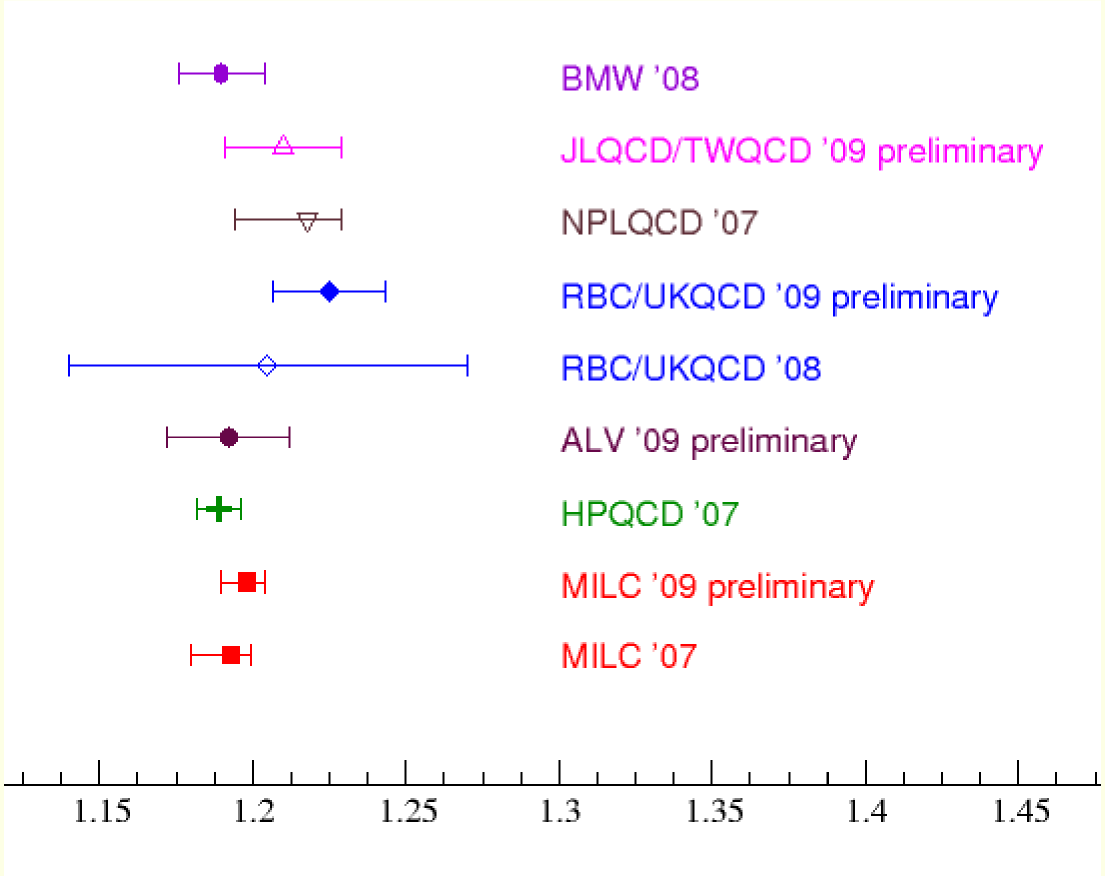
Experimental averages:

$$|\frac{V_{us}}{V_{ud}}|\times\frac{f_K}{f_\pi} = 0.27599\ (59)$$

$$|V_{us}|\times f_+^{K\pi}(0) = 0.21661\ (47)$$



f_K/f_π



open symbols: results based on one lattice spacing

Note: all groups also determine f_K separately

BMW '08:
L. Lellouch, Lattice 2008, arXiv:0902.4545

JLQCD/TWQCD '09:
J. Noaki, Lattice 2009 **preliminary**

NPLQCD '07:
Phys. Rev. D75, 094501 (2007)

RBC/UKQCD '09:
R. Mawhinney and C. Kelly, Lattice 2009 **preliminary**

RBC/UKQCD '08:
Phys. Rev. D78, 114509 (2008)

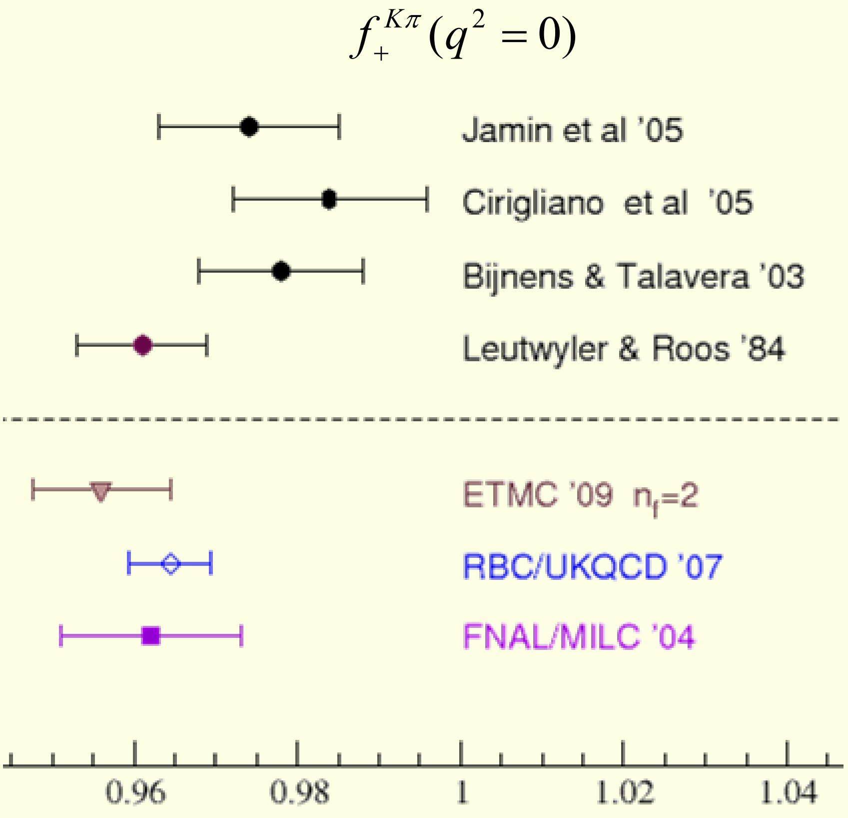
ALV '09:
J. Laiho, Chiral Dynamics 2009 **preliminary**

HPQCD '07:
Phys. Rev. Lett. 100, 062002 (2008)

MILC '09:
U. Heller, Lattice 2009 **preliminary**

MILC '07:
Lattice '07, arXiv:0710.1118

form factor for $K \to \pi l \nu$



Black circles: continuum estimates

FNAL/MILC '04: M. Okamoto, hep-lat/0412044

- calculate $f_0(q^2_{\max})$, extrapolate to $q^2=0$ using shape from experiment

RBC/UKQCD '07: Phys. Rev. Lett. 100, 141601 (2008)

- one lattice spacing, $a \approx 0.11$ fm
- $m_\pi \geq 330$ MeV, $m_\pi L > 4$
- use pole dominance to interpolate to $q^2 = 0$

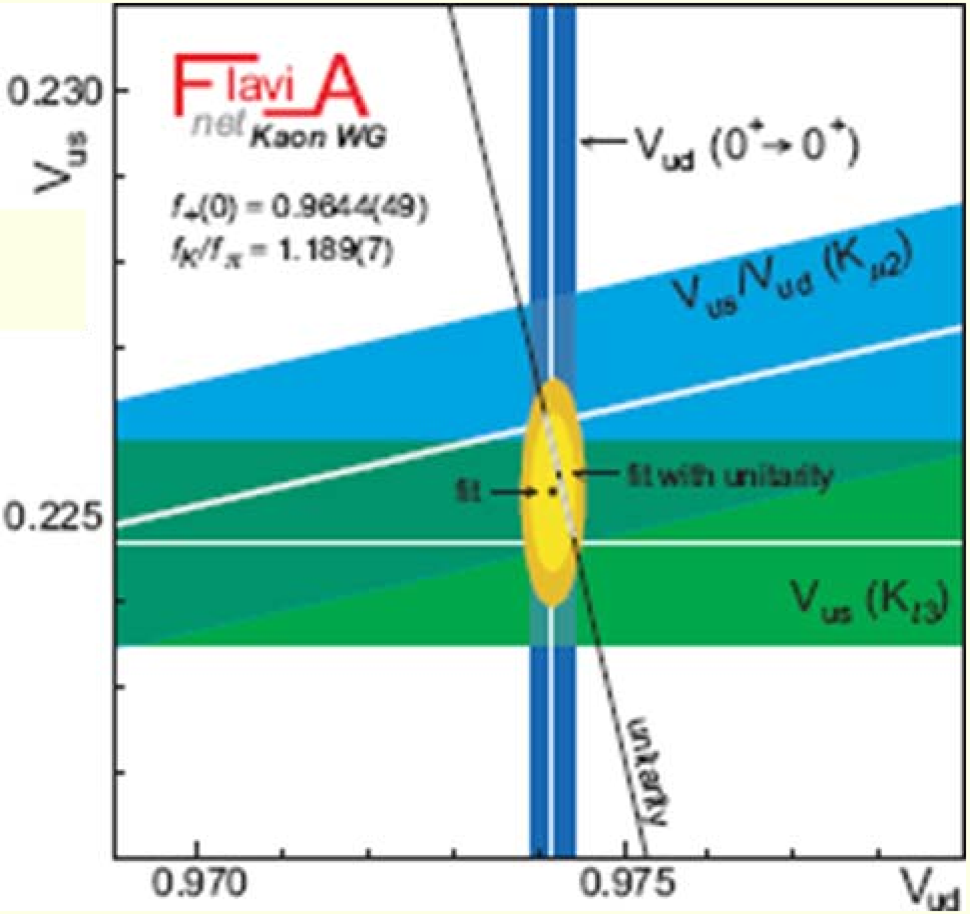
ETMC '09: arXiv:0906.4728

- $\boldsymbol{n_f = 2}$
- but analysis includes an estimate of the error due to quenching strange quark
- two lattice spacings, $a \approx 0.088, 0.07$
- $m_\pi \geq 260$ MeV, $m_\pi L \geq 3.7$
- use pole dominance to interpolate to $q^2= 0$.

Also: use twisted boundary conditions to calculate Kl3 form factor directly at $q^2 = 0$.
(RBC/UKQCD @ Lattice 2008, arXiv:0812.4265)

f_K/f_π and $f_+^{K\pi}(0)$ $\Rightarrow$ $|V_{ud}|$ and $|V_{us}|$

Flavianet: http://www.lnf.infn.it/wg/vus/



- f_K/f_π from HPQCD ’07
- $f_+^{K\pi}(0)$ from RBC/UKQCD ’08
- plus experimental averages

$$\Rightarrow \begin{array}{l} V_{us} = 0.22461\ (124) \\ V_{us}/V_{ud} = 0.23211\ (145) \end{array}$$

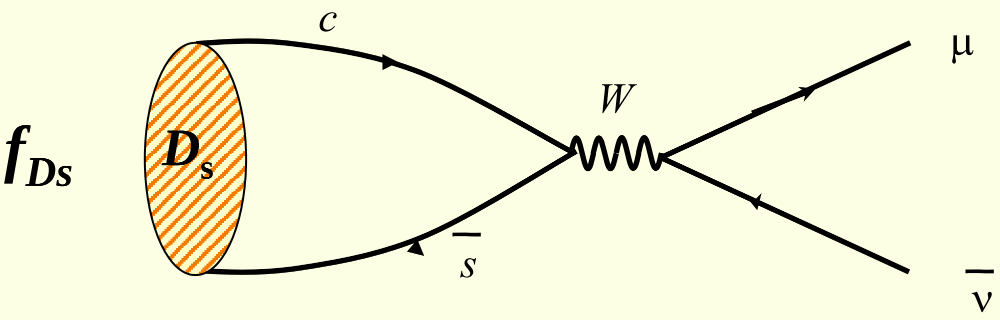
Combine with:

$V_{ud}| = 0.97418\ (26)$ from nuclear β-decay

$$\Rightarrow \begin{array}{l} V_{us} = 0.2253\ (9) \\ V_{ud} = 0.97416\ (26) \end{array}$$

Leptonic D and D_s decays

example: $D_s \to \mu\nu$



$$\Gamma(D_s \to \mu\nu) = (known)\,|V_{cs}|^2\, f_{D_s}^2$$

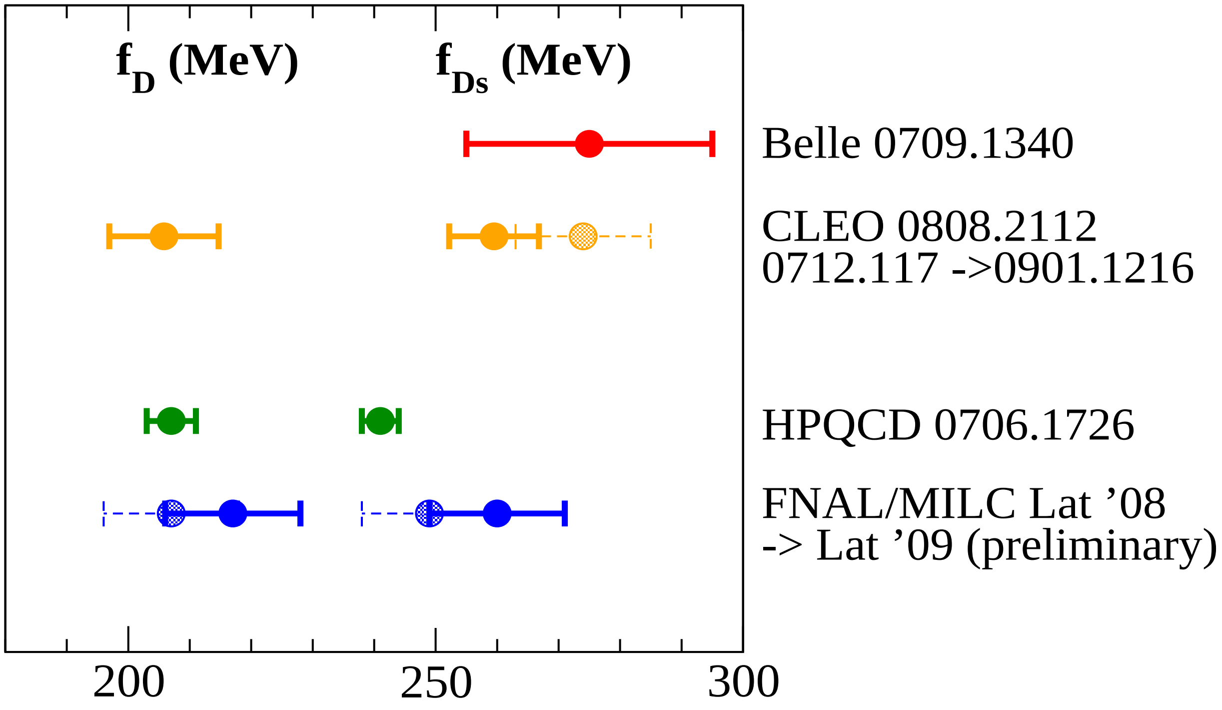
test lattice QCD:

- take V_{cs} and V_{cd} from other sources

⇒extract f_{Ds} and f_D from experiment

- initial tests (lattice QCD predictions at 7-9% level) were fine
- for Lattice QCD, f_{Ds} is easier than f_D : strange valence quark instead of light
- new physics unlikely, or so we thought …

f_{Ds} and f_D



R. van de Water @ Lattice 2009

2008: $3.6\ \sigma$ discrepancy for f_{Ds} between HPQCD and experiment but agreement for f_D

Jan 2009: new CLEO results $\Rightarrow$ discrepancy reduced to $3\ \sigma$

July 2009: new, prelimary results from FNAL/MILC at Lattice 2009 (J. Simone)

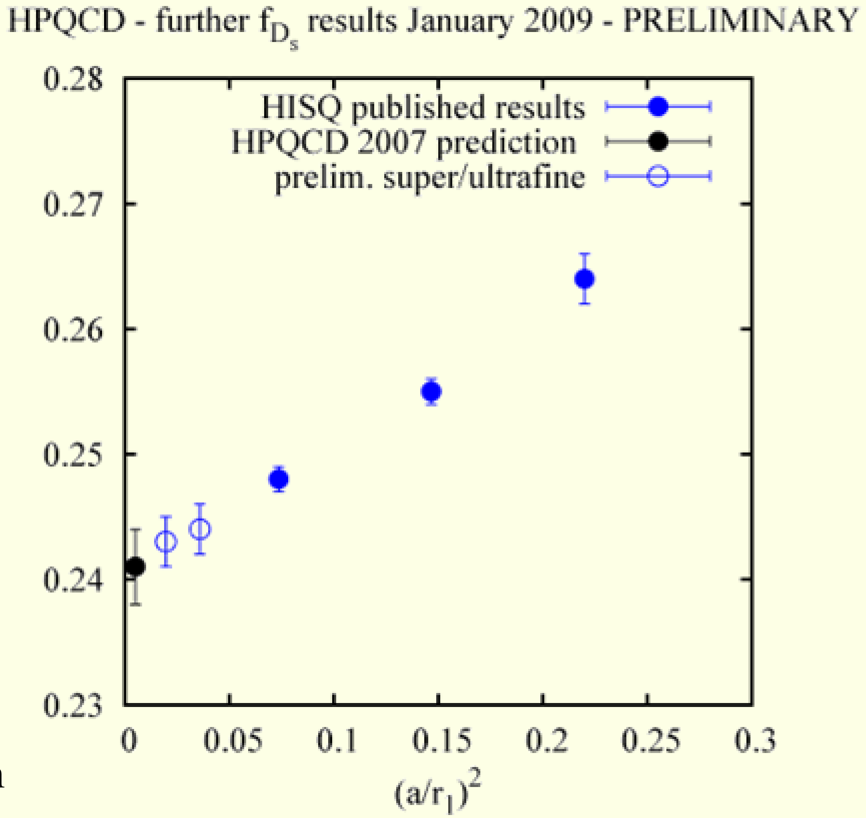
HPQCD's f_{D_s} calculation

HPQCD '07: Phys. Rev.Lett. 100, 062002 (2008)

- 3 lattice spacings, $a = 0.09, 0.12, 0.15$ fm
- HISQ action for valence quarks + MILC sea
- dominant systematic errors:
  - » scale setting uncertainty
  - » discretization errors $\sim a^2$
- results for $f_D, f_K, f_\pi, m_D, m_{Ds}, m(J/\psi - \eta_c), \ldots$ agree with experiment.

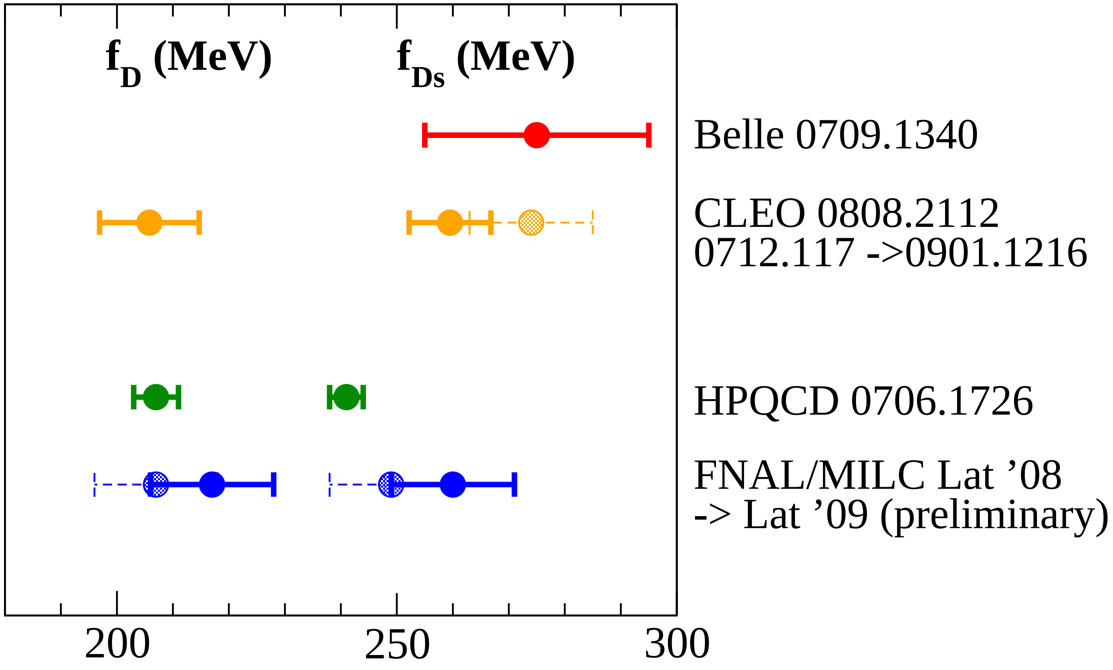
HPQCD '09: PRELIMINARY

- 2 additional lattice spacings, $a = 0.06, 0.045$ fm
- agrees with HPQCD '07 results



f_{Ds} and f_D

R. van de Water @ Lattice 2009



2008:
3.6 σ discrepancy for f_{Ds} between HPQCD and experiment but agreement for f_D

Jan 2009:
new CLEO results $\Rightarrow$ discrepancy reduced to $3\ \sigma$

July 2009:

- new, prelimary result from FNAL/MILC at Lattice 2009
- new, preliminary results from HPQCD at $a \approx 0.06, 0.045$

FNAL/MILC ’08: Lattice 2008, arXiv:0904.1895

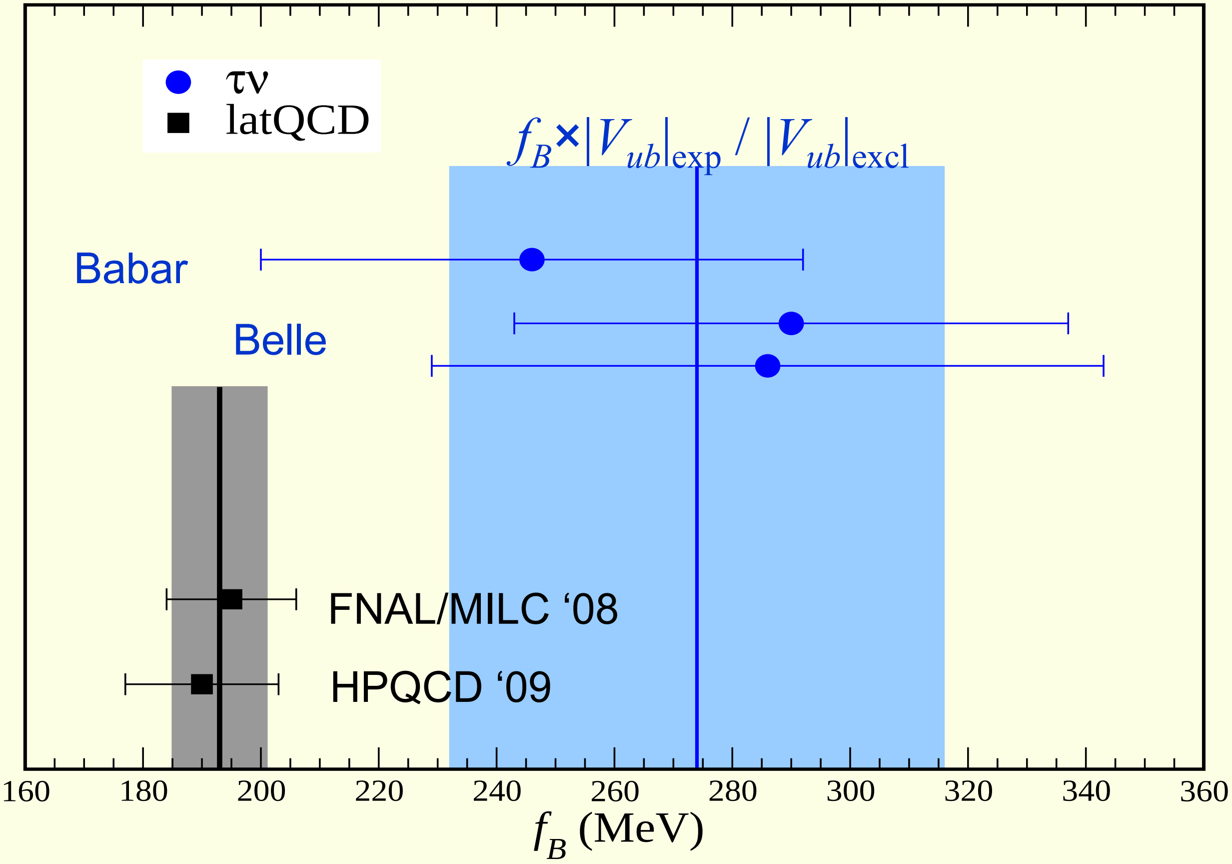
- 2 lattice spacings, $a = 0.09, 0.12$ fm
- valence quarks: Fermilab heavy + Astad light
- dominant sys. error:
  - » discretization errors
  - » inputs (m_{charm}, scale)

FNAL/MILC ’09: J. Simone, Lattice ’09 PRELIMINARY

- better statistics at $a = 0.09$
- changes in inputs:
  - » scale setting $\Upsilon \to f_\pi$
  - » better determination of charm mass

f_B lattice QCD vs. experiment

A. Kronfeld @ Pheno 2009



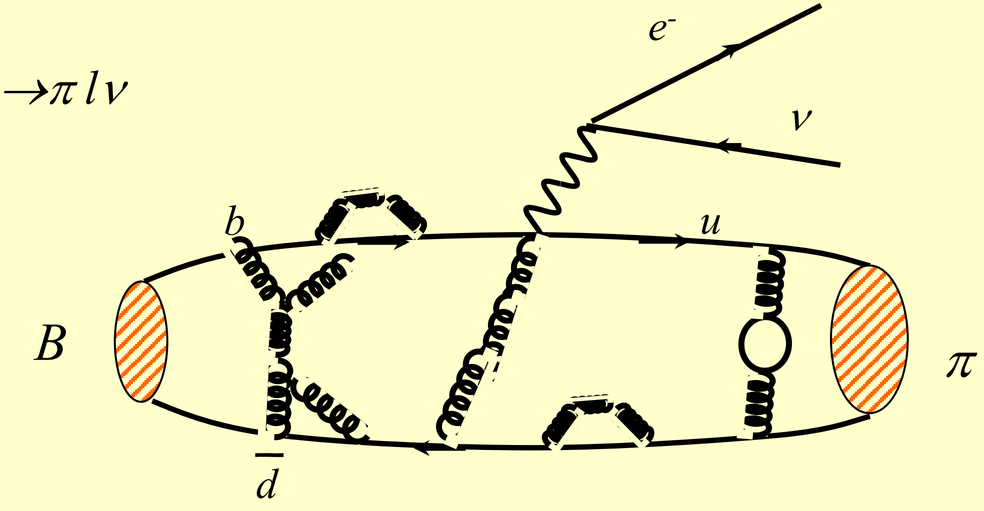
FNAL/MILC '08:
Lattice 2008, arXiv:0904.1895

HPQCD '09:
arXiv:0902.1815

- experimental measurements of $\mathrm{Br}(B \to \tau\nu) \Rightarrow f_B \times |V_{ub}|_{\rm exp}$
- $|V_{ub}|_{\rm excl}$ from exp. measurement of rate of $B \to \pi l \nu$ + lattice QCD form factors (see slides on $B \to \pi l \nu$)

semileptonic B decays

Example: $B \to \pi l \nu$



parameterize the matrix element in terms of form factors

$$\langle \pi | J_\mu | B \rangle \propto f_+(q^2), f_0(q^2)$$

$$\Rightarrow \quad \frac{d\Gamma}{dE_\pi} = (known) |V_{ub}|^2 |f_+(q^2)|^2$$

form factors for $B \to \pi\, l\, \nu$ and V_{ub}

- $p_\pi\,(q^2)$ dependence: $\langle \pi | V_\mu | B \rangle^{\text{lat}} = \langle \pi | V_\mu | B \rangle^{\text{cont}} + O(ap_\pi)^n$

  $\Rightarrow \quad p_\pi \lesssim 1\text{GeV}$ improved actions help (keep n large)

- poor overlap between lattice (high q^2) and exp. (low q^2)

  $\Rightarrow$ increased model dependence (shape of form factor)

  $\Rightarrow$ larger error on V_{ub}

HPQCD '06: Phys. Rev. D73, 074502 (2006)

- MILC ensembles at two lattice spacings, a = 0.09, 0.12 fm
- NRQCD b quark + Asqtad light
- use staggered ChPT
- use BK and BZ parameterization to interpolate in q^2
- calculate partial decay rate between $16\ \text{GeV}^2 \le q^2 < q^2_{max}$

FNAL/MILC '08: arXiv:0811.3640

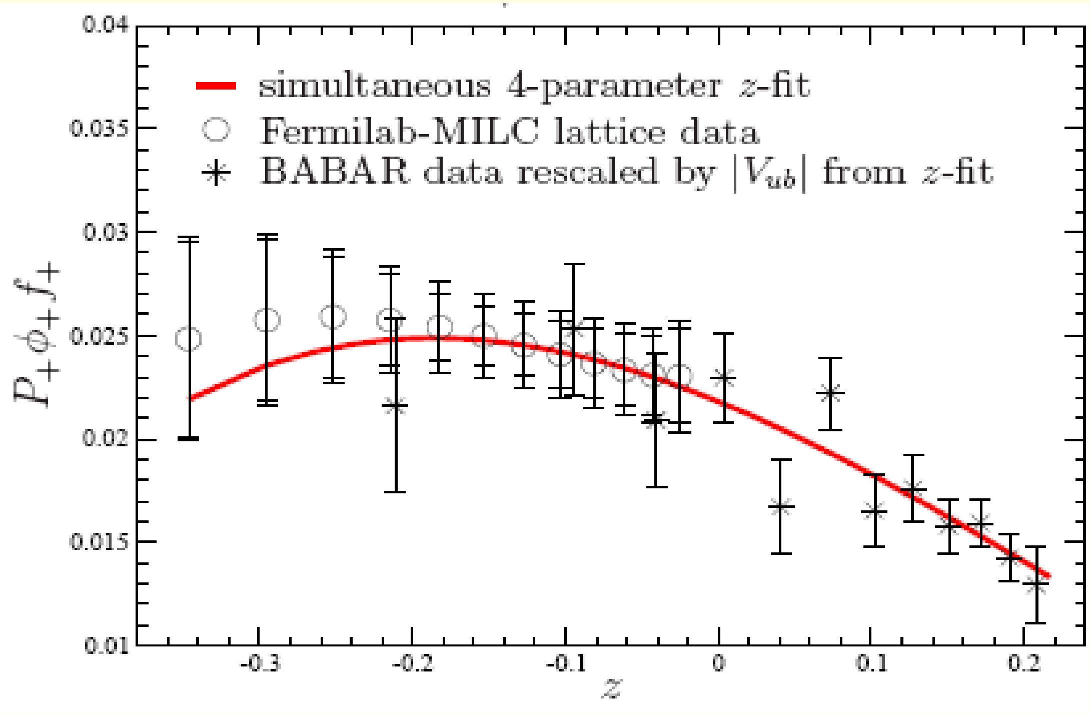
- MILC ensembles at two lattice spacings, a = 0.09, 0.12 fm
- Fermiab b quark + Asqtad light
- use staggered ChPT to simultaneously fit all m_l, E_p points
- use z-expansion (model independent parameterization of shape) for simultaneous fit to lattice and exp. data

  $\Rightarrow$ eliminates model dependence in the shape of form factor

  $\Rightarrow$ reduced error on V_{ub}

Form factors for $B \to \pi\, l \nu$ and V_{ub} cont'd

FNAL/MILC '08:



V_{ub} determined from combined fit to lattice and BaBar data:

$V_{ub} = (3.38 \pm 0.36)\ 10^{-3}$

HPQCD '06 + HFAG '08 average:

$V_{ub} = 3.40 \pm 0.20_{\text{exp}} \pm^{0.59}_{0.39\,\text{thy}}$

Form factor for $B \to D, D^* l\, \nu$ and V_{cb}

- At zero recoil, the rate for $B \to D^* l \nu \sim |V_{cb}\, h_{A1}(1)|^2$
- experimental errors are smaller for $B \to D^* l \nu$ than for $B \to D\, l \nu$

FNAL/MILC '08: Phys.Rev.D79, 014506 (2009)

- 3 lattice spacings, $a = 0.09,\ 0.12,\ 0.15$
- Fermilab heavy quark + Asqtad light
- staggered chiral PT
- new double ratio method:

$$|h_A(1)|^2 = \frac{\langle D^* | \bar{c}\gamma_j\gamma_5 b | \bar{B}\rangle \langle \bar{B} | \bar{b}\gamma_j\gamma_5 c | D^* \rangle}{\langle D^* | \bar{c}\gamma_4 c | D^* \rangle \langle \bar{B} | \bar{b}\gamma_4 b | \bar{B}\rangle}$$

  - » statistical and systematic errors cancel in ratio
  - » computationally more efficient than previous method

- first unquenched calculation of $h_{A1}(1)$

FNAL/MILC '04: hep-lat/0409116

- one lattice spacing, $a = 0.12$ fm
- first unquenched result for $h_+(1)$

Form factor for $B \to D^* l \nu$ and V_{cb}, cont'd

$h_{A1}(1) = 0.921 \pm 0.013_{\text{stat}} \pm 0.020_{\text{sys}}$

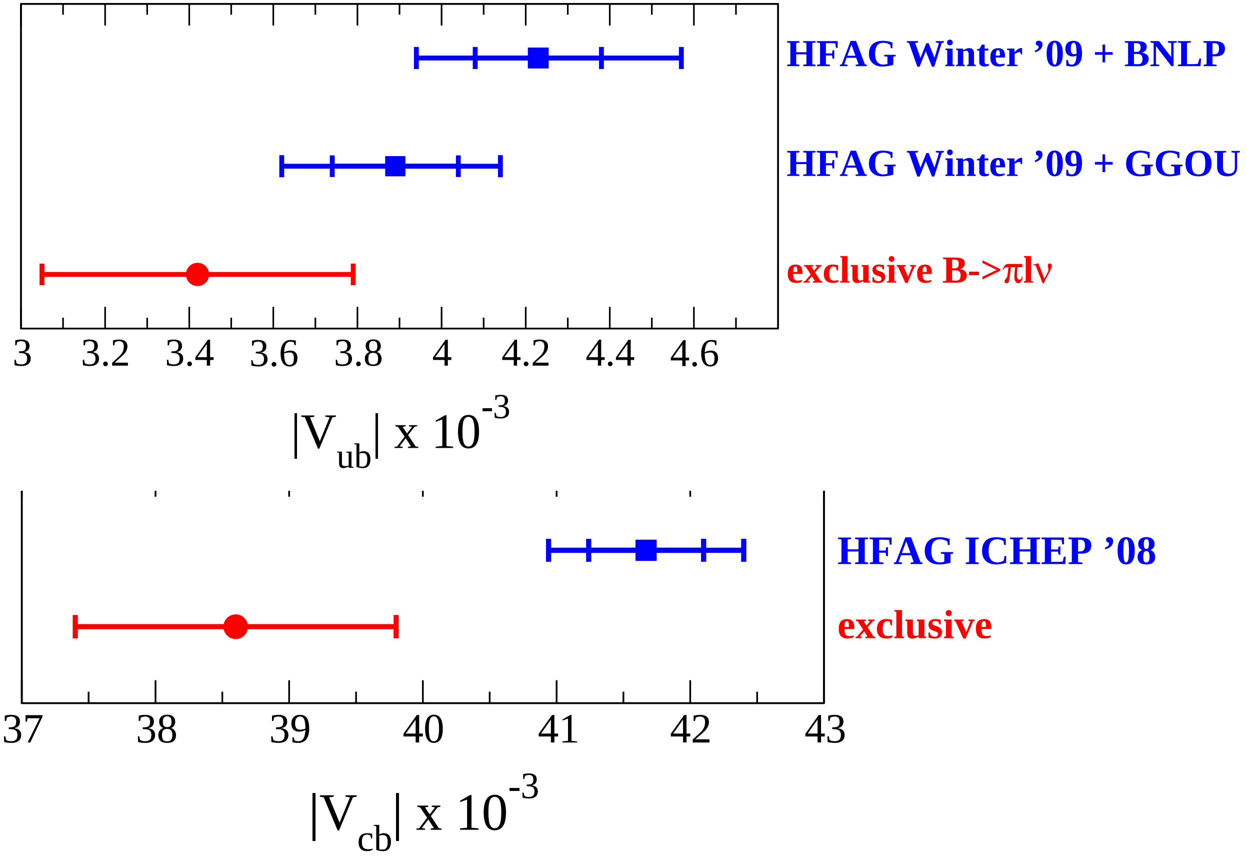
uncertainty (%)	$h_{A1}(1)$
statistics	1.4
g_{DD^*}	0.9
chiral fits	0.9
discretization errors	1.5
kappa tuning	0.7
perturbation theory	0.3
u_0 tuning	0.4
total	2.6

Using $|V_{cb}|\, F(1) = (35.41 \pm 0.52)\, 10^{-3}$ (HFAG, 2008)

we find $|V_{cb}| = (38.7 \pm 0.6_{\text{exp}} \pm 1.0_{\text{thy}})\, 10^{-3}$ c.f. inclusive det. $|V_{cb}| = (41.6 \pm 0.6_{\text{tot}})\, 10^{-3}$

V_{ub} and V_{cb} in comparison: exclusive vs. inclusive

R. van de Water @ Lattice 2009

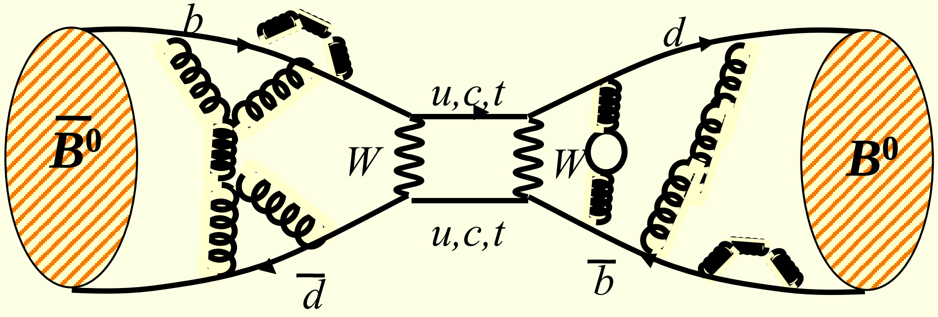


V_{ub}: significant variation between inclusive determinations depending on theoretical methods

V_{cb}: 2σ discrepancy between exclusive and inclusive determinations

Neutral *K* and *B* meson mixing

Example: $B^0 - \overline{B^0}$ mixing



$$\Delta m_d = (known) \times |V_{td}^* V_{tb}|^2 \quad \langle \overline{B^0} | \mathcal{O}_{\Delta B=2} | B^0 \rangle$$

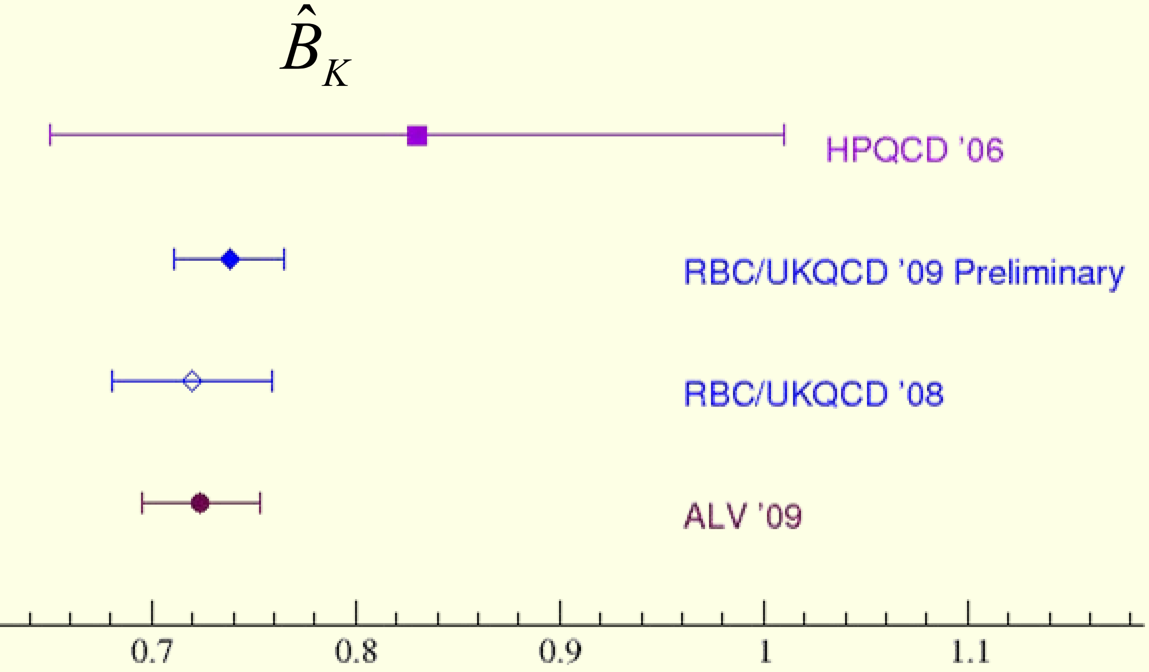
$$\rightarrow \tfrac{8}{3} m_B^2 f_B^2 B_B$$

experimental averages

$\varepsilon_K = (2.229 \pm 0.012) \times 10^{-3}$ (PDG)

$\Delta m_d = (0.507 \pm 0.005)\ \mathrm{ps}^{-1}$ (HFAG '08)
$\Delta m_s = (17.77 \pm 0.10 \pm 0.07)\ \mathrm{ps}^{-1}$

$K-\overline{K}$ mixing



HPQCD ’06:
Phys. Rev. D73, 114502 (2006)

RBC/UKQCD ’08:
Phys. Rev.Lett. 100, 032001 (2008)

RBC/UKQCD ’09:
C. Kelly, R. Mawhinney, Lattice 2009
PRELIMINARY

ALV ‘09:
arXiv: 0905.3947

$B_s^0 - \overline{B_s^0}$ and $B_d^0 - \overline{B_d^0}$ mixing

- focus on the ratio $\frac{\Delta m_s}{\Delta m_d} = \frac{m_{B_s}}{m_{B_d}} \xi^2 \left|\frac{V_{ts}}{V_{td}}\right|^2$ where $\xi = \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}}$
- ξ can be calculated more precisely than $f_B\sqrt{B}$ (discretization errors and other systematic errors cancel in the ratio)
- $n_f=2+1$ results from two groups (using MILC ensembles):
  HPQCD '09 (arXiv:0902.1815) FNAL/MILC '08 (POS (Lattice 2008) 052 (2008))
- 2 lattice spacings, $a = 0.09, 0.12$ fm
- NRQCD action (HPQCD) Fermilab action (FNAL/MILC) for heavy quarks

HPQCD '09: ξ = 1.258 ± 0.025 ± 0.021
(stat) (sys)

FNAL/MILC '08 (preliminary): ξ = 1.205 ± 0.036 ± 0.037
(stat) (sys)

impact on the unitarity triangle

- Lattice QCD results with realistic sea quark effects ($nf = 2+1$) and control over systematic errors are here!

- use them in unitarity triangle analyses

- Lunghi, Laiho & van de Water '09 (paper in preparation):
    - include lattice results for B_K, f_K, ξ, V_{ub}, V_{cb}
    - averaging procedure includes correlations
    - average V_{cb} exclusive and inclusive a la PDG
    - for V_{ub} use only exclusive (inclusive varies significantly)
    - new,preliminary results from Lattice 2009 not included
    - inputs:

$$\hat{B}_K = 0.725 \pm 0.026$$

$$f_K = (155.8 \pm 1.7) \text{ MeV}$$

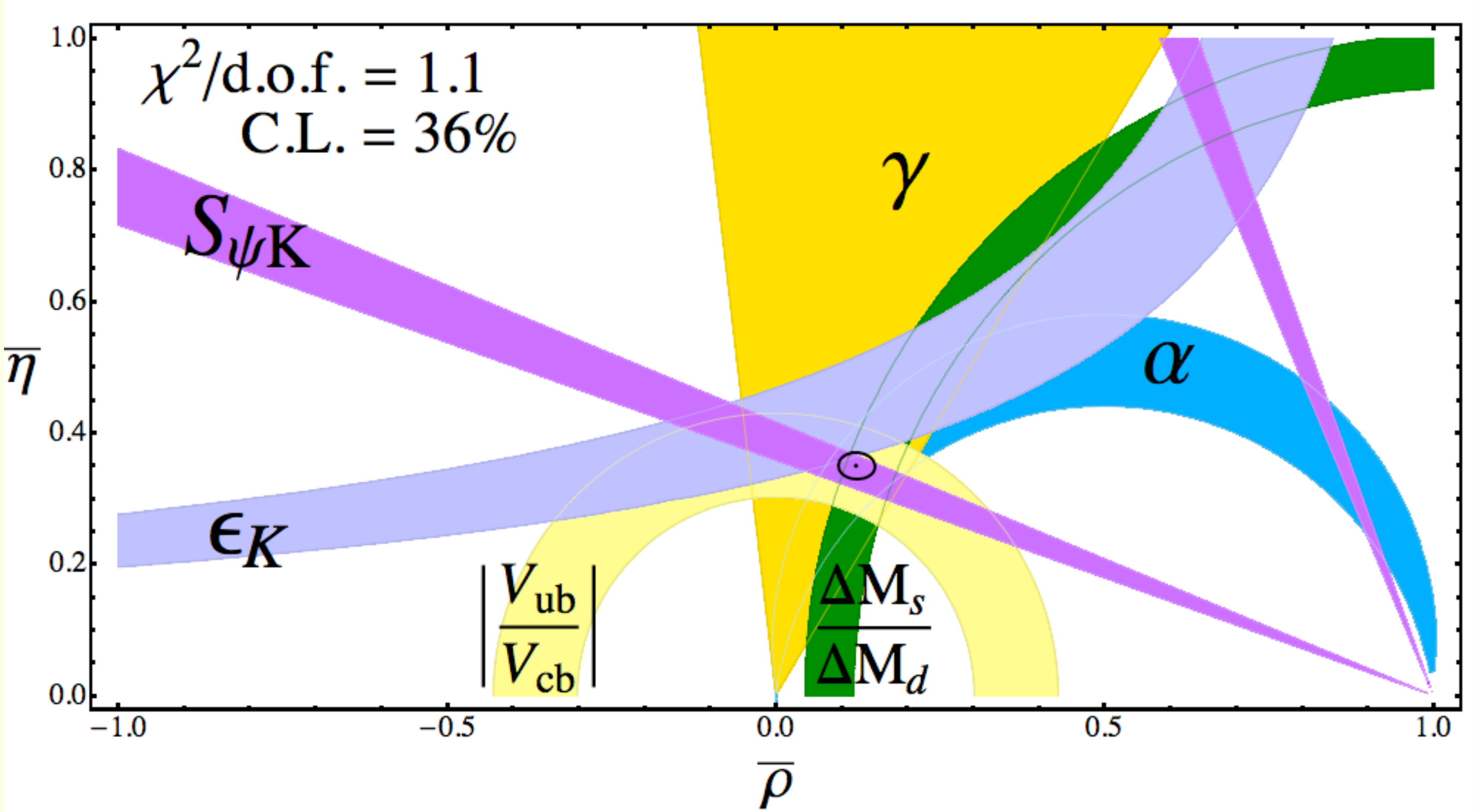
$$\xi = 1.243 \pm 0.028$$

$$|V_{ub}|_{\text{excl}} = (3.42 \pm 0.37) \times 10^{-3}$$

$$|V_{cb}|_{\text{excl}} = (38.6 \pm 1.1) \times 10^{-3}$$

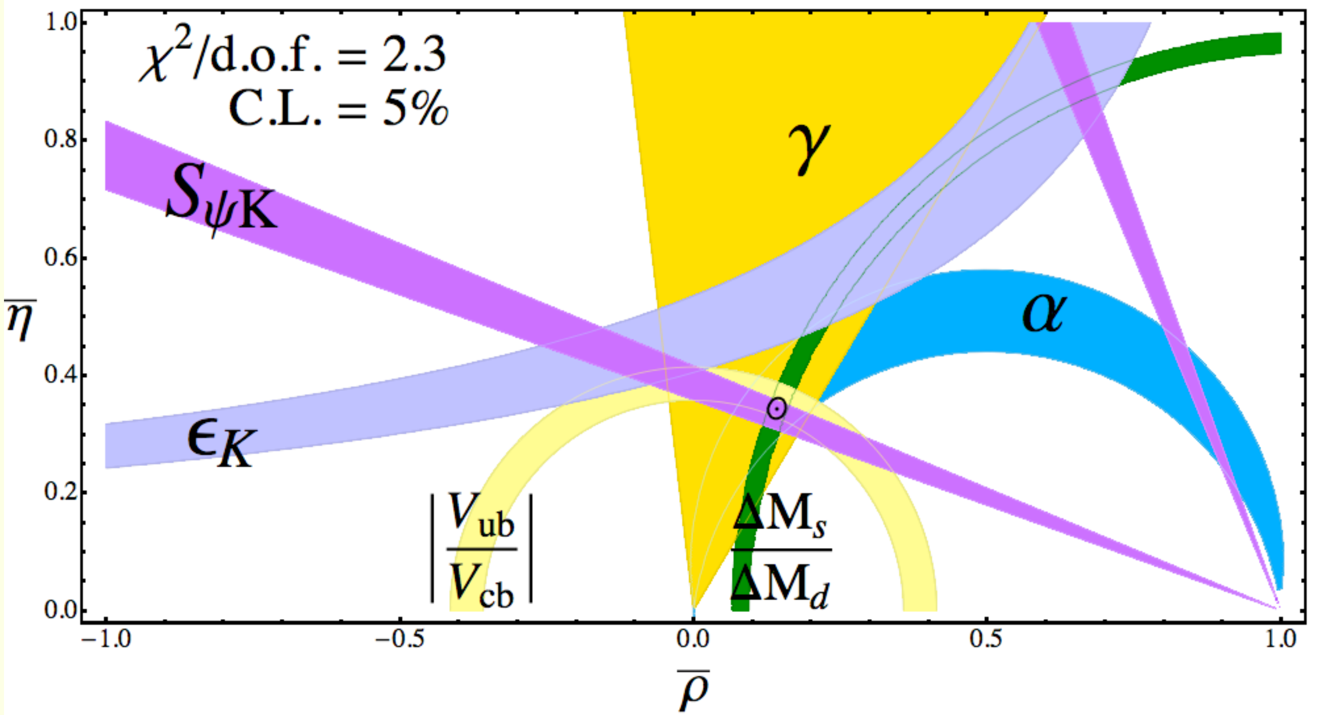
UT triangle constraints

Lunghi, Laiho, van de Water (in preparation)



the $\varepsilon_K, \Delta m_s / \Delta m_d, |V_{ub}/V_{cb}$ constraints are limited by lattice QCD uncertainties when lattice QCD uncertainties become smaller…..

Lunghi, Laiho, van de Water (in preparation)



for this plot:

- lattice QCD errors are reduced to 1% (keeping central values fixed)
- keep current experimental uncertainties unchanged
- use only exclusive V_{cb}

… then we might see new physics!

UT triangle constraints

Notes:

- with the error on $B_K < 5\%$:
  - » ε_K constraint (hyperbole) in UT plot is now dominated by the error on V_{cb}
  - » Buras & Gudagnoli (arXiv:0805.3887):
    - ♣ need to include corrections, κ_ε
    - ♣ need lattice QCD calculation of $\mathrm{Im}(A_2)$ (currently quenched only)

- there are already hints ($2.7\,\sigma$ tension) for new physics in UT triangle constraints, see for example, Lunghi & Soni (arXiv:0803.4340):

Conclusions

- Lattice QCD is now a reliable tool for nonperturbative QCD
- several important nonperturbative parameters are now known from Lattice QCD with an uncertainty of < 5%:

  $m_c, f_K, f_K/f_\pi, B_K, f_{Ds}, h_{A1}(1), \xi, \ldots$
  - » for light quark quantities: results from several groups using different sea quarks and different valence quarks
  - » for heavy quark quantities: MILC sea only, but different valence quarks
- impacts UT triangle constraints and search for new physics in the flavor sector
- complementary to LHC effort

Outlook

- near future:
  - » FNAL/MILC & HPQCD: analyze MILC ensembles at $a = 0.06, 0.045$
    + new HISQ ensembles
    + other improvements ⇒ reductions in errors
  - » RBC/UKQCD: have started a B physics program (+ DWF light quarks)
  - » other groups will start similar analyses for heavy and light quark quantities (BMW, PACS-CS, JLQCD, …)

  ⇒ Lattice results will continue to improve; goal: 1%

- further ahead:
  - » beyond simple quantities (has already started) (resonances, weak hadronic decays, …)
  - » beyond QCD (also already started)

Thanks

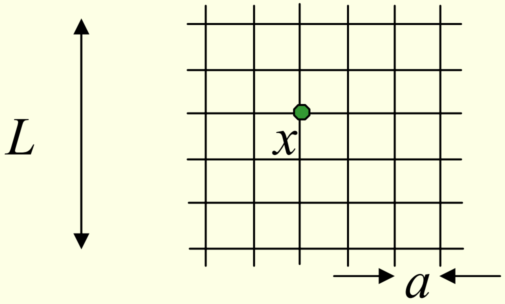
to DoE and NSF for support for computing resources for Lattice QCD

to Aubin, Bernard, Follana, Gamiz, Hashimoto, Kronfeld, Laiho, Lellouch, Lubicz, Lunghi, Mawhinney, Noaki, Onogi, Scholz, Sharpe, Shigemitsu, Simone, Soni, van de Water

for useful information, discussions, answering questions while preparing this talk.

Appendix

Introduction to Lattice QCD



discretize the QCD action (Wilson, …)
e.g. discrete derivative

$$\partial_\mu \psi(x) \to \Delta_\mu \psi(x) = \frac{1}{2a}[\psi(x+a\hat{\mu}) - \psi(x-a\hat{\mu})]$$

in general: $\langle \mathcal{O} \rangle^{\text{lat}} = \langle \mathcal{O} \rangle^{\text{cont}} + O(ap)^n \qquad n \geq 1$

errors scale with the typical momenta of the particles,
e.g. $(\Lambda_{\text{QCD}}\, a)^n$ for gluons and light quarks $\Longrightarrow$ keep $1/a \gg \Lambda_{\text{QCD}}$

typical lattice spacing $a \sim 0.1$ fm or $1/a \sim 2$ GeV

in practice: need to consider a range of a's

Improvement: add more terms to the action to make n large

Introduction to Lattice QCD, cont’d

$$\langle \mathcal{O} \rangle \sim \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A \, \mathcal{O}(\psi, \bar{\psi}, A) \, e^{-S} \qquad S = \int d^4x \left[\bar{\psi}(\not{D} + m)\psi + \frac{1}{4}(F^a_{\mu\nu})^2 \right]$$

use monte carlo methods (importance sampling) to evaluate the integral.

Note: integrating over the fermion fields leaves $\det(\not{D} + m)$ in the integrand.
the correlation functions, $\mathcal{O}$, are then written in terms of $(\not{D}+m)^{-1}$ and gluon fields

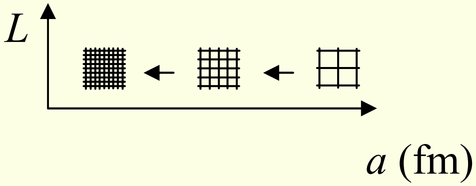
steps of a lattice QCD calculation:

1. generate gluon field configurations according to $\det(\not{D}+m)\, e^{-S}$
2. calculate quark propagators, $(\not{D}+m_q)^{-1}$, for each valence quark flavor and source point
3. tie together quark propagators into correlation functions (usually 2 or 3-pt functions)
4. statistical analysis to extract hadron masses, energies, hadronic matrix elements, …. from correlation functions
5. systematic error analysis

errors, errors, errors, …

✓ statistical errors: from monte carlo integration
also need to include errors from fit procedures

➢ finite lattice spacing, a: $\langle \mathcal{O} \rangle^{lat} = \langle \mathcal{O} \rangle^{cont} + O(ap)^n$
take continuum limit:
computational effort grows like ~ $(L/a)^{5\text{-}6}$



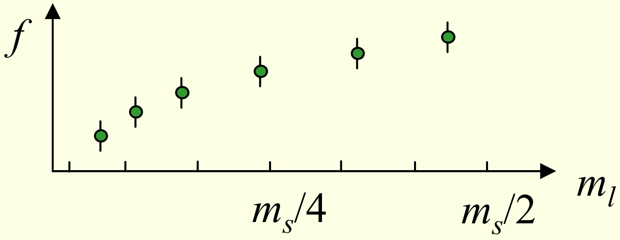
✓ finite volume: keep $m_\pi L > 4$

➢ m_l dependence: chiral extrapolation

in numerical simulations, $m_l > m_{ud}$ because the computational cost grows as m_l^{-p}.

⇒ use chiral perturbation theory to extrapolate to m_{ud}

need a range of values with $m_l < m_s/2$



✓ n_f dependence: realistic sea quark effects: use $n_f = 2+1$ or $n_f = 2+1+1$

❖ renormalization: $\langle J_\mu^{\text{cont}} \rangle = Z^{\text{lat}} \langle J_\mu^{\text{lat}} \rangle$

use lattice perturbation theory: $Z = z^{(0)} + z^{(1)}\alpha_s + z^{(2)}\alpha_s^2 + O(\alpha_s^3)$
⇒need to include PT errors

nonperturbative methods: also need to include errors

Glossary - Light Quark Methods

- Asqtad (improved staggered):
  errors: ~ $O(\alpha_s a^2)$, $O(a^4)$, but large due to taste-changing interactions
  has chiral symmetry; uses square root of the determinant in sea
  computationally efficient

- HISQ (Highly Improved Staggered Action): also similar: HYP smeared
  errors: ~ $O(\alpha_s a^2)$, $O(a^4)$, ×1/3 smaller than Asqtad
  comp. cost: efficicient, ×2 Asqtad

- improved Wilson (Clover, …): also Stout link smeared
  errors: ~ $O(\alpha_s a)$, if tree-level (tadpole) imp.; $O(a^2)$ if nonpert. imp.
  Wilson term breaks chiral symmetry
  comp. cost: ×4 Asqtad for $m_{\text{light}} \sim m_{\text{strange}}$, but less efficient at small quark masses

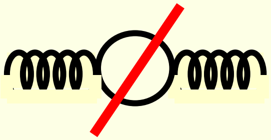
- twisted mass Wilson (tmQCD):
  errors: ~ $O(a^2)$
  twisted mass term for quark masses at chiral limit
  comp. cost: ×4 Asqtad

- Domain Wall Fermions (DWF):
  errors: ~ $O(a^2)$, $O(m_{\text{res}} a)$
  almost exact chiral symmetry; breaking ~ $m_{\text{res}} \sim 3 \times 10^{-3}$
  comp. cost: ×L_5 Asqtad, $L_5 \sim 16 - 20$

- Overlap Fermions:
  errors: ~ $O(a^2)$
  exact chiral symmetry
  comp. cost: ×5-10 DWF

Glossary – sea quarks

- quenched approximation: no sea quarks, $n_f = 0$
  $\det(\not{D}+m) = \text{const.}$ $\Rightarrow$ computational cost reduced by factor ~ 100–1000
  but systematic errors ~ 10-30% (for π's K's, … particles without decay thresholds)



- unquenched: $n_f \neq 0$
  simulation includes sea quarks, $\det(\not{D}+m)$ included in integration
- $n_f = 2$
  two degenerate flavors of light quarks (for up and down) in sea, generally with $m_l > m_{ud}$
  strange quark is still quenched
- $n_f = 2+1$
  two degenerate flavors (for u and d) plus one heavier sea quark (for s) with mass $\approx m_s^{phys}$
- $n_f = 2+1+1$
  two degenerate flavors (for u and d) plus one heavier sea quark (for s) with mass $\approx m_s^{phys}$
  plus one heavy sea quark (for c) with mass $\approx m_c^{phys}$
- partially quenched: $n_f \neq 0$ with $m_{sea} \neq m_{valence}$
  sea quarks are computationally much more expensive than valence quarks
  $\Rightarrow$ one often generates several light valence quarks on each sea quark ensemble
  use partially quenched ChPT; extremely useful for determining chiral parameters
- full QCD: $n_f \neq 0$ with $m_{sea} = m_{valence}$
  sometimes used synonymous with unquenched
- mixed action: sea quark action $\neq$ valence quark action
  use mixed action partially quenched ChPT

Glossary con’td

- rooted staggered quarks

doubling problem ⇒ 4 “tastes” (degenerate lattice quark flavors) for every continuum flavor

in the sea: $\sqrt{\det(\not{D}+m)}$ ⇒ two remaining tastes = two degenerate continuum flavors (u,d)

$\sqrt[4]{\det(\not{D}+m)}$ ⇒ one remaining taste = one flavor (s)

- Is rooted staggered lattice QCD = QCD ?

$\sqrt{\det(\not{D}+m)}$ is nonlocal at $a \neq 0$ (Bernard, Golterman, Shamir)

but there is a lot of evidence that nonlocality ~ a^2
based on renormalization group analysis (Shamir) and ChPT analysis (Bernard)
also a growing body of numerical checks (Dürr& Hoelbling, Follana, Hart & Davies, MILC, ..)

- rooted staggered chiral perturbation theory

accounts for the taste violations in the rooted staggered sea
⇒ includes leading discretization effects, which can then be removed in continuum limit

What are the "easy" lattice calculations ?

stable (or almost stable) hadrons, masses and amplitudes
with no more than one initial (final) state hadron,
for example:

- π, K, D, D_s, B, B_s mesons
  masses, decay constants, weak matrix elements for mixing, semileptonic and rare decay form factors
- charmonium and bottomonium ($\eta_c, J/\psi, h_c, \ldots, \eta_b, \Upsilon(1S), \Upsilon(2S), ..$) states below open *D*/*B* threshold
  masses, leptonic widths, electromagnetic matrix elements

This list includes most of the important quantities for CKM physics. Excluded are ρ, K^* mesons and other resonances.

The Players

Collaborations which generate n_f = 2+1 ensembles

BMW: generate n_f = 2+1 tree improved stout smeared Wilson sea
Dürr, Fodor, Frison, Hoelbling, Hoffmann, Katz, Krieg, Kurth, Lellouch, Lippert, Ramos, Szabo, Vulvert

ETMC: generate n_f = 2, 2+1+1 twisted mass Wilson sea
Blossier, Dimopoulos, Frezzotti, Haas, Herdoiza, Jansen, Lubicz, Mescia, Palao, Shindler, Simula, Tarantino, Urbach, Wenger

HSC: generate nf = 2+1 anisotropic Clover sea
Bulava, Cohen, Dudek, Edwards, Engelson, Foley, Joo, Juge, Lin, Mathur, Morningstar, Orginos, Peardon, Richards, Ryan, Thomas, Thomas, Wallace

JLQCD+TWQCD: generate n_f = 2, 2+1 Overlap sea
Aoki, Aoyama, Chiu, Fukaya, Hashimoto, Hsieh, Ikeda, Ishizuka, Kanaya, Kaneko, Kuramashi, Matsufuru, Noaki, Ogawa, Ohki, Okawa, Onogi, Shintani, Takeda, Taniguchi, Ukawa, Yamada, Yamazaki, Yoshie

PACS-CS: generate n_f = 2+1 NP improved Wilson sea
Aoki, Ishii, Ishikawa, Ishizuka, Izubuchi, Kadoh, Kanaya, Kuramashi, Murano, Namekawa, Okawa, Taniguchi, Ukawa, Ukita, Yoshie

MILC: generate n_f = 2+1 Asqtad sea and n_f = 2+1+1 HISQ sea
Bazavov, Bernard, DeTar, Du, Freeman, Gottlieb, Heller, Hetrick, Laiho, Levkova, Oktay, Osborn, Sugar, Toussaint, Van de Water

RBC+UKQCD: generate n_f = 2+1 DWF sea
RBC: Aoki, Blum, Christ, Cohen, Dawson, Ishikawa, Izubuchi, Jin, Jung, Kim, Li, Li, Lightman, Lin, Liu, Mawhinney, Ohta, Sasaki, Scholz, Soni, Van de Water, Witzel, Yamazaki

UKQCD: Allton, Antonio, Boyle, Clark, Donellan, Flynn, Hart, Jüttner, Kelly, Kennedy, Kenway, Maynard, Pedroso de Lima, Pendleton, Sachrajda, Tweedie, Wennekers, Zanotti

QCDSF: generate nf = 2+1 stout smeared NP Clover (SLiNC)
Al-Haydari, Ali Khan, Braun, Collins, Cundy, Göckeler, Horsley, Kaltenbrunner, Kennedy, Lacagnina, Nakamura, Panero, Perlt, Pleiter, Rakow, Schäfer, Schierholz, Schiller, Streuer, Stüben, Zanotti

The Players cont'd

also: Collaborations which generate(d) $n_f = 2$ ensembles

most of the groups listed on previous slide
+ ALPHA, CLS, CERN-ToV, …

The Players, cont'd

Users:

ALV: use MILC n_f = 2+1 sea + DWF valence
Aubin, Laiho, Van de Water

BSW: use MILC n_f = 2+1 sea + HYP smeared improved staggered
Bae, Kim, Kim, Kim, Jung, Lee, Yoon, Sharpe

χQCD: use PACS-CS n_f = 2+1 sea
Alexandru, Chen, Dong, Doi, Draper, Horvath, Joo, Kathuria, Lee, Lewis,Li, Liu, Lujan, Mankame, Mathur, Moerschbacher, Pelissier, Streuer, Thacker, Wang, Zhang,

Fermilab Lattice: us: MILC n_f = 2+1 sea + Asqtad valence + Fermilab heavy quarks
Di Pierro, AXK, Gottlieb, Kronfeld, Mackenzie, Simone, Di Pierro, Gottlieb, Bailey, Freeland, Gamiz, Laiho, Scholz, Van de Water, Evans, Bouchard

HPQCD: use MILC n_f = 2+1 sea + HISQ valence (light and charm) + NRQCD b quark
Allison, Dalgic, Davies, Follana, Gamiz, Gregory, Horgan, Hornbostel, Kendall, Lepage, McNeile, Shigemitsu, Trottier, Woloshin

LHPC: use MILC n_f = 2+1 sea + DWF valence
Edwards, Engelhardt, Fleming, Hagler, Lin, Lin, Meyer, Musch, Negele, Orginos, Pochinsky, Procura, Renner, Richards, Schroers, Syritsyn, Walker-Loud

NPLQCD: use MILC n_f = 2+1 sea + DWF valence and HSC n_f = 2+1 sea
Beane, Detmold, Lin, Thomas, Orginos, Parreno, Savage, Torok, Walker-Loud

RBC+UKQCD: use n_f = 2+1 DWF sea + DWF valence + rel. heavy quark action or static

Back-up slides

Heavy Quark Methods

$m_Q \gg \Lambda_{\mathrm{QCD}}$ and $am_Q \not\ll 1$:

lattice NRQCD (Lepage, et al., Caswell+Lepage) :

- discretize NRQCD lagrangian: valid when $am_Q > 1$
- errors: $\sim (ap)^n$, $(p/m_Q)^n$
- good for b quarks, but not charm

Fermilab (Kronfeld, Mackenzie, AXK):

- rel. Wilson action has the same heavy quark limit as QCD
- add improvement: preserve HQ limit
- smoothly connects light and heavy mass limits, valid for all am_Q
- errors: $\sim \alpha_s(a\Lambda), (a\Lambda)^2$ or $\sim \alpha_s\, \Lambda/m_Q, (\Lambda/m_Q)^n$
- good for for charm and beauty
- also similar approach but determine improvement parameters nonperturbatively (Christ, Li, Lin)

HISQ (Follana, Hart, Davies, Follana et. al):

- errors: $\sim \alpha_s\, (am_c)^2,\ (am_c)^4$
- good for charm, but beauty needs very small a's