

# LECTURE 3.

## EXTRA DIMENSIONS

- HIERARCHY PROBLEM:

"WHY IS GRAVITY SO MUCH WEAKER THAN PARTICLE PHYSICS"  $(M_{\text{uv}} \ll M_{\text{pl}})$

- USUAL RESOLUTION

CHANGE PARTICLE PHYSICS AT TeV SCALE ( $\sim$  SUSY TECHNICOLOR)

- DIFFERENT APPROACH:

CHANGE GRAVITY OR BOTH

- HOW TO CHANGE GRAVITY?

INTRODUCE EXTRA DIMENSIONS...

# MAIN FEATURE OF EXTRA DIMENSIONS: "KALUZA-KLEIN TOWER"

5D FIELD  $\Psi(x)$  WITH CIRCLE RADIUS  $r$ :

$$\Psi(x, y) = \sum_n \Psi_n(x) e^{i \frac{ny}{r}}$$

4D  $\Psi(x)$  EXTRA COORDINATE  $y$

FROM 4D POINT OF VIEW

$$\square_{(5)} \Psi = \sum_n \left[ \square_{(4)} \Psi_n - \frac{n^2}{r^2} \Psi_n \right]$$

↑ HIGHER DIMENSIONAL FIELD  $\equiv$

TOWER OF MASSIVE 4D FIELDS



KK-TOWER

MOMENTUM ALONG EXTRA DIM  $\sim$  4D MASS

# MATCHING OF 4D THEORY

OUR OBSERVED WORLD : 4D

ANY HIGHER DIMENSIONAL EXTEN.  
MUST EFFECTIVELY GIVE 4D  
AT LARGE DISTANCES (SMALL E).

- MEANING :
- KK MODES SHOULD BE UNOBSERVABLE (HEAVY)
  - ORDINARY 4D LAWS OF PHYSICS SHOULD BE RECOVERED

EXAMPLE : GAUSS'S LAW IN 5D

1.) SMALL DISTANCE  $\int \vec{E} \cdot d\vec{F} = d_{5D} Q$

$$|E| = \frac{d_{5D} \cdot Q}{2\pi r^3} \sim \frac{1}{r^3}$$

2.) LONG DISTANCE  $\int \vec{E} \cdot d\vec{F} = 2\pi R \int \vec{E} \cdot d\vec{F}$

$$|E| = \frac{d_{5D} Q}{2\pi R \cdot 4\pi r^2} \sim \frac{1}{r^2}$$

→  $\frac{d_{5D}}{8\pi^2 R} = \frac{1}{4\pi \epsilon_0}$

$d_{5D} = \frac{2\pi R}{\epsilon_0}$

MATCHING

## REASON FOR SMALL EXTRA DIM:

1.) NEED TO MATCH PARAMETERS OF HIGHER DIM. THEORY TO OBSERVED 4D

$M_*$  : FUND'L PLANCK SCALE

$n$  : NUMBER OF EXTRA DIMS.

$r$  : RADIUS OF EXTRA DIMS.:

MATCHING OF GRAVITY:

$$M_{Pl}^2 = M_*^{n+2} r^n$$

MATCHING OF GAUGE COUPLINGS:

$$\frac{1}{g^2} \sim M_*^n r^n$$

$$M_* \sim M_{Pl}$$

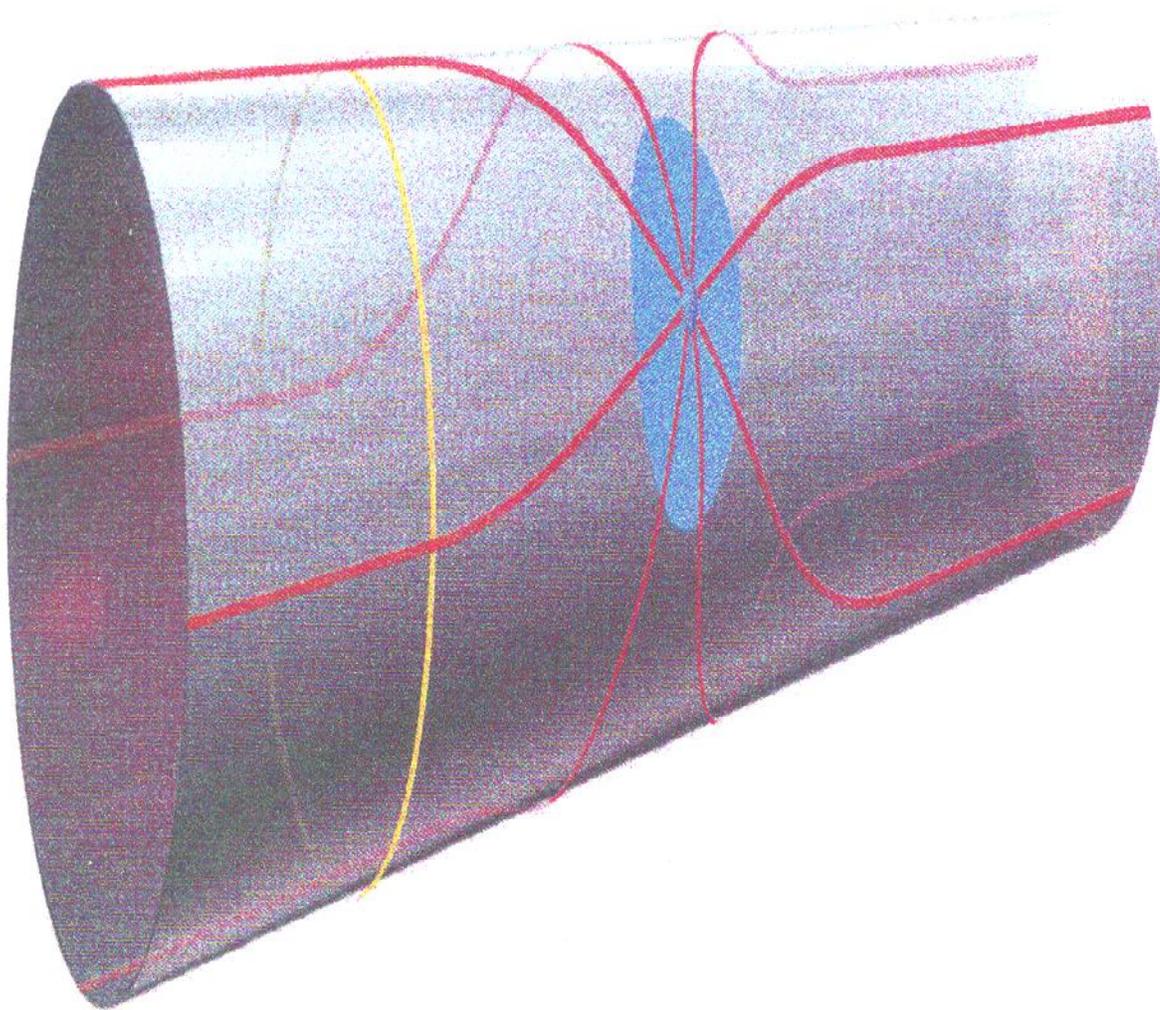
$$r \sim \frac{1}{M_{Pl}}$$

EXTREMELY TINY

2.) HIGHER DIMS. FIELDS APPEAR AS TOWER OF MASSIVE 4D STATES

$$M_n^{(4)} \sim \frac{n}{r}$$

$$r \leq \frac{1}{\text{TeV}} \sim 10^{-17} \text{ cm}$$



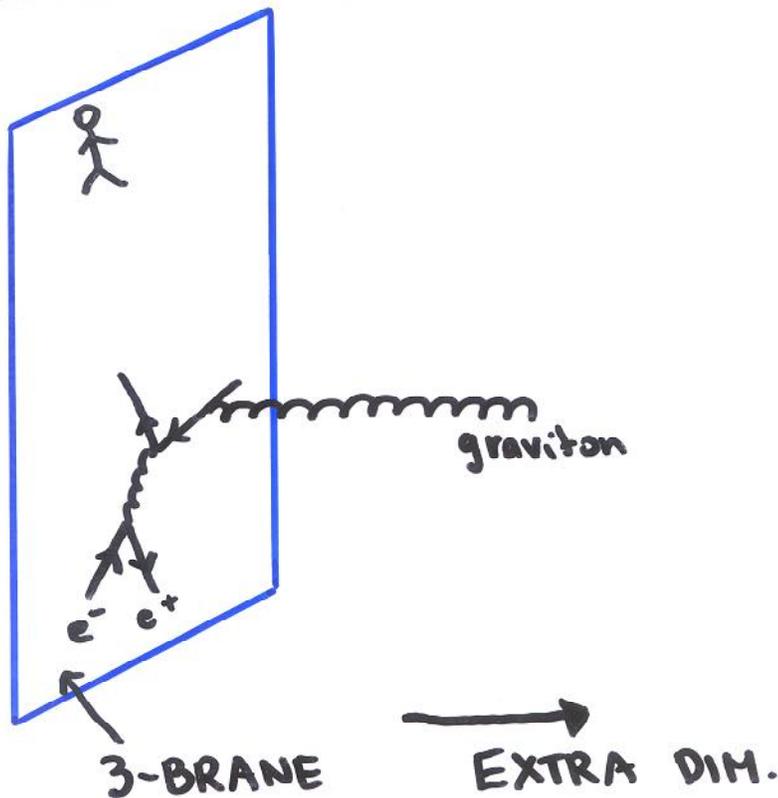
**SMALL EXTRA DIMENSION** wrapped in a circle (*circumference of tube*) modifies how gravity (*red lines*) spreads in space. At distances smaller than the circle radius (*blue patches*), the lines of force spread apart rapidly through all the dimensions. At much larger distances (*yellow circle*), the lines have filled the extra dimension, and it has no further effect on them.

**PLOT FROM SCIENTIFIC AMERICAN  
ARKANI-HANED ET AL.**

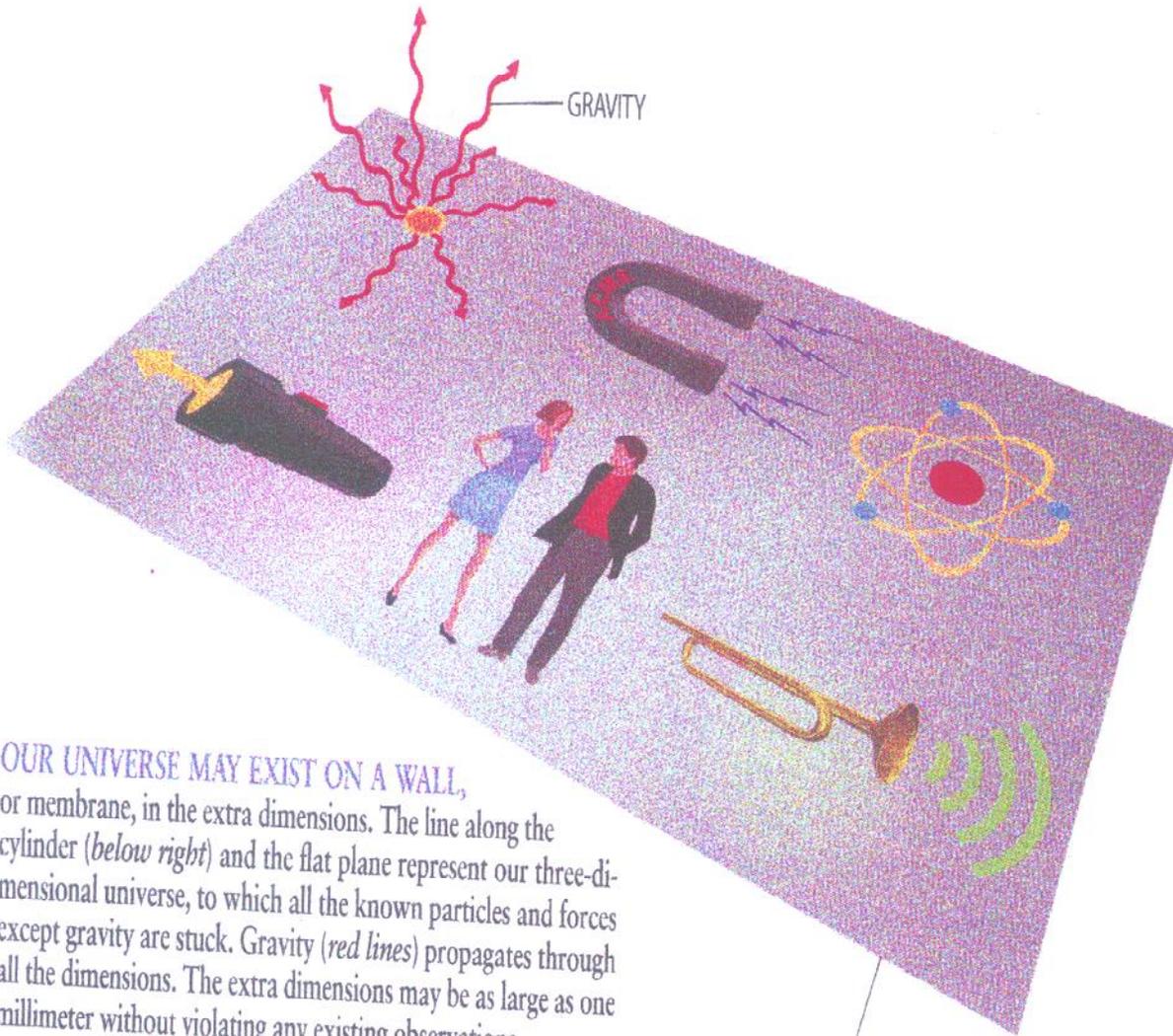
BUT: THESE ARGUMENTS ASSUME THAT SM FIELDS PROPAGATE IN X-D'S.

DIFFERENT POSSIBILITY:

- SM FIELDS CONFINED TO "BRANE" ( $\sim$  MEMBRANE, BUT MORE SPATIAL DIRECTIONS)
- GRAVITY (& OTHER NON-SM FIELDS) PROPAGATE IN BULK



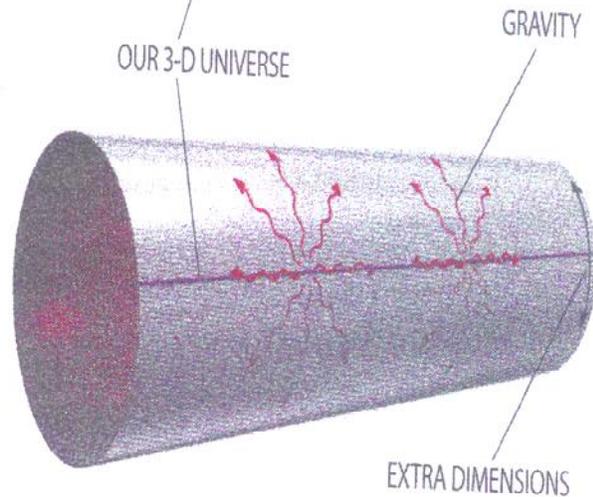
FROM ARKANI-HAMED  
ET AL.  
SCIENTIFIC AMERICAN



OUR UNIVERSE MAY EXIST ON A WALL, or membrane, in the extra dimensions. The line along the cylinder (below right) and the flat plane represent our three-dimensional universe, to which all the known particles and forces except gravity are stuck. Gravity (red lines) propagates through all the dimensions. The extra dimensions may be as large as one millimeter without violating any existing observations.

the full higher-dimensional space. In high-energy particle collisions, we expect to observe missing energy, the result of gravitons escaping into the extra dimensions.

Although it may seem strange that some particles should be confined to a wall, similar phenomena are quite familiar. For instance, electrons in a copper wire can move only along the one-dimensional space of the wire and do not travel into the surrounding three-dimensional space. Likewise, water waves travel primarily on the surface of the ocean, not throughout its depth. The specific scenario we are describing, in which all particles except gravity are stuck to a wall, can arise naturally in string theory. In fact one of the major insights...



- IN THIS CASE SIZE OF EXTRA DIM. ONLY CONSTRAINED BY GRAVITY

- $M_* \ll M_{PL}$  POSSIBLE  $\rightarrow$   
4D PLANCK SCALE ONLY EFF. SCALE

- $M_* \cong 1 \text{ TeV} \rightarrow$  NO HIERARCHY IN THE FUNDAMENTAL SCALES

- $$r = \frac{1}{M_*} \left( \frac{M_{PL}}{M_*} \right)^{2/n} \sim 10^{-17 + \frac{30}{n}} \text{ cm}$$

$r \gg \frac{1}{M_*}$  LARGE EXTRA DIMENSIONS

(ARKANI-HAMED, DIMOPOULOS, DVALI)

$n=1$  EXCLUDED

$n=2$   $r \sim 1 \text{ mm}$  BARELY EXCLUDED

$\vdots$

- $n > 2$  NOT EXCLUDED, BUT TESTABLE AT LHC

# EXPERIMENTAL CONSEQUENCES OF LARGE EXTRA DIM'S

## 1.) GRAVITATIONAL MEASUREMENTS:

IF  $n=2$ :  $r \sim 1 \text{ mm}$

GRAVITATIONAL LAW WOULD  
CHANGE FROM  $\frac{1}{r^2}$  TO  $\frac{1}{r^4}$   
AT THIS DISTANCE SCALE!

RECENT RESULTS OF EÖT-WASH  
GROUP : GRAVITY UNCHANGED AT  
 $r \sim 200 \mu\text{m} \rightarrow M_*^{n=2} \gtrsim 3 \text{ TeV}.$

## 2.) PARTICLE PHYSICS & COSMOLOGY

INDUCED METRIC:

$$g_{\mu\nu}(x) = G_{\mu\nu}(x, y^a=0)$$
$$G_{MN} = \eta_{MN} + \frac{H_{MN}}{(M_*^{n+2})^{1/2}}$$

WHERE  $H_{MN}$  ARE THE CAN.  
NORMALIZED BULK GRAVITON MODES.

SM COUPLING: (FROM THE  
FACT THAT  $S^{SM} = \int d^4x \mathcal{L}$ ,  
BUT  $\mathcal{L}$  CONTAINS THE INDUCED  
METRIC WHICH DEPENDS ON  $H$ !)

$$\int d^4x T^{\mu\nu} \frac{H_{\mu\nu}(x, y=0)}{(M_*^{n+2})^{1/2}}$$

INTERACTION VIOLATES TRANSL.  
INVARIANCE IN BULK, SO  
BULK MOM. NOT CONSERVED!  
(BUT ENERGY IS!)

= WALL CAN ABSORB ANY  
EXTRA DIM'S MOMENTUM...

KK EXPANSION:

$$H_{\mu\nu}(x, y=0) = \sum_n \frac{1}{\sqrt{r_n}} H_{\mu\nu}^{n a}$$

# THE RULE OF THUMB TO CALCULATE :

COUPLING:  $\frac{1}{\sqrt{M_*^{n+2}}}$  IN AMPLITUDE

IN RATE  $\frac{1}{M_*^{n+2}} E^n$  ; E AVAILABLE ENERGY RANGE

SAME FROM KK DECOMPOSITION:

$$\frac{1}{M_{pl}^2} \cdot (E r_n)^n$$

$\uparrow$  COUPLING OF SINGLE MODE       $\leftarrow$  # OF KK MODES

$$= \frac{1}{M_*^{n+2} r_n^n} (E r_n)^n = \frac{E^n}{M_*^{n+2}} \dots$$

## HOW WOULD GRAVITY BEHAVE?

LONG DISTANCE (  $R > r = 10^{-17 + \frac{30}{n}} \text{ cm}$  )

$$F \sim \frac{1}{r^2}$$

ORDINARY NEWTON  
LAW

SHORT DISTANCE (  $R < r$  )

$$F \sim \frac{1}{r^{2+n}}$$

SEVERAL EXPERIMENTS LOOK FOR  
MODIFICATION OF GRAVITY AT  
SHORT DISTANCES.

$n=2$        $r \sim 1 \text{ mm}$   
TESTABLE , ANSWER SEEMS NEGATIVE  
 $n=3$        $r \sim 10^{-7} \text{ cm} \dots$

# THE EÖT-WASH EXPERIMENT

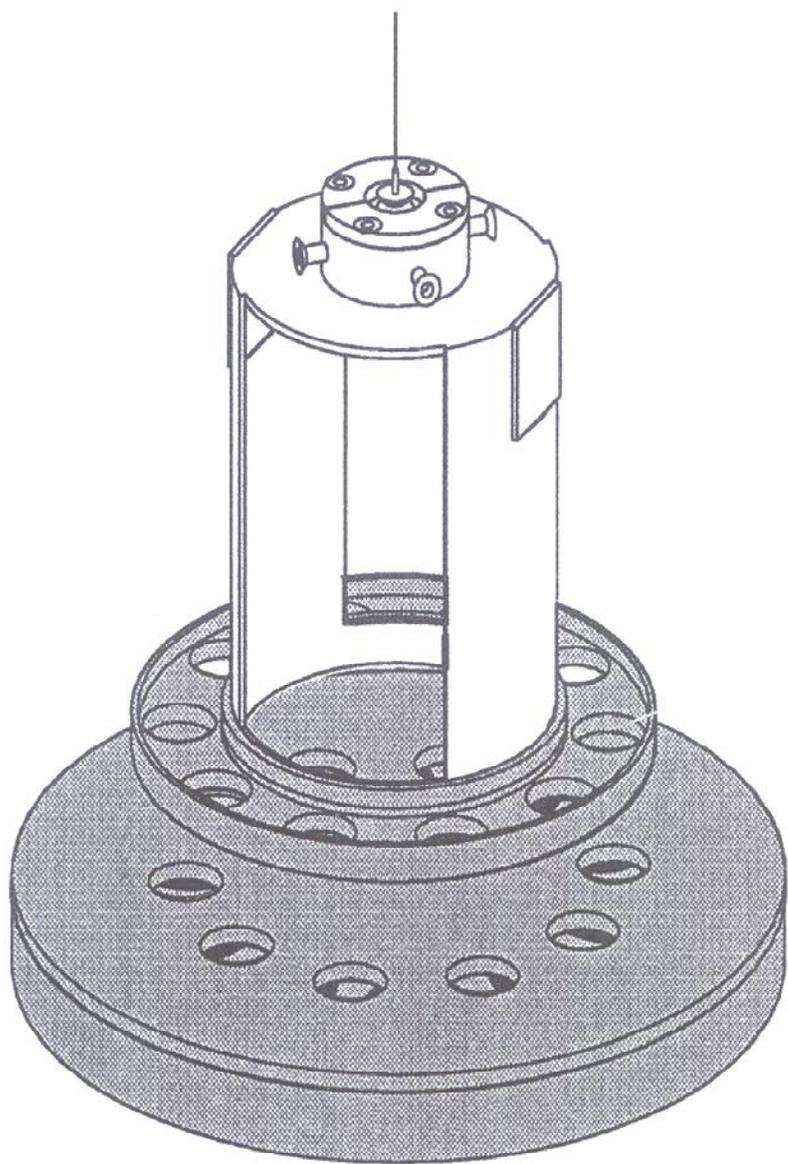


FIG. 1. Scale drawing of the torsion pendulum and rotating attractor. The active components are shaded. For clarity, we show an unrealistically large 1.5 cm vertical separation between pendulum and attractor, and omit the BeCu membrane and the attractor drive mechanism. The 4 horizontal screws were adjusted to make the pendulum precisely level.

# RESULTS OF THE EÖT-WASH EXP. HEP-PH/0011014

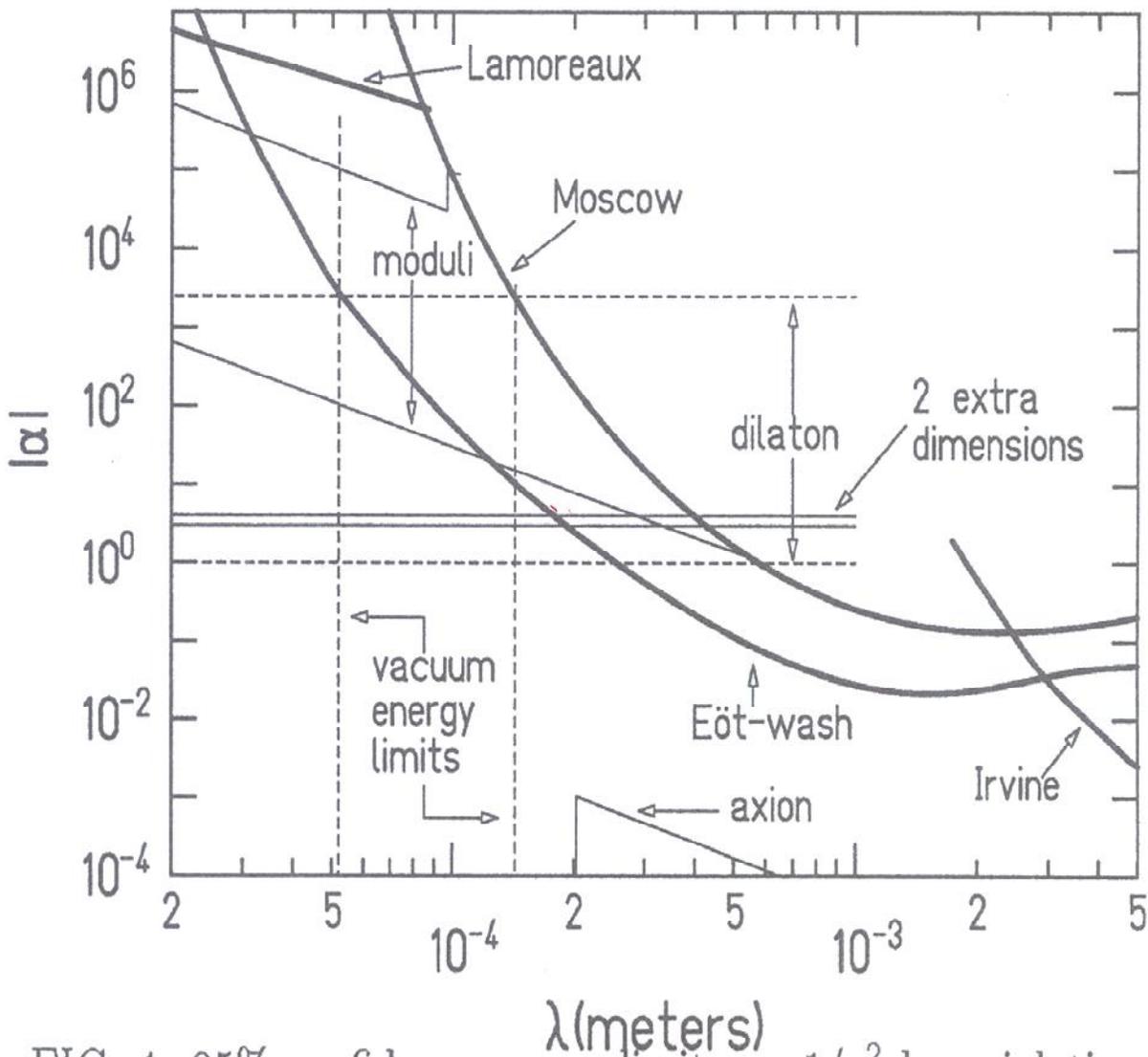
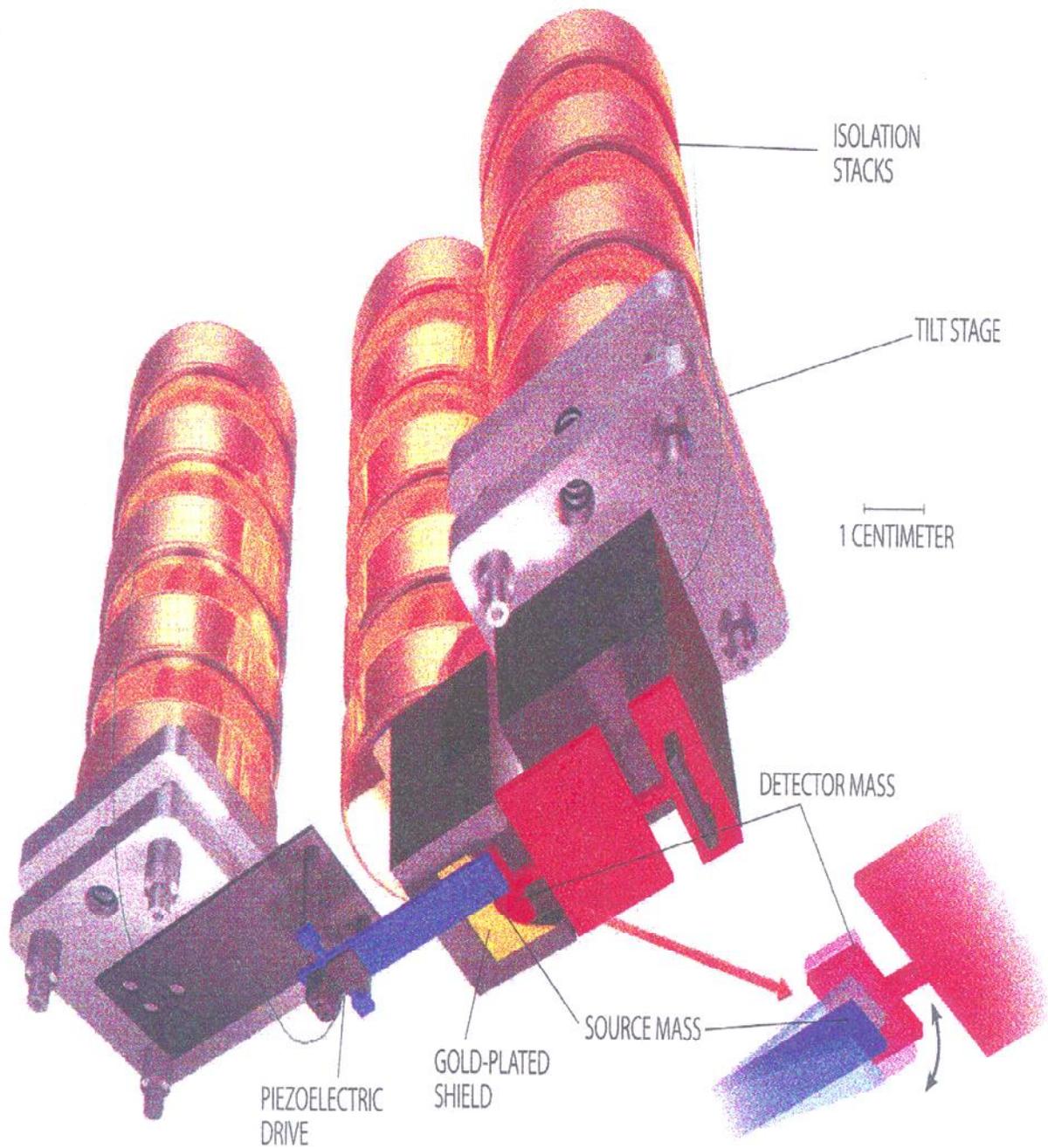


FIG. 4. 95% confidence upper limits on  $1/r^2$ -law violating interactions of the form given by Eq. 2. The region excluded by previous work [2,3,20] lies above the heavy lines labeled Irvine, Moscow and Lamoreaux, respectively. The data in Fig. 3 imply the constraint shown by the heavy line labeled Eöt-Wash. Constraints from previous experiments and the theoretical predictions are adapted from Ref. [8], except for the dilaton prediction which is from Ref. [14].

$r \lesssim 0.2 \text{ mm}$

# THE COLORADO EXPERIMENT



**TORSION OSCILLATOR** at the University of Colorado looks for changes in gravity from 0.05 to 1.0 millimeter. Piezoelectrics vibrate the tungsten source mass (*blue*) like a diving board. Any forces acting between the source mass and the tungsten detector (*red*) produce twisting oscillations of the detector (*inset; oscillations are exaggerated*), which are sensed by electronics. A gold-

plated shield (*yellow*) suppresses electrostatic forces, and suspension from brass isolation stacks stops vibrations from traveling from the source to the detector. Electrostatic shields enclosing the apparatus are not shown. Results at room temperature (300 kelvins) are expected this year. For maximum sensitivity, liquid helium will cool the apparatus to four kelvins.

# POSSIBLE BOUNDS :

## 1.) MACROSCOPIC GRAVITY

SUN : INTERPARTICLE SEPARATION  
 $< 1\text{mm}$ .

DOES IT CHANGE GRAVITATIONAL  
ENERGY?

ADD CALCULATED  $\downarrow$  size of extra  
dims.

$$\frac{\Delta E_{\text{grav}}}{E_{\text{grav}}} \sim \frac{r_n^n}{r_{\text{min}}^{n-2} R^2}$$

inter particle sep.  $\uparrow$   $r_{\text{min}}$   $\leftarrow$  radius of sun.  $\leftarrow$   $R^2$

NEGLECTIBLE EVEN FOR NEUTRON  
STARS.

## 2.) COMPOSITENESS BOUNDS

IF  $M_* \sim \text{TeV}$  : NON-RENORMALIZABLE  
OP'S SUPPRESSED BY TeV SCALE.

OF COURSE GENERIC OP'S DISASTROUS

- PROTON DELAY
- FLAVOR VIOLATIONS IN KAON

DANGEROUS OP'S MUST BE SOMEHOW SUPPRESSED (ALWAYS PROBLEM WITH

NEWPHYSICS AT TeV SCALE, e.g. SUSY NEEDS R-PARITY, ...)

$$\mathcal{O}_{4\text{-fermi}} = \frac{2\pi^2}{\Lambda^2} (\bar{\Psi}\Psi)^2$$

IN LEPTON SECTOR FROM LEP:  $\Lambda \gtrsim 3.5 \text{ TeV}$

IF  $\frac{1}{M_*^2} (\bar{\Psi}\Psi)^2 \rightarrow \text{OK FOR } M_* \gtrsim 1 \text{ TeV}$

### 3. COSMIC RAYS

HIGH ENERGY NUCLEONS  $\sim 10^{20} \text{ eV}$   
 $= 10^8 \text{ TeV}$ .

DOES THIS GIVE ANY BOUNDS?  
HIGH CM ENERGY NOT ENOUGH!  
NEED SEPARATION (MOMENTUM TRANSFER)  
 $\sim (\text{TeV})^{-1}$  BUT NUCLEON-NUCLEON ONLY  
USUALLY GIVES  $q \approx 6 \text{ GeV}$

## OTHER CONSTRAINTS :

### 4.) PRECISION OBSERVABLES

$$S_{KK} < m_\pi \quad \sim \left( \frac{m_\pi}{M_*} \right)^{n+2}$$

$$S_{m_{KK} > m_\pi} \quad \sim \left( \frac{m_\pi}{M_*} \right)^4$$

### 5.) RARE DECAYS TO GRAVITONS

E.G.  $K \rightarrow \pi + \text{grav.}$

$$\Gamma_{K \rightarrow \pi + \text{grav.}} = \frac{1}{16\pi} \frac{m_K^5}{M_*^4} \cdot \left( \frac{m_K}{M_*} \right)^{n+2}$$

$$B(n=2) = 10^{-12}$$

### 6.) COOLING OF SUN, SUPERNOVAE :

ESTIMATE:

$$\frac{1}{f_a^2} \rightarrow \frac{T^n}{M_*^{n+2}}$$

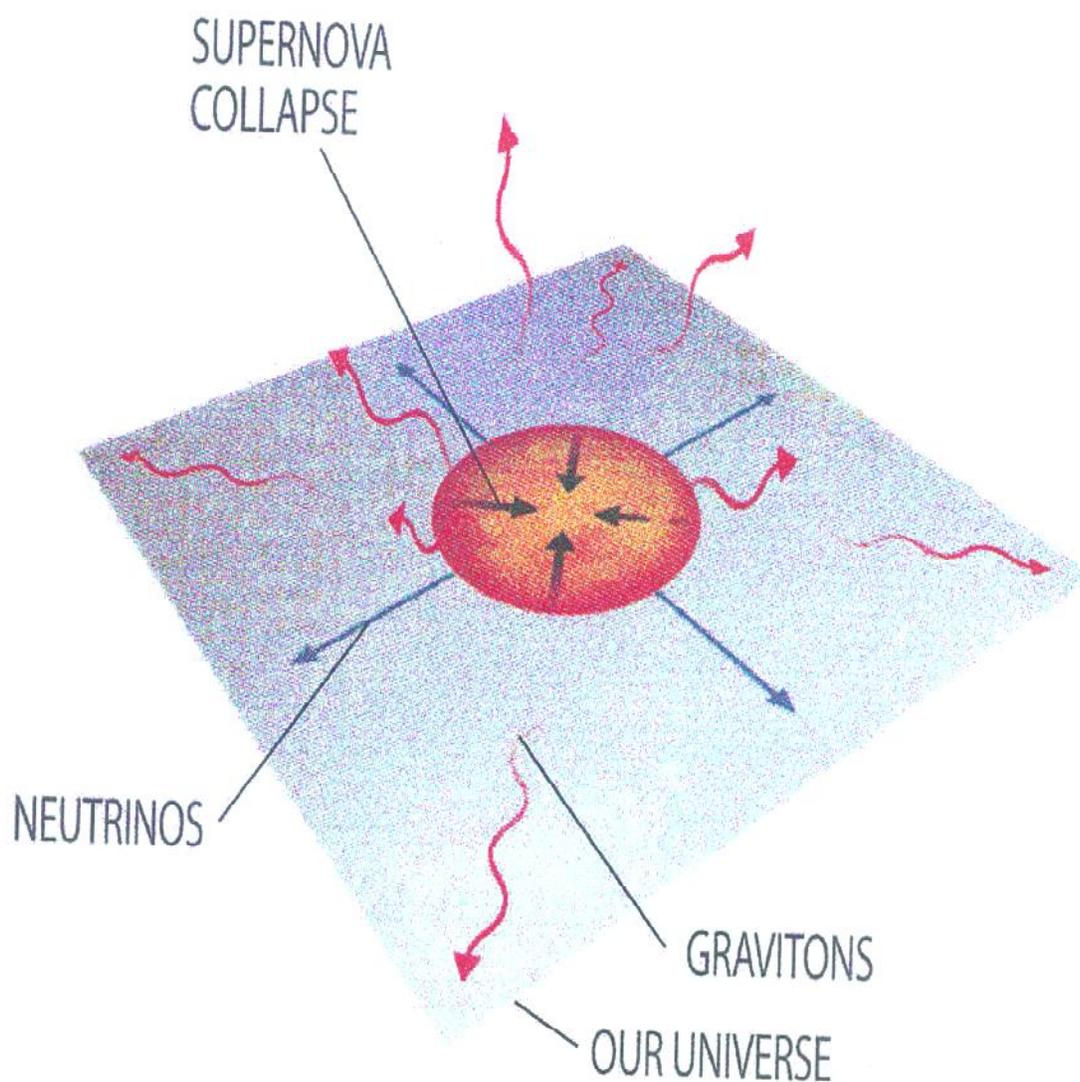
SN1987A for  $n=2$ :  $M_* \geq 30 \text{ TeV!}$

(Hall & Smith :  $M_* \gtrsim 100 \text{ TeV}$ .)

FAIR TO SAY : RULED OUT ...)

A PROCESS CONSTRAINING  $N=2$  EXTRA  
DIM'S :

$M_* \gtrsim 100 \text{ TeV}$



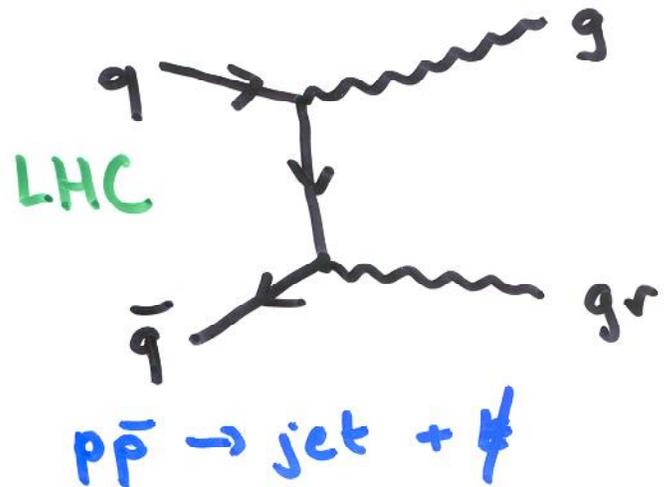
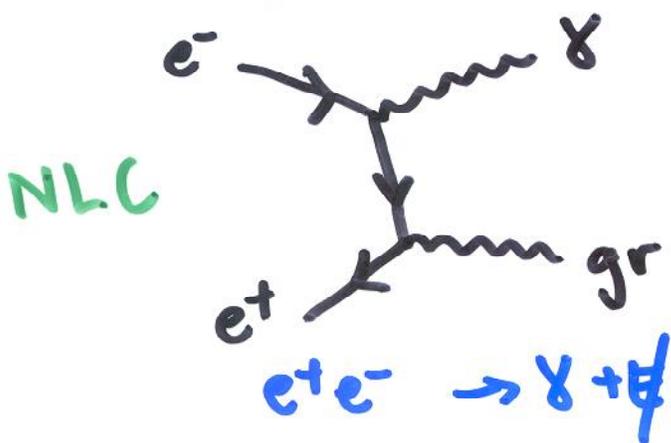
SCIENTIFIC AM.  
ARKANI-HAMED ET AL.

FAIR TO SAY:  $n \geq 3$  ALIVE.

POSSIBLE COLLIDER TESTS

1.) IF  $M_{*} = 1 \text{ TeV}$ , LHC WILL DIRECTLY PROBE THE THEORY OF QUANTUM GRAVITY (STRING THEORY, 2 (ANALYSIS OF PESKIN ET AL.))

2.) NEW EVENTS WITH NO BACKGROUND:

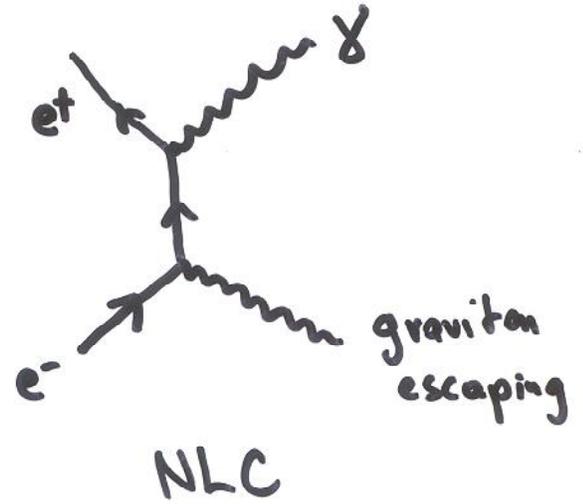
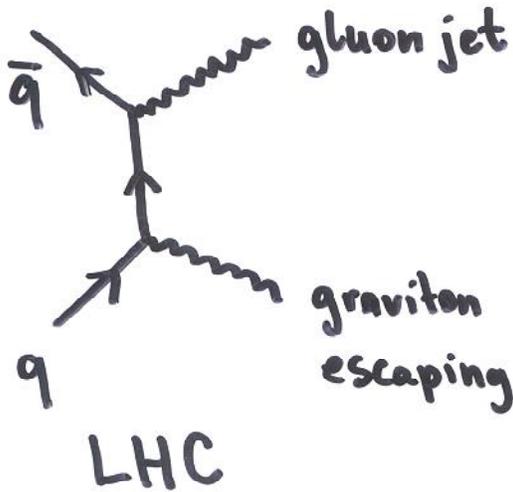


DETAILED ANALYSIS : GIUDICE, RATAZZI, WELLS

LHC WILL BE ABLE TO TEST  $M_{*}$  UP TO FEW TeV ( $\sim 3-10$ )

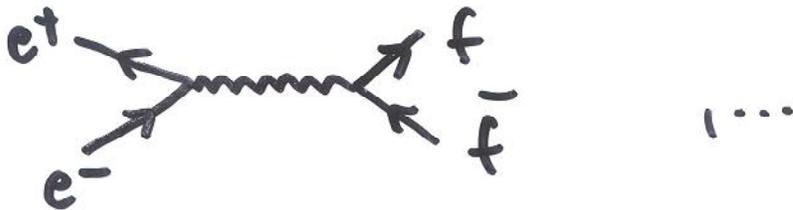
# EXPERIMENTAL SIGNATURES

## DIRECT



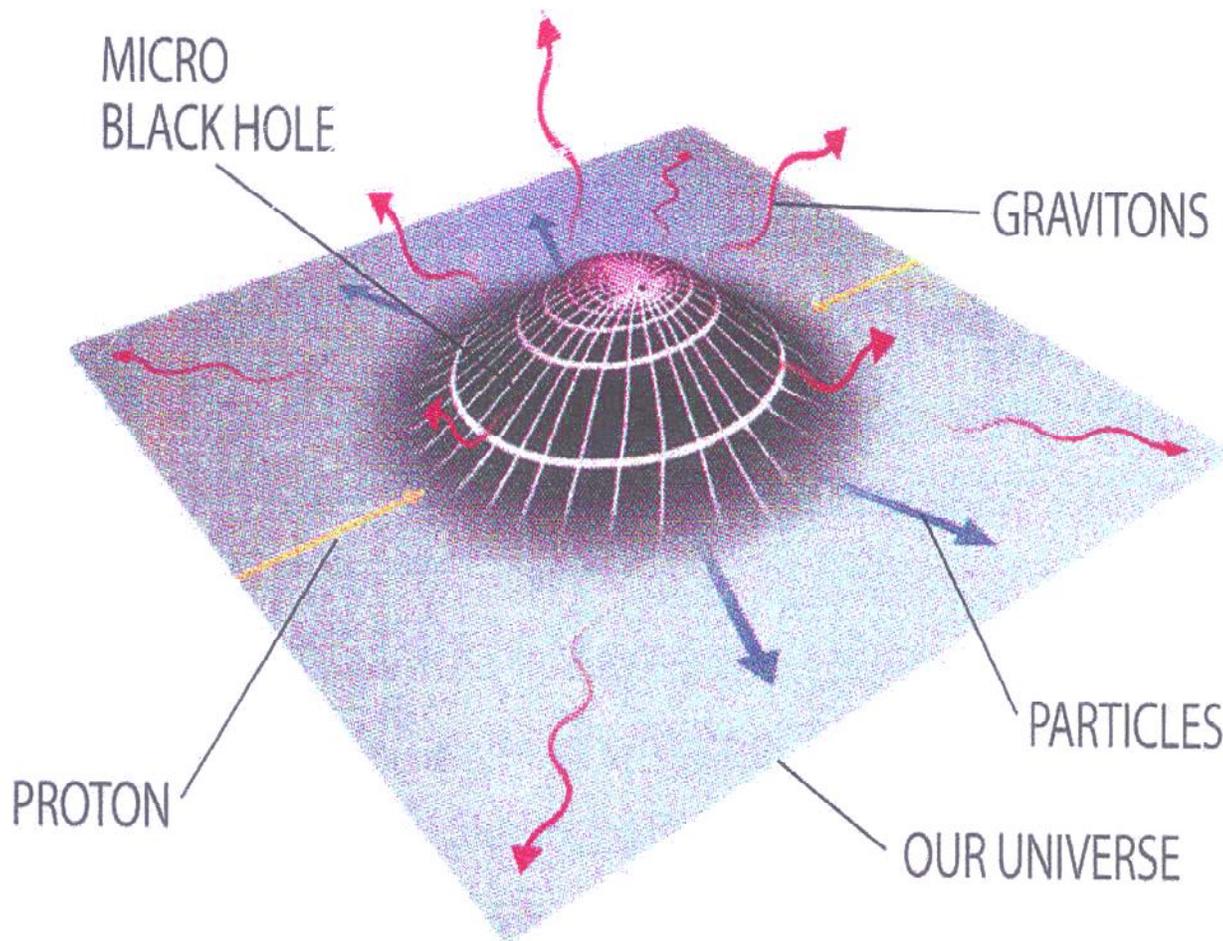
## INDIRECT

CHANGE IN CROSS SECTIONS  
DUE TO EXCHANGE OF KK  
GRAVITONS:



HOWEVER DUE TO  $r \gg \frac{1}{M_*}$  NOT  
REALLY SOLUTION TO HIERARCHY PROBLEM ...

# BLACK HOLE PRODUCTION AT THE LHC?



SCIENTIFIC AM.  
ARKANI-HAMED ET AL.

# WARPED EXTRA DIMENSIONS

CRUCIAL DIFFERENCE BETWEEN  
A 4D AND A HIGHER DIM (e.g. 5D)

THEORY:

4D: VACUUM ENERGY  $V \rightarrow$   
EXPONENTIALLY EXPANDING  
UNIVERSE

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_N}{3} V$$

5D: POSSIBLE TO BALANCE 4D  
ENERGY DENSITY  $V_{4D}$  AGAINST  
BULK COSM. CONST.  $\Lambda_{5D}$

$$V_{\text{eff}} = V_{4D} - \sqrt{\frac{-3}{2} \frac{\Lambda_{5D}}{k^2}}$$

CAN MAKE  $V_{\text{eff}} = 0$  BY APPROPRIATE  
TUNING OF  $V_{4D}$  AND  $\Lambda_{5D}$

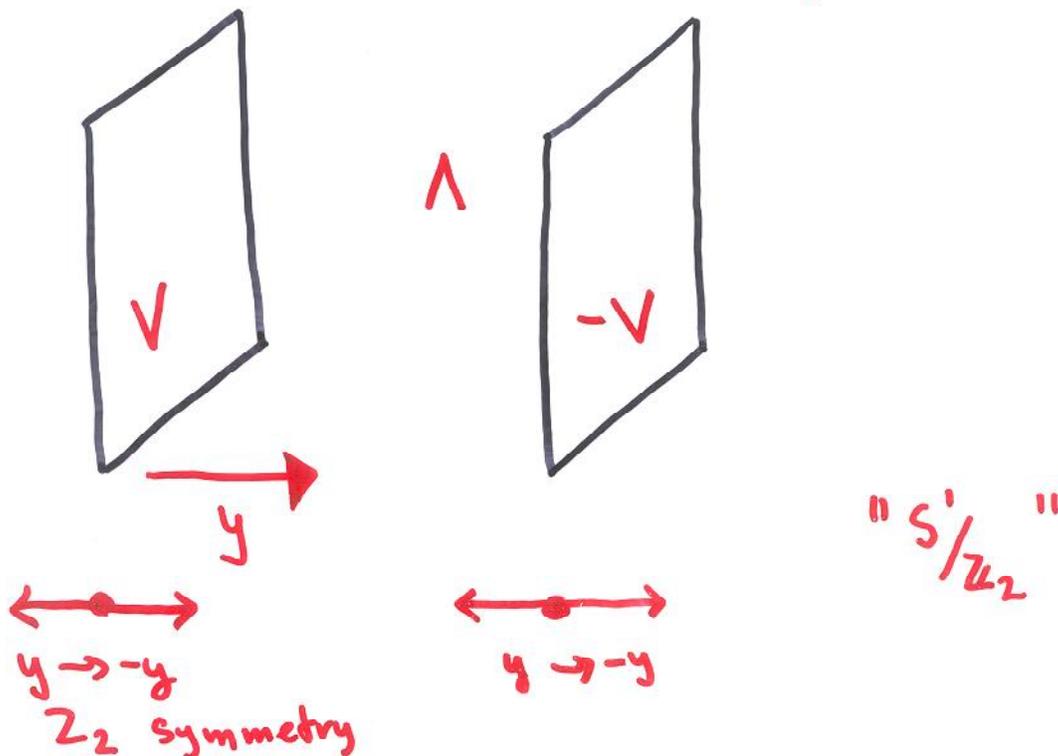
(RUBAKOV & SHAPOSHNIKOV, '83)

# WHY IS THIS INTERESTING?

- NON-TRIVIAL GEOMETRY ALONG 5<sup>th</sup> DIM. WITH FLAT 4D SECTIONS → MASS SCALES VARY ALONG 5<sup>th</sup> DIM
- GRAVITY WILL NOT BE ORDINARY 5 DIM GRAVITY DUE TO NON-TRIV. BACKGROUND
- COSMOLOGICAL CONSTANT PROBLEM MAY BE REINTERPRETED.

## THE BASIC MODEL

(RANDALL, SUNDRUM '99)



- LOOK FOR 4D FLAT SOLUTIONS:

$$ds^2 = a^2(y) \eta_{\mu\nu} dx^\mu dx^\nu + dy^2 \cdot b^2$$

"WARPED METRIC" : DISTANCE SCALES  
GET WARPED BY  $a(y)$  WARP FACTOR

- FROM EINSTEIN'S EQUATION:

55 COMPONENT:

$$6 \left(\frac{a'}{a}\right)^2 = -k^2 b^2 \Lambda$$



$a(y) = e^{-|y|mb}$  ; WHERE:  
 $6m^2 = -k^2 \Lambda$   
5D AdS space glued at  
the brane.

- TO SOLVE EQUATION AT THE BRANE:  
NEED TO TUNE  $V$  AND  $\Lambda$

$$6m = k^2 V$$

$$6m^2 = -k^2 \Lambda$$

- TO MAKE SPACE FINITE : NEED  
SECOND BRANE

$$V' = -V, \quad k^2 V^2 = -6\Lambda$$

# MASS SCALES

GRAVITY:

$$M_{\text{pl}}^2 = M_{(5)}^3 \int_{-\frac{1}{2}}^{\frac{1}{2}} dy \sqrt{g}$$

$$M_{\text{pe}}^2 = \frac{M_{(5)}^3}{m_0} (1 - e^{-m_0 b_0})$$

ONLY SLIGHT  
DEPENDENCE  
ON SIZE OF  
EXTRA DIM.

PARTICLE PHYSICS (ON - TENSION BRANE):

$$g_{\mu\nu}^{(\text{ind})} = e^{-m_0 b_0} \eta_{\mu\nu} \quad \text{IF}$$

$$S = \int d^4x \sqrt{g^{(\text{ind})}} (g_{(\text{ind})}^{\mu\nu} D_\mu \phi D_\nu \phi - M_0^2 \phi^2)$$

CANONICALLY NORMALIZED:

$$\phi \rightarrow e^{-m_0 b_0 / 2} \phi$$

$$M_\phi^2 = M_0^2 e^{-m_0 b_0}$$

RATIO, IF ALL MASSES  $M_{(5)}, m_0, M_\phi = \mathcal{O}(M_{\text{pe}})$ :

$$\frac{M_\phi}{M_{\text{pl}}} = \mathcal{O}(e^{-m_0 b_0 / 2})$$

# THE GRAVITON KK SPECTRUM

TO FIND MASSIVE SPECTRUM, NEED TO SOLVE EINSTEIN'S EQUATIONS AROUND BACKGROUND:  $(K \equiv m \equiv m_0)$   
KK gravitons

$$ds^2 = e^{-2Kb|y|} (\eta_{\mu\nu} + h_{\mu\nu}) dx^\mu dx^\nu + b^2 dy^2$$

ASSUME  $h_\mu{}^\mu = \partial_\mu h^{\mu\nu} = 0$  (RS gauge)

SEPARATE VARIABLES:

$$h_{\mu\nu}(x,y) = e^{ipx} \underbrace{h_{\mu\nu}(y)}_{\text{WAVE FUNCTION OF KK MODE}}$$

WAVE FUNCTION OF KK MODE

$$-\frac{1}{b^2} \frac{d}{dy} \left[ e^{-4Kyb} \frac{d h_{\mu\nu}(y)}{dy} \right] = m_n^2 e^{-2Kyb} h_{\mu\nu}(y)$$

$$p^2 = m_n^2$$

SOLUTION

$$m_n^2 = K e^{-Kb/2} x_n$$

$$J_1(x_n) = 0.$$

KK SPECTRUM HAS TeV SPACING!

## COUPLING OF KK MODES

$$\mathcal{L}_{INT} = \frac{1}{\Lambda} T_{SM}^{\alpha\beta} \sum_n h_{\alpha\beta}^{(n)}$$

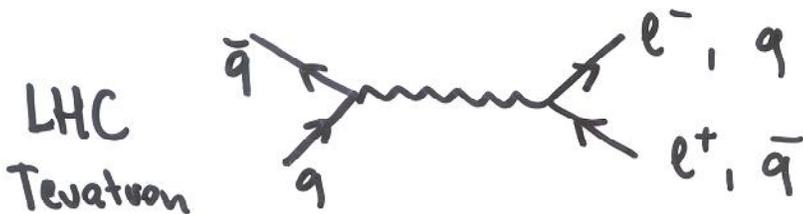
$$\Lambda = M_{pe} e^{-kb/2} \sim \text{TeV}$$

## KK SPECTRUM:

- SPACING  $\propto (\text{TeV})$
- COUPLING  $\propto \left(\frac{1}{\text{TeV}}\right)$
- INDIVIDUALLY DETECTABLE

SIGNATURE : GRAVITON RESONANCE PRODUCTION

(DAVOUDI, SL,  
HEWITT, RIZZO)



VERY CLEAN SIGNALS AT  $e^+e^-$  COLLIDERS

FROM

DAVOUDIASHVILI, HEWITT, RIZZO

hep-ph/9909255

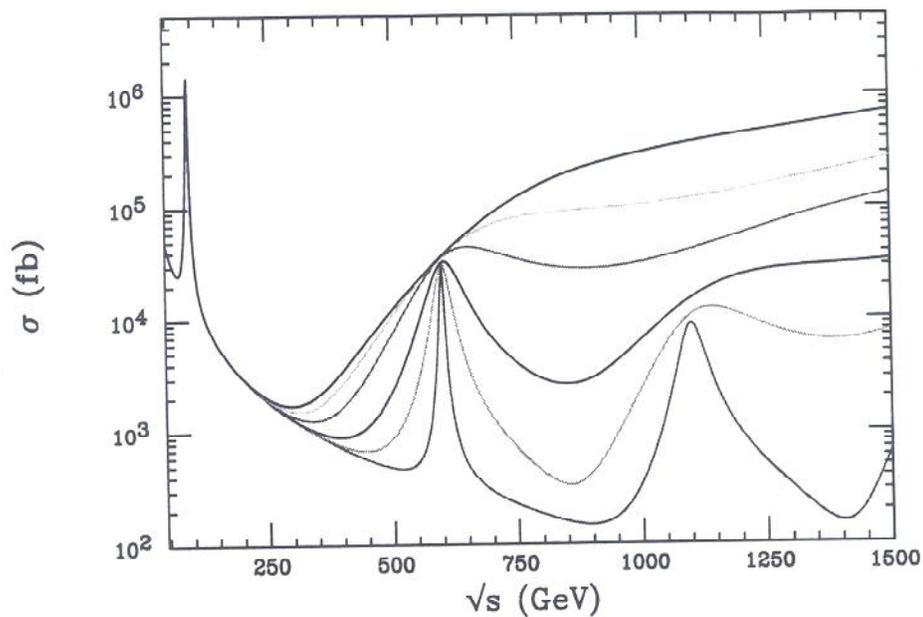
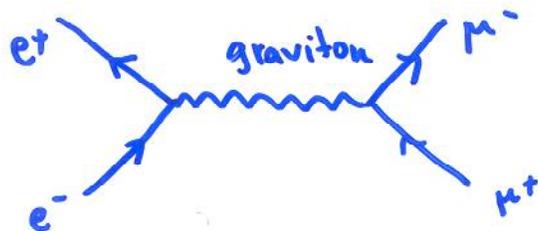
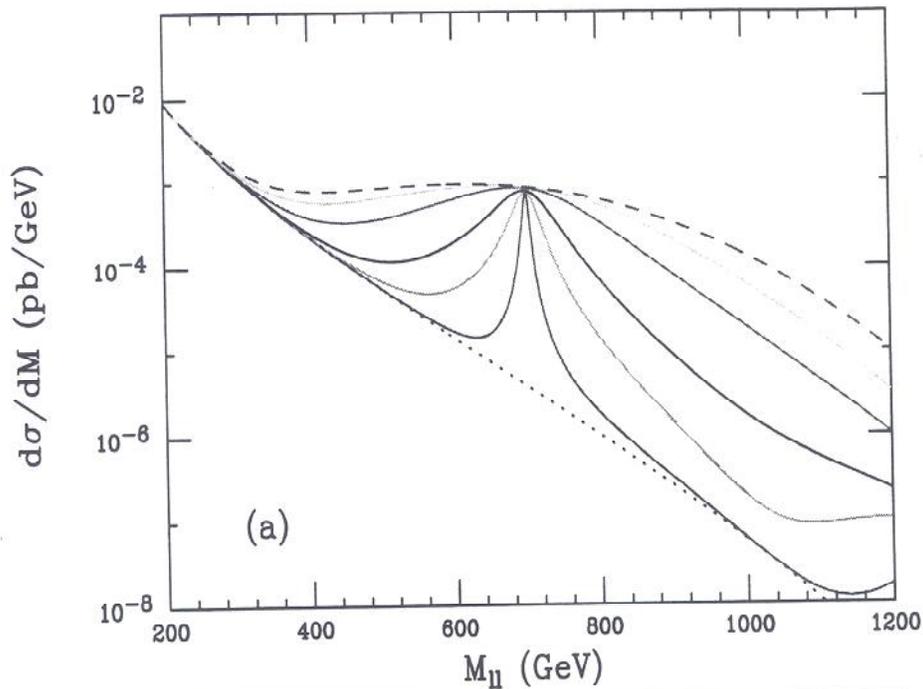


Figure 2: The cross section for  $e^+e^- \rightarrow \mu^+\mu^-$  including the exchange of a tower of KK gravitons, taking the mass of the first mode to be 600 GeV, as a function of  $\sqrt{s}$ . From top to bottom the curves correspond to  $k/\overline{M}_{Pl} = 1.0, 0.7, 0.5, 0.3, 0.2, 0.1$ .

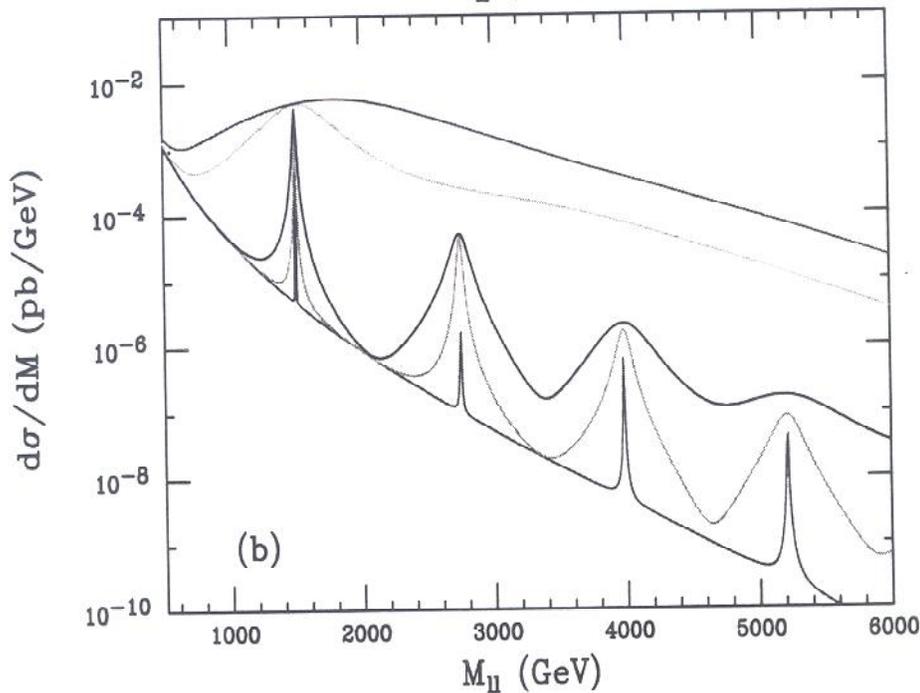


FROM DAVOUDI ASL, HEWETT, RIZZO

hep-ph/0006041



Tevatron  
 $M_g \sim 700 \text{ GeV}$



LHC  
 $M_g \sim 1.5 \text{ TeV}$

Figure 17: Drell-Yan production of a (a) 700 GeV KK graviton at the Tevatron with  $k/\overline{M}_{Pl} = 1, 0.7, 0.5, 0.3, 0.2,$  and  $0.1,$  respectively, from top to bottom; (b) 1500 GeV KK graviton and its subsequent tower states at the LHC. From top to bottom, the curves are for  $k/\overline{M}_{Pl} = 1, 0.5, 0.1, 0.05,$  and  $0.01,$  respectively.

# OTHER ASPECTS OF WARPED SPACETIMES

## 1. LOCALIZATION OF GRAVITY

- ALREADY IMPLICITLY PRESENT IN DISCUSSION WITH 2 BRANES
- REMOVE - TENSION BRANE (INFINITE SPACE)
- CALCULATE FLUCTUATIONS IN  $\infty$  SPACE (CONTINUUM SPECTRUM, SEEMS BAD)

$$ds^2 = e^{-2k|y|} (\eta_{\mu\nu} + h_{\mu\nu}) dx^\mu dx^\nu + dy^2$$

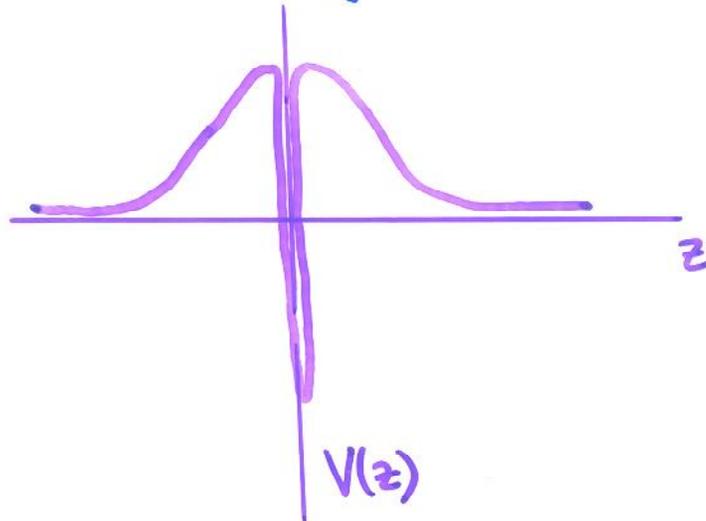
ASSUME  $\partial^\mu h_{\mu\nu} = 0, h^\mu{}_\mu = 0$

GO TO CONFORMALLY FLAT BASIS

$$z = \frac{1}{k} (e^{k|y|} - 1), \quad \hat{h} = h e^{-3/2 k z},$$
$$\hat{h} = \psi(z) e^{i p x}$$

$$-\psi'' + \left[ \frac{15k^2}{8(kz+1)^2} - \frac{3k}{2} \delta(z) \right] \psi = m^2 \psi$$

- YOU OBTAIN AN ORDINARY SCHRÖDINGER EQUATION FOR  $\psi$
- THE POTENTIAL ("VOLCANO")



- ESSENTIAL FEATURES OF POTENTIAL
  - SINGLE BOUND STATE ("4D GRAVITON")
 
$$\psi_0 = k^{-1} \frac{1}{(k|z| + 1)^{3/2}} \quad \text{WITH 0 ENERGY}$$
  - MASSIVE MODES ("KK MODES") CAN BE FOUND ANALYTICALLY
 

FAR FROM  $z=0$  : PLANE WAVES ( $v \rightarrow 0$ ).  
USING PLANE WAVE NORMALIZATION:

$$\psi_m(z) \propto m^{1/2}$$
  - AMPLITUDE OF LIGHT KK MODES VERY SUPPRESSED AT  $z=0$ , DUE TO THE TUNNELING THROUGH THE POTENTIAL

- WHAT IS THE POTENTIAL BETWEEN STATIC SOURCES ON THE BRANE?

FROM 0 MODE:

$$\frac{\langle \Psi_0 | \Psi_0 \rangle}{M_*^3} \frac{m_1 m_2}{r}$$

THIS JUST DEFINES EFFECTIVE 4D NEWTON-CONST.

FROM KK MODES

$$\frac{1}{M_*^3} \int_0^\infty dm \frac{m_1 m_2}{r} e^{-mr} \Psi_m(0)^2$$

- THE TOTAL CONTRIBUTION

$$V(r) = \frac{G_N m_1 m_2}{r} \left( 1 + \frac{1}{r^2 k^2} \right)$$

$$G_N = \frac{k}{M_*^3}$$

AND WE USED  $\Psi_m(0)^2 \propto \frac{m}{k}$

- AT LONG ( $r > \frac{1}{k}$ ) DISTANCES THE 4D NEWTON POTENTIAL IS REPRODUCED

# COSMOLOGICAL CONSEQUENCES

MODEL WITH 2 BRANES TAKEN AT  
FACE VALUE:

GIVES UNCONVENTIONAL COSMOLOGICAL  
EXPANSION

(C.C., GRAESSER, KALDA,  
TERNING, CLINE et al.,  
BINETRUZ et al.)

ON - TENSION BRANE:

$$\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 = \frac{4\pi G_N}{3} (3p - \rho) e^{mb} + \mathcal{O}(\rho^2)$$

WRONG SIGN + EXTRA FACTOR

ON + TENSION BRANE

$$\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 = -\frac{4\pi G_N}{3} (3p - \rho) + \mathcal{O}(\rho^2)$$

CORRECT EQUATION

PUZZLED PEOPLE WHY THIS HAPPENS.

RESOLUTION: THERE IS AN ADDITIONAL  
MASSLESS FIELD IN THE THEORY RADION

- BUT: IF X-D REALLY PHYSICAL, HOW DOES IT AFFECT COSMOLOGY?

## IN WARPED SPACES (C.C., GRAESSER, RANDALL, TERNING)

- SIZE OF EXTRA DIMENSION  $b$  WILL APPEAR AS MASSLESS FIELD IN 4D THEORY  $\equiv$  RADION
- IF RADION PRESENT  $\rightarrow$  COSMOLOGY UNCONVENTIONAL ( $\equiv$  EXCLUDED)
- NEED TO STABILIZE RADIUS ( $\equiv$  GIVE MASS TO RADION)  $\rightarrow$  COSMOLOGY CONVENTIONAL
- AT EARLY TIMES RADION WILL BE IMPORTANT ...

- RADION ALSO COUPLES TO SM MATTER:

$$\mathcal{L}_{int} = \gamma \frac{\phi}{v} T_{\mu}^{\mu}$$

$\phi$ : CANONICALLY NORM. RADION

$\gamma$ : RADION COUPLING  $\sim \frac{246 \text{ GeV}}{\sqrt{6} \text{ TeV}} \sim 0.1$

$v$ : HIGGS VEV = 246 GeV

(C.C. GRAESSER, RANDALL, TERNING)

- BEHAVES SIMILARLY TO SM HIGGS EXCEPT

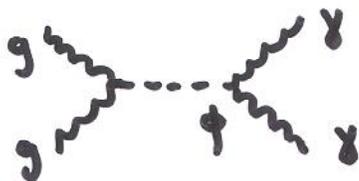
- SUPPRESSION BY  $\gamma$

- ENHANCEMENT OF gluon COUPLING

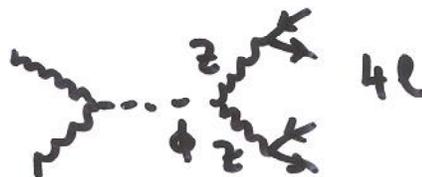
$\text{TeV} > m_r \gtrsim 10 \text{ GeV}$

LIGHTEST MODE IN RS MODEL

- LHC:



FOR  $m_r \lesssim 150 \text{ GeV}$



FOR  $m_r \gtrsim 150 \text{ GeV}$

- NOTE: RADIUS ORDINARY 4D SCALAR FIELD

- IF NOT STABILIZED:  
EXPANDS JUST AS A BRANS-DICKE FIELD, BUT COUPLING FIXED BY GR (COMPONENT OF  $g_{\mu\nu}$ )

- PHYSICAL "CONSTANTS" DEPEND ON RADIUS!

eg.  $M_{pe}^2 = M_*^{2+n} r^n$  IN LED

IF  $r = r(t) \rightarrow M_{pe}^2 = M_{pl}^2(t) = M_*^{2+n} r(t)^n$

SCALE OF GRAVITY  
t-dependent

$M_{weak} = M_{pe} e^{-kr(t)}$  IN RS

WEAK SCALE  
t-dependent

- KNOW, THAT THESE CONSTANTS HAVE NOT CHANGED A LOT SINCE BBN AT BBN RADIUS STABILIZED
- COULD STILL BE A SMALL TIME DEPENDENCE DUE TO EXPANSION OF UNIVERSE, SINCE  $\rho(t)$  IS CONTRIBUTING TO RADION POTENTIAL
- FOR EXAMPLE IN RS

$$\frac{\Delta r}{r} = \frac{1}{3} \frac{1}{\underbrace{m_0 b_0}_{\sim 30}} \frac{(\rho_v - 3\rho_r)}{m_r^2 (\text{TeV})^2}$$

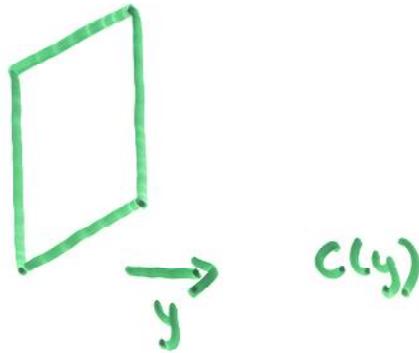
SHIFT IN MINIMUM DUE TO MATTER

$$\rho = \rho(t) \rightarrow \Delta r = \Delta r(t)$$

FOR REALISTIC MODELS TOO SMALL TO EXPLAIN  $\alpha(t)$  OBSERVATION.

- EXTRA DIMENSIONS CAN GIVE VERY UNEXPECTED PHENOMENA

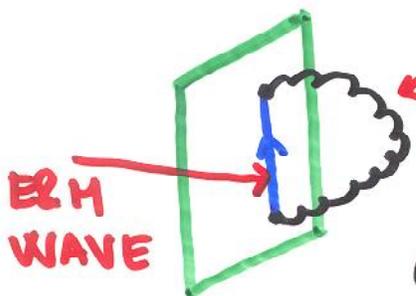
- LOCAL SPEED OF LIGHT MAY BE CHANGING ALONG X-D:



$$ds^2 = a^2(y) dt^2 - b^2(y) dx^2 - c^2(y) dy^2$$

- PARTICLE PHYSICS COMPLETELY 4D LORENTZ INVARIANT, BUT BULK MAY BREAK IT!

- PROP. OF GRAVITATIONAL WAVES: LIKE FERMAT'S PRINCIPLE



GRAV. WAVE

$c_{\text{grav}} > c_{\text{em}}$  POSSIBLE  
 (TESTABLE AT LIGO, LISA,...)  
 (C.C., ERLICH, GROJEAN; CHUNG, KOLB, RIZTO)

- POSSIBLE TO FIND SUCH SPACETIMES  
 ADS BLACK HOLE OR  
 ADS REISSNER-NORDSTRØM  
 BULK  
 (CC, ERLICH, GROJEAN)

- IN SUCH SPACETIMES

$$c_{\text{grav}} - c_{\text{EM}} \geq 0$$

- LIGO WOULD SEE THESE EFFECTS (CHUNG, KOLB, RIOTTO; CC, GROJEAN, ERLICH)

$$\text{FOR } \frac{\delta c}{c} \geq 10^{-17}, \Delta t \sim 0.1 \text{ sec}$$

- POSSIBLE PROBLEM: ALREADY FOR

$$\frac{\delta c}{c} \sim 10^{-7} \quad \Delta t \sim 5 \text{ years}$$

FOR TYPE II SUPERNOVAE 20 Mpc AWAY

$$\frac{\delta c}{c} \sim 10^{-3} \quad \Delta t \sim 5 \text{ yrs}$$

FOR TYPE I SUPERNOVAE WITHIN THE GALAXY

- ASYMMETRICALLY WARPED SPACES MAY HAVE UNCONVENTIONAL 4D EFF. THEORY, MAY EVADE WEINBERG'S NO-GO FOR CC

# SUMMARY OF EXTRA DIMENSIONS

- COULD SOLVE THE HIERARCHY PROBL.:

LARGE EXTRA DIMENSIONS

WARPED EXTRA DIMENSIONS

- DO HAVE EXPERIMENTAL PREDICTIONS

GRAVITY EXP'S.

MISSING ENERGY

KK RESONANCE

RADION

GRAVITATIONAL WAVES?

- COULD GET US CLOSER TO COSM. CONSTANT PROBLEM, ACCELERATING UNIVERSE?

TOO SOON TO TELL WHICH OF THESE IDEAS WILL REMAIN, BUT THERE ARE PLENTY OF THEM...

# CONCLUSIONS

- THE SM LIKELY NOT ULTIMATE THEORY OF EVERYTHING
- THERE SHOULD BE NEW PHYSICS AT THE TeV SCALE ACCESSIBLE TO LHC (OR TEVATRON)
- LEADING THEORIES
  - SUSY
  - EXTRA DIMENSIONS
  - ???
- MANY REASONS TO BELIEVE THAT COMING YEARS WILL BE VERY EXCITING IN PARTICLE PHYSICS