The Statistical Properties of Large Scale Structure

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Outline

- Lecture 1:
  - *Basic cosmological background*
  - *Growth of fluctuations*
  - *Parameters and observables*
- Lecture 2:
  - *Statistical concepts and definitions*
  - *Practical approaches*
  - *Statistical estimators*
- Lecture 3:
  - *Applications to the SDSS*
  - *Angular correlations*
  - *Real-space power spectrum*

Lecture #1

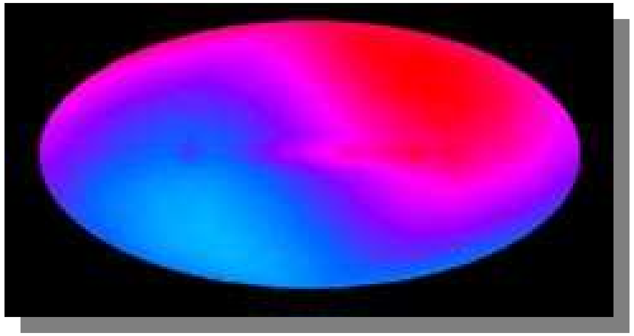
- Cosmological background – the big picture
  - *The expanding Universe*
  - *The small fluctuations*
  - *The Parameters of the Universe*
  - *Observables*
  - *Large-scale surveys*

The Expansion

- The Universe started in a hot Big Bang
- It is expanding, it is extremely uniform
- It is filled with black-body radiation
- The expansion is described by a small number of parameters
- There are small ripples on top of the smooth background, leading to the observed large-scale structure (LSS) in the Universe

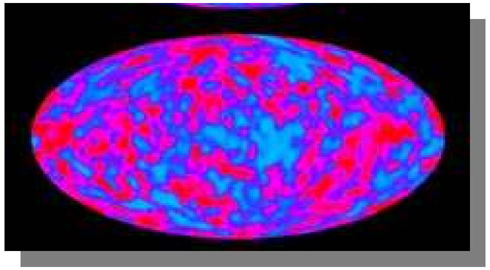
The Microwave Background

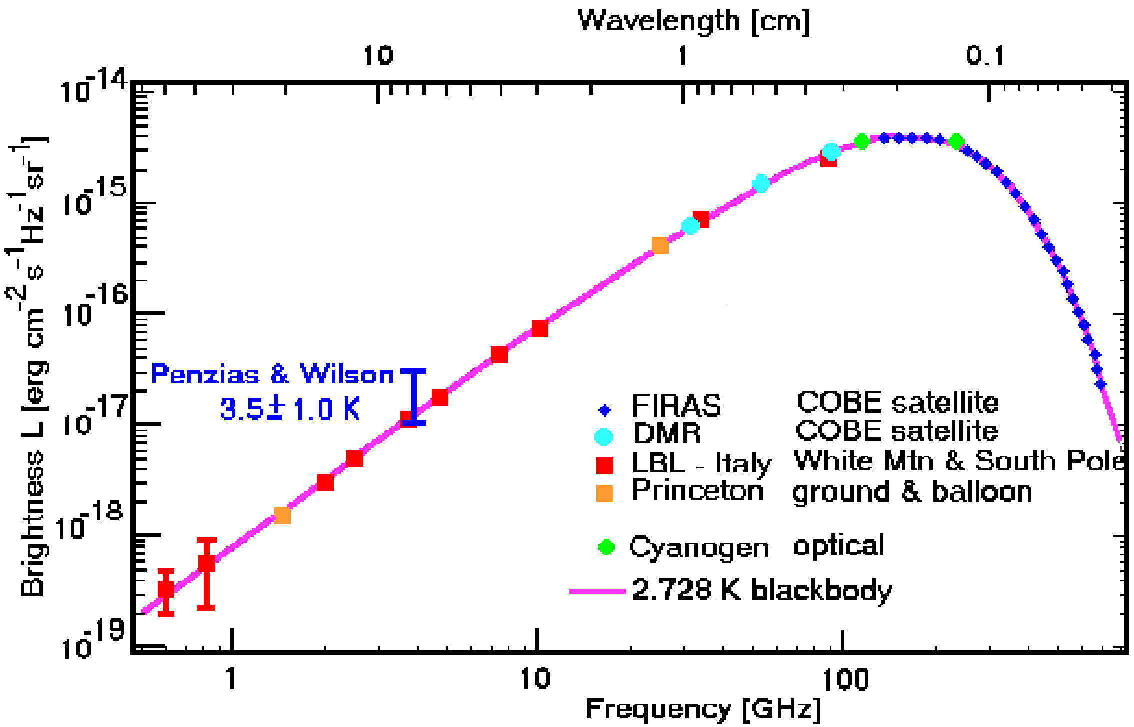
- Uniform black-body radiation, detected by Penzias and Wilson
- Temperature T_0=2.728°K today
- Earlier:



$$T = T_0(1+z)$$

Ripples at 10^{-5}





The Einstein Equations

- The expansion equation, and the equation of state:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{4\pi G\rho}{3} - \mathrm{K}\frac{c^2}{a^2}$$

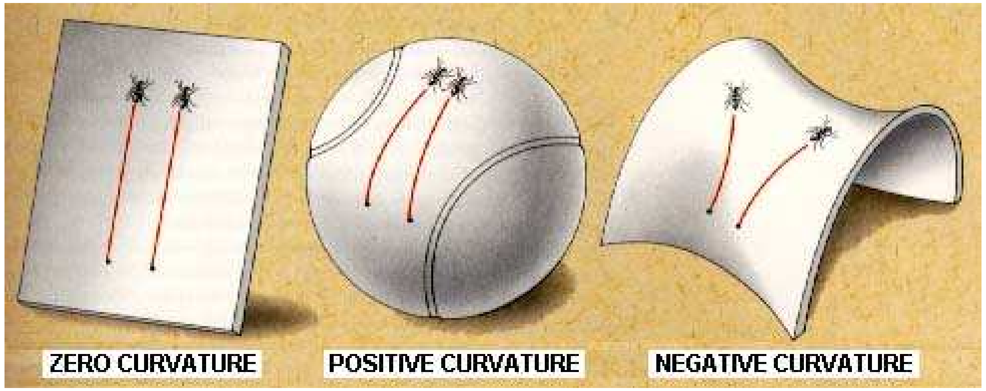
$$\left(\frac{\ddot{a}}{a}\right) = -\frac{4\pi G}{3}\left[\rho + 3P\right]$$

- Curvature depends on the density

- Today:

$$\frac{\dot{a}}{a} = H_0$$

$$a(t_0) = 1$$



The Critical Density

- Density is expressed in terms of critical density:

$$\Omega = (\rho / \rho_{crit})$$

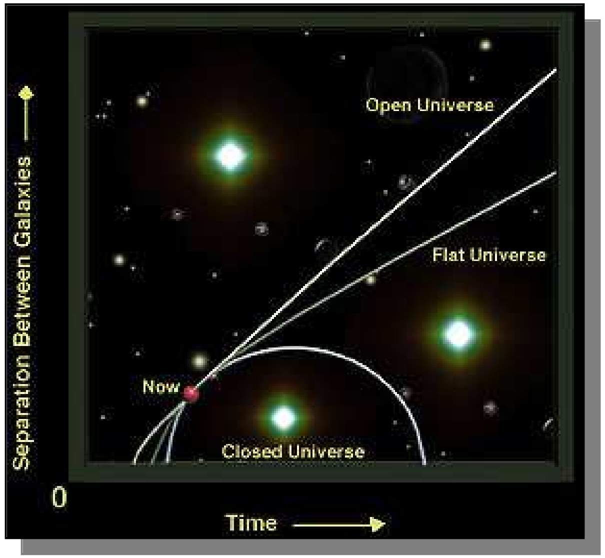
$$\rho_{crit} = \frac{3H_0^2}{8\pi G}$$

- The density has different components, which differ in their equation of state:
  - *Matter (dark matter, baryons)* *$P = 0$*
  - *Radiation* *$P = \rho/3$*
  - *Cosmological constant (dark energy)* *$P = -w\rho$*

The Expansion vs Ω

$$\left(\frac{\dot{a}}{a}\right)^2 = H(t)^2 = H_0^2\left[\frac{\Omega_m}{a^3} + \frac{\Omega_r}{a^4} + \Omega_\Lambda + \frac{\Omega_\kappa}{a^2}\right]$$

$$\Omega_\kappa = 1 - \Omega_m - \Omega_\Lambda \cancel{- \Omega_r} = -K$$

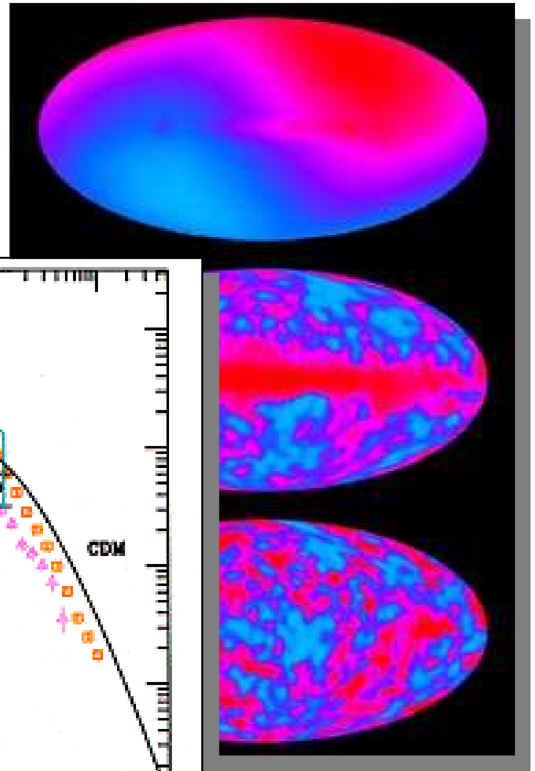


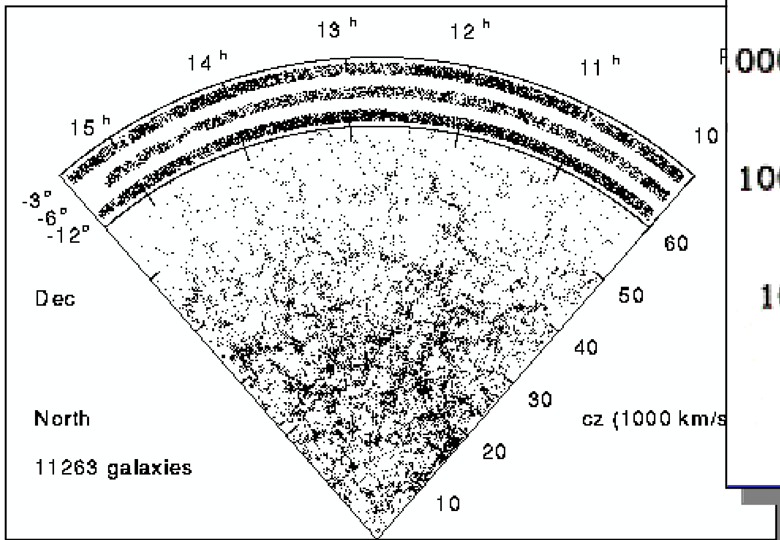
Density	Expansion	Curvature
high	closed	positive
low	open	negative
critical	parabolic	zero

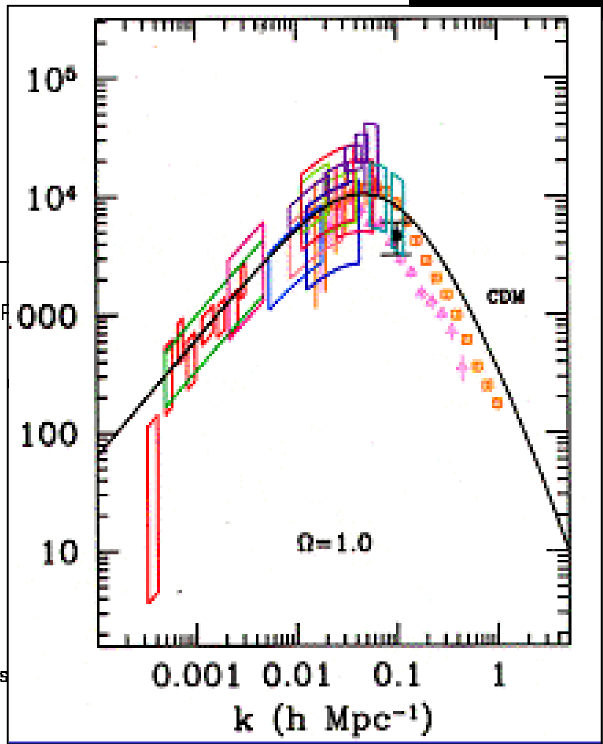
The Fluctuations

The Universe has small ripples, originating shortly after from the Big Bang. The gravitational potential is modified these small fluctuations.

P(k): *fluctuation spectrum*







Shape of the Power Spectrum

- Tied to the Big Bang
  - *Early horizon size is much smaller than any relevant scale today*
  - *Must be scale invariant => power law: $P(k) = A\,k^n$*
- What is the exponent?
  - *Poisson's equation* $\nabla^2 \chi = 4\pi G \rho \delta$
  - *The power spectrum of the potential fluctuations is*

$$P_\chi(k) \propto \frac{1}{k^4} P(k) = A k^{n-4}$$

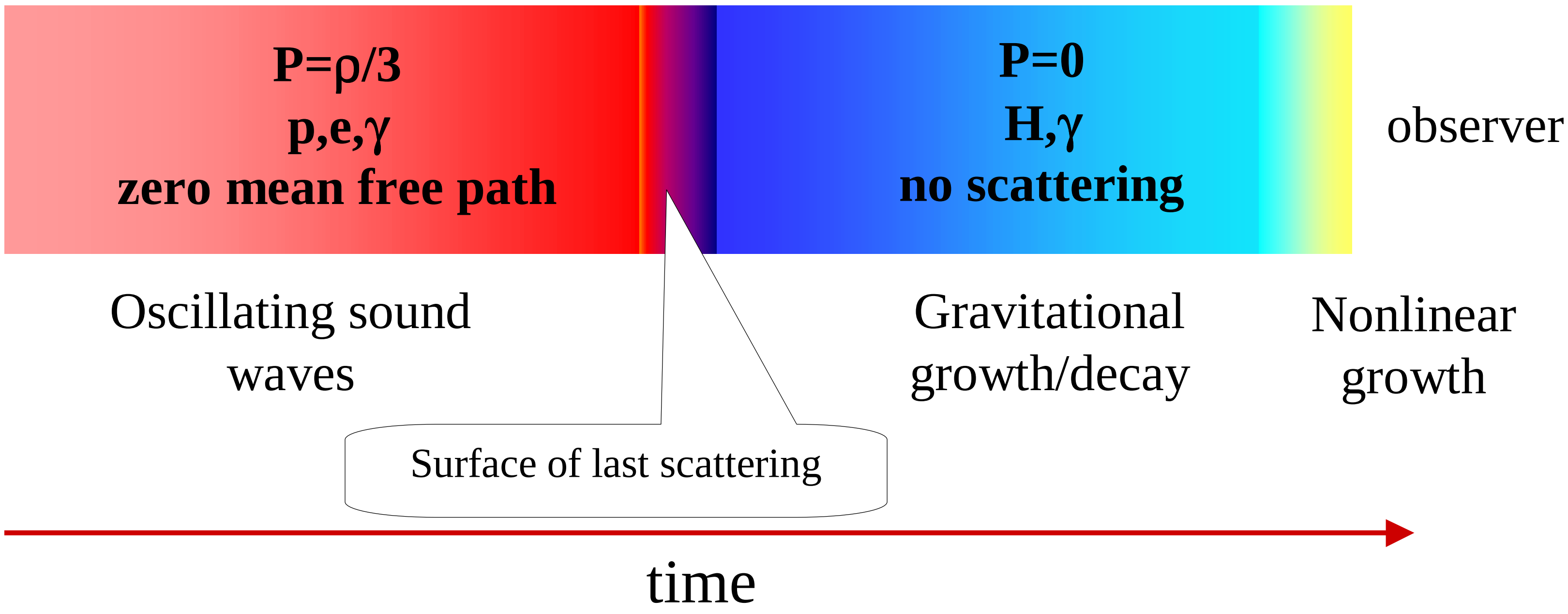
  - *The variance per logarithmic interval is*

$$k^3 P_\chi(k) = A k^{n-1}$$

  - *The* ***n=1*** *spectrum is "double scale-invariant" (Harrison-Zeldovich)*

The Growth of Fluctuations

- Jeans (1902): gravitational instability in a P=0 fluid
- When the pressure is large, sound waves
- At around recombination:
  - *Phase interference => modulation of fluctuations*
  - *Behaves like a resonant cavity*



Transfer Function

- Subsequent evolution changes the power spectrum
  - *Linear growth:* $D(t)$
    - The same for each wave number, self-similar
  - *Transfer phenomena:* $T(k)$
    - Different for different wavenumbers, but modes independent
    - Includes damping, suppressed growth

$$P(k) = Ak^n T(k)^2 D(t)^2$$

  - *Non-linear growth*
    - Nonlinear interactions, mode-mode coupling
    - Happening mostly on small scales
    - Difficult to describe analytically

Evolution of Fluctuations

- Components:
  - *Dark energy, dark matter, baryons, photons, neutrinos*
- At z>1300 strong coupling between baryons and photons

$$\ddot{\delta}_B + 2\left(\frac{\dot{a}}{a}\right)\dot{\delta}_B = (4\pi G\rho\Omega_B - k^2 c_s^2)\delta_B + 4\pi G\rho\Omega_m\delta_m$$

$$\ddot{\delta}_m + 2\left(\frac{\dot{a}}{a}\right)\dot{\delta}_m = 4\pi G\rho(\Omega_B\delta_B + \Omega_m\delta_m)$$

- Several effects present:
  - *"Friction": the expansion rate of the universe, depends on the effective equation of state of the whole universe*
  - *"Speed of sound": the back reaction of pressure gradients, depends on the equation of state of the baryons*

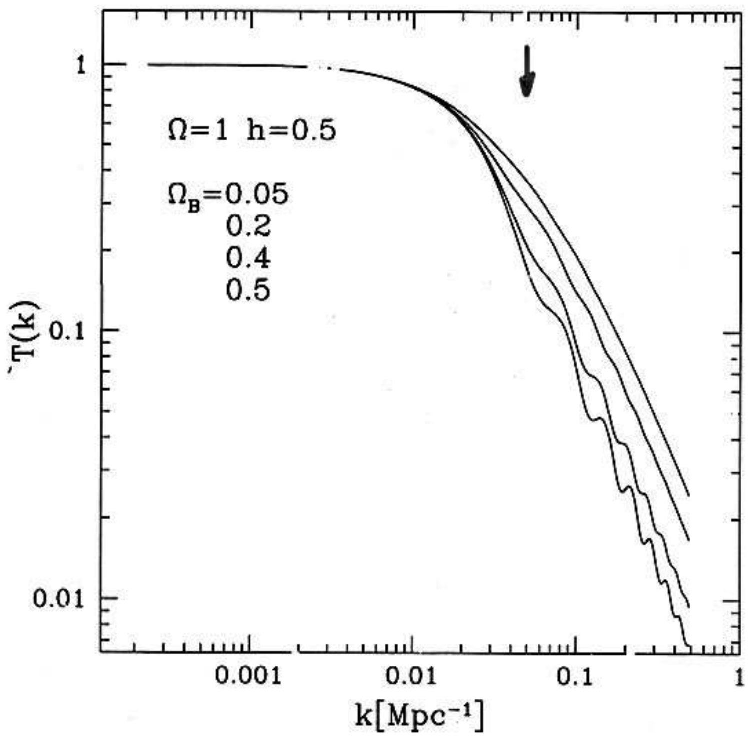
Changing Equations of State

- Sharp changes in the equations of state:
  - *Equality* $\rho_m(t_{eq}) = \rho_r(t_{eq})$ $k_{eq} = \dfrac{2\pi}{\lambda_{eq}}$
  - *Recombination* $c_s^2 = \dfrac{c^2}{3} \rightarrow 0 \quad \text{at } t_{rec}$
- All happens between z=1300 and z=1000
- Radiation drag acting on the baryons until z=200

The Effect of Sound Waves

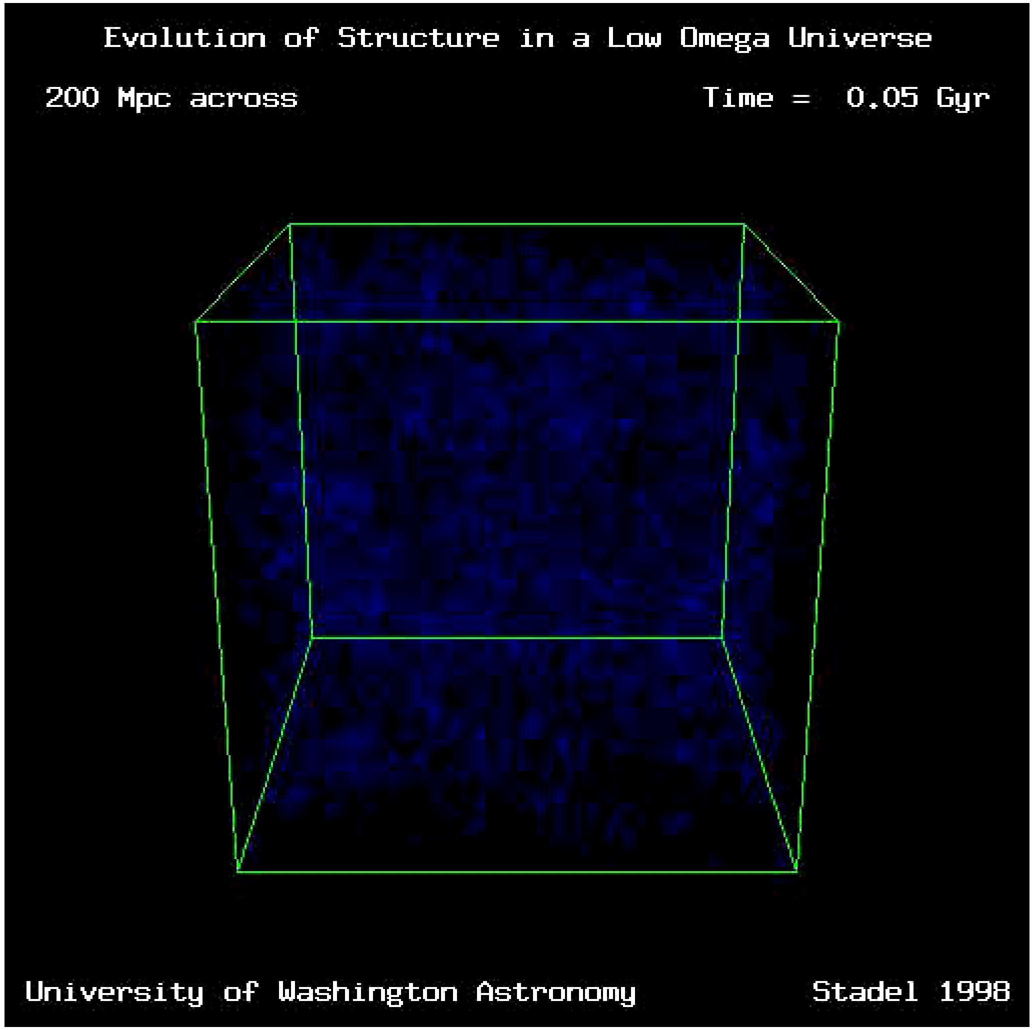
- The remnants of early sound waves show up as small bumps in the LSS P(k)
- They fully affect the CMB
- Wavelength given by the sound horizon (Sakharov)
- Amplitude of LSS bumps depends on

$$f_B = \Omega_B / \Omega_{tot}$$



W. Hu (1992)

Non-linear Growth



Relevant Scales

Distances measured in Mpc [megaparsec]

1 Mpc = 3×10^{24} cm

5 Mpc = distance between galaxies

3000 Mpc = scale of the Universe

if λ >200 Mpc

fluctuations have a PRIMORDIAL shape

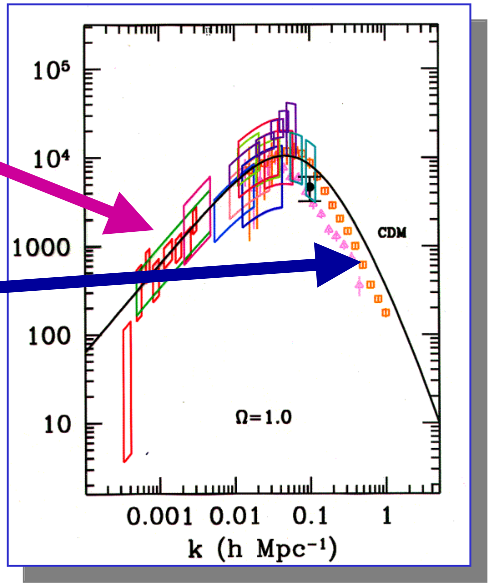
if λ <100 Mpc

gravity creates sharp features, like walls, filaments and voids

Biasing

conversion of mass into light is nonlinear

light is much more clumpy than the mass



Parameters of the Universe

H_0	Hubble constant	71 km/s/Mpc
Ω_0	density parameter	0.95-1.05
t_0	age of the Universe	13-14 Gyr
Ω_k	curvature	0 ±0.05
Ω_m	density of matter	0.35 ±0.05
Ω_Λ	cosmological constant	0.65 ±0.05
Ω_B / Ω_m	baryon fraction	0.17 ±0.05
Ω_ν	density of neutrinos	?

After WMAP

Observables

We can measure:

- Fluxes – luminosity distance
- Angles and distances – angular diameter distance
- Galaxy velocities – the density of matter
- Galaxy 2D positions – the shape of the fluctuations
- Galaxy 3D positions – anisotropies in the fluctuations

Angular Diameter Distance

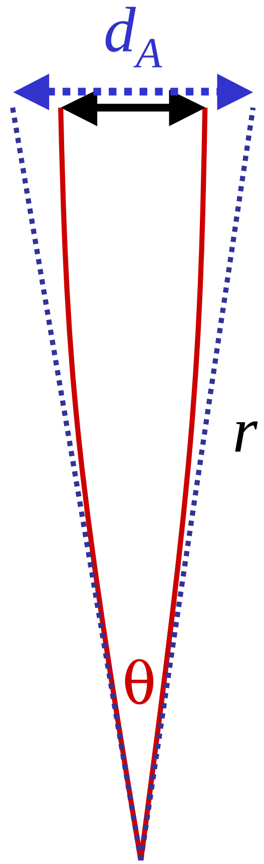
- The light travels along geodesics:

“Angular diameter distance”

$$d_A = r\theta = \frac{r}{1+z}\,\frac{\sin n\,|\Omega_\kappa|^{1/2}\,r}{|\Omega_\kappa|^{1/2}\,r}$$

$$\sin n = \begin{cases} \sin & \text{if} \quad \mathrm{K} > 0 \\ \sinh & \text{if} \quad \mathrm{K} < 0 \end{cases}$$

$$r = \int_0^z dz'\left[(1+z')^2(1+\Omega_m z') - z'(2+z')\Omega_\Lambda\right]^{-1/2}$$



Luminosity Distance

- The Euclidian distance based on its observed flux

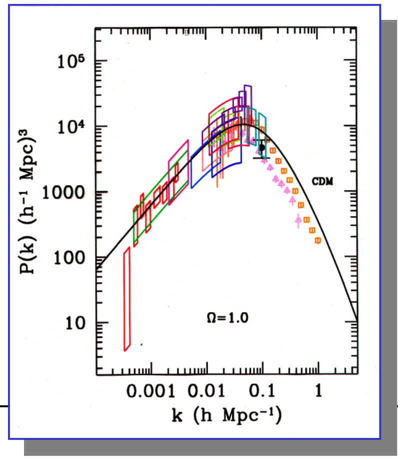
$$f = \frac{L}{4\pi d_L^2}$$

- Related to the angular diameter distance

$$d_L = (1+z)^2 d_A$$

The Shape of the Fluctuations

Overall spectral index (n)	(tilt)
Amplitude (σ_8)	(up-down shift)
Matter-radiation equality ($\Omega_m h$)	(left-right shift)
Remnants of sound waves (f_B)	(bumps)
Neutrino mass (Ω_ν)	(damping)
Separation of light from the mass (b)	(biasing)
Redshift distortion ($\beta=\Omega_m^{0.6}/b$)	(radial anisotropy)
Dark Energy (Ω_Λ)	(xverse anisotropy)
Nonlinear effects	(high-k tail)

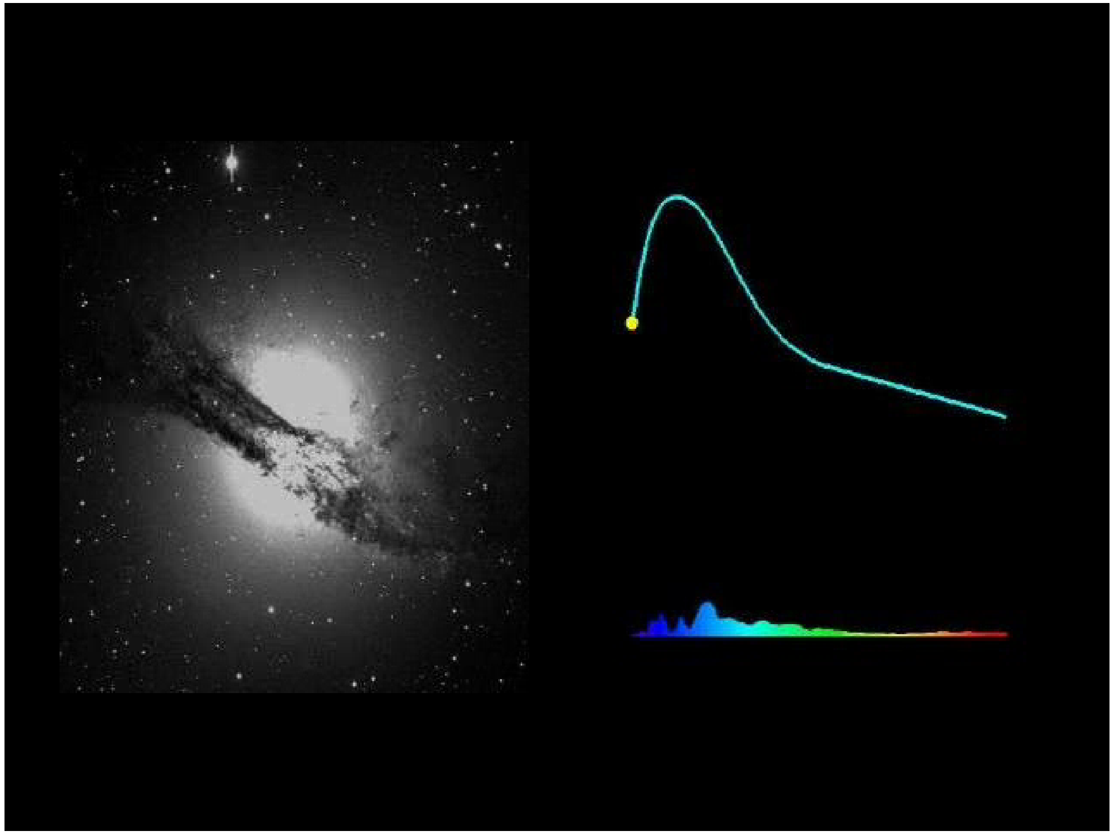


The Basic Data Sets

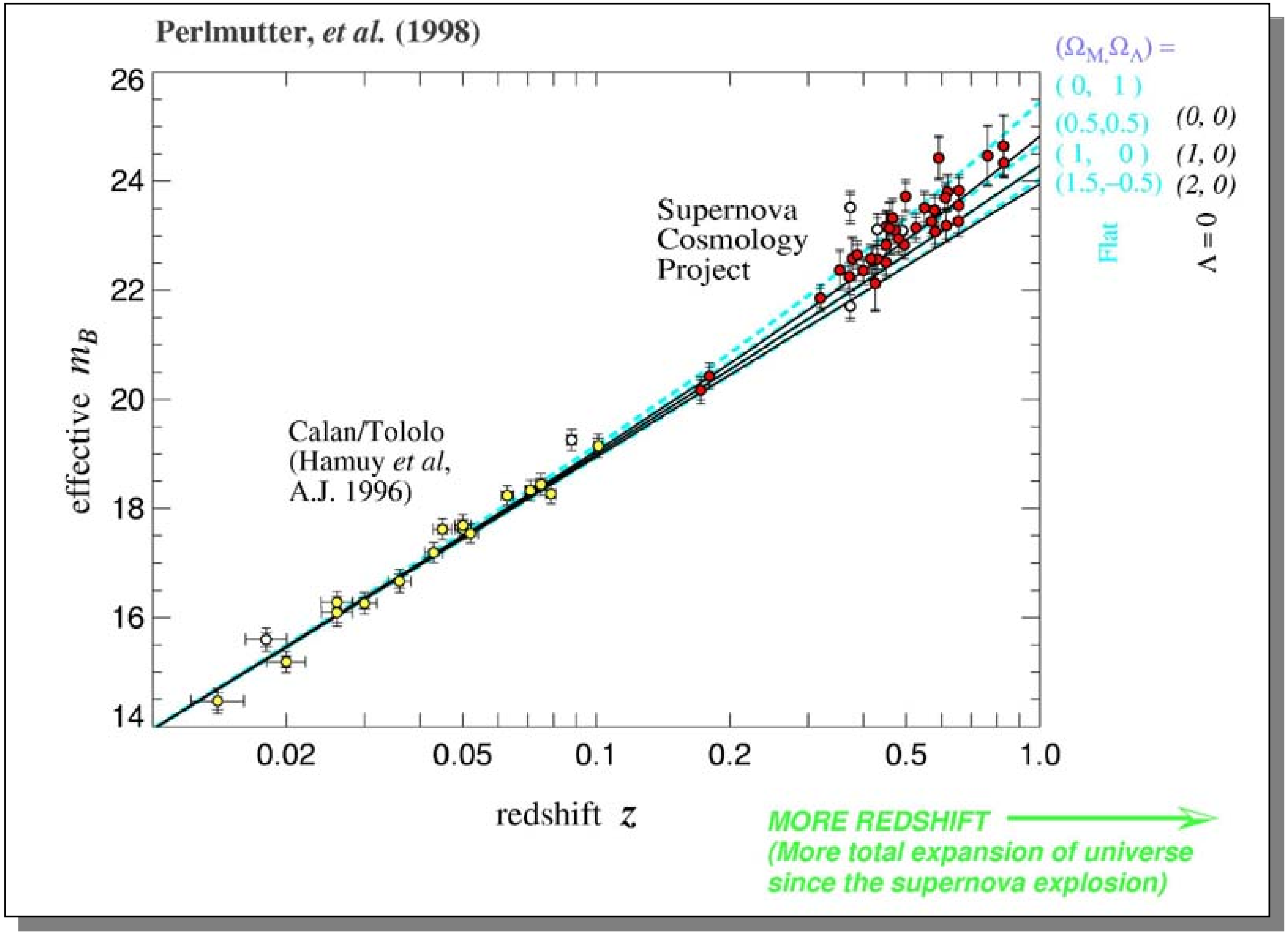
- Redshifts of supernovae – standard candles
- Ripples in the Microwave background – yardsticks
- Galaxy clustering – cosmological parameter fitting
- Galaxy photometry – careful sample selection, evolution

Distant Supernovae

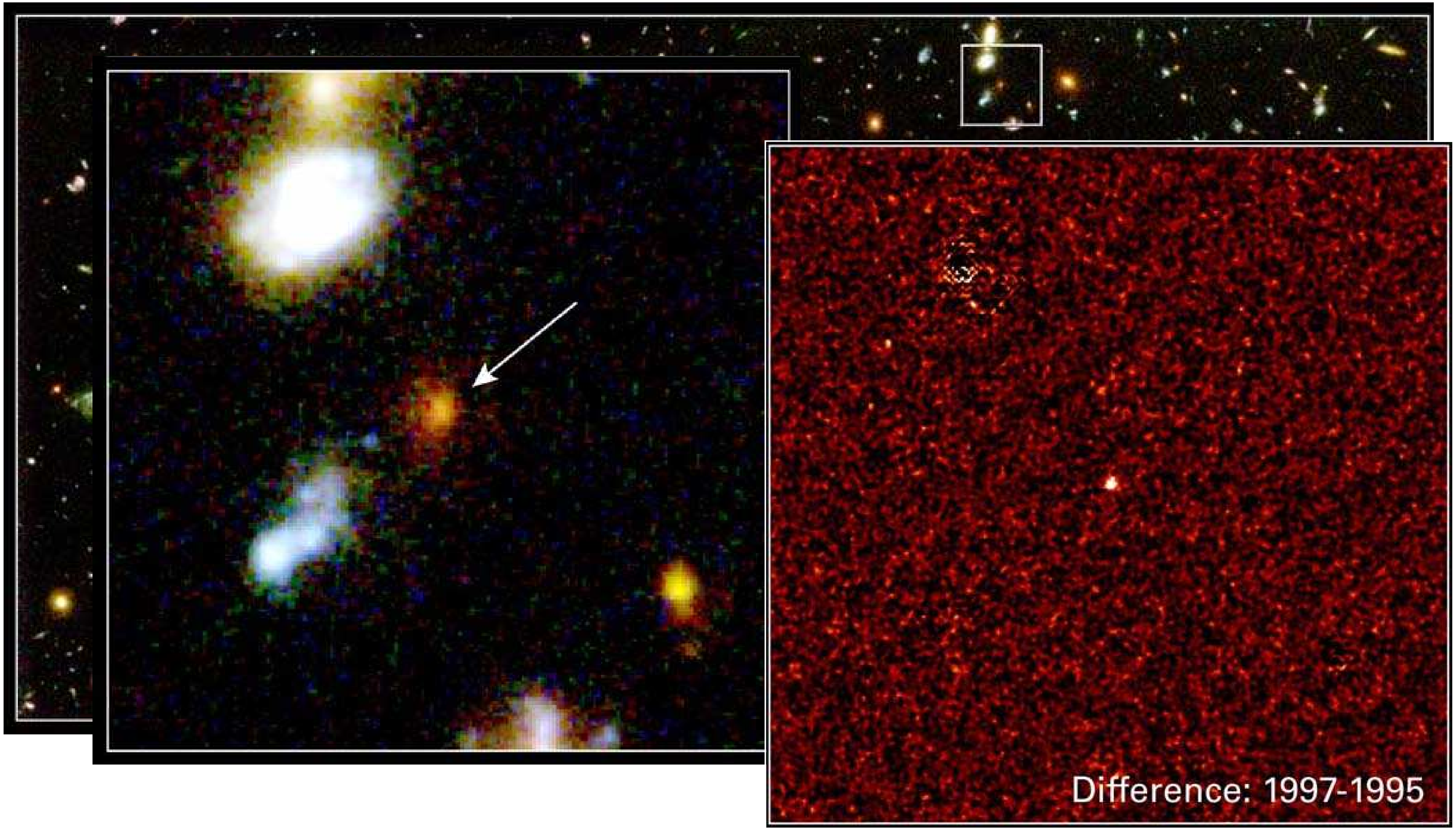
- The maximum luminosity of supernovae is constant
- If spotted in time => distance
- Brightness/Redshift: Hubble diagram
- Deviations from Hubble's law: curvature



Hubble-diagram

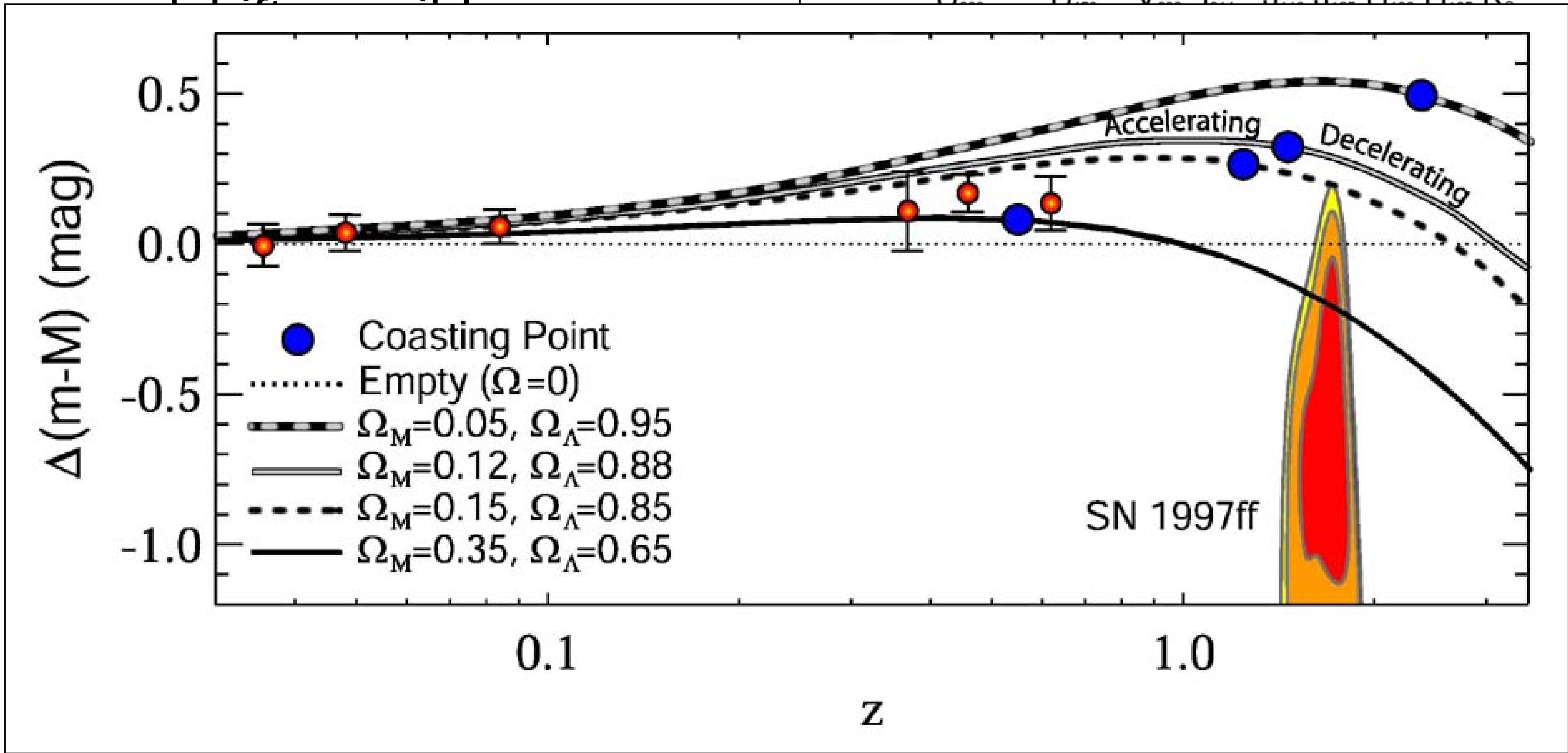


SN 1997ff



Fitting a Redshift to the SN

- Only photometric

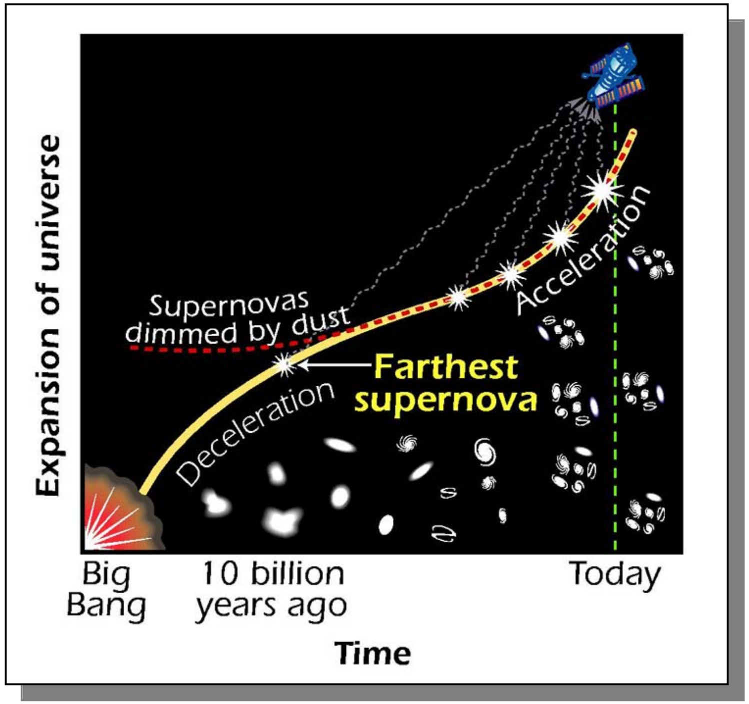


Comments: Accepted to the Astrophysical Journal 38 pages, 15 figures, Pretty version available at this http URL

We present photometric observations of an apparent Type Ia supernova (SN Ia) at a redshift of ~1.7, the farthest SN observed to date.

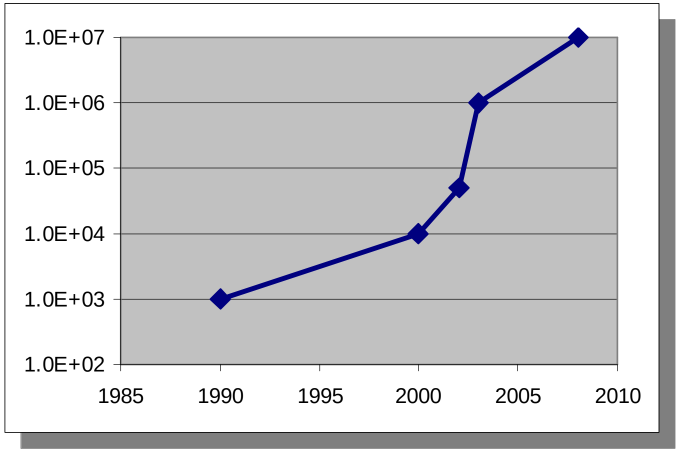
Cosmological Constant

- The redshift and brightness of the supernova suggested a nonzero cosmological constant!
- The Universe will expand forever



CMB Maps

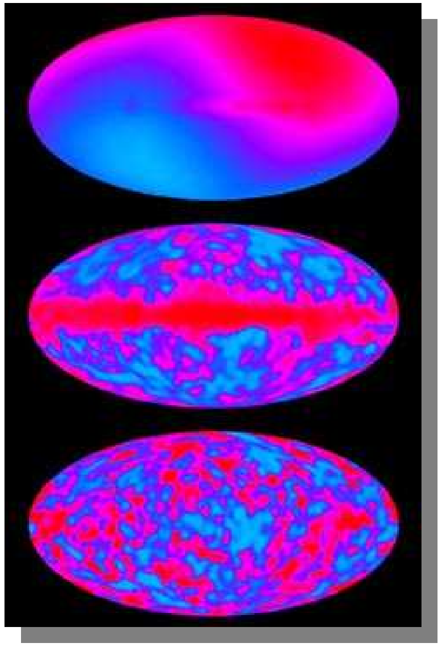
• 1990	COBE	1000
• 2000	Boomerang	10,000
• 2002	CBI	50,000
• 2003	WMAP	1 Million
• 2008	Planck	10 Million

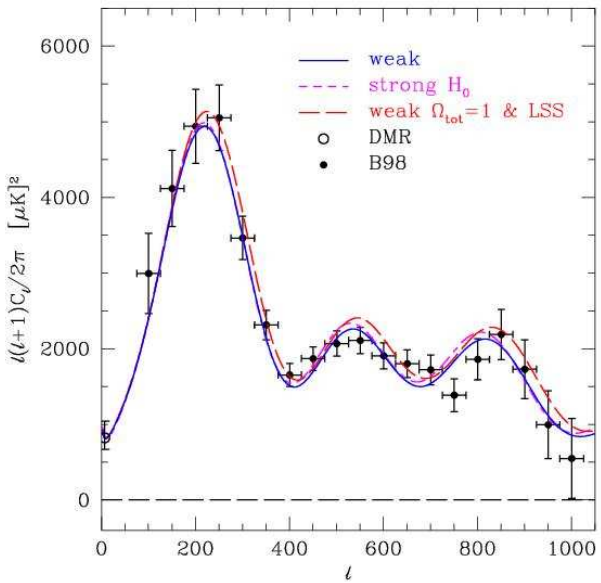


Cosmic Microwave Background

- Snapshot of the surface of last scattering
- Physical size of ripples given by sound horizon
- Angular size measured from harmonic analysis

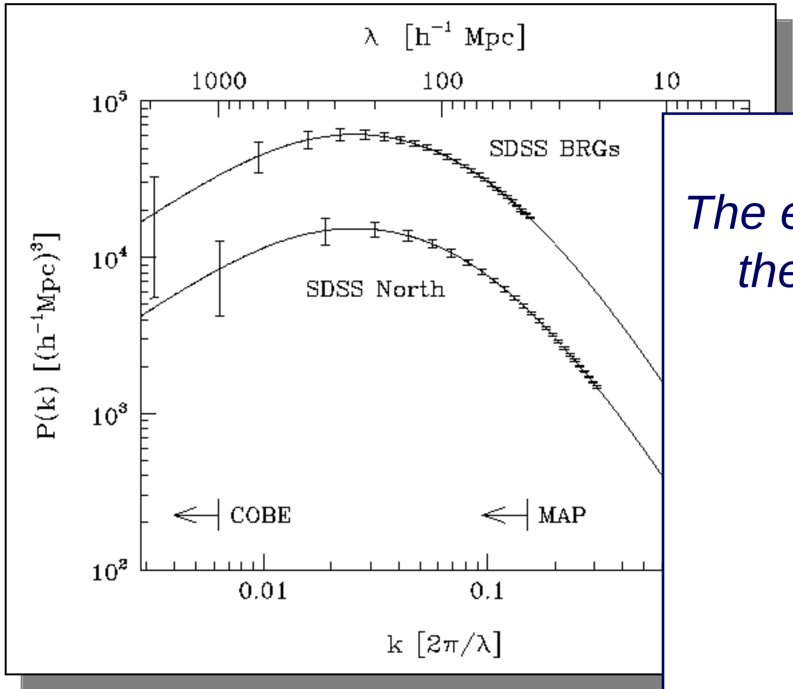
  => Angular diameter distance gives curvature

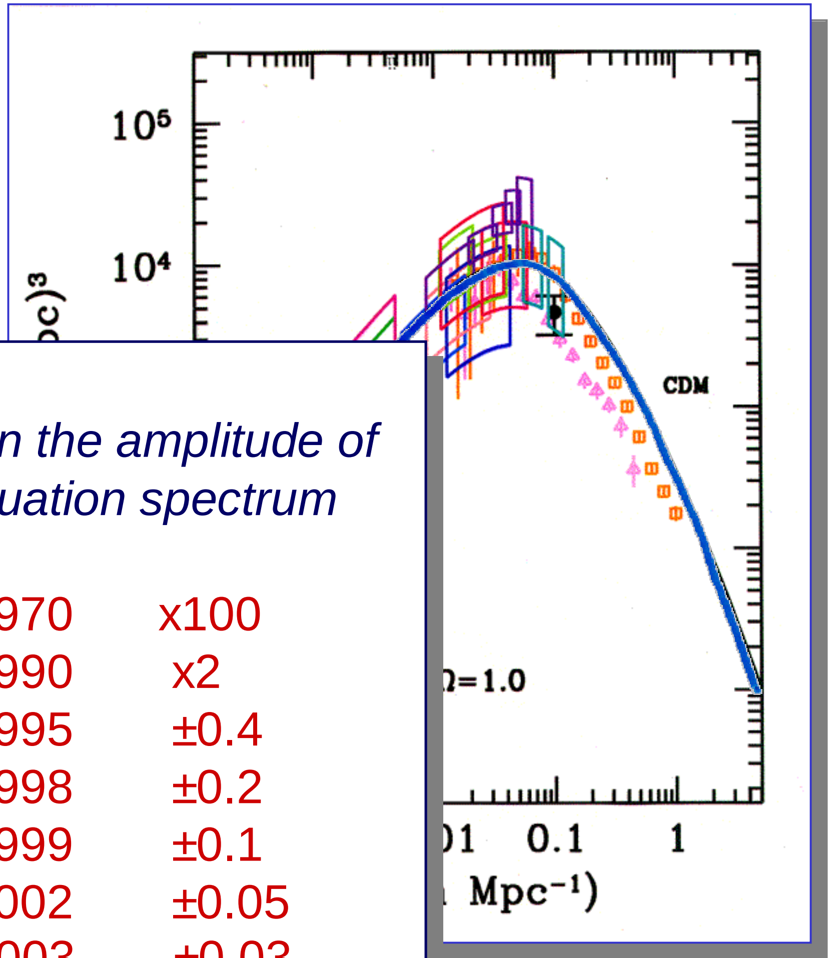




Clustering of Galaxies

Measure the spectrum of the density fluctuations today, even on very large scales





The error in the amplitude of the fluctuation spectrum

1970	x100
1990	x2
1995	±0.4
1998	±0.2
1999	±0.1
2002	±0.05
2003	±0.03

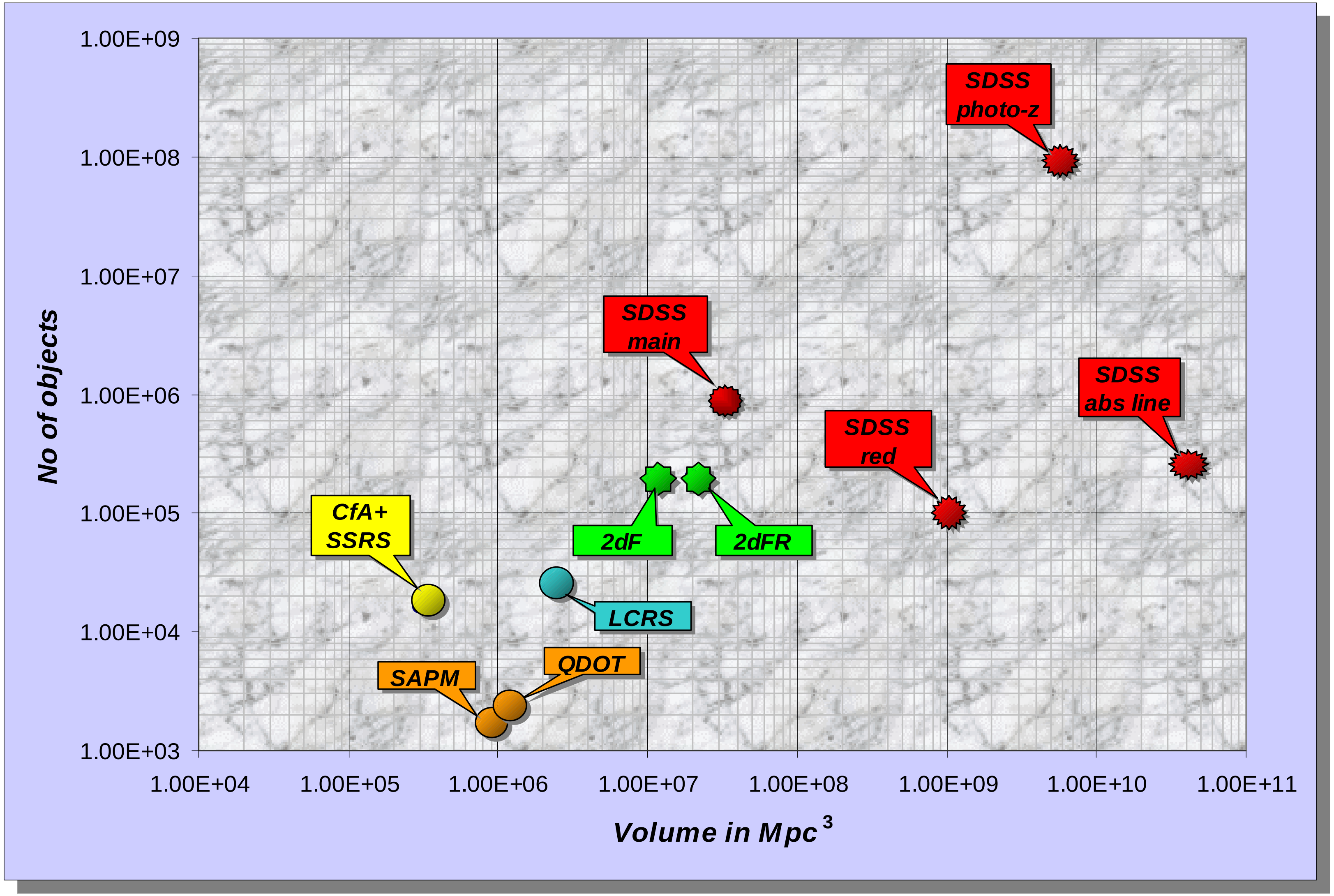
Angular Surveys

1970	Lick	1M
1990	APM	2M
2005	SDSS	200M
2008	LSST	2000M

Local Redshift Surveys

1986	CfA	3500
1996	LCRS	23000
2003	2dF	250000
2005	SDSS	800000

Area and Size of Redshift Surveys



Status Today

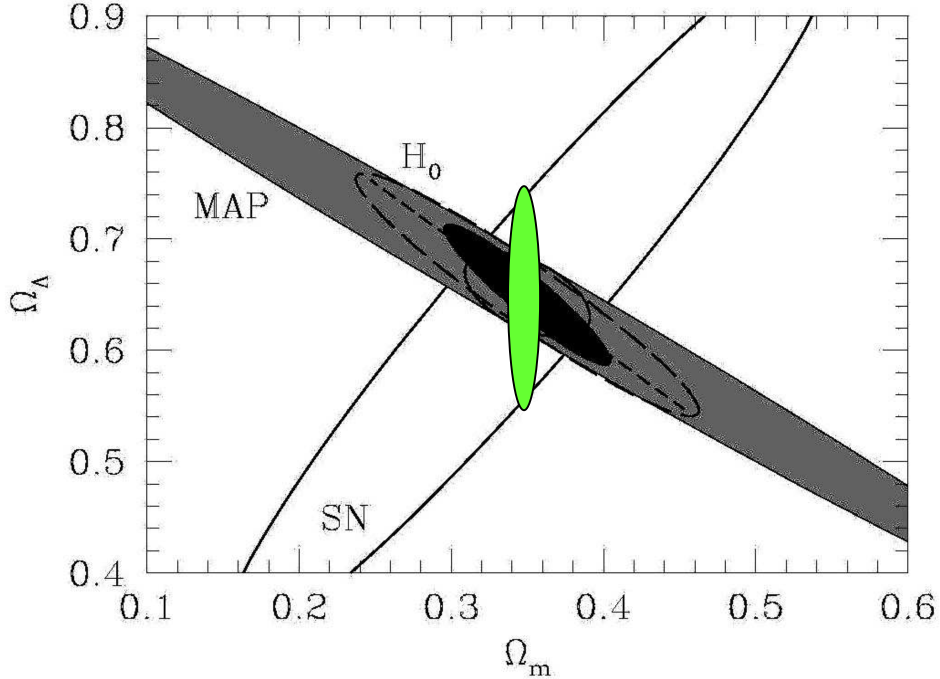
- Large samples, selected on rest-frame criteria
- Power spectrum measured longward of the peak
- Global shape of power spectrum fully understood
- Advanced analysis techniques working as planned
- Degeneracies almost orthogonal to CMB
- Excellent agreement between CMB, LSS and SN

Near Future

- Challenges:
  - *Baryon bumps, cosmological constant, equation of state*
- Cosmological constant is a special case of a more general possibility: **DARK ENERGY**
- Equation of state: $P = w\rho$
- Cosmological constant: $w = -1$
- New data around the corner:
  - *Very large redshift samples (SDSS-LRG)*
  - *Very large supernova samples (SNAP)*
  - *Very large CMB maps (MAP, PLANCK)*
- Combined analysis

Combining Methods

- Eisenstein and Hu (1992)
- Different methods have different degeneracies
- Combinations at the likelihood level yield huge improvements



Summary

- Cosmology became an experimental science,
  we are performing physical experiments
- Many of the outstanding questions are statistical
- Sky surveys provide huge, uniform data sets
- The geometric properties of the Universe are known
- The primordial fluctuation spectrum is ‘shaping up’
- Lots of very hard work is needed to reach precision!
- We still have outstanding challenges

Precision Cosmology

- Over the next three years redshift surveys will
  - *Detect the baryon bumps*
  - *Constrain the dark energy*
  - *Constrain the neutrino mass*
  - *Contribute substantially to constraints when combined with CMB*
- The end of an era…

- Martin Rees (2001):
  "soon cosmology will bifurcate"…
  - *'brane' cosmology*
  - *'environmental' cosmology*