Thermalization in quantum fields

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Outline

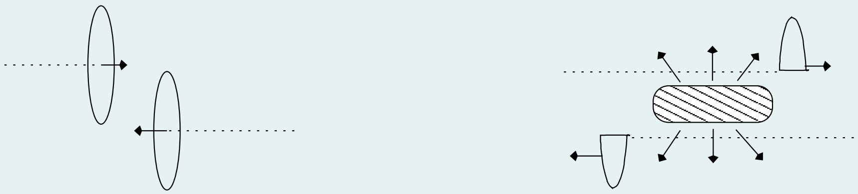
Motivation
2PI effective action formalism
Lorentz structure of the fermionic two-point functions
Discretization
Numerical results

(J. Berges, Sz. B., J. Serrau, hep-ph/0212404)

Applications of nonequilibrium field theory

1. Dynamics of early universe fields
   - Reheating dynamics *(mainly classical, Hartree)*
   - Formation of topological structures *(classical)*
   - Development of the radiation dominated universe

2. Heavy ion collisions



$$t_{freeze-out} \approx 5fm$$

   - Thermalization time scale
   - Rethermalization after critical behaviour

3. Statistical field theory
   - Equilibration of a many-body system
   - Emergence of the Bose-Einsten and Fermi-Dirac statistics
   - Renormalization . . .

What is thermalization?

Thermal equilibrium:

$$\hat{\rho} = e^{-\beta \hat{H}} / \mathrm{Tr} e^{-\beta \hat{H}} \qquad \left\langle \hat{X} \right\rangle = \mathrm{Tr} \hat{X} \hat{\rho}$$

Is thermalization possible in closed nonlinear system?

- Ch. Wetterich: Equilibrium is a fixed point of the evolution
- $\rho \not\longrightarrow e^{-\beta \hat{H}} / \mathrm{Tr} e^{-\beta \hat{H}}$ Unitarity!
- $\langle \hat{H} \rangle$=const. uniquely determines the equilibrium ensemble.

  But: $\langle \hat{H}^2 \rangle$, $\langle \hat{H}^3 \rangle$, $\ldots$ conserved *(initial conditions)*
- The quantum ensemble cannot converge to equilibrium!
- Still, the quantum average of some selected observables may converge to the equilibrium value:

$$\langle \Phi(x)\Phi(y) \rangle_{\text{noneq}} \longrightarrow \langle \Phi(x)\Phi(y) \rangle_{\text{thermal}}\,, \quad \text{as } x_0, y_0 \to \infty$$

Toy model: Chiral quark model, symmetric phase

Quantum initial value problem

$$\langle T_c \Phi(y) \Phi(x) \rangle = ?$$
$$\left\langle T_c \bar{\Psi}(y) \Psi(x) \right\rangle = ?$$

The higher connected n-point functions are dropped

A convenient assumtion:
The density operator at $t = 0$ is quadratic

- Switching on the interaction at $t = 0$
- Problems with coupling renormalization:

either the final equilibrium or the initial state is singular

- Considerably simplifies our equations

Task: – Set up explicite equations of motion for the scalar and fermionic two-point functions
– Solve them numerically in the two-time-variable plane (x_0, y_0)

Approximation schemes

Boltzmann equation

1. Small occupation numbers, delute gas
2. On-shell processes only

Classical field theory

1. High occupation numbers (cosmological applications)
2. Nonperturbative dynamics (particle production, domain formation)
3. No quantum effects, failure of the UV description
4. Classical equilibrium $\neq$ quantum equilibrium

Naive strategy for solving the quantum dynamics

1. Equations for the n-point functions built of Heisenberg operators

2a Collisionless approximation:
Connected 4-point functions are neglected $\longrightarrow$ no scattering, no thermalization

2b Inclusion of the 4-point functions, truncation at the 6-point functions:
$\longrightarrow$ solution blows up

Way out: The truncation of the hierachy should be carried out on the level of the two-particle irreducible (2PI) effective action.

Propagator resummation

$\longrightarrow$ stable dynamics

2PI effective action for the fermionic fields

Classical action:

$$S = \int d^4x \left(\bar{\psi}_i(x)[i\partial\!\!\!/ - m_f]\psi_i(x) + V(\bar{\psi}, \psi)\right)$$

Effective action:

$$\Gamma[D] = -i\text{Tr}\ln D^{-1} - i\text{Tr}D_0^{-1}D + \Gamma_2[D] + const$$

$$iD^{-1}_{0,ij}(x,y) = (i\partial\!\!\!/ - m_f)\delta_c^4(x-y)\delta_{ij}$$

Equation of motion:

$$\frac{\delta\Gamma[D]}{\delta D_{ij}}(x,y) = 0$$

Self energy and the propagators (Schwinger–Dyson):

$$D^{-1}_{ij}(x,y) = D_{0,ij}(x,y)^{-1} - \Sigma_{ij}(x,y;D)$$

$$\Sigma_{ij}(x,y;D) = -i\frac{\delta\Gamma_2[D]}{\delta D_{ji}(y,x)}$$

EOM for the propagator:

$$(i\partial\!\!\!/_x - m_f)D_{ij}(x,y) - i\int_z \Sigma_{ik}(x,z;D)D_{kj}(z,y) = i\delta_c^4(x-y)\delta_{ij}$$

Σ contains the infinite power series of the propagators $\longrightarrow$ truncation

Time explicite equations

We consider the real and imaginary part of the propagator:

$$D_{ij}(x,y) = F_{ij}(x,y) - \frac{i}{2}\rho_{ij}(x,y)\mathrm{sgn}_c(x^0,y^0)$$

For fermionic fields:

$$\begin{aligned}(i\partial\!\!\!/ - m - \Sigma_0)\,F(x,y) &= \int^{x_0} dz \Sigma^\rho(x,z)F(z,y) - \int^{y_0} dz \Sigma^F(x,z)\rho(z,y)\\ (i\partial\!\!\!/ - m - \Sigma_0)\,\rho(x,y) &= \int_{y_0}^{x_0} dz \Sigma^\rho(x,z)\rho(z,y)\end{aligned}$$

For scalar fields:

$$\begin{aligned}\left(\partial_x^2 + m^2 + \Sigma_{0,i}(x)\right) F_{ij}(x,y) &= \int^{y_0} dz \Sigma_{ik}^F(x,z)\rho_{kj}(z,y) - \int^{x_0} dz \Sigma_{ik}^\rho(x,z)F_{kj}(z,y)\\ \left(\partial_x^2 + m^2 + \Sigma_{0,i}(x)\right) \rho_{ij}(x,y) &= \int_{x_0}^{y_0} dz \Sigma_{ik}^\rho(x,z)\rho_{kj}(z,y)\end{aligned}$$

The collision terms appear in the forms of memory kernels.

The equations are time-reversal symmetric *(Energy is conserved)*

Lorentz structure

$$\rho = \rho_S + i\gamma_5\rho_P + \gamma_\mu\rho_V^\mu + \gamma_\mu\gamma_5\rho_A^\mu + \tfrac{1}{2}\sigma_{\mu\nu}\rho_T^{\mu\nu}$$ and similarly for F

Symmetry requirements imposed on the initial conditions:

- Space reflection and rotation: $\longrightarrow \rho_A^0 = 0$, $\rho_A = 0$
- CP symmetry

$$\rho_V^0(x^0,y^0;p) = \rho_V^0(y^0,x^0;p), \quad \rho_V(x^0,y^0;p) = -\rho_V(y^0,x^0;p),$$
$$F_V^0(x^0,y^0;p) = -F_V^0(y^0,x^0;p), \quad F_V(x^0,y^0;p) = F_V(y^0,x^0;p),$$

$\longrightarrow \rho_V$, F_V are real, ρ_V^0, F_V^0 are imaginary

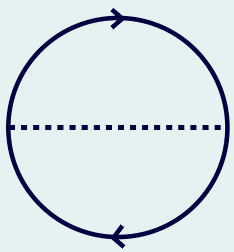
Discretiation:

- First order space derivative can be transformed out
  $\longrightarrow$ *no spatial fermion doubling occurs*
- The time-like lattice spacing is much less than the spatial one
  $\longrightarrow$ *no time-like doubling either*
- The discretized equations for ρ_V, ρ_V^0 and F_V^0, F_V are reminiscent of the standard Leap-frog prescription for the canonica co-ordinate and momentum.
  $\longrightarrow$ *stable numerics*

The formulation of the equations by means of the two-point functions gives a way to avoid the fermion doubling problem.

The order of approximation

Self energies form a coupling constant expansion:

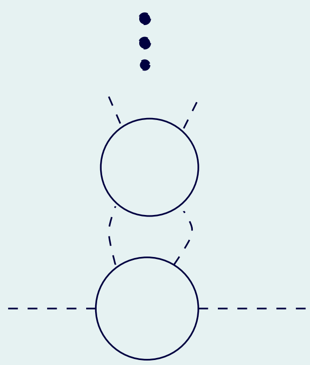


$$\Sigma_\phi^\rho(x^0,y^0;\vec{p}) = -8g^2N_f \int \frac{\mathrm{d}^3q}{(2\pi)^3}\,\rho_V^\mu(x^0,y^0;\vec{q})\,F_{V,\mu}(x^0,y^0;\vec{p}-\vec{q})\,,$$

$$\Sigma_\phi^F(x^0,y^0;\vec{p}) = -4g^2N_f \int \frac{\mathrm{d}^3q}{(2\pi)^3}\Big[F_V^\mu(x^0,y^0;\vec{q})\,F_{V,\mu}(x^0,y^0;\vec{p}-\vec{q}) - \frac{1}{4}\rho_V^\mu(x^0,y^0;\vec{q})\,\rho_{V,\mu}(x^0,y^0;\vec{p}-\vec{q})\Big]\,,$$

$$\Sigma_V^{\rho,\mu}(x^0,y^0;\vec{p}) = -g^2N_s \int \frac{\mathrm{d}^3q}{(2\pi)^3}\Big[F_V^\mu(x^0,y^0;\vec{q})\,\rho_\phi(x^0,y^0;\vec{p}-\vec{q}) + \rho_V^\mu(x^0,y^0;\vec{q})\,F_\phi(x^0,y^0;\vec{p}-\vec{q})\Big]\,,$$

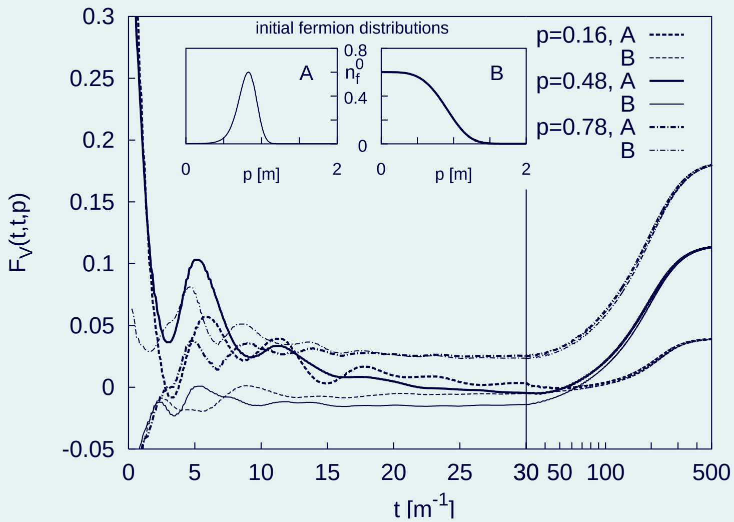
$$\Sigma_V^{F,\mu}(x^0,y^0;\vec{p}) = -g^2N_s \int \frac{\mathrm{d}^3q}{(2\pi)^3}\Big[F_V^\mu(x^0,y^0;\vec{q})\,F_\phi(x^0,y^0;\vec{p}-\vec{q}) - \frac{1}{4}\rho_V^\mu(x^0,y^0;\vec{q})\,\rho_\phi(x^0,y^0;\vec{p}-\vec{q})\Big]\,.$$

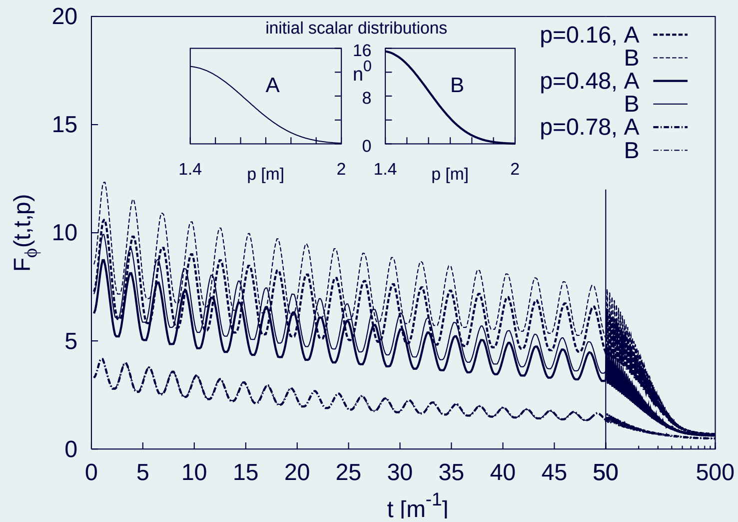


Infinite ladder diagrams are summed

Loss of initial information

- Different initial fermion number distribution
- Equal (conserved) energy density
  (uniquely determines the final temperature)
- The equations obey the time-reflection symmetry





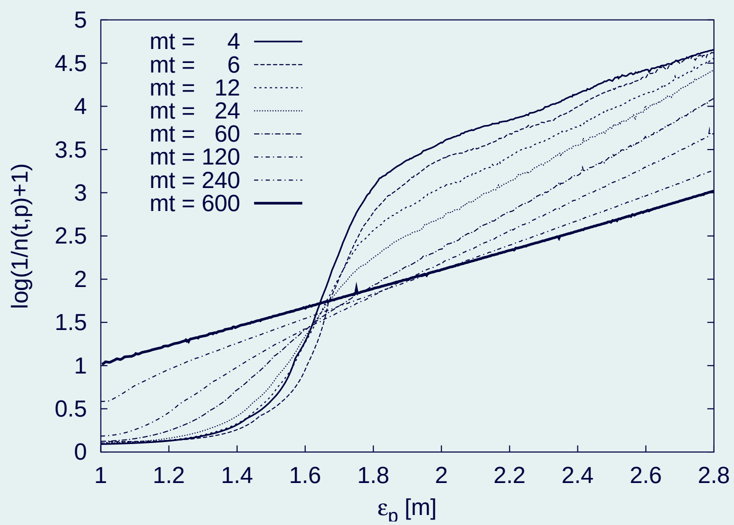
Evolution of the particle distribution

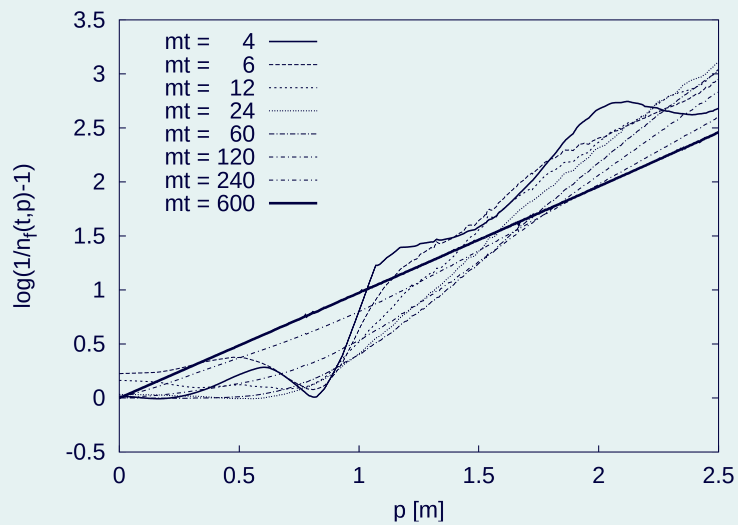
- Particle number distribution from the quasiparticle picture:

$$F_V(t,t';p) = \left(\frac{1}{2} - n^f_{qp}(p)\right)\cos[p(t-t')];$$

$$F_\phi(t,t';p)\big|_{t=t'=\text{now}} = \frac{1}{\epsilon_0}(p)\left[n^s_{qp}(p) + \frac{1}{2}\right],$$

$$\partial_t\partial_{t'}\, F_\phi(t,t';p)\big|_{t=t'=\text{now}} = \epsilon_0(p)\left[n^s_{qp}(p) + \frac{1}{2}\right],$$





- The emerging distribution is thermal,

  its β parameter defines the *quasiparticle temperature*

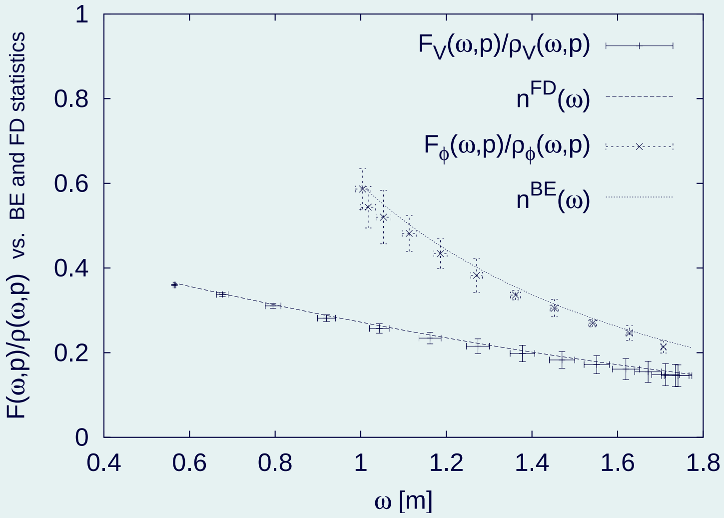
Final equilibrium

Exact equilibrium relation:

$$F_{S/F}^{thermal}(\omega,p) = \left(n_{BE/FD} \pm \tfrac{1}{2}\right) \rho_{S/F}^{thermal}(\omega,p),$$

with $n_{BE} = \frac{1}{e^{\beta\omega}-1}$

and $n_{FD} = \frac{1}{e^{\beta\omega}+1}$



Numerical solution from first principles:

Initially:

$F_{S/F}$ and $\rho_{S/F}$ are independent

The dynamics leads to

$$F_{S/F} \sim \rho_{S/F}$$

With the measured quotient

$$n_{S/F}^{\text{experiment}} \pm \frac{1}{2}$$

being equal (within error) to

$$n_{S/F}^{\text{experiment}} = n_{BE/FD}$$

1. First numerical evindence for thermalization in $3+1$ dimensions
2. First observation of the formation of the Fermi-Dirac statistics
3. Estimate for thermalization time (RHIC) $\tau^{thermalization} \lesssim 1\text{fm}$
4. *Fermionic preheating . . .*