Backreaction effects about pure de Sitter

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Outline of Talk

- Why look at de Sitter (dS)?
  - Slow-Roll inflation vs. dS
  - Linear vs. nonlinear quantum fluctuations
- Our approximation: passive quantum gravity
  - Spacetime fluctuations induced only by stress-energy quantum fluctuations
- Classical metric and matter fluctuations in dS
  - Gauge fixing procedure
  - Background slicing conditions
  - Global constraints on the linear fluctuations
    - Example of these global constraints: Einstein static metric and matter backreactions

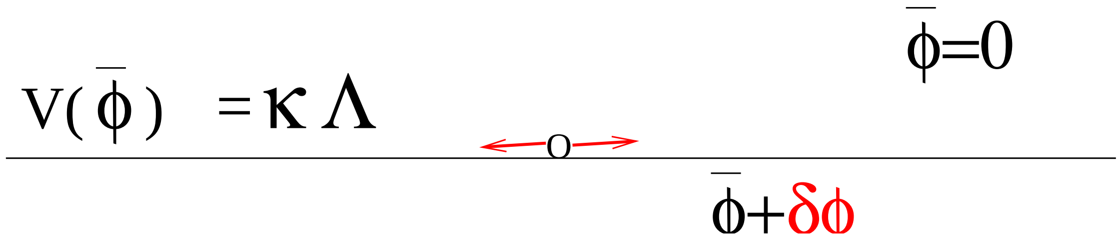
Outline of Talk (cont'd)

- Quantum issues: QFT in dS
- Summary and conclusion

Slow-Roll vs. de Sitter

- Slow-roll:

- de Sitter:



- We want to study quantum perturbation effects in dS (which are quadratic) and ultimately compare them to the linear fluctuations in inflation.

Passive quantum gravity

- In a full quantum theory spacetime will fluctuate of its own accord.
- To the extent that matter fields source gravity, we study the gravity fluctuations induced only by matter fluctuations.
- One replaces the classical T_{ab} with the quantum expectation value $< T_{ab} >$ to obtain

$$G_{ab} + \underbrace{\Lambda_0 g_{ab} + A^{(1)} H_{ab} + B^{(2)} H_{ab}}_{\text{Counterterms}} = 8\pi G_0 < T_{ab} >$$

- By adjusting G_0, Λ_0, A, B we renormalize $< T_{ab} >$.
- But first we need to study the classical metric and matter fluctuations.

Classical matter, metric fluctuations (dS)

- Consider perturbing the field equations of gravity with scalar matter ϕ and metric g:

$$G_{ab}(g_{ab}) = \kappa T_{ab}(g_{ab}, \phi)$$

- In deSitter, $\bar{\phi} = 0$, so the leading order equations are of the character

$$\begin{aligned} \mathcal{L}_{ab}(\delta^2 g_{ab}) &= \kappa \mathcal{Q}_{ab}(\delta\phi)(\delta\phi), \quad \text{where} \\ g_{ab} &= \bar{g}_{ab} + \delta^2 g_{ab} \\ \phi &= \bar{\phi} + \delta\phi \end{aligned}$$

- Here, $\delta^2 g_{ab}$ represents the leading order gravitational perturbation due to the quadratic matter fluctuation $(\delta\phi)(\delta\phi)$
- Which gauge should we pick for $\delta^2 g_{ab}$ and $\delta\phi$?

Picking a gauge for the metric

- At linear order, a gauge transformation is

$$\delta^2 \tilde{g}_{ab} = \delta^2 g_{ab} + \pounds_\zeta \bar{g}_{ab}$$

- One can find ζ^a such that

$$\bar{\nabla}^b \delta^2 \tilde{g}_{ab} = \frac{1}{2} \bar{\nabla}_a [\bar{g}^{\ell m} \delta^2 \tilde{g}_{\ell m} - \frac{\kappa}{2} \underbrace{(\delta\phi)^2}_{\delta\phi = \delta\tilde{\phi}}]$$

- Then the *trace* of the field equations is

$$\bar{g}^{ab} \left[-(\bar{\nabla}^\ell \bar{\nabla}_\ell - 2) \delta^2 \tilde{g}_{ab} + \frac{\bar{g}_{ab}}{2} (\bar{\nabla}^\ell \bar{\nabla}_\ell + 2) \delta^2 \tilde{g} \right] = \bar{g}^{ab} \tilde{\mathcal{Q}}_{ab}$$

(i.e., no matter dependence) $= 0$

- $\Rightarrow$ *Further* set $\bar{g}^{ab} \delta^2 \tilde{g}_{ab} = 0, \delta^2 \tilde{g}_{0i} = 0$ consistently!

Solving the perturbation equations

- In our gauge we can write the metric perturbation as

$$\delta^2\tilde{g}_{ab} = \begin{pmatrix} -\left(\frac{1}{\bar{g}^{00}}\right)\bar{g}^{ij}\delta^2\tilde{g}_{ij} & 0 \\ 0 & \delta^2\tilde{g}_{ij} \end{pmatrix}$$

- Therefore only the $\delta^2\tilde{g}_{ij}$ are independent in this gauge, but note they are neither traceless nor transverse since

$$\bar{\nabla}^j\delta^2\tilde{g}_{ij} = -\frac{\kappa}{4}\bar{\nabla}_i(\delta\phi)^2 \neq 0$$

- Solve the field equations for $\delta^2\tilde{g}_{00}$, we find:

$$\delta^2\tilde{g}_{00} = \frac{\kappa}{2(L_g(L_g+2)+2)}\mathcal{F}[(\delta\phi)(\delta\phi)],$$

Background slicing conditions

- We can use compact S^3 slices: the covers all of dS with

$$ds^2 = -dt^2 + \cosh^2(t)(d\chi^2 + \sin^2(\chi)d\Omega_2^2),$$

for $-\infty < t < \infty, 0 \le \chi \le \pi, 0 \le \theta \le \pi, 0 \le \phi \le 2\pi$

- Noncompact flat $\Re^3$ slices: this covers only half of dS

$$ds^2 = -dt^2 + e^t \left(\delta_{ij} dx^i dx^j\right)$$

- Compact spherical slicing: only covers half of dS

$$ds^2 = -(1 - r^2)dt^2 + \frac{1}{1 - r^2} dr^2 + r^2 d\Omega_2^2$$

- Choosing a compact slicing implies quadratic global constraints on the linear metric and matter fluctuations.

Einstein static (Es) global constraint

- How does one formally solve the (Es) second order (backreaction) **Hamiltonian constraint**

$$\hat{\mathcal{L}}_{00}(\delta^2 g_{ab}; \delta^2 \phi_{ab}) = {}^{(2)}\mathcal{S}_{00}\left((\delta g_{ab})^2; (\delta \phi_{ab})^2\right) \quad ?$$

- First (nonlocally) decompose the perturbations $\delta^n g_{ab}$:

$$\delta^n g_{ab} = \underbrace{\delta^n g_{ab}^{(TT)} + \hat{\mathcal{B}}_{ab}\delta^n g^{(Tr)} + \bar{g}_{ab}\delta^n \mathcal{A}}_{\text{Transverse}} + \underbrace{\delta^n g_{ab}{}^{(L)}}_{\text{Longitudinal}}$$

- Then we have to invert $\hat{\mathcal{L}}_{00}$, which is elliptic and self-adjoint (because of spatial compactness).

Invertibility condition $\Longrightarrow$

$$\int_{S^3} \underbrace{H}_{\hat{\mathcal{L}}_{00} H = 0} {}^{(2)}\mathcal{S}_{00}\sqrt{|\bar{g}|}\, d^3x = 0$$

Backreactions in Einstein Static (cont'd)

- The situation to linear order: **Unstable** to linear homogeneous scalar perturbations. **Stable** (neutrally) to inhomogeneous scalar perturbations. **Stable** to tensor and vector perturbations on all scales.
- What linear modes are needed for the backreactions to be integrable?
- The integrability condition for backreactions is

$$\int_{S^3} \underbrace{A[(\delta g_{ab}^{(TT)})^2, (\delta g^{(Tr)})^2]}_{\text{inhomogeneous}} - \underbrace{B[(\delta \mathcal{A})^2]}_{\text{homogeneous}} \, d^3x = 0$$

- For $B = 0$ there are no nontrivial δg_{ab}.
- Thus the backreaction equations have **no** solution unless we include the destabilizing modes.

Quantum aspect of dS global constraints

- For dS, $\exists$ similar constraints both classical and quantum.
- At the quantum level in vacuum dS, they impose SO(4,1) invariance on all linear *vacuum* graviton states.
- Generally, there are no SO(4,1) invariant *scalar* states without infrared divergences. But here ok since dS is closed.
- Do we use just SO(4,1) invariant states?

Summary and Conclusion

- About dS generated by some VEV of ϕ ,we study the leading order gravitational fluctuations induced by the *quadratic* ϕ fluctuations .
  - dS generated by VEV of ϕ means that Λ is not a hand-picked constant but is *determined* by the vacuum energy of ϕ
- We gauge fix the gravitational sector to 1 scalar mode plus gravitational waves, the matter fluctuations are gauge invariant at linear order.

Summary and Conclusion (cont'd)

- Within this gauge we solve the classical Hamiltonian constraint.
- To solve the semiclassical Hamiltonian constraint globally, we choose a background slicing such that $M = S^3 X \Re^1$
- This background choice in turn imposes *global* constraints on the class of allowable states for the gravitational radiation and matter fluctuations.
- Since the spacetime is closed we have only one unitary equivalence class of states to worry about and now we have a 'preferred' SO(4,1) invariant vacuum
- What do the semiclassical solutions tell us for VEV's taken in these states?

Acknowledgements

- NSERC Canada
- I also gratefully acknowledge the generous financial support of the Eotovos Graduate Course and Workshop in Physics