Merging Matrix Elements and Parton Showers at LO

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Outline :

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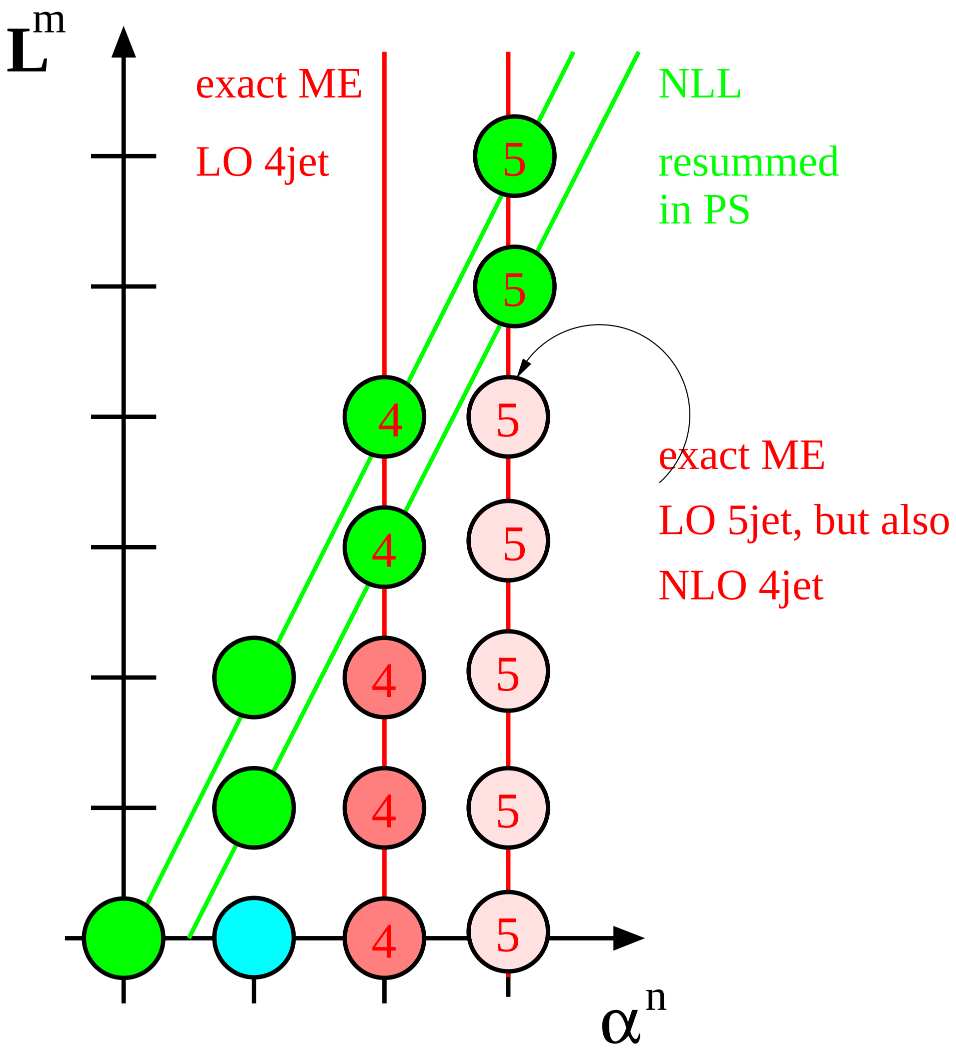
   - Matrix Elements vs. Parton Shower
2. Merging ME & PS for e^+e^-
   - Catani-Krauss-Kuhn-Webber
3. Merging in Hadron Collision
4. Results for Sherpa

Matrix Elements vs. Parton Shower

- ME is **exact** at some given order in the coupling constant α_s etc..
  $\Longrightarrow$ ME is "exclusive".

- PS **resums** all orders of $\alpha_s^n(L^{2n} + L^{2n-1})$.
  L are large logs like $\ln s/Q_0^2$
  $\sqrt{s}$ is cm energy, Q_0^2 is some infrared scale.
  $\Longrightarrow$ PS is "inclusive".



Merging ME & PS for e^+e^-

S. Catani, F. Krauss, R. Kuhn, B. Webber, JHEP 0111:063,2001

Aim:

- Good description of soft and hard region
- Universality of fragmentation (energy dependent)

Merging algorithm:

1. Produce jets (energetic particles, large angles) by ME
2. Evolve jets by PS down to fragmentation scale
3. Divide phase space into jet production & jet evolution by the Durham jet algorithm
$$2 \cdot \min\{E_i^2,\ E_j^2\} \cdot (1 - \cos\theta_{ij}) \gtrless y_{\rm jet} E_{\rm cm}^2$$
4. Weight ME's and Veto on PS.

Determine NLL-weight for MEs

Consider jet rates at next-to leading log accuracy with $Q_{\text{jet}} = y_{\text{jet}} E_{\text{cm}}$:

$$\mathcal{R}_2(Q_{\text{jet}}, E_{\text{cm}}) = \left[\Delta_q(Q_{\text{jet}}, E_{\text{cm}})\right]^2$$

$$\mathcal{R}_3(Q_{\text{jet}}, E_{\text{cm}}) = 2\left[\Delta_q(Q_{\text{jet}}, E_{\text{cm}})\right]^2 \int_{Q_{\text{jet}}}^{Q} dq\, \Gamma_q(q, E_{\text{cm}})\, \Delta_g(Q_{\text{jet}}, q)$$

where

$\Delta(p,q)$ = No branch between p and q (Sudakov form factor),

$\Gamma(q)$ = one branch at q including coupling constant etc.

Sudakov Form factor in NLL

$\Longrightarrow$ Sudakov form factor in NLL

$$\Delta_q(Q_{\min}, Q_{\max}) = \exp\left(- \int_{Q_{\min}}^{Q_{\max}} dq\, \Gamma_q(q, Q_{\max}) \right)$$

$$\Delta_g(Q_{\min}, Q_{\max}) = \exp\left(- \int_{Q_{\min}}^{Q_{\max}} dq\, \left[\Gamma_g(q, Q_{\max}) + \Gamma_f(q)\right] \right)$$

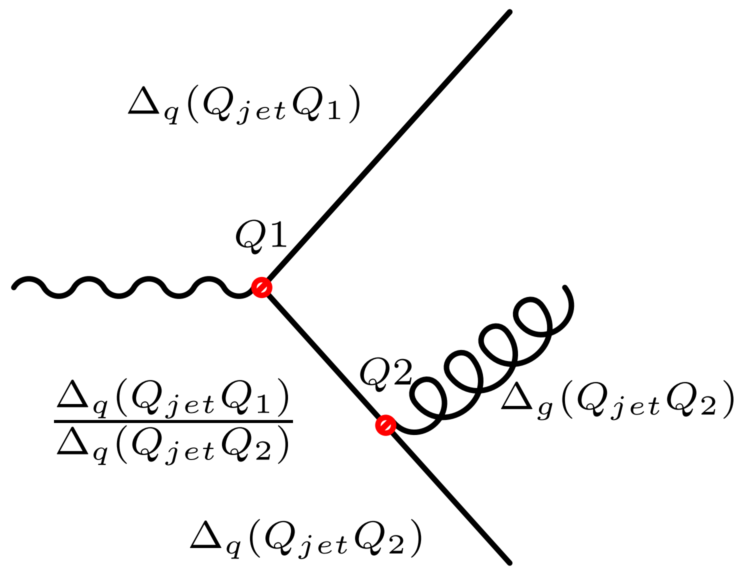
with new kernel

$$\begin{aligned}
\Gamma_q(q, Q) &= \frac{2C_F}{\pi} \frac{\alpha_s(q)}{q} \left(\log \frac{Q}{q} - \frac{3}{4} \right) \\
\Gamma_g(q, Q) &= \frac{2C_A}{\pi} \frac{\alpha_s(q)}{q} \left(\log \frac{Q}{q} - \frac{11}{12} \right) \\
\Gamma_f(q) &= \frac{N_f}{3\pi} \frac{\alpha_s(q)}{q}
\end{aligned}$$

Determine Matrix Element Kinematics

1. Choose vectors p_i according to M.E. with strong coupling α_s at $Q_{\rm jet}$.
2. Look for "shower" kinematics :
   - Merge $p_i + p_j = p_{ij}$ with smallest y_{ij}.
   - Repeat "parton–merging" until core $2 \to 2$ process.
3. Generate weight with factors
   - $\Delta_{q,g}(Q_{\rm jet}, Q_{\rm prod})$ for outgoing partons
   - $\Delta_{q,g}(Q_{\rm jet}, Q_{\rm prod})/\Delta_{q,g}(Q_{\rm jet}, Q_{\rm dec})$
4. Reweight for different scales in α_s by factors

$$\frac{\alpha_s(Q_i)}{\alpha_s(Q_{\rm jet})}$$



$$w = \Delta_g(Q_{jet}Q_2)[\Delta_q(Q_{jet}Q_1)]^2 \frac{\alpha_s(Q_2)}{\alpha_s(Q_{\rm jet})}$$

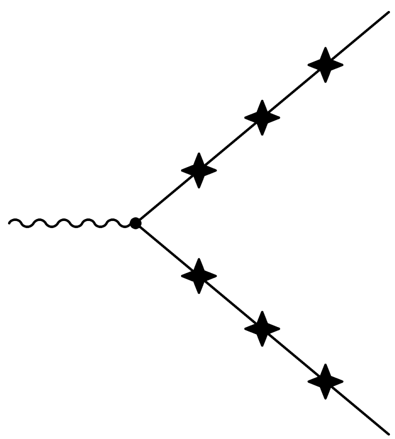
Vetoed showers

Start scale of shower:

- Naive guess $Q_{\rm jet}$.
- Consider $\mathcal{R}_2(Q_0 < Q_{\rm jet})$.

  Starting from $Q_{\rm jet}$: $\left[\Delta_q(Q_0, Q_{\rm jet})\Delta_q(Q_{\rm jet}, Q)\right]^2$

  instead of : $\left[\Delta_q(Q_0, Q)\right]^2$
- Better choice : Start shower at Q and veto all emissions that would lead to an extra jet, i.e. $q > Q_{\rm jet}$.



Sum for one quark–line :

$$\Delta_q(Q_0, Q)\Delta_q(Q_{\rm jet}, Q)\left[1 + \int_{Q_{\rm jet}}^{Q} dq\, \Gamma_q(q, Q) + \dots\right]$$

$$\Longrightarrow \frac{\Delta_q(Q_0, Q)\Delta_q(Q_{\rm jet}, Q)}{\Delta_q(Q_{\rm jet}, Q)} = \Delta_q(Q_0, Q).$$

Merging ME & PS in Hadron Collision

F. Krauss,JHEP 0208:015,2002

Algorithm

1. Determination of jet rates at an initialization scale
2. Distribute the jet momenta according to the ME
3. Backward cluster the partons
4. Apply a reweighting (Sudakov form factors and coupling constants)
5. Initialization of the parton shower from initial values obtained during step 3

Determination of jet rates at the initialization scale

Cross section for the production of a n-jet event:

$$\sigma_n = \int dx_1\, dx_2\, f_1(x_1, Q^2)\, f_2(x_2, Q^2)\, \sigma(x_1\, x_2\, E^2_{\text{cms}})\Big|_{y_{ij} > y_{\text{cut}}}$$

Jet measure : covariant E-scheme

$$y_{ij} = \frac{\min(p^2_{\perp i}, p^2_{\perp j}) \cdot R_{ij}}{E^2_{\text{cms}}} \qquad \text{or} \qquad y_{\text{B}j} = \frac{p^2_{\perp j}}{E^2_{\text{cms}}}$$

$$R_{ij} = 2\,[\cosh(\eta_i - \eta_j) - \cos(\varphi_i - \varphi_j)]$$

- strong coupling α_S entering the ME fixed at the jet scale Q_{jet}

Initialization jet rates given by

$$j_n = \frac{\sigma_i}{\sum_i \sigma_i}$$

Note, these jet rates are modified by the Sudakov rejection.

Backward Clustering

Aim: determine hard 2 to 2 process, and equivalent PS history

1. possible clusterings are given by all possible Feynman diagrams (`AMEGIC++`)
2. choose the pair of partons with minimum y_{ij}
   (in a frame where both incoming are aligned with the z-axis)
3. favor clusterings with the beam where $p_i^z \cdot p_j^z > 0$
4. repeat all steps until 2 to 2 process is reached

Sudakov rejection weights

1. for every reconstructed splitting of one parton into two

$$\mathcal{W}_{\mathrm{corr.}} = \frac{\Delta_j(Q_{\mathrm{jet}}, Q_{\mathrm{prod}})}{\Delta_j(Q_{\mathrm{jet}}, Q_{\mathrm{dec}})} \cdot \frac{\alpha_s(Q_{\mathrm{dec}})}{\alpha_s(Q_{\mathrm{jet}})}$$

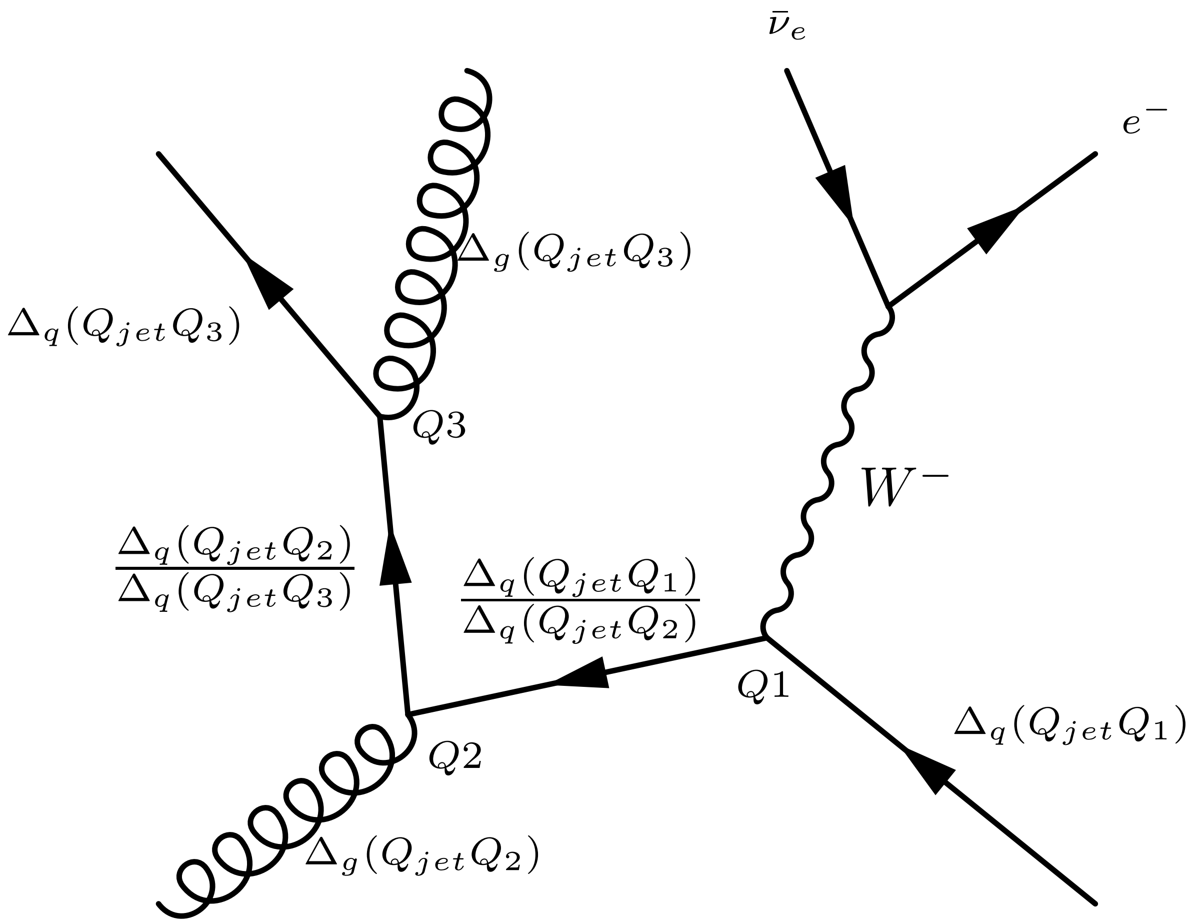
2. for each outgoing particle

$$\mathcal{W}_{\mathrm{corr.}} = \Delta_j(Q_{\mathrm{jet}}, Q_{\mathrm{prod}})$$

the scales are given by

$$Q = \sqrt{y_{ij}} \cdot E_{\mathrm{cms}}$$

Sudakov rejection weights - Example



Initialization of the parton shower

1. Colour flow is determined on the level of the hard $2 \to 2$ matrix element (as obtained by the Backward Clustering)
2. Colours of other partons are fixed by the shower procedure
3. Starting scale of the parton shower is given by the virtuality of the production parton

$$Q^2_{\mathrm{start}} = p^2_{\mathrm{prod}}$$

4. Angular constraints are derived from angle to colour partner

Veto on every jet with $p_\perp^2 > p^2_{\perp\,\mathrm{ini}} = y_{\mathrm{cut}} \cdot E^2_{\mathrm{cms}}$!

Results

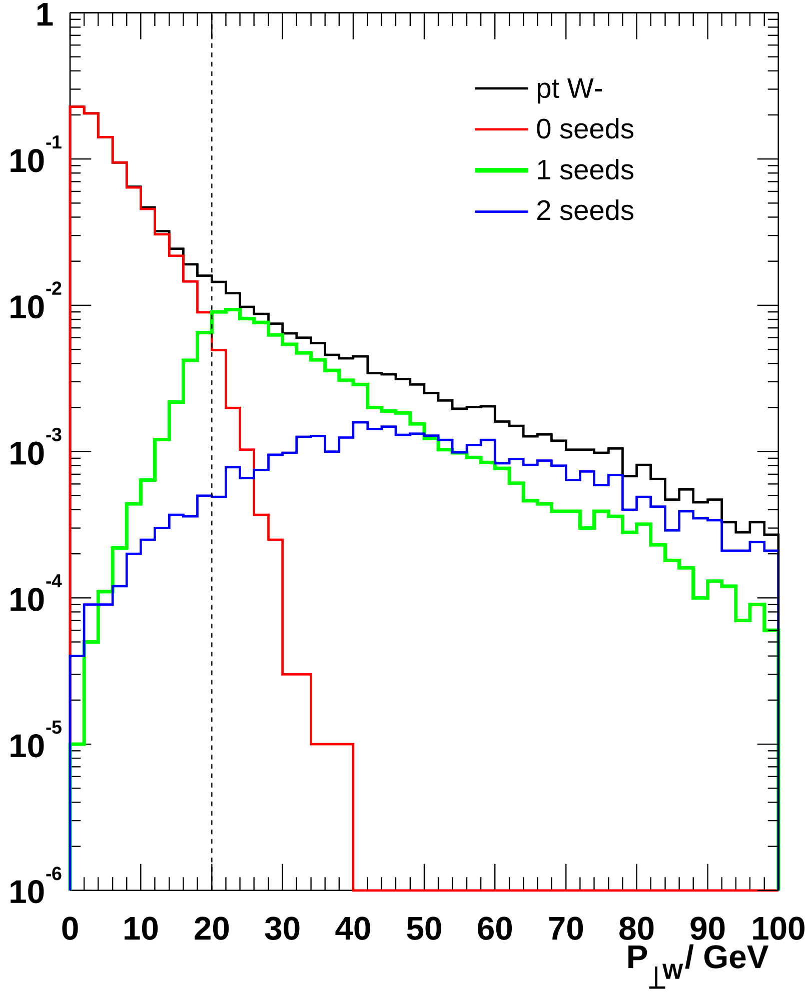
Process:

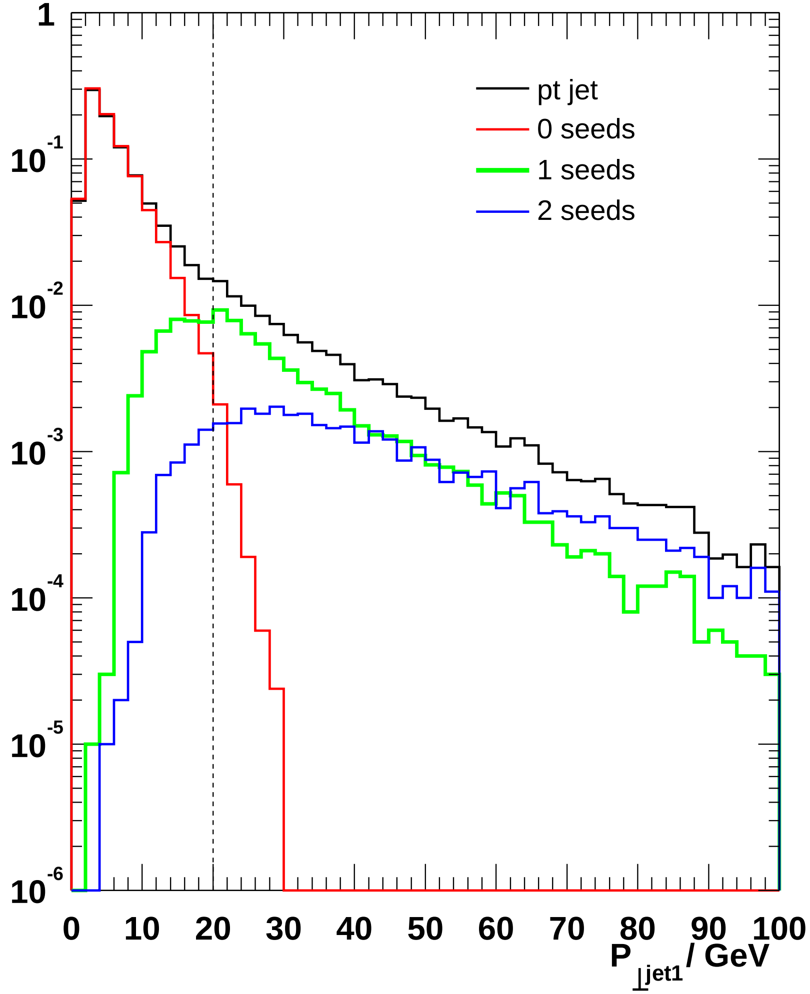
W Production at Tevatron

Variables

- p_T distribution of the W boson
- p_T distribution of the hardest jet
- differential jet rates

W Production at Tevatron





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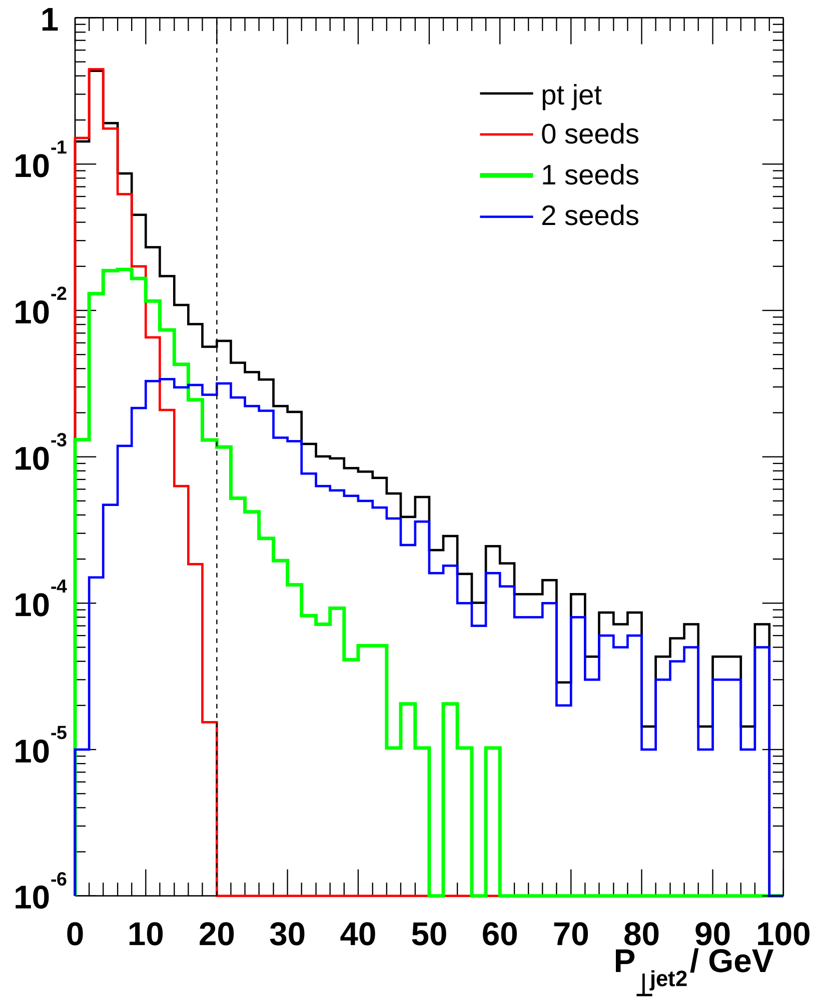

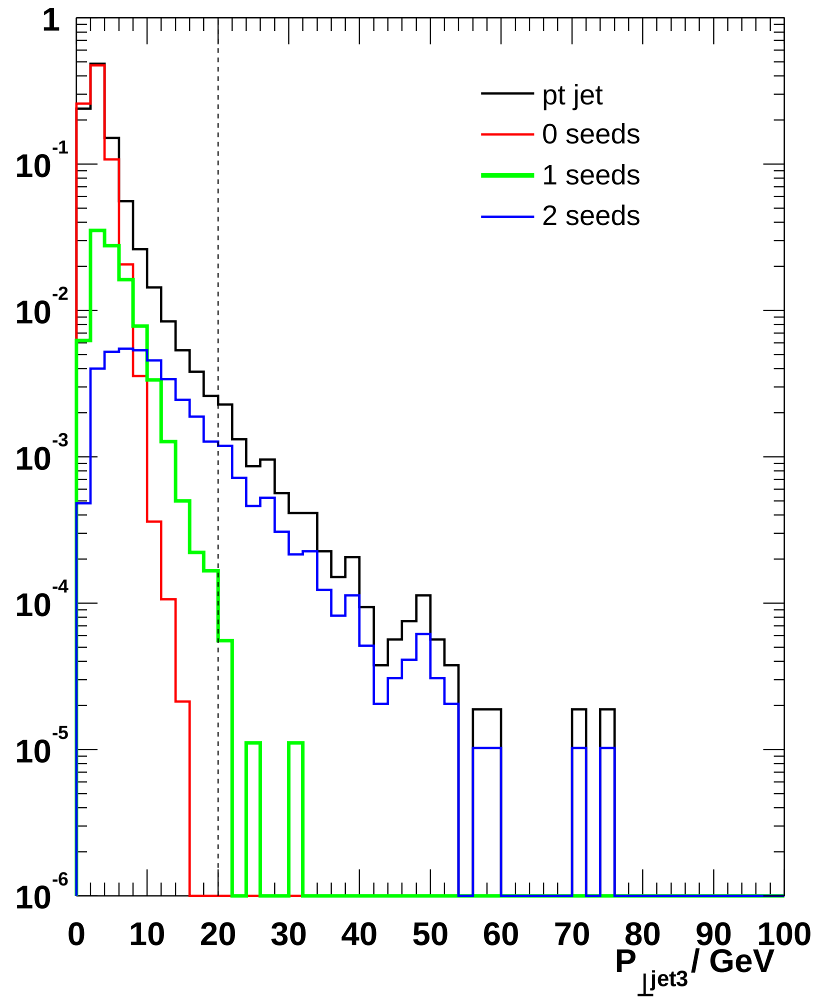

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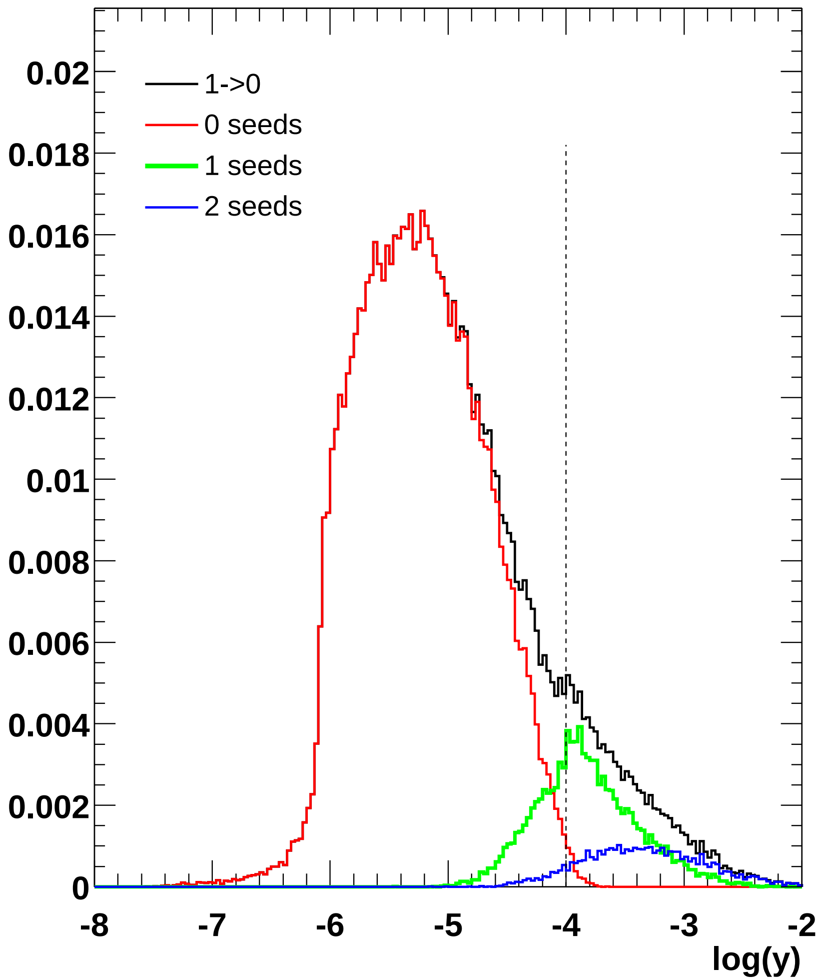

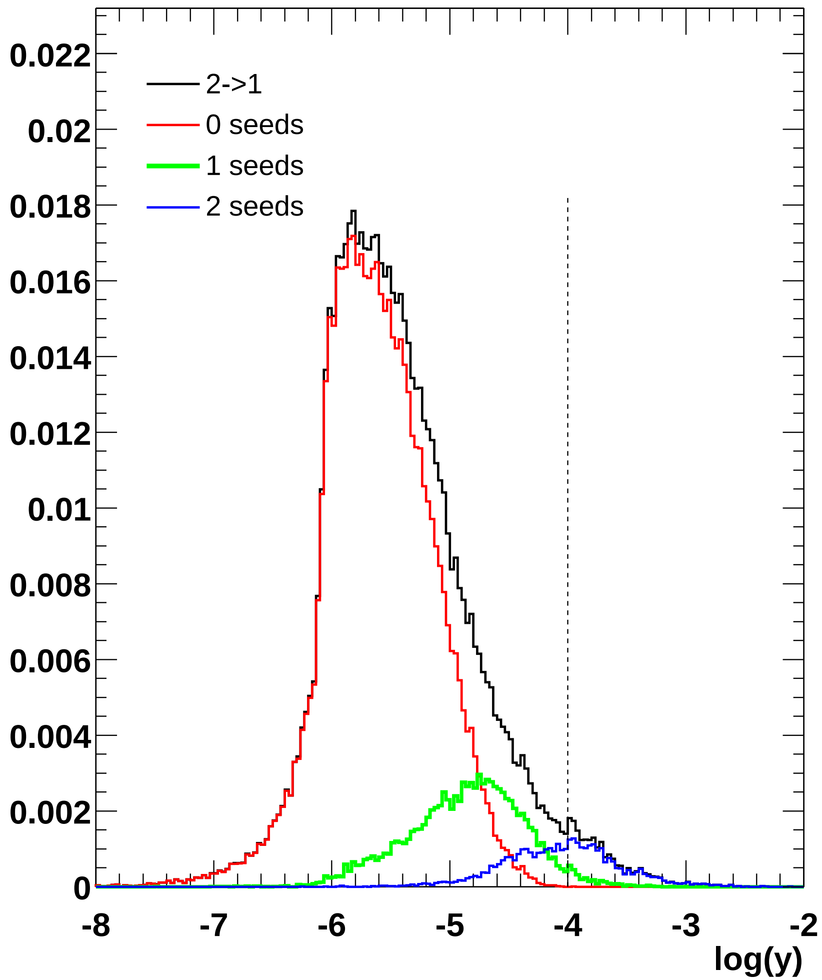

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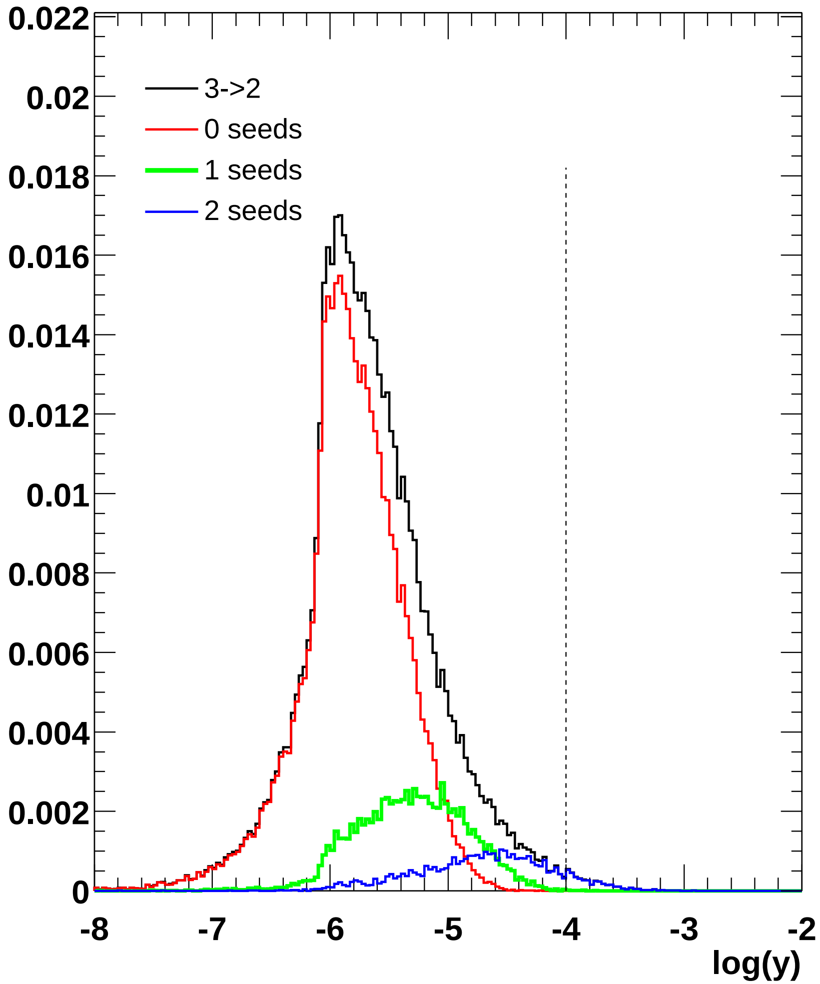

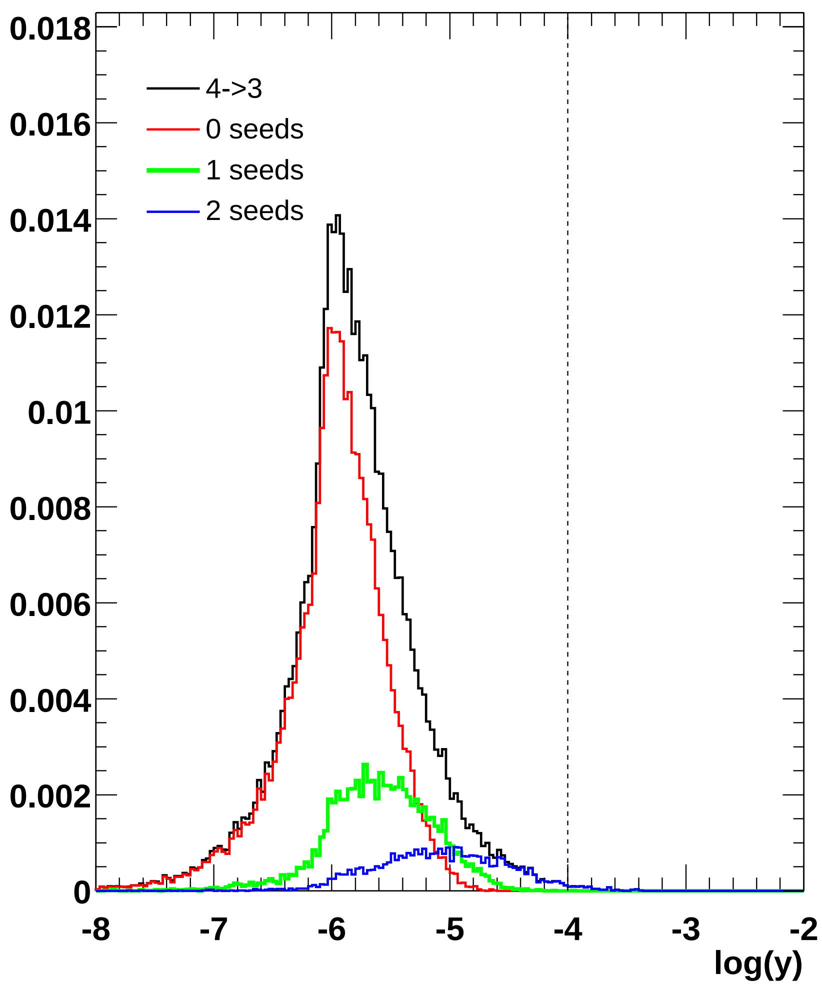

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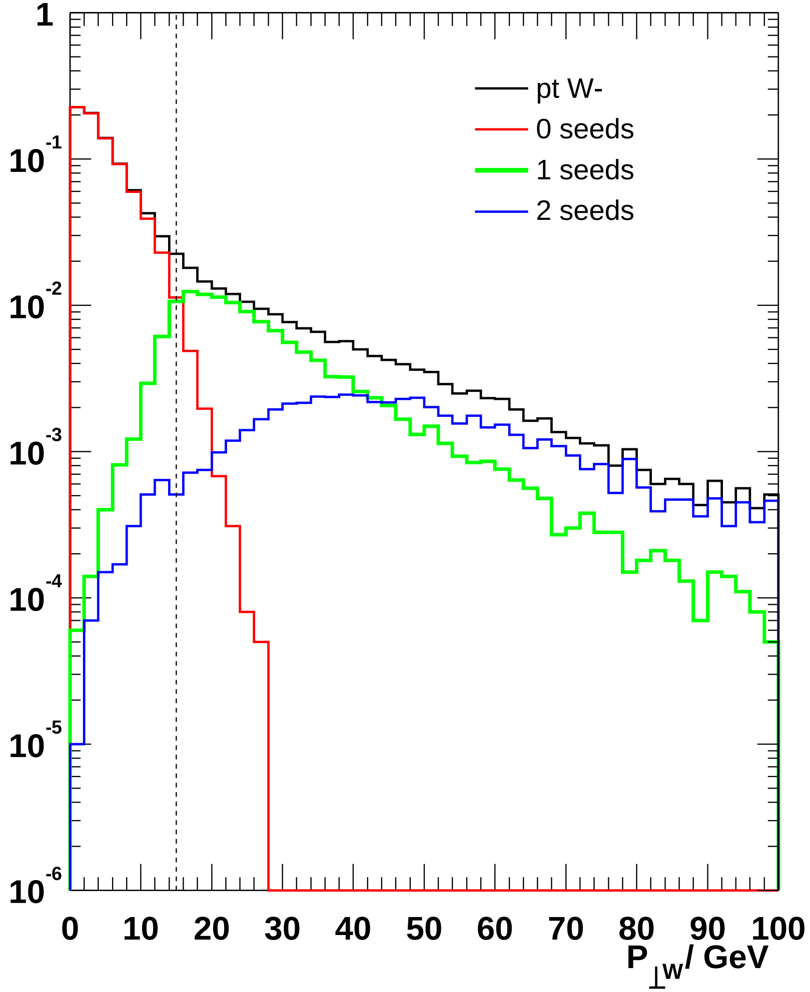

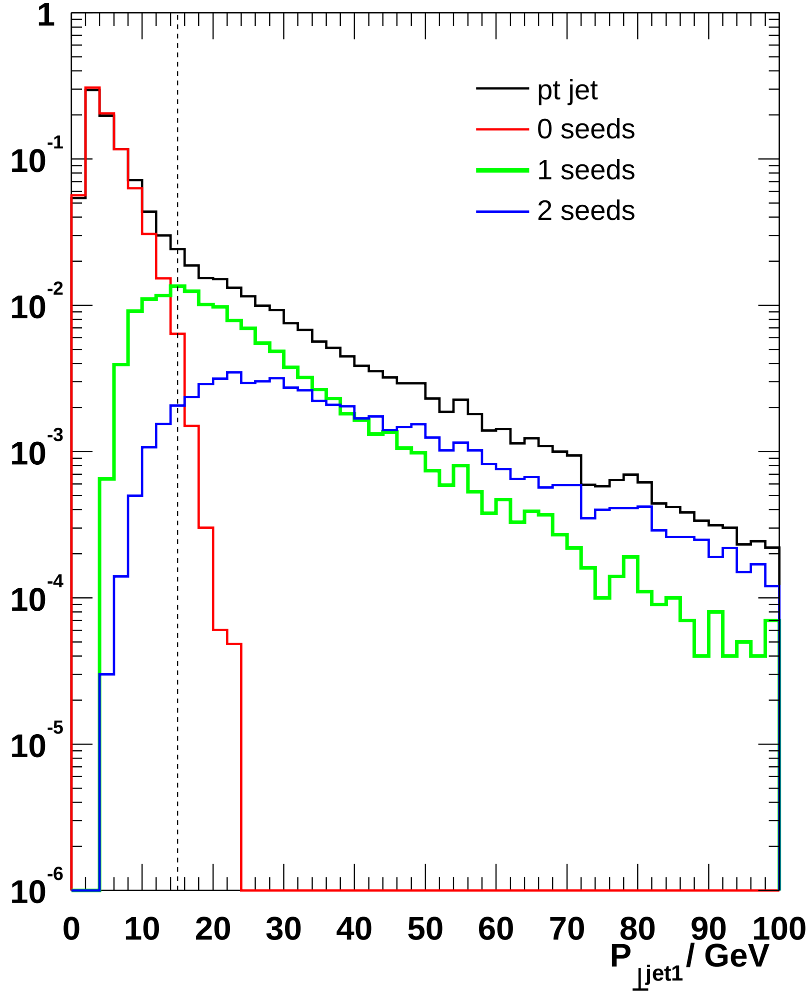

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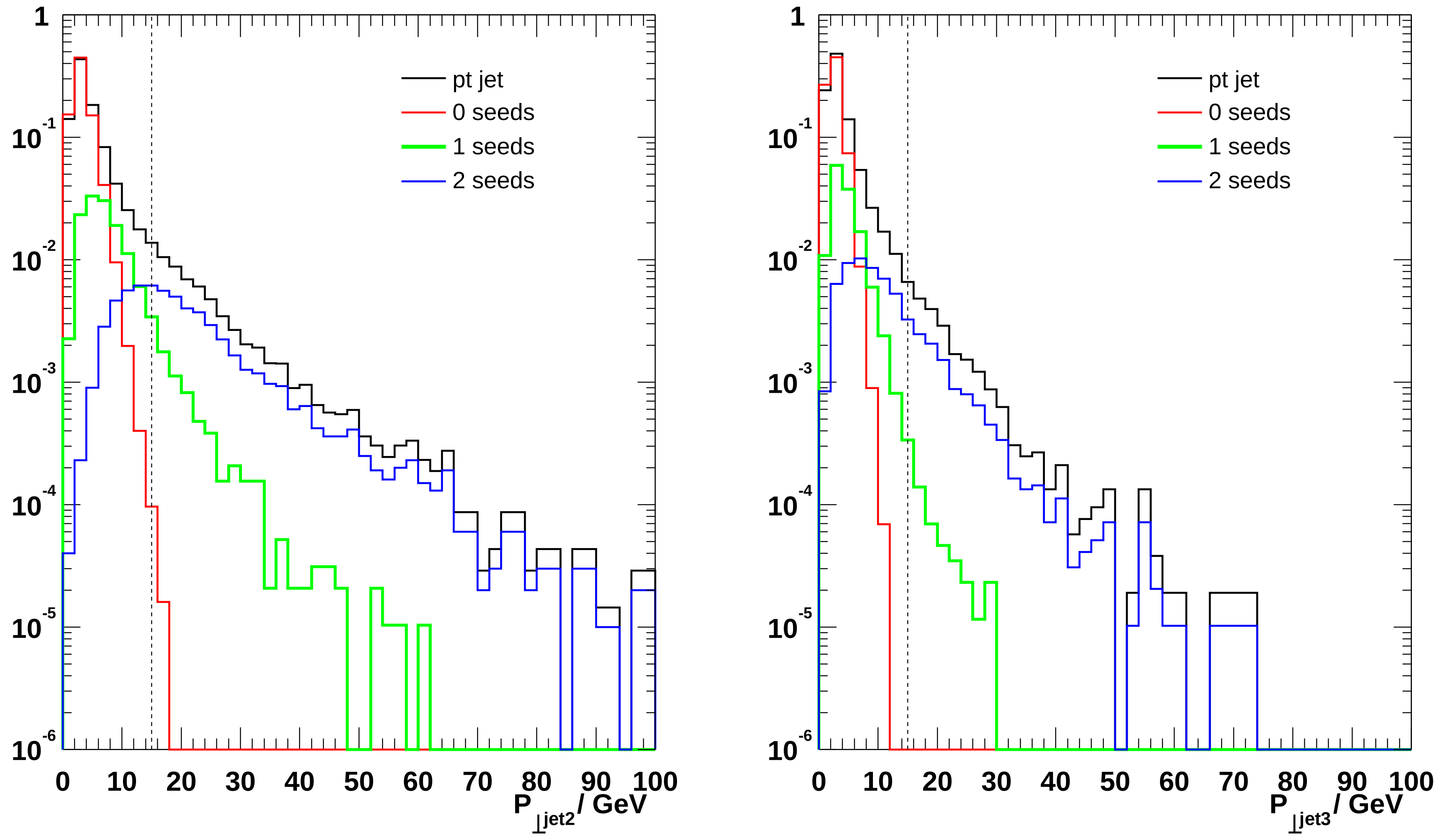

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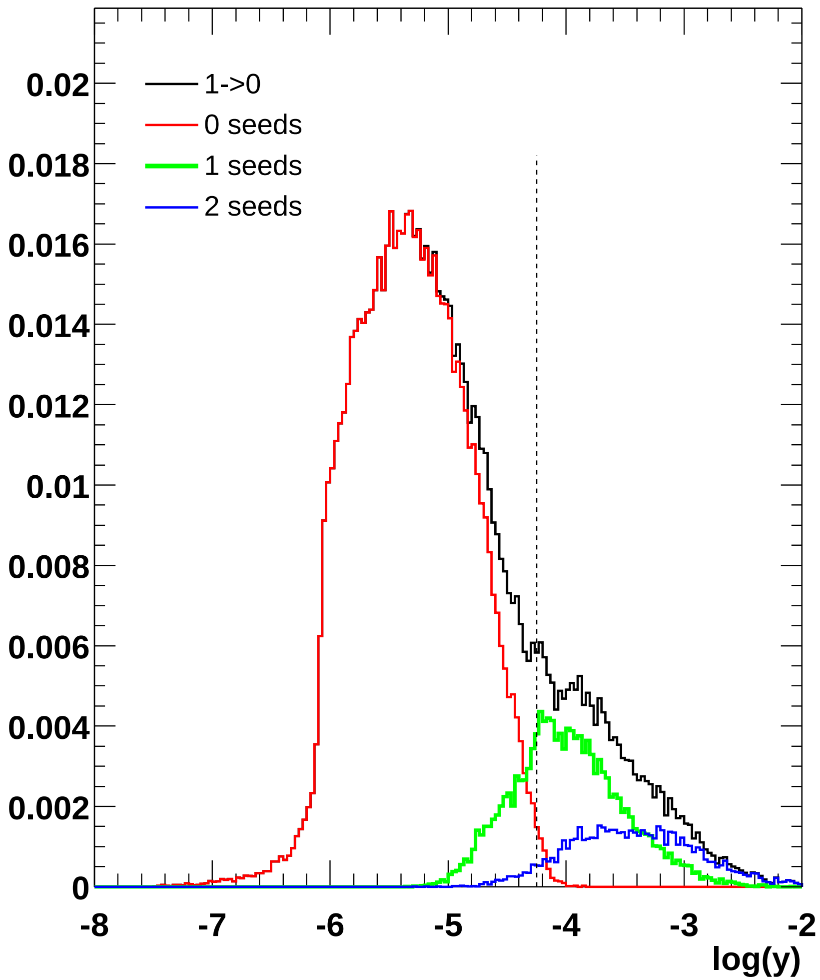

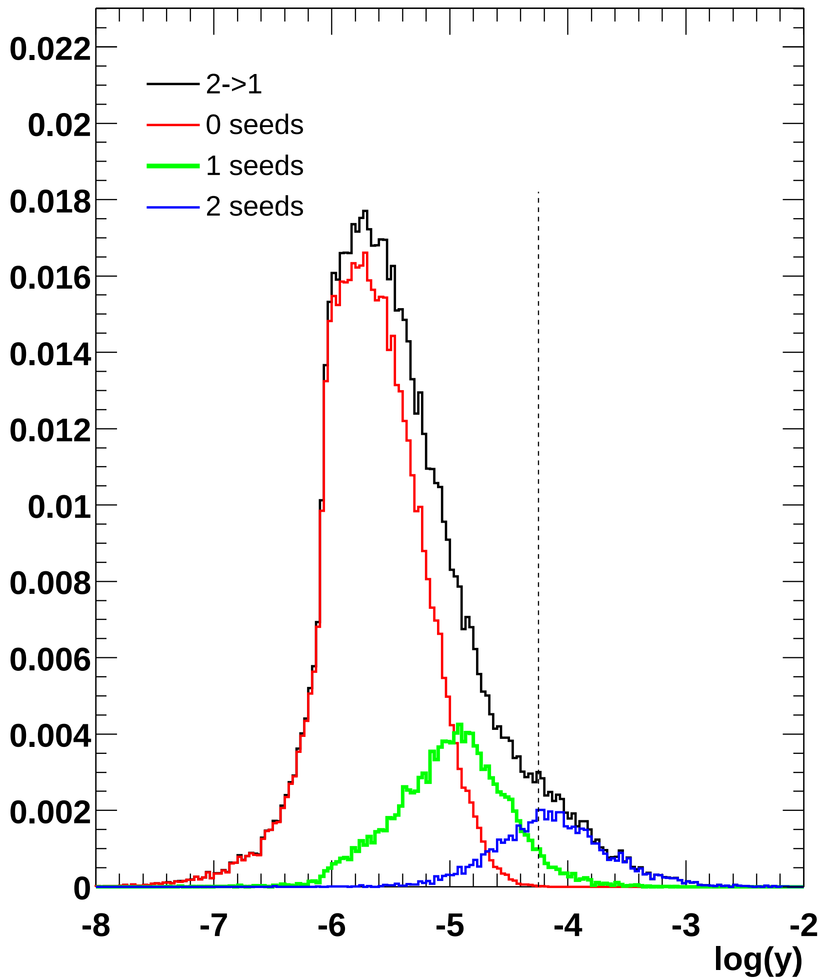

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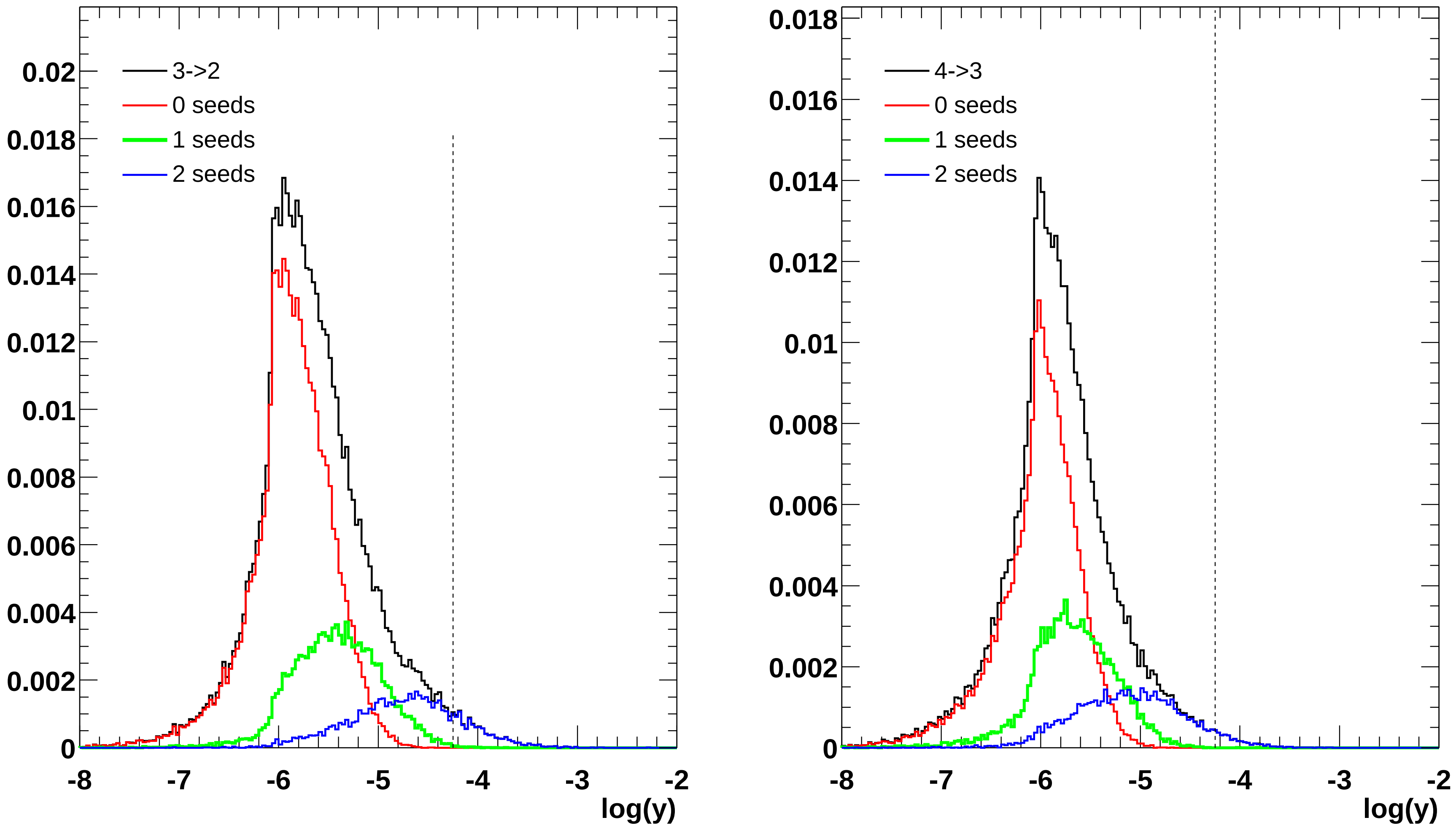

The End.