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MC@NLO progress report

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SF & Bryan Webber, JHEP 0206:029,2002 [hep-ph/0204244]
SF, Paolo Nason & Bryan Webber [hep-ph/0305252]

QCD higher-order computations are tough business

Unfortunately, we do need them:

- Reliable prediction of cross sections and final-state distributions is important not only as a test of QCD but also for the design of collider experiments and new particle searches
- Systematic approaches to this problem are based on perturbation theory, usually truncated at NLO
- For the description of exclusive hadronic final states, perturbative calculations have to be combined with a model for the conversion of partonic final states into hadrons
- Such (realistic) models operate on partonic states with high multiplicity and low relative transverse momenta, obtained from a parton shower *approximation* to QCD dynamics and not from fixed-order calculations

Perturbative N^{k}LO computations and MC simulations have complementary strengths

Are NLO computations and MC simulations compatible?

Well, they *must* be, since they are both derived from perturbation theory

We therefore aim to develop a practical method for combining parton shower MC programs with NLO perturbative calculations (MC@NLO)

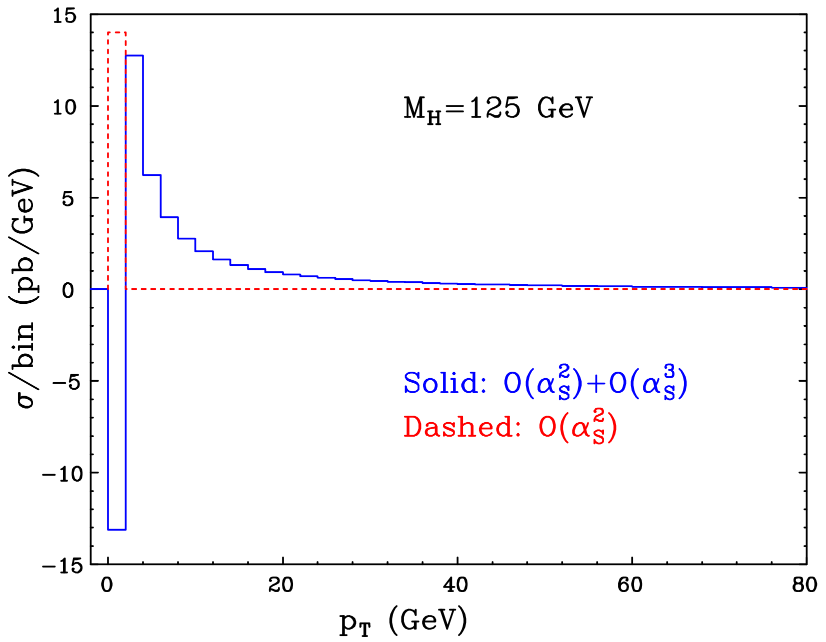
We require MC@NLO to have the following characteristics:

- The output is a set of events, which are fully exclusive
- Total rates are accurate to NLO
- NLO results are recovered upon expansion of MC@NLO results in α_S.
  In other words: there is no double counting in MC@NLO
- Hard emissions are treated as in NLO computations
- Soft/collinear emissions are treated as in MC
- The matching between hard- and soft/collinear-emission regions is smooth
- MC hadronization models are adopted

MC@NLO improves matrix elements corrections: virtual terms are included

What does NLO mean, actually?

Consider Higgs production:



$$\frac{d\sigma}{dp_T} = \left(A\alpha_S^2 + B\alpha_S^3\right)\delta(p_T) + C(p_T)\alpha_S^3$$

$$\int_{p_T^{min}}^{\infty} dp_T \frac{d\sigma}{dp_T} = \mathcal{C}_3\alpha_S^3, \qquad p_T^{min} > 0$$

$$= \mathcal{D}_2\alpha_S^2 + \mathcal{D}_3\alpha_S^3, \quad p_T^{min} = 0$$

$p_T^{min} > 0 \Rightarrow$ LO, $p_T^{min} = 0 \Rightarrow$ NLO

The answer depends on the process, on the observable, and even on the kinematic range considered. So this definition cannot be adopted in the context of event generators

- N^{k}LO accuracy in event generators is defined by the number k of extra gluons (either virtual or real) wrt the LO contribution (hopefully we all agree on LO definition)

This coincides with the former definition, for fully-inclusive quantities

The N^{k}LO accuracy in event generators does not imply the computation of K factors, since no one is talking about integrating anything

What does MC mean?

(Apart from 10^n lines of Fortran/C++ code, with n large)

In spite of the somewhat drastic approximations they contain, MC's have been proved to be fundamental tools for high energy physics, providing accurate and realistic descriptions of nature (which cannot be said for many theorists – realistic, I mean)

We should not overlook

- The amount of debugging which went into it
- The tunings performed by the experiments – a lot of work on the perturbative/non-perturbative interplay
- The human factor: each of us tends to use not the best tool, but the one he's most familiar with

If you can't beat them, join them

- MC@NLO must build upon existing Monte Carlo's

Major requirements for MC@NLO

A) Must be built upon an existing Monte Carlo

B) Must be as good as the MC in the soft/collinear regions

C) Must be as good as the NLO computations in the hard regions

Which imply

A) Less suffering

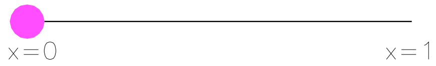
B) If the MC implements soft/collinear effects to a certain logarithmic accuracy, MC@NLO will not spoil it. MC@NLO should perform as the MC in the comparison with analytic resummations

C) MC@NLO, apart from reproducing the NLO results for infrared-safe observables, includes some higher-order effects, and gives a fully realistic description of the final state (correct multiplicities, etc.)

A simple way to understand MC@NLO

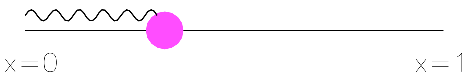
A system S moves along a line between 0 and 1. It can radiate "photons", whose energy we denote with x. S can undergo several further emissions; on the other hand, one photon cannot branch

$$\left(\frac{d\sigma}{dx}\right)_B = B\delta(x) \qquad \longleftrightarrow$$



$$\left(\frac{d\sigma}{dx}\right)_V = \alpha_S \left(\frac{B}{2\epsilon} + V\right)\delta(x) \qquad \longleftrightarrow$$

$$\left(\frac{d\sigma}{dx}\right)_R = \alpha_S \frac{R(x)}{x} \qquad \longleftrightarrow$$



It doesn't need to be QCD, but must behave the same. Thus:

$$\frac{d\sigma}{dO} = \lim_{\epsilon\to 0}\int_0^1 dx\, x^{-2\epsilon}\delta(O - O(S,x))\left[\left(\frac{d\sigma}{dx}\right)_B + \left(\frac{d\sigma}{dx}\right)_V + \left(\frac{d\sigma}{dx}\right)_R\right]$$

is finite, provided that $\lim_{x\to 0} R(x) = B$, as usual in QCD. Note the kinematics:

$$\boxed{\text{B\&V} \Longrightarrow O(S,0),\ \text{R} \Longrightarrow O(S,x)}$$

NLO $\oplus$ MC $\longrightarrow$ MC@NLO?

A: In standard MC's $(S, 0)$ happens to be the initial condition for the shower

B: At the NLO, kinematic configurations (S, x) and $(S, 0)$ are generated, and used to fill the histogram bins

Try the same at the NLO as at the LO: $(S, 0)$ and (S, x) are MC initial conditions

$$\mathcal{F}_{\text{MC@NLO}} = \int_0^1 dx \left[\mathcal{F}_{\text{MC}}(S, x) \frac{\alpha_S R(x)}{x} + \mathcal{F}_{\text{MC}}(S, 0) \left(B + \alpha_S V - \frac{\alpha_S B}{x} \right) \right]$$

It doesn't work:

- Cancellations between (S, x) and $(S, 0)$ contributions occur after the shower: hopeless from the practical point of view (unweighting impossible)
- $(d\sigma/dO)_{MC@NLO} - (d\sigma/dO)_{NLO} = \mathcal{O}(\alpha_S)$. In words: double counting

The problem is a fundamental one: KLN cancellation is achieved in standard MC's through unitarity, and embedded in Sudakovs. This is no longer possible: IR singularities do appear in hard ME's

MC@NLO: modified subtraction

Get rid of the MC $\mathcal{O}(\alpha_S)$ contributions by an extra subtraction of the $\mathcal{O}(\alpha_S)$ term in the expansion of the Sudakov, $\alpha_S Q(x)/x$

$$\mathcal{F}_{\text{MC@NLO}} = \int_0^1 dx \left[\mathcal{F}_{\text{MC}}(S,x) \frac{\alpha_S [R(x) - BQ(x)]}{x} + \mathcal{F}_{\text{MC}}(S,0) \left(B + \alpha_S V + \frac{\alpha_S B [Q(x) - 1]}{x} \right) \right]$$

provided that the new terms are chosen sensibly:

$$\left(\frac{d\sigma}{dx} \right)_{MC} = \alpha_S B \frac{Q(x)}{x} + \mathcal{O}(\alpha_S^2)$$

$Q(x)$ is MC-dependent (i.e., Pythia's and Herwig's differ), but $Q(x) \to 1$ for $x \to 0$ always holds

By explicit computation, $(d\sigma/dO)_{MC@NLO} - (d\sigma/dO)_{NLO} = \mathcal{O}(\alpha_S^2)$, and therefore there is no double counting

By-product: w_{EV} and w_{CNT} converge: unweighting is now possible

Modified subtraction in QCD

Strategy: Take the toy model seriously, and literally translate it in QCD language

$$\mathcal{F}_{\text{MC@NLO}} = \sum_{ab} \int dx_1\, dx_2\, d\phi_3\, f_a(x_1) f_b(x_2)$$

$$\Big[\mathcal{F}_{\text{MC}}^{(2\to3)} \left(\mathcal{M}_{ab}^{(h)}(x_1,x_2,\phi_3) - \mathcal{M}_{ab}^{(\text{MC})}(x_1,x_2,\phi_3) \right) +$$

$$\mathcal{F}_{\text{MC}}^{(2\to2)} \left(\mathcal{M}_{ab}^{(b,v,c)}(x_1,x_2,\phi_2) - \mathcal{M}_{ab}^{(c.t.)}(x_1,x_2,\phi_3) + \mathcal{M}_{ab}^{(\text{MC})}(x_1,x_2,\phi_3) \right) \Big]$$

Since it is literal...

$$\frac{\alpha_S R(x)}{x} \leftrightarrow \mathcal{M}_{ab}^{(h)}, \qquad \frac{\alpha_S B Q(x)}{x} \leftrightarrow \mathcal{M}_{ab}^{(\text{MC})}$$

it works only if $\mathcal{M}_{ab}^{(\text{MC})}$ is a local counterterm of $\mathcal{M}_{ab}^{(h)}$. This is not the case: large-angle soft gluon emission in MC's is not described by eikonals

Fortunately, the problem is not a serious one: we can still use existing MC's. Formally, some observables will get extra power-suppressed contributions; practically, the effects are invisible

MC@NLO 2.0

IPROC	Process
–1705	$H_1 H_2 \to b\bar{b} + X$
–1706	$H_1 H_2 \to t\bar{t} + X$
–2850	$H_1 H_2 \to W^+ W^- + X$
–2860	$H_1 H_2 \to Z^0 Z^0 + X$
–2870	$H_1 H_2 \to W^+ Z^0 + X$
–2880	$H_1 H_2 \to W^- Z^0 + X$

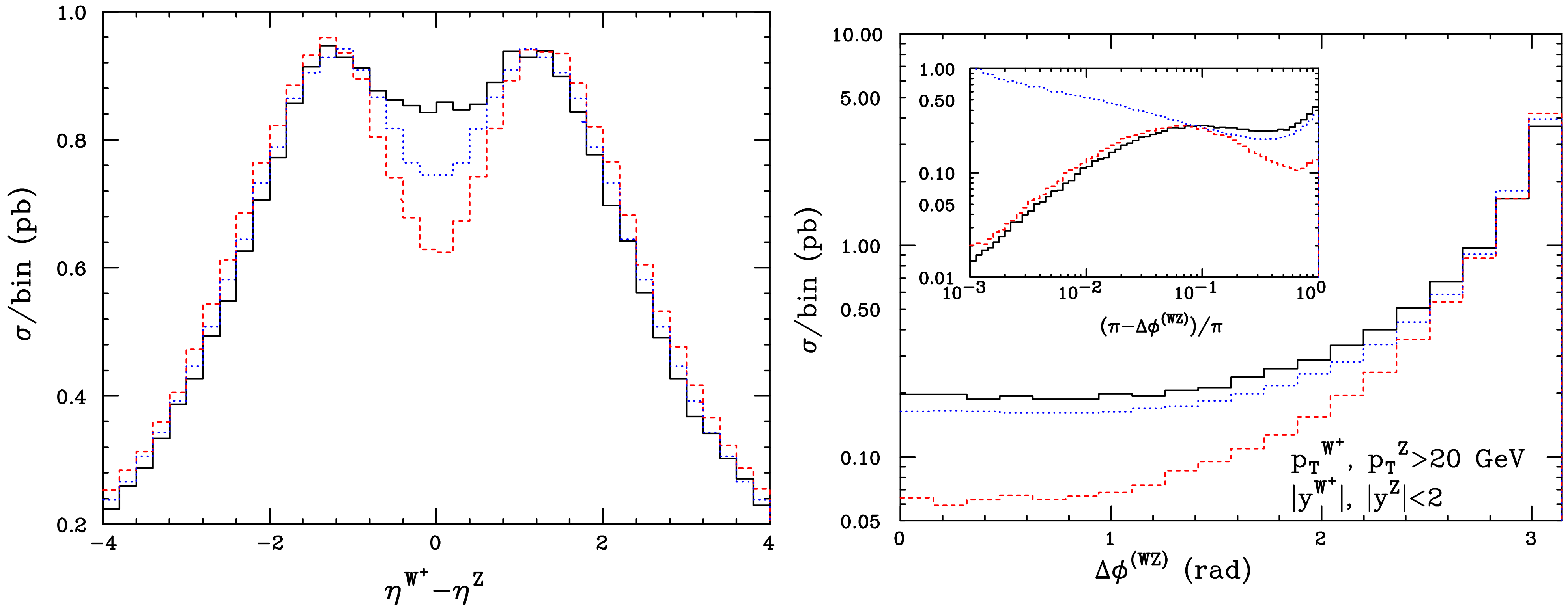
- Works identically to HERWIG: the very same analysis routines can be used
- Reads shower initial conditions from an event file (as in ME corrections)
- Exploits Les Houches accord for process information and common blocks
- LHAPDF will come next

The first application [hep-ph/0204244]: vector boson pairs. Subsequent application [hep-ph/0305252] to heavy flavour pairs demonstrates that the formalism can be used for fairly complex processes

NEW! Higgs production now running (still being tested)

Give us a call (+0041 22 767 2300) if you want to implement your favourite process!!!

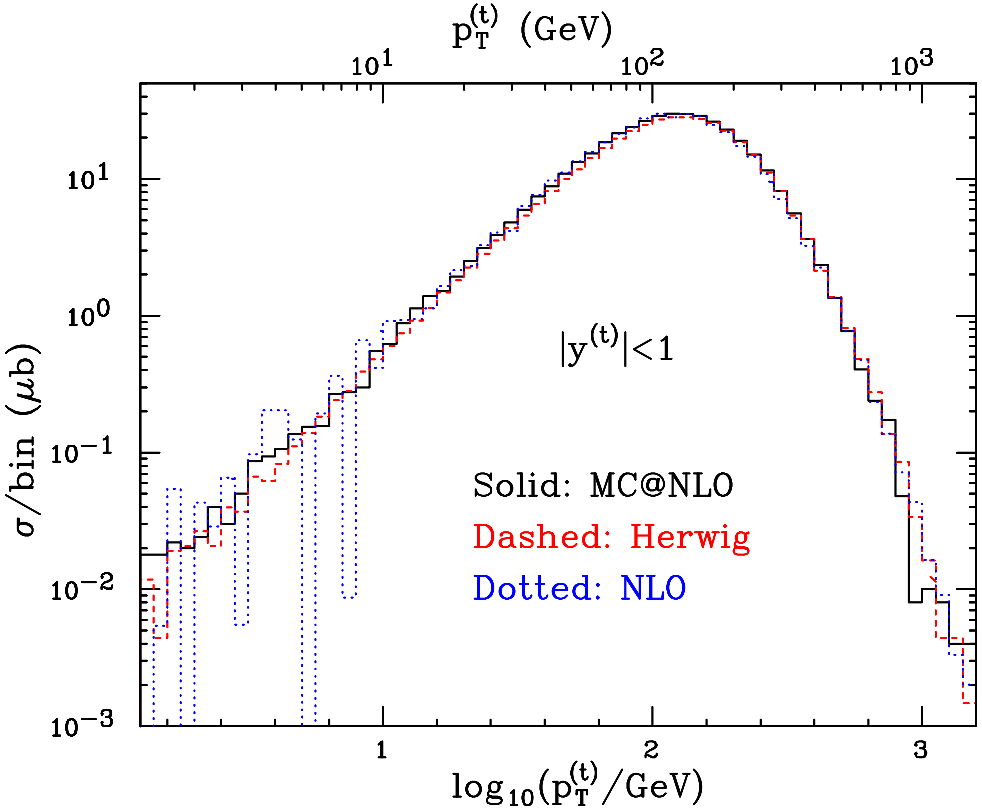
W^+Z observables

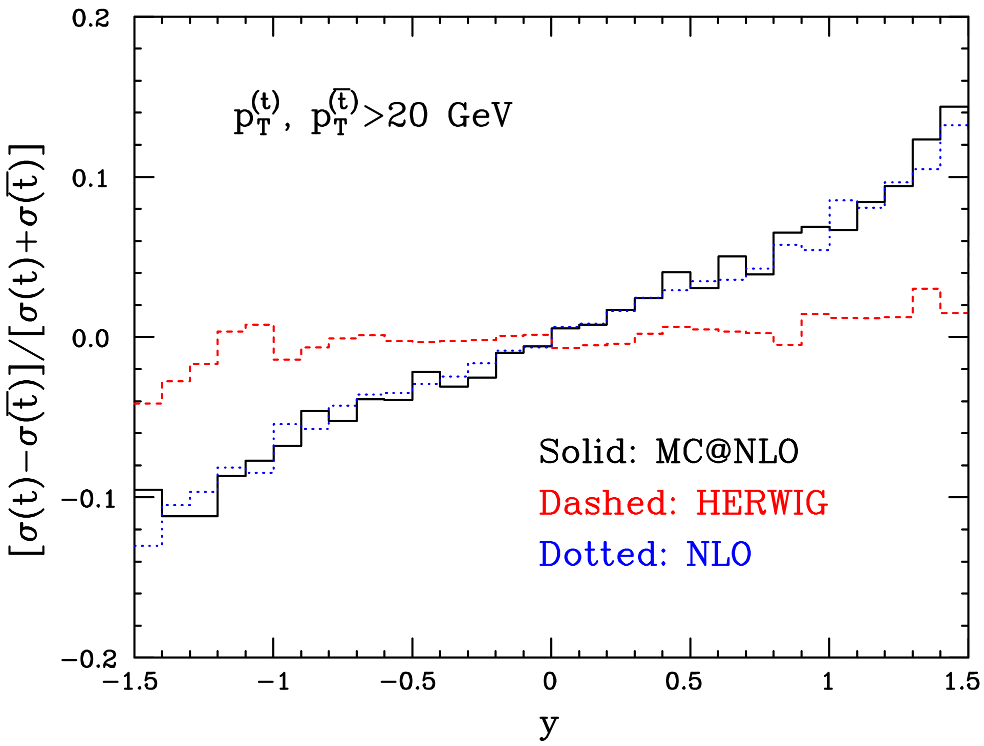


It is interesting that the MC@NLO fills further the kinematic dip at $\eta_{W^+} - \eta_Z = 0$. The difference between MC@NLO and MC is enhanced by the cuts in the $\Delta\phi$ tail

Solid: MC@NLO
Dashed: HERWIG$\times\frac{\sigma_{NLO}}{\sigma_{LO}}$
Dotted: NLO

t, $\bar{t}$ observables





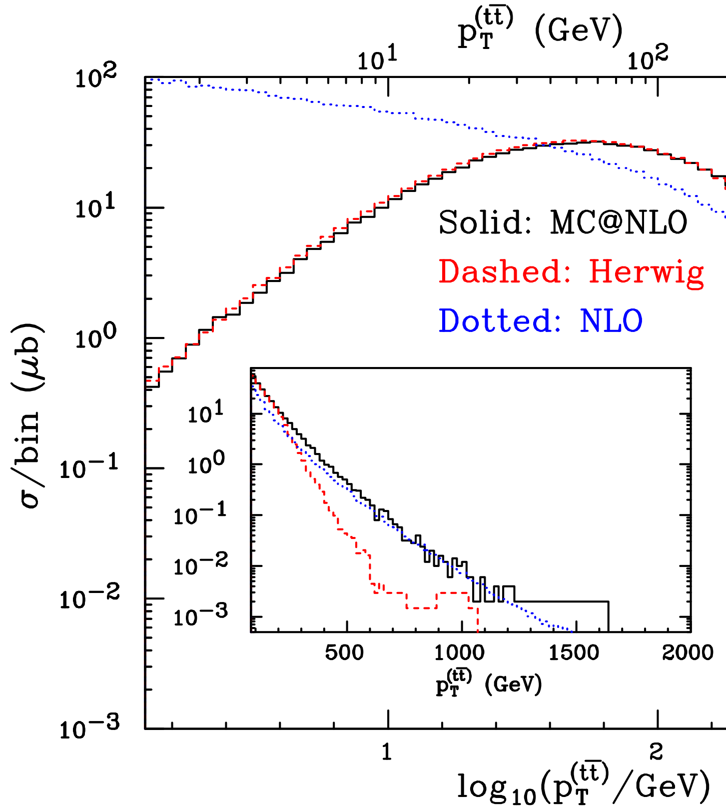
- MC@NLO is numerically more stable than NLO
- F/B asymmetry at the Tevatron is absent in MC's

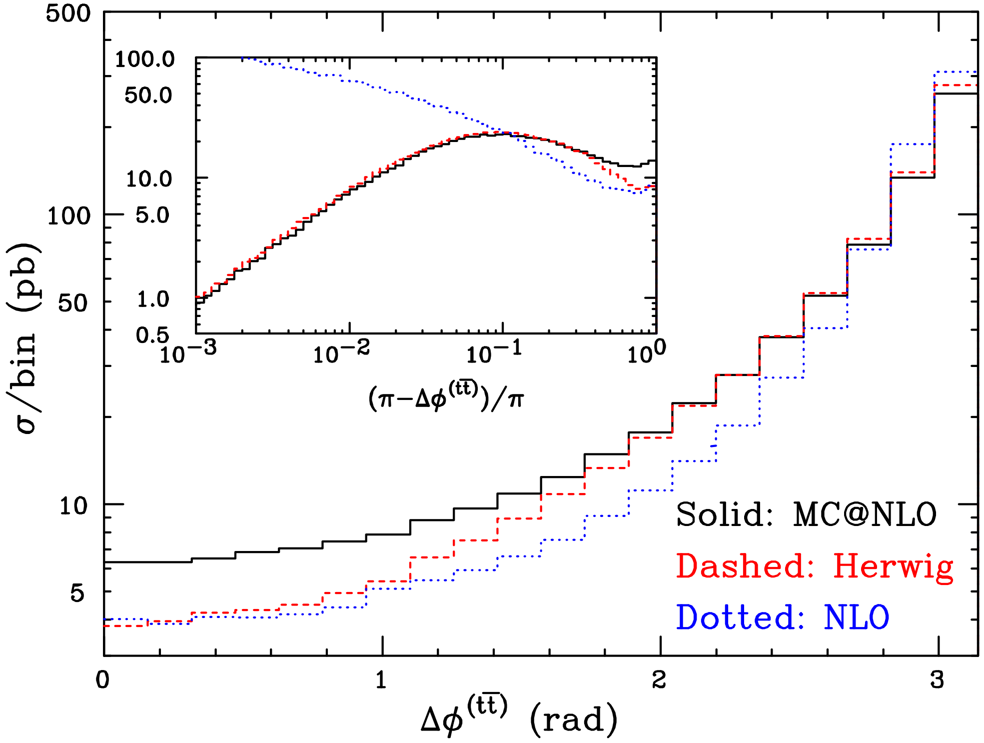
Solid: MC@NLO

Dashed: HERWIG$\times\frac{\sigma_{NLO}}{\sigma_{LO}}$

Dotted: NLO

$t\bar{t}$ correlations at the LHC





These correlations are problematic: the soft and hard emissions are both relevant. MC@NLO does well, resumming large logarithms, and yet handling the large-scale physics correctly

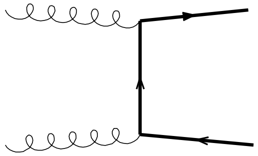
Solid: MC@NLO

Dashed: HERWIG$\times\frac{\sigma_{NLO}}{\sigma_{LO}}$

Dotted: NLO

Bottom production is much nastier than top

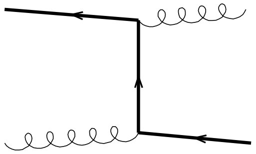
MC rule: if we aim to study any physical system, we start by producing it in the hard process $\Longrightarrow$



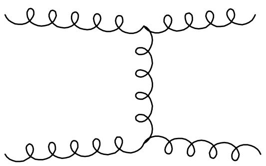
Flavour CReation

This is going to underestimate the rate by a factor of 4 (which is not so important), and to miss key kinematic features (which is crucial)

So break the rule and add other hard processes



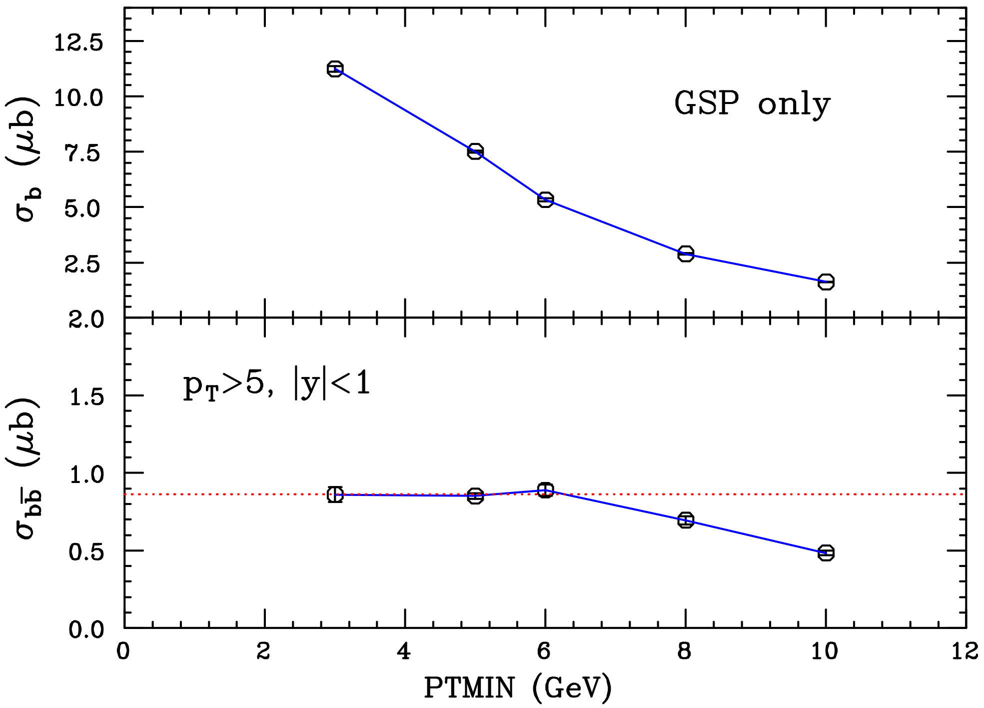
Flavour EXcitation

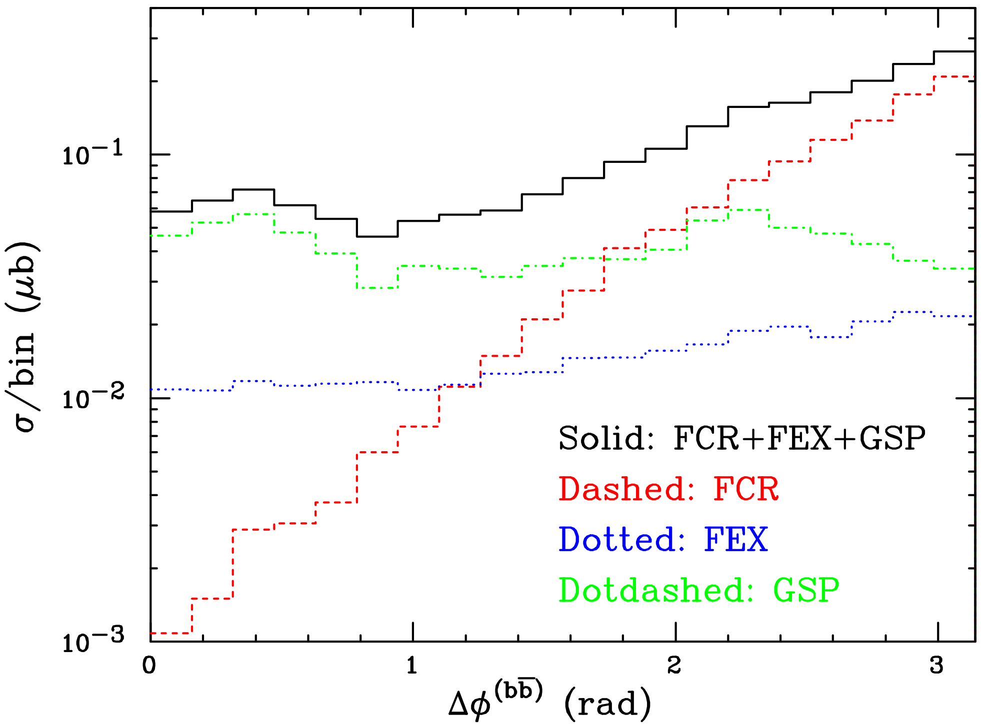


Gluon SPlitting

- In FEX, the missing b or $\bar{b}$ results from initial-state radiation. A cutoff PTMIN avoids divergences in the matrix element
- In GSP, the b and $\bar{b}$ result from final-state gluon splitting; PTMIN is again necessary to obtain finite results

b production with HERWIG

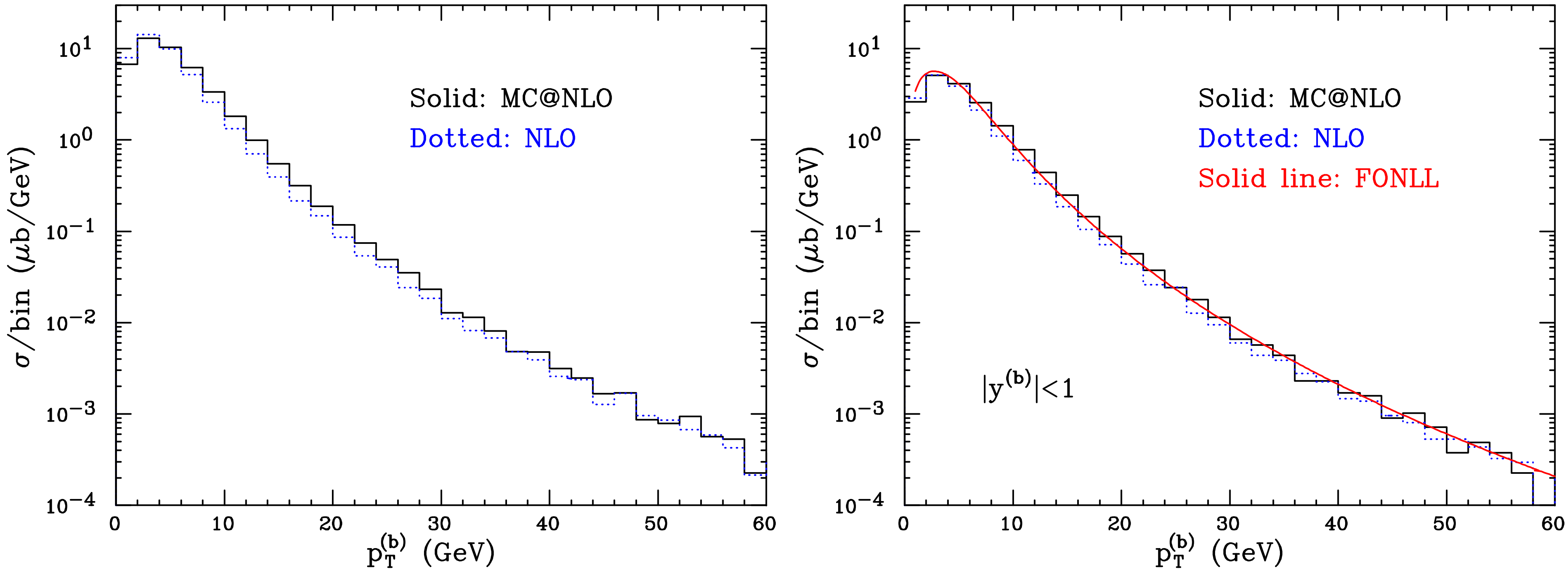




- The PTMIN dependence is worrisome in the case of single-inclusive observables
- FCR, FEX and GSP are complementary, and must be all generated
- GSP efficiency is extremely poor: 10^{-4} within cuts for correlations

Reliability and efficiency rapidly degrade for smaller p_T cuts. In FEX, the dependence on bottom PDF is problematic

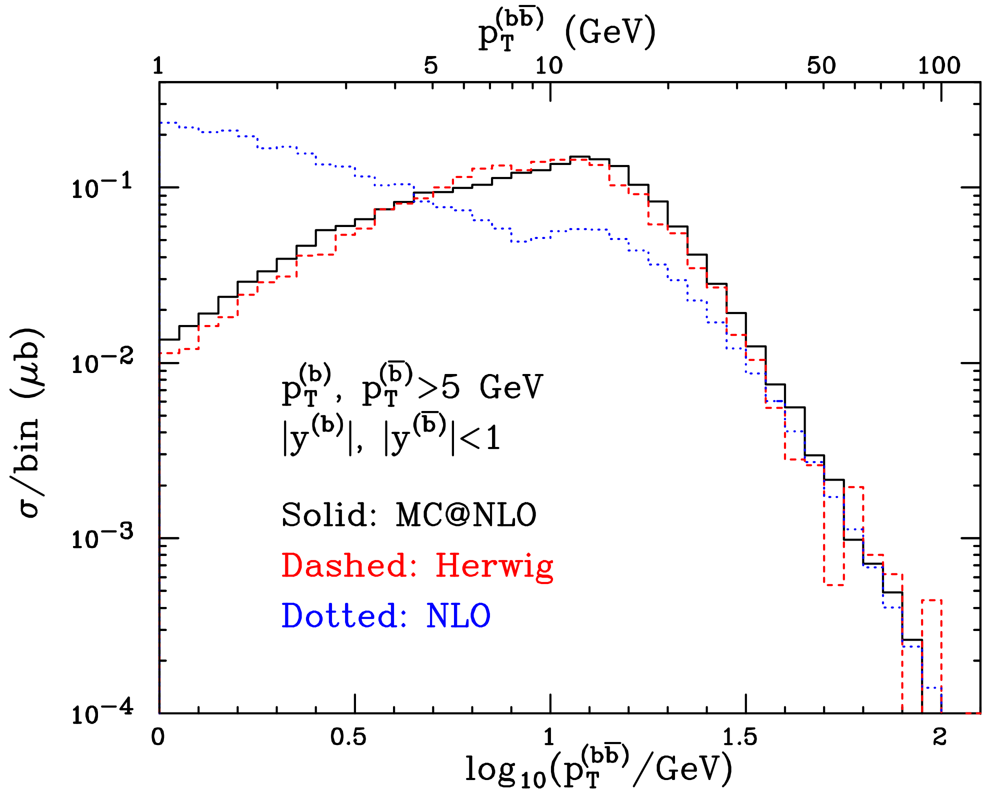
Single-inclusive b with MC@NLO

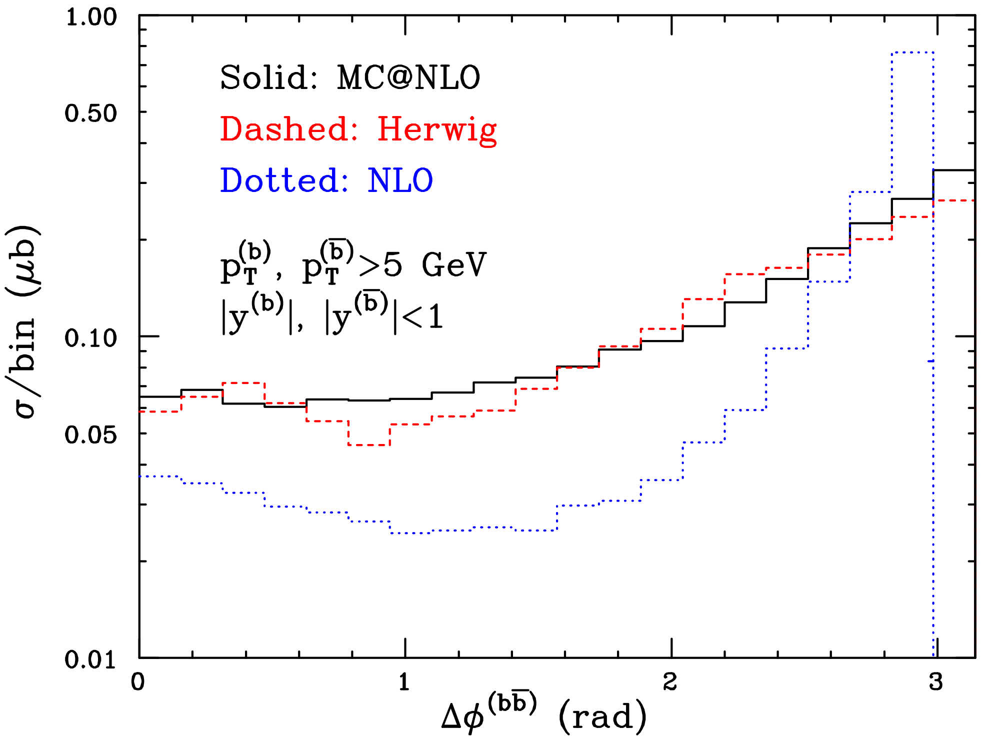


- We can consider predictions all the way down to $p_T = 0$
- Rapidity predicted by MC@NLO is marginally broader than at NLO
- MC@NLO is in better agreement with FONLL than with NLO
- At small p_T, the enhancement due to initial-state emissions is visible

The impact of initial-state emissions can be estimated by giving a k_T-kick to NLO: it shows up precisely where MC@NLO is larger than FONLL

$b\bar{b}$ correlations with MC@NLO

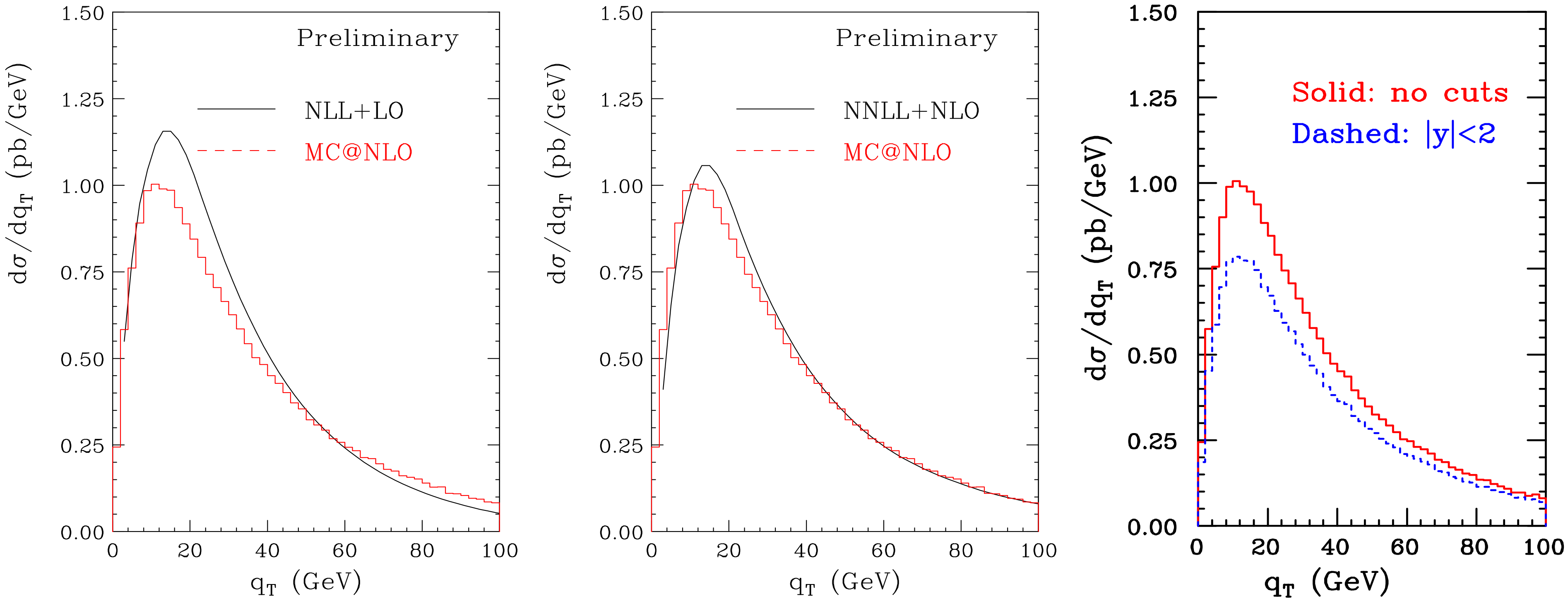




HERWIG does surprisingly well, but needs quite a lot of CPU (14 millions events – 1 million for MC@NLO). The hard emission effects are huge for b production, and cannot be neglected

Solid: MC@NLO
Dashed: HERWIG
Dotted: NLO

Higgs in MC@NLO (preliminary)



The transverse momentum spectrum (without any cuts) can be compared to state-of-the-art resummed results (Bozzi, Catani, de Florian, Grazzini).

The matching condition is similar to MC@NLO, in that it conserves the total rate

- HERWIG does a very good job in resumming large logs
- MC@NLO does not spoil this feature, improves in the hard region, and computes the NLO rate (i.e., no need of rescaling)
- We can now apply any cuts we like (decay products, recoiling system)

Outlook

A practical proof exists that NLO corrections can be incorporated in a parton shower formalism: MC@NLO runs and works well

It is remarkable that the numerical stability of MC@NLO is superior to that of the NLO computations from which it is derived. Other highlights:

- Final- and initial-state emissions simultaneously treated
- Highly non-trivial kinematics due to heavy quark mass
- Several colour structures
- Non-trivial structure of the matrix elements in the soft limit

Some issues we are now working on (btw, we need manpower):

- Spin correlations for vector boson pairs
- W and Z production
- Jet and dijet production

`http://www.hep.phy.cam.ac.uk/theory/webber/MCatNLO`