HELAC/PHEGAS

MC generator for arbitrary parton-level scattering process

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CERN Monte Carlo Workshop July 2003

- **Reliable cross section computation and event generation for multiparticle processes, with $\sim$ 10-12 particles in the final state.**

- - **HELAC:** A.Kanaki and C.G.Papadopoulos, CPC 132 (2000) 306, hep-ph/0002082.
    Matrix element computation algorithm, based on Dyson-Schwinger equations, including: EWK, QCD, fermion masses, reliable arithmetic, running couplings and masses ....
  - **PHEGAS:** C.G.Papadopoulos, CPC 137 (2001) 247, hep-ph/0007335
    Monte-Carlo phase space integration/generation based on optimized multichannel approach.

hep-ph/0012004 and Tokyo 2001,(CPP2001) Computational particle physics, p. 20-25

- Give the process

$$e^- e^+ \to \mu\ \bar{\nu}_\mu\ u\ \bar{d}\ \gamma\ \gamma$$

- Define the cuts

$$E_i > E_0 \quad \cos\theta_i < c_0 \quad \cos\theta_{ij} < c_1$$

- The cross section is:

$$\sigma = 1.73(5)\ \text{fb}\ \ \text{at}\ \sqrt{s} = 500\ \text{GeV}$$

- **... plus any other kinematical distribution !**

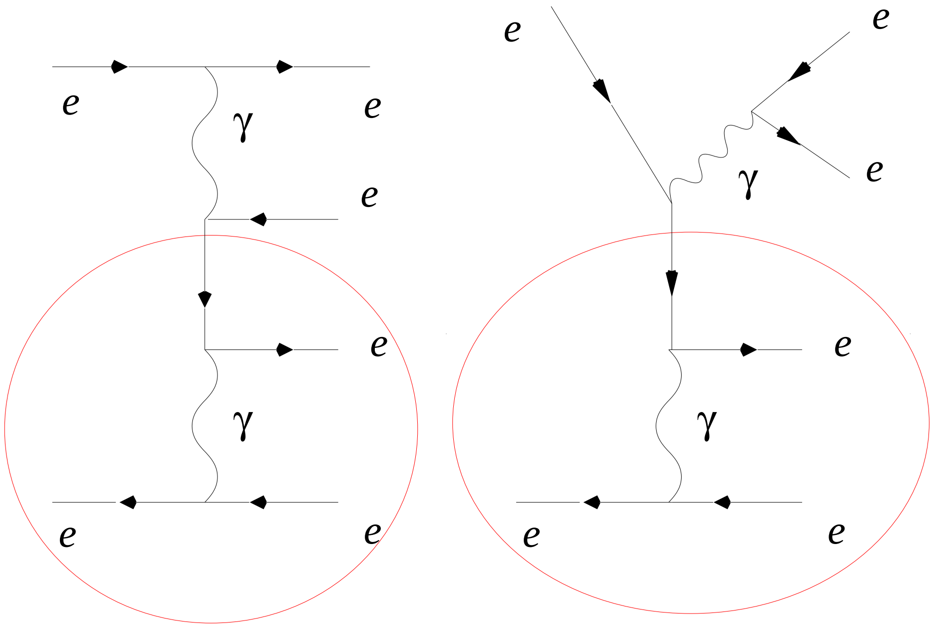
Old **Feynman graphs $\rightarrow$ computational cost $\sim n!$**

New **Dyson-Schwinger $\rightarrow$ computational cost $\sim 3^n$**

F. Caravaglios and M. Moretti, Phys. Lett. B358 (1995) 332

P.Draggiotis, R.H.Kleiss and C.G.Papadopoulos, Phys. Lett. B439 (1998) 157

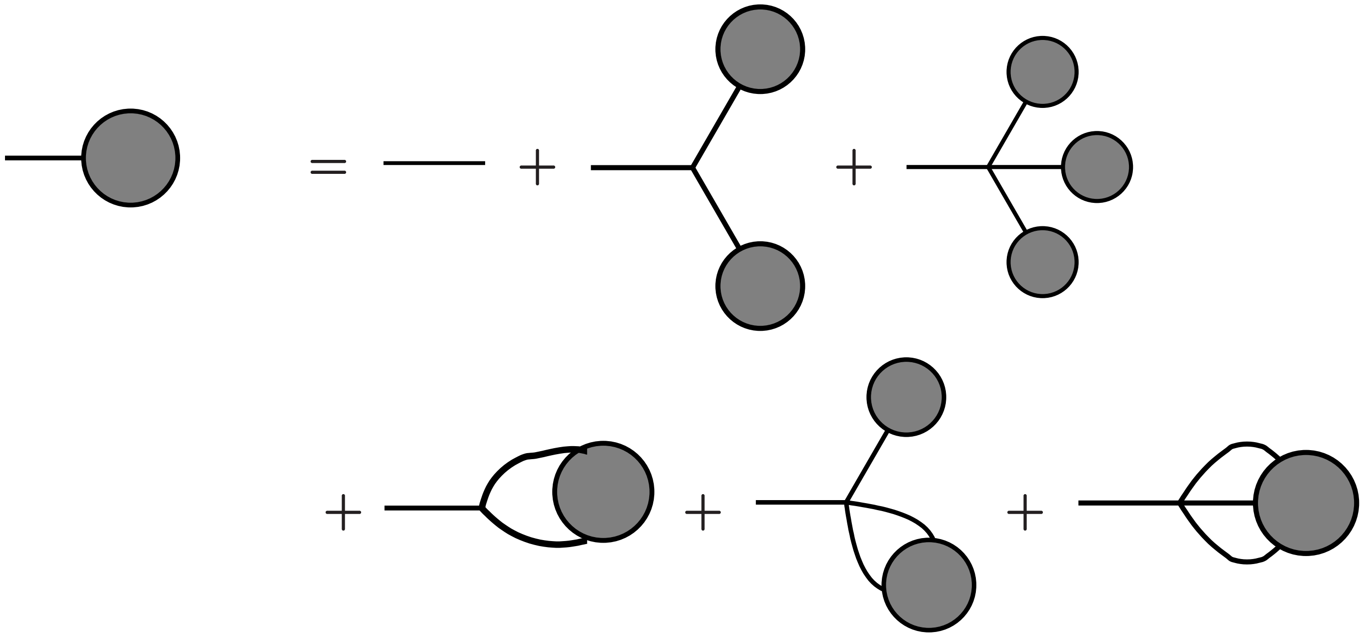
- ***Example:*** $e^-e^+ \rightarrow e^-e^+e^-e^+$ **in QED:**



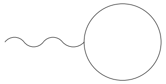
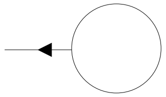
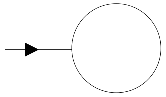
The Dyson-Schwinger

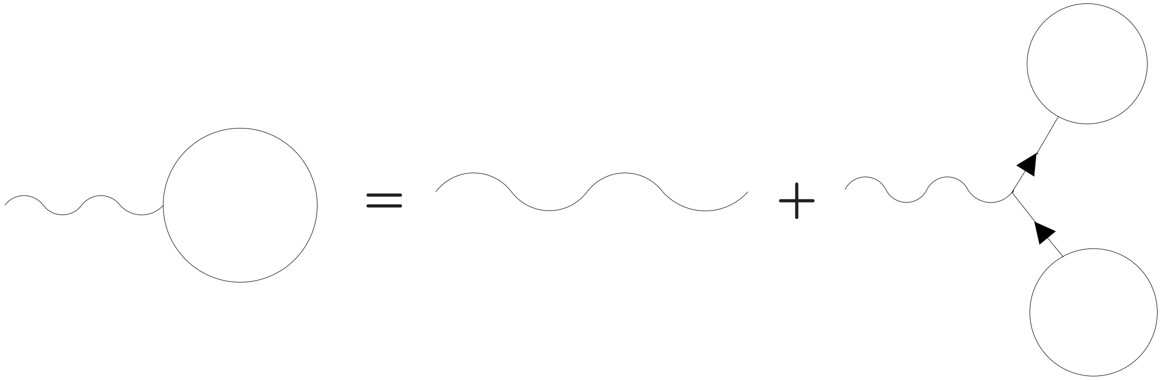
Imagine a theory with 3- and 4- point vertices and just one field.

Then it is straightforward to write an equation that gives the amplitude for $1 \to n$

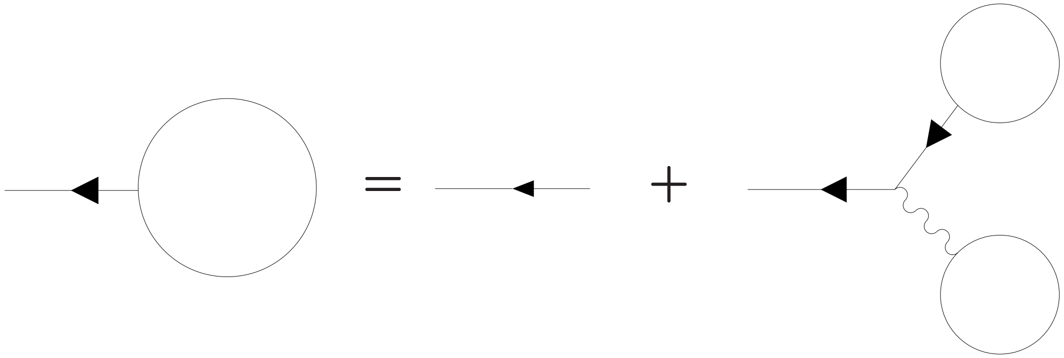


⇒ **Systematic approach**:

$b_\mu(P) =$  $\psi(P) =$  $\bar{\psi}(P) =$ 



$$b^\mu(P) = \sum_{i=1}^{n} \delta_{P=p_i} b^\mu(p_i) + \sum_{P=P_1+P_2} (ig)\Pi^\mu_\nu \, \bar{\psi}(P_2)\gamma^\nu \psi(P_1)\epsilon(P_1, P_2)$$



$$\psi(P) = \sum_{i=1}^{n} \delta_{P=p_i}\psi(p_i) + \sum_{P=P_1+P_2} (ig)\not{\epsilon}(P_2)\frac{(\not{P}_1 + m)}{P_1^2 - m^2}\psi(P_1)\epsilon(P_1, P_2)$$

$$V^A = (V^0 + V_z, V^0 - V_z, V_x + iV_y, V_x - iV_y)\ , \quad A = 1, \ldots, 4\ .$$

Polarization state-vectors are given by

$$\begin{aligned}
\epsilon_-^A &= \left(\frac{-p_T}{\sqrt{2}|\vec{p}|}, \frac{p_T}{\sqrt{2}|\vec{p}|}, \frac{(p_x + ip_y)(|\vec{p}| + p_z)}{\sqrt{2}|\vec{p}|p_T}, \frac{(p_x - ip_y)(-|\vec{p}| + p_z)}{\sqrt{2}|\vec{p}|p_T}\right) \\
\epsilon_+^A &= \left(\frac{p_T}{\sqrt{2}|\vec{p}|}, \frac{-p_T}{\sqrt{2}|\vec{p}|}, \frac{(p_x + ip_y)(|\vec{p}| - p_z)}{\sqrt{2}|\vec{p}|p_T}, \frac{(p_x - ip_y)(-|\vec{p}| - p_z)}{\sqrt{2}|\vec{p}|p_T}\right) \\
\epsilon_0^A &= \left(\frac{|\vec{p}|}{\sqrt{p^2}} + \frac{p_z p_0}{|\vec{p}|\sqrt{p^2}}, \frac{|\vec{p}|}{\sqrt{p^2}} - \frac{p_z p_0}{|\vec{p}|\sqrt{p^2}}, \frac{(p_x + ip_y)p_0}{|\vec{p}|\sqrt{p^2}}, \frac{(p_x - ip_y)p_0}{|\vec{p}|\sqrt{p^2}}\right)
\end{aligned}$$

As for the Dirac matrices we are using the chiral representation. The wave functions which describe massive spinors are given by:

$$u_+(p) = \begin{pmatrix} r/c \\ a(p_x + ip_y)/r \\ -mb/r \\ -m(p_x + ip_y)/r \end{pmatrix} \qquad \bar{u}_+(p) = \begin{pmatrix} mb/r \\ m(p_x - ip_y)/r \\ -r/c \\ -a(p_x - ip_y)/r \end{pmatrix}$$

$$u_-(p) = \begin{pmatrix} m(p_x - ip_y)/r \\ -mb/r \\ -a(p_x - ip_y)/r \\ r/c \end{pmatrix} \qquad \bar{u}_-(p) = \begin{pmatrix} a(p_x + ip_y)/r \\ -r/c \\ -m(p_x + ip_y)/r \\ mb/r \end{pmatrix}$$

$$v_+(p) = \begin{pmatrix} -m(p_x - ip_y)/r \\ mb/r \\ -a(p_x - ip_y)/r \\ r/c \end{pmatrix} \qquad \bar{v}_+(p) = \begin{pmatrix} a(p_x + ip_y)/r \\ -r/c \\ m(p_x + ip_y)/r \\ -mb/r \end{pmatrix}$$

$$v_-(p) = \begin{pmatrix} r/c \\ a(p_x + ip_y)/r \\ mb/r \\ m(p_x + ip_y)/r \end{pmatrix} \qquad \bar{v}_-(p) = \begin{pmatrix} -mb/r \\ -m(p_x - ip_y)/r \\ -r/c \\ -a(p_x - ip_y)/r \end{pmatrix}$$

where:

$$a = p_0 + |\vec{p}|, \quad b = p_z + |\vec{p}|, \quad c = 2|\vec{p}|, \quad r = \sqrt{abc}$$

For a massless particle the spinors are

$$u_R(p) = \begin{pmatrix} \sqrt{p_0 + p_z} \\ (p_x + ip_y)/\sqrt{p_0 + p_z} \\ 0 \\ 0 \end{pmatrix} \qquad \bar{u}_R(p) = \begin{pmatrix} 0 \\ 0 \\ -\sqrt{p_0 + p_z} \\ -(p_x - ip_y)/\sqrt{p_0 + p_z} \end{pmatrix}$$

$$u_L(p) = \begin{pmatrix} 0 \\ 0 \\ -(p_x - ip_y)/\sqrt{p_0 + p_z} \\ \sqrt{p_0 + p_z} \end{pmatrix} \qquad \bar{u}_L(p) = \begin{pmatrix} (p_x + ip_y)/\sqrt{p_0 + p_z} \\ -\sqrt{p_0 + p_z} \\ 0 \\ 0 \end{pmatrix}$$

$$V_\mu = \bar{\psi}(P_1)\gamma_\mu \left(g_R\omega_R + g_L\omega_L\right)\psi(P_2)$$

turns out to be

$$V^A = \begin{pmatrix} -g_R\psi_1\bar{\psi}_3 - g_L\psi_4\bar{\psi}_2 \\ -g_R\psi_2\bar{\psi}_4 - g_L\psi_3\bar{\psi}_1 \\ -g_R\psi_2\bar{\psi}_3 + g_L\psi_4\bar{\psi}_1 \\ -g_R\psi_1\bar{\psi}_4 + g_L\psi_3\bar{\psi}_2 \end{pmatrix}$$

where $\psi_i(\bar{\psi}_i), i = 1, \ldots, 4$ are the components of the spinor $\psi(P_2)$ $\left(\bar{\psi}(P_1)\right)$ and

$$\omega_L = \frac{1}{2}(1 - \gamma_5) \;,\quad \omega_R = \frac{1}{2}(1 + \gamma_5) \quad .$$

On the other hand, the spinor

$$u = (\not{P} + m)\not{b}(P_1)\omega_R\psi(P_2)$$

can be reduced to

$$u = \begin{pmatrix} (-b_2p_1 + b_3p_4)\psi_1 + (b_4p_1 - b_1p_4)\psi_2 \\ (b_3p_2 - b_2p_3)\psi_1 + (-b_1p_2 + b_4p_3)\psi_2 \\ m(b_2\psi_1 - b_4\psi_2) \\ m(-b_3\psi_1 + b_1\psi_2) \end{pmatrix}$$

where ψ_i, b_i, p_i, $i = 1, \ldots, 4$ are the components of P, $b(P_1)$ and $\psi(P_2)$ respectively.

- Let n external particles with momenta $p_i^\mu, i = 1\ldots, n$, and define the momentum P^μ

$$P^\mu = \sum_{i\in I} p_i^\mu \ , \qquad I \subset \{1, \ldots, n\}$$

- the binary vector $\vec{m} = (m_1, \ldots, m_n)$, where its components take the values 0 or 1 :

$$P^\mu = \sum_{i=1}^{n} m_i \, p_i^\mu \ .$$

- Moreover this binary vector can be uniquely represented by the integer

$$m = \sum_{i=1}^{n} 2^{i-1} m_i \ , \quad 0 \leq m \leq 2^n - 1$$

- Replace

$$b_\mu(P) \to b_\mu(m) \ .$$

- Convenient ordering of integers in binary representation $\Rightarrow$ level l, defined by

$$l = \sum_{i=1}^{n} m_i \; .$$

- external momenta are of level 1
- the total amplitude corresponds to the unique level n integer $2^n - 1$

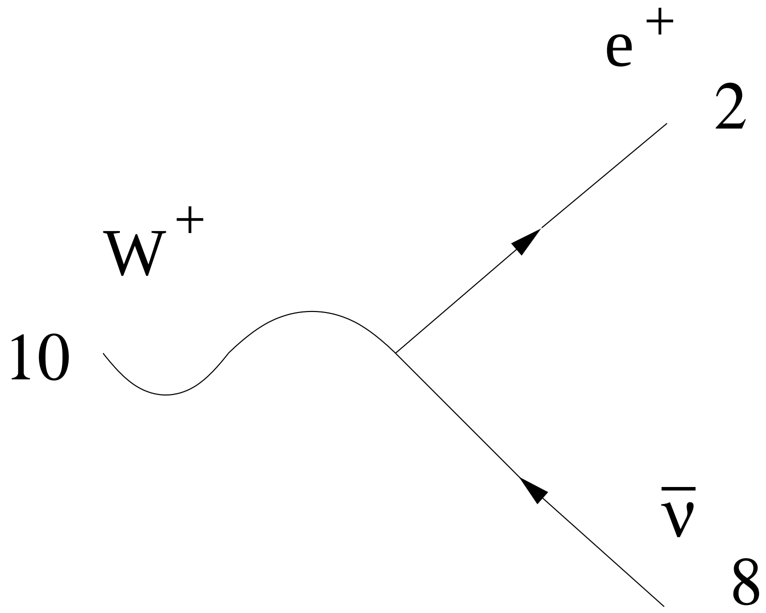
$$\mathcal{A} = b(1) \cdot b(2^n - 2)$$

This ordering dictates the natural path of the computation : starting with level-1 sub-amplitudes, we compute the level-2 ones using the Dyson-Schwinger equations and so on up to level $n - 1$

The solution

$$e^-(1)\ \ e^+(2) \rightarrow e^-(4)\ \ \bar{\nu}_e(8)\ \ u(16)\ \ \bar{d}(32)$$

1	10	33	2	-2	8	1
1	12	33	4	-2	8	1
1	48	34	16	-3	32	4
2	26	-4	10	33	16	-3
			...			
2	62	-2	10	33	52	-1
2	62	-2	12	33	50	-1
2	62	-2	58	31	4	-2
2	62	-2	58	32	4	-2
2	62	-2	60	31	2	-2
2	62	-2	60	32	2	-2



e+
2
W+
10
ν̄
8

- Dirac algebra simplification: 2-dim vs 4-dim and chiral representation, including $m_f \neq 0$.
- The sign factor:

$$\epsilon(P_1, P_2) \to \epsilon(m_1, m_2)$$

  we define

$$\epsilon(m_1, m_2) = (-1)^{\chi(m_1, m_2)}$$

$$\chi(m_1, m_2) = \sum_{i=n}^{2} \hat{m}_{1i} \left(\sum_{j=1}^{i-1} \hat{m}_{2j} \right)$$

  where hatted components are set to 0 if the corresponding external particle is a boson.

- Full EWK theory, both Unitary and Feynman gauges.

A. Denner, Fortsch. Phys. 41, 307 (1993).

Colour Configuration - EWK$\oplus$QCD

- ⋆ quarks $1 \ldots n$
- ⋆ antiquarks $\sigma_i(1 \ldots n)$ and
- ⋆ gluons $= q\bar{q}$

Each color-configuration amplitude is proportional to

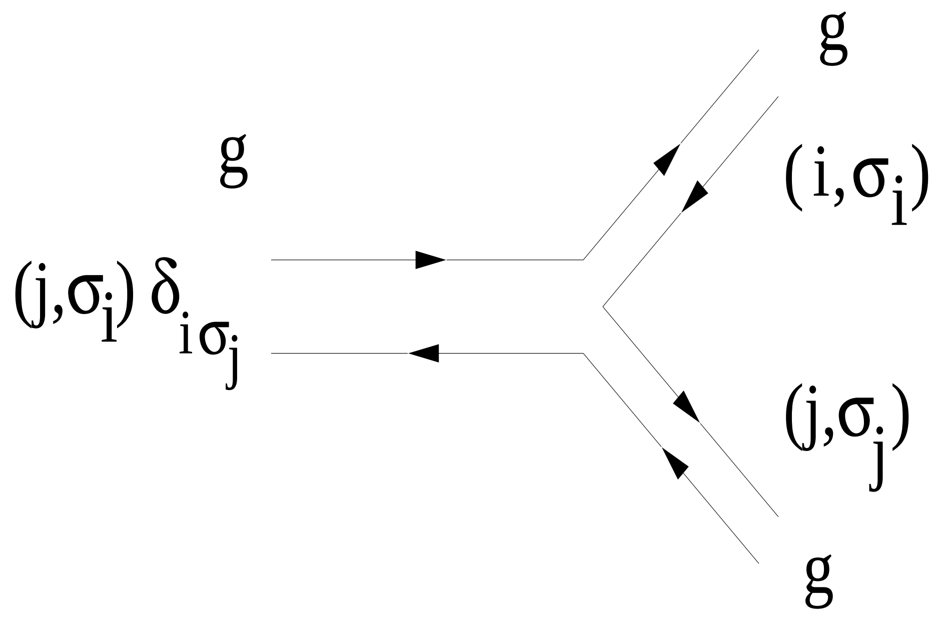
$$D_i = \delta_{1,\sigma_i(1)}\delta_{2,\sigma_i(2)} \cdots \delta_{n,\sigma_i(n)}$$

where σ_i represents the i-th permutation of the set $1, 2, \ldots, n$.

$$\mathcal{C}_{ij} = \sum D_i D_j = N_c^{\alpha} \; , \quad \alpha = \langle \sigma_1, \sigma_2 \rangle$$

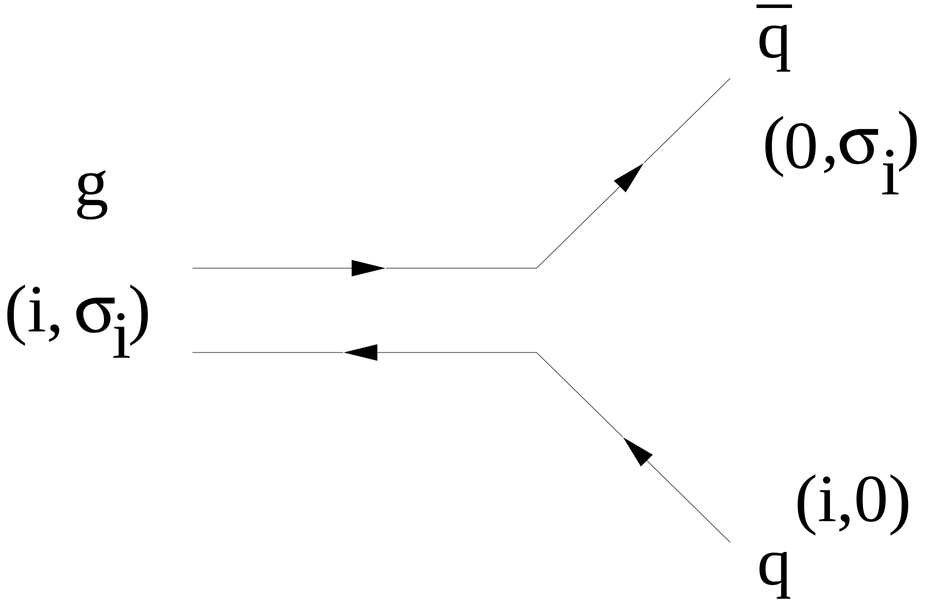
♠ exact color treatment $\Rightarrow$ low color charge

Problem: number of colour connection configurations: $\sim n!$ where n is the number of gluons or $q\bar{q}$ pairs. $\Rightarrow$ Monte-Carlo over continuous colour-space.



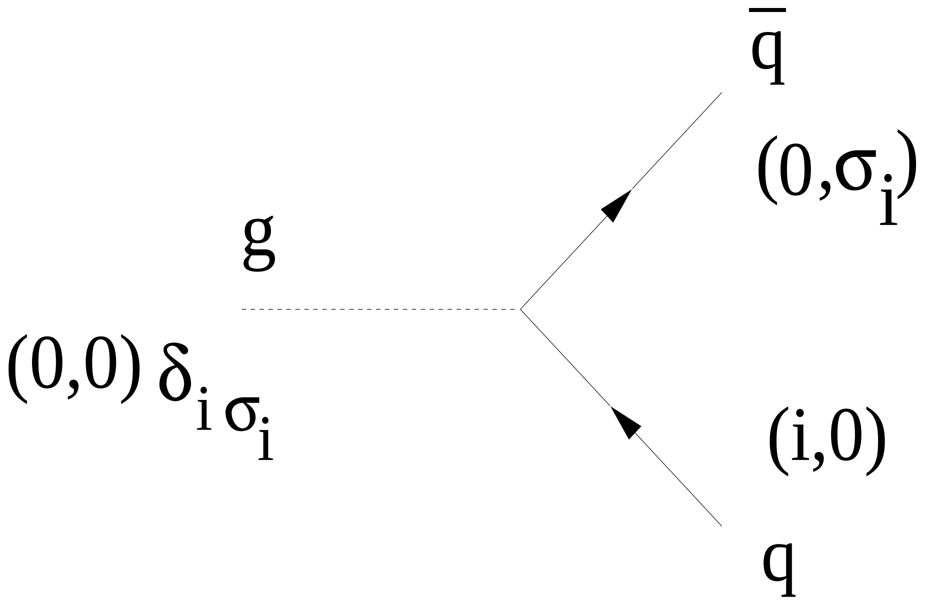
$$\sum f^{abc} t^a_{AB} t^b_{CD} t^c_{EF} = -\frac{i}{4}(\delta_{AD}\delta_{CF}\delta_{EB} - \delta_{AF}\delta_{CB}\delta_{ED})$$

$$\delta_{1\sigma_2}\delta_{2\sigma_3}\delta_{3\sigma_1}$$



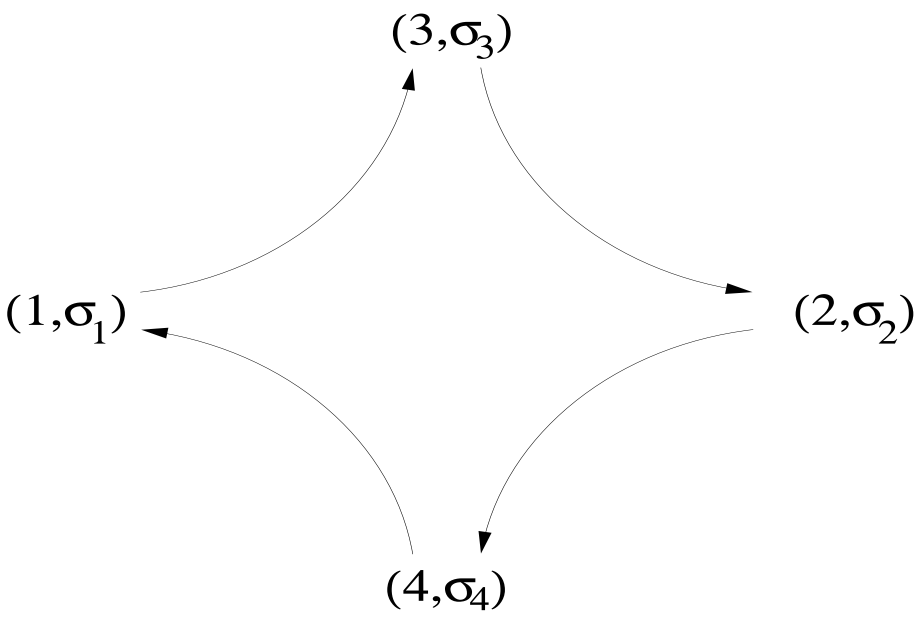
$$\sum t^a_{AB} t^b_{CD} = \frac{1}{2}(\delta_{AD}\delta_{CB} - \frac{1}{N_c}\delta_{AB}\delta_{AC})$$

$$\frac{1}{\sqrt{2}}$$



$$\sum t^a_{AB} t^b_{CD} = \frac{1}{2}(\delta_{AD}\delta_{CB} - \frac{1}{N_c}\delta_{AB}\delta_{AC})$$

$$\frac{1}{\sqrt{2N_c}}$$



$$\delta_{1\sigma_3}\delta_{3\sigma_2}\delta_{2\sigma_4}\delta_{4\sigma_1}$$

$$2g_{12}g_{34} - g_{13}g_{24} - g_{14}g_{23}$$

The $\frac{1}{N_c}$ expansion

$$\mathcal{C}_{ij} = \sum D_i D_j = N_c^{\alpha} \ , \ \ \alpha = \langle \sigma_1 , \sigma_2 \rangle$$

The leading term $\sigma_1 = \sigma_2$, N_c^n

The subleading terms: how many δ's survive after contraction, which in Combinatorial Analysis are known to be the Stirling numbers

$$(-)^{n-m} S_n^{(m)}$$

where n is the number of 'objects' and m the number of 'surviving' δ's, called cycles.

For instance the $\frac{1}{N_c}$-term is related to $\frac{n(n-1)}{2}$ permutations !

HELAC - the code

- A two-phase computation
  - phase A: selection and tree-construction (integer arithmetic)
  - phase B: computation (high-precision arithmetic)
- Comparisons: full technical agreement
  - EXCALIBUR $m_f = 0$ comparable speed
  - MADGRAPH $m_f \neq 0$ comparable speed for $n < 7$.
- As expected HELAC shows an exponential (instead of factorial) CPU-time growth. F.A.Berends, C.G.Papadopoulos and R.Pittau, hep-ph/0002249.
- There is no a priori limitation for the number of particles that HELAC can treat, the only restrictions being that of memory allocation.

Numerical reliability

- Problem due to huge 'gauge' cancellations !
- In order to have a taste of a multi-precision computation we have computed the squared amplitude for the process

$$e^-(p_1)e^+(p_2) \to e^-(p_3)e^+(p_4)e^-(p_5)e^+(p_6)$$

at two phase space points.

  - Phase space point (A) is just a randomly generated one by the phase-space generator `RAMBO`.
  - Phase space point (B)

$$p_1^0/\ \mathrm{GeV} = 100,\ \vec{p}_1 + \vec{p}_2 = 0,$$
$$p_3^0/p_1^0 = 0.9,\ \theta_3 = 0,$$
$$(p_5+p_6)^2/(p_4+p_5+p_6)^2 = 0.1,\quad \theta_4 = \phi_4 = \theta_5 = \phi_5 = 0$$

with $m_e = 0.511 \times 10^{-3}$ GeV.

- **Results are provided for these points, by using the `real*8` (DP), `real*16` (QP) and 34-digit multi-precision version of the code.**

David M. Smith, Transactions on Mathematical Software 17 (1991) 273.

(A)	(B)
1.539728523150595E-008	1.256276706229023E+023
1.539728523150588541567630028250I3D-08	3.07162601093710915134136924973089D+22
1.5397285231505885415676300282501I853M-8	3.07162601093710915127950109241770808M+22

- **In the `MP`-version of the code the precision is defined by the user.**

PHEGAS

- Phase space

$$d\Phi_n = (2\pi)^{4-3n} \prod_{i=1}^{n} \frac{d^3 p_i}{2E_i} \delta\left(\sum E_i - w\right) \delta^3\left(\sum \vec{p}_i\right)$$

- RAMBO, VEGAS-based nice but completely inefficient!

$$d\sigma_n = \mathrm{FLUX} \times |\mathcal{M}_{2\to n}|^2 d\Phi_n$$

  need appropriate mappings of peaking structures, plus optimization!

- Efficiency $\Rightarrow$ to a large number of generators, each one for a specific class of processes.

Multichannel approach

$$\mathcal{I} = \int f(\vec{x})d\mu(\vec{x}) = \int \frac{f(\vec{x})}{p(\vec{x})}p(\vec{x})d\mu(\vec{x})$$

$$p(\vec{x}) = \sum_{i=1}^{M_{ch}} \alpha_i \, p_i(\vec{x}) \qquad \sum_{i=1}^{M_{ch}} \alpha_i = 1$$

$$\mathcal{I} \to \left\langle \frac{f(\vec{x})}{p(\vec{x})} \right\rangle \qquad \mathcal{E}^2 N \to \left\langle \left(\frac{f(\vec{x})}{p(\vec{x})}\right)^2 - \mathcal{I}^2 \right\rangle$$

$\star$ **Optimize $\alpha_i \Rightarrow$ Minimize $\mathcal{E}$** $\star$

R.Kleiss and R.Pittau, Comput. Phys. Commun. 83, 141 (1994).

New **Dyson-Schwinger equations: subamplitude is a combination of several peaking structures!**

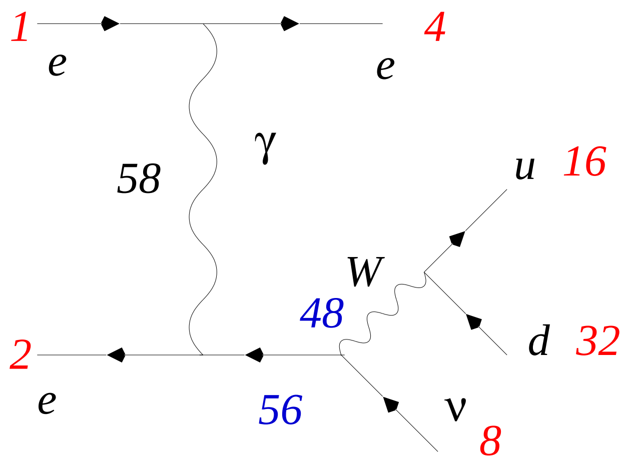
problem unsolved

P.D.Draggiotis, A.van Hameren and R.Kleiss, hep-ph/0004047.

Old **Feynman graphs: exhibit single peaking structure!**

problem solved

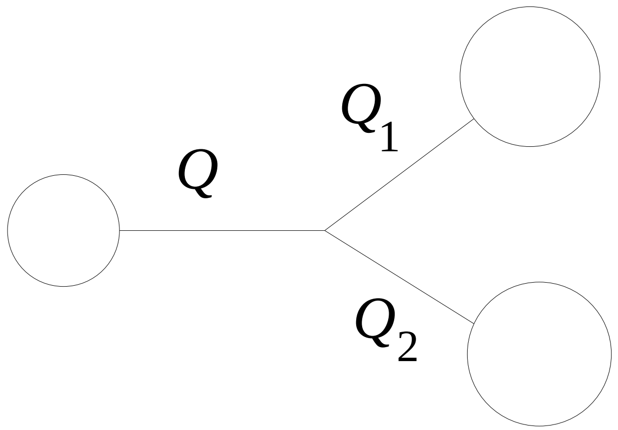
Back to Feynman graphs:



The corresponding intrinsic representation looks like

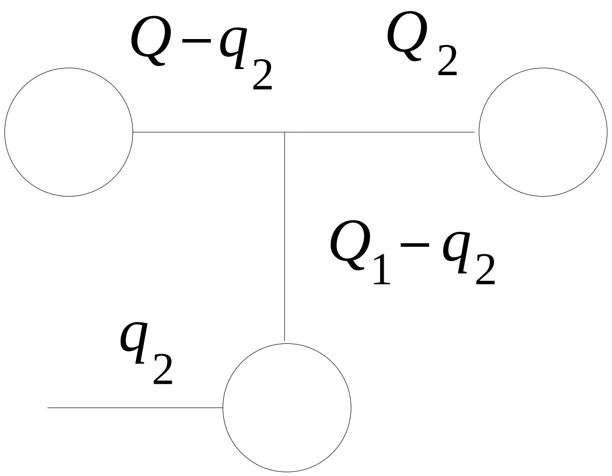
62	-2	4	-2	58	31
58	31	2	-2	56	2
56	2	48	33	8	1
48	33	16	-3	32	4

Time-like momenta $q^2 \geq 0$



$$
\begin{aligned}
d\Phi_n &= \ldots \frac{dQ_1^2}{2\pi}\,\frac{dQ_2^2}{2\pi}\, d\Phi_2(Q \to Q_1, Q_2)\ldots \\
&= \ldots \frac{dQ_1^2}{2\pi}\,\frac{dQ_2^2}{2\pi}\, d\cos\theta\, d\phi\, \frac{\lambda^{1/2}(Q^2, Q_1^2, Q_2^2)}{32\pi^2\, Q^2}\ldots
\end{aligned}
$$

Space-like momenta



$$d\Phi_n = \ldots \frac{dQ_1^2}{2\pi}\frac{dQ_2^2}{2\pi}\, d\Phi_2(Q \to Q_1, Q_2) \ldots$$
$$= \ldots \frac{dQ_1^2}{2\pi}\frac{dQ_2^2}{2\pi}\, dt\, d\phi\, \frac{1}{32\pi^2\, Q\, |\vec{q}_2|} \ldots$$

$$t = (Q_1 - q_2)^2 = m_2^2 + Q_1^2 - \frac{E_2}{Q}(Q^2 + Q_1^2 - Q_2^2) + \frac{\lambda^{1/2}}{Q}|\vec{q}_2| \cos\theta$$

- **Find limits of $t(Q_1^2, \cos\theta)$:**

$$t_{\pm} = m_2^2 + Q_1^2 - \frac{E_2}{Q}(Q^2 + Q_1^2 - Q_2^2) \pm \frac{\lambda^{1/2}}{Q}|\vec{q}_2|$$

In order to find the maximum of t_+ we study the function $\partial t_+/\partial Q_1^2$ in the region $Q_{1,min}^2 < Q_1^2 < (Q - Q_2)^2$. Since

$$\frac{\partial^2 t_+}{\partial (Q_1^2)^2} = -4Q^2 Q_2^2 \lambda^{-3/2} \frac{|\vec{q}_2|}{Q} \leq 0$$

and

$$\partial t_+/\partial Q_1^2|_{Q_1^2=(Q-Q_2)^2} \to -\infty$$

we just consider two cases ($|\vec{q}_2| \neq 0$):

1. $\partial t_+/\partial Q_1^2|_{Q_1^2=Q_{1,min}^2} < 0$ in which case $t_{max} = t_{+,max} = t_+(Q_1^2 = Q_{1,min}^2)$, and
2. $\partial t_+/\partial Q_1^2|_{Q_1^2=Q_{1,min}^2} > 0$ in which case one can easily derive

$$t_{max} = t_+(Q_1^2 = x_-) \text{ with}$$

$$x_- = Q^2 + Q_2^2 - 2\,Q\,Q_2 \frac{1 - E_2/Q}{\sqrt{\alpha}}\,, \quad \alpha = \left(1 - \frac{E_2}{Q}\right)^2 - \left(\frac{|\vec{q}_2|}{Q}\right)^2 > 0$$

♠ **Q_1^2-limits:**

The limits for the Q_1^2-integration for given t can now be fixed by the condition $|\cos\theta| \leq 1$ or equivalently

$$\Pi(Q_1^2) \leq 0$$

with

$$\Pi(Q_1^2) = \left(t - Q_1^2 - m_2^2 + \frac{E_2}{Q}(Q^2 + Q_1^2 - Q_2^2)\right)^2 - \left(\frac{|\vec{q}_2|}{Q}\right)^2 \lambda$$

If $y_1 \leq y_2$ are the two roots of the polynomial $\Pi(Q_1^2)$ then we have

1. For $a > 0$, $y_- < Q_1^2 < y_+$, with $y_- = \max(y_1, Q^2_{1,min})$ and $y_+ = \min(y_2, Q^2_{1,max})$
2. For $a < 0$ we have to satisfy two conditions $Q_1^2 < y_1$ or $y_2 < Q_1^2$ and $Q^2_{1,min} < Q_1^2 < Q^2_{1,max}$

- **At the end we get:**

$$d\Phi_n \to \prod ds_i \; p_i(s_i) \prod dt_j \; p_j(t_j) \prod d\phi_k \prod d\cos\theta_l$$

- $p(x)$ **are chosen so that 'singularities' are smoothed out!**

  - ♦ $(s-m^2)^2 + m^2\Gamma^2$ **for massive unstable particles,** $W^{\pm}$, Z.
  - ♦ s^{ν} **for time-like massless propagators, e.g.** γ, **gluons, fermions.**
  - ♦ $|t|^{\nu}$ **for space-like massless propagators.**

Final states	number of FG	$\sqrt{s}$ (GeV)	Cross section (fb)
$u\ \bar{d}\ s\ \bar{c}\ \gamma$	90(74)	200	199.75 (16)
$e^-\ \bar{\nu}_e\ \mu^+\ \nu_\mu\ \gamma$	108(100)	200	29.309 (25)
$\mu^-\ \bar{\nu}_\mu\ u\ \bar{d}\ \gamma\ \gamma$	587(210)	500	1.730 (58)
$\mu^-\ \bar{\nu}_\mu\ u\ \bar{d}\ c\ \bar{c}$	209(102)	500	0.1783 (20)
$\mu^-\ \bar{\nu}_\mu\ u\ \bar{d}\ c\ \bar{c}\ \gamma$	2142(339)	500	0.02451 (65)

6 (and more) fermion production

- **Input parameters and cuts as defined in hep-ph/0206070 and the WG**
- **DS (Dyson-Schwinger) elements, HC (helicity configurations) and CC (colour connections) determine the matrix element computational cost.**
- **FG (Feynman graphs) determine the phase-space generation cost. This is drastically reduced by using 'removing channels' techniques.**

FS	CC	DS	FG	HC	σ(fb)	T
$b\bar{b}u\bar{u}d\bar{d}$	2	139+256	821	16	49.05(9)	
	6	981/6	1417	16	49.92(12)	
$b\bar{b}u\bar{u}gg$	-	-	-	-	-	
	22	3276/22	1145	32	5.939(51)	
$b\bar{b}gggg$	-	-	-	-	-	
	65	6112/65	856	64	71.21(72)	
$b\bar{b}u\bar{d}e^-\bar{\nu}_e$	1	215	420	6	17.210(20)	
$b\bar{b}e^+\nu_e e^-\bar{\nu}_e$	1	434	1170	12	5.866(9)	
$b\bar{b}\mu^+\nu_\mu e^-\bar{\nu}_e$	1	200	360	6	5.822(7)	
$b\bar{b}\mu^+\nu_\mu \mu^-\bar{\nu}_\mu$	1	241	405	8	5.809(5)	
$b\bar{b}\mu^+\nu_\mu \tau^-\bar{\nu}_\tau$	1	121	180	4	5.798(4)	
$b\bar{b}u\bar{d}\mu^-\bar{\nu}_\mu$	1	130	210	4	17.107(18)	
	2	145+47	242	4	17.204(18)	

FS=final states CC=colour connections DS=Dyson-Schwinger FG=Feynman graphs HC=helicity configurations Input parameters: hep/ph 0206070 *:average number of DS per colour connection

FS	CC	DS	FG	HC	σ(fb)	T
$b\bar{b}u\bar{u}d\bar{d}$	2	147 268	880	32	50.55(26)	
	6	202 343 83 148 148 83	1484	32	50.20(13)	
$b\bar{b}u\bar{u}gg$	-	-	-	-	-	
	22	3361/22	1168	64	8.832(43)	
$b\bar{b}gggg$	-	-	-	-	-	
	65	6112/65	856	64	22.72(13)	
$b\bar{b}u\bar{d}e^-\bar{\nu}_e$	1	229	464	12	17.353(31)	
	2	254 84	528	12	17.492(41)	
$b\bar{b}e^+\nu_e e^-\bar{\nu}_e$	1	458	1306	24	5.963(11)	
$b\bar{b}\mu^+\nu_\mu e^-\bar{\nu}_e$	1	214	402	12	5.868(10)	
$b\bar{b}\mu^+\nu_\mu\mu^-\bar{\nu}_\mu$	1	255	452	16	5.839(12)	

Input parameters: Working Group

FS	CC	DS	FG	HC	σ(fb)	T
$e^- e^+ u\bar{u}d\bar{d}$	2	403+235	1640	24	1.212(22)	
$e^- e^+ u\bar{u}e^- e^+$	1	749	3474	40	0.0064(3)	
$e^- e^+ u\bar{u}\mu^- \mu^+$	1	403	1158	24	0.0086(5)	
$\nu_e \bar{\nu}_e u\bar{d}d\bar{u}$	2	150+246	618	8	1.60(20)	
$\nu_e \bar{\nu}_e u\bar{d}e^- \bar{\nu}_e$	1	303	614	3	0.88(6)	
$\nu_e \bar{\nu}_e u\bar{d}\mu^- \bar{\nu}_\mu$	1	142	196	2	0.77(5)	

Input parameters: Working Group

FS	CC	DS	FG	HC	σ(fb)	T
$\mu^+\mu^-\mu^+\nu_\mu e^-\bar{\nu}_e$	1	334	716	9	0.0373(6)	
$\mu^+\mu^- u\bar{d}e^-\bar{\nu}_e$	1	213	418	6	0.1073(9)	
$\mu^+\mu^-\mu^+\mu^- e^- e^+$	1	648	2316	36	0.00281(16)	
$\mu^+\mu^- u\bar{u}e^- e^+$	1	403	1158	24	0.0099(8)	
$\mu^+\mu^- u\bar{u}d\bar{d}$	2	138+256	820	16	0.2525(33)	
$\mu^+\mu^- u\bar{u}u\bar{u}$	2	256+256	1158	24	0.00587(13)	

Input parameters: Working Group

FS	CC	DS	FG	HC	σ(fb)	T
$\mu^- \mu^+ b\bar{b}b\bar{b}$	2	256+256	1158	24	0.00827(18)	
$b\bar{b}b\bar{b}u\bar{d}\mu^- \nu_\mu$	2	938+938	23116	18	0.001576(17)	
$u\bar{d}e^- \bar{\nu}_e \gamma$	1	119	108	6	10.87(18)	

Input parameters: Working Group

$$g\,g \to b\,\bar{b}\,b\,\bar{b}\,W^-W^+$$

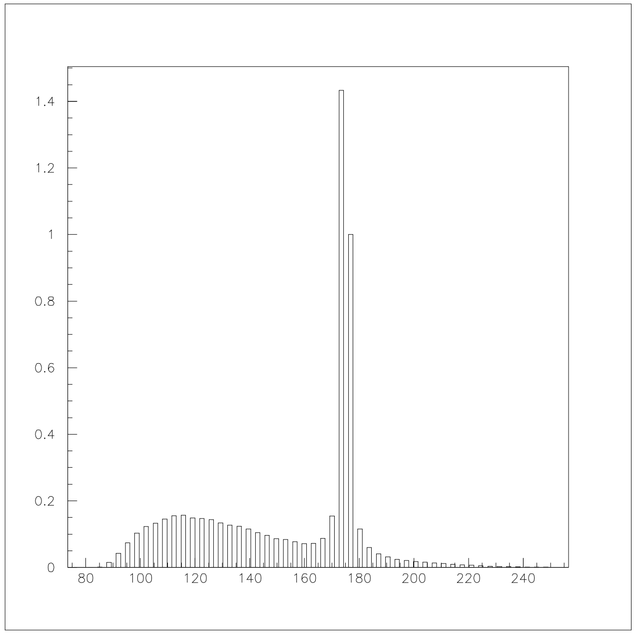
- challenging process, from a computational point of view
- a nice example to demonstrate the ability of PHEGAS/HELAC to deal with QCD processes.
- background of $t\bar{t}H$ production

MC points $w > 0$	result (fb)	error (fb)	efficiency (%)	efficiency $w > 0$ (%)
99442	4.716	0.024	3.3	33

- energy $\sqrt{s} = 500$ GeV and to 1×10^6 MC points.
- Feynman graphs for this process is 960, with 4! colour configurations, without taking into account electroweak contributions from Z and γ intermediate states.

Parameters used are $g_{QCD} = 1$, $m_{top} = 175$ GeV and $\Gamma_{top} = 1.5$ GeV. Moreover the following set of cuts has been applied:

$$M_{q,q'} > 20\mathrm{GeV}, \quad E_q > 20\mathrm{GeV}, \quad |\cos\theta(q, \mathrm{beam})| < 0.9,$$



$$p\,p \to t\,\bar{t}\,b\,\bar{b}\,b\,\bar{b}$$

- **Another challenging process, from a computational point of view**
- **A nice example to demonstrate the ability of** `PHEGAS/HELAC` **to deal with QCD processes in a realistic setup.**
- **A background of $t\bar{t}HH$ production, which seems interesting in a high-luminosity LHC version for HHH coupling.**
- **Feynman graphs for this process is 1454 (gg), with 5! colour configurations.**
- **A two-phase implementation has been set up.**
- **Srtucture functions and α_s from** `PDFLIB`**, CTEQ-4L (LO).**
- **Kinematical decays of $t \to bW^+$ has been implemented**
- **Cuts: $p_T^b > 20GeV$, $|\eta_b| < 2.5$, $\Delta R > 0.5$**

The result is 1.053 ± 0.073 (fb) @ LHC

- $e^- \, e^+ \to e^- \, e^+ \, \mu^- \, \mu^+$

σ_{tot} **(in nb)**

$\sqrt{s}$(GeV)	BDK	NEXTCALIBUR
20	98.9 ± 0.6	99.20 ± 0.98
35	131.4 ± 2.2	131.03 ± 0.88
50	154.4 ± 0.9	152.33 ± 0.83
100	205.9 ± 1.2	204.17 ± 1.73
200	—	263.50 ± 1.31
200 (all)	—	265.58 ± 1.44

- $e^- \, e^+ \to e^- \, e^+ \, e^- \, e^+$

$\boldsymbol{\sigma_{tot}}$ **(in nb $\times 10^7$)**

$\sqrt{s}$(GeV)	BDK	NEXTCALIBUR
20	**0.920 ± .011**	**0.905 ± .011**
35	**1.070 ± .015**	**1.079 ± .014**
50	**1.233 ± .018**	**1.214 ± .016**
100	**1.459 ± .025**	**1.485 ± .020**
200	—	**1.776 ± .019**
200 (all)	—	**1.787 ± .030**

The high-colour processes

The idea is to replace colour summation with integration and then follow a MC approach

$$G^{\mu}_{AB}(P_i) = \sum_{a=1}^{8} G^{a}(P_i)\eta^{a}(z) = \sqrt{6}\left(z_{iA}z^{*}_{iB} - \frac{1}{3}\delta_{AB}\right)\epsilon^{\mu}_{\lambda}(P_i)$$
$$\psi_A(P_i) = \sqrt{3}\,u(P_i)\,z_{iA}$$
$$\bar{\psi}_A(P_i) = \sqrt{3}\,\bar{u}(P_i)\,z^{*}_{iA}$$

In such a representation the amplitude can be seen

$$\mathcal{M}(z_1, z_2, \ldots) = \sum z_1 \cdot z_{\sigma(i)_1} z_2 \cdot z_{\sigma(i)_2} \ldots \mathcal{A}_i$$

and the MC is over

$$\int [dz] \equiv \int \left(\prod_{i=1}^{3} dz_i dz_i^* \right) \delta(\sum_{i=1}^{3} z_i z_i^* - 1)$$

where

$$\int [dz] G_{AB} G_{CD} = \int [dz] \sqrt{6} \left(z_A z_B^* - \frac{1}{3}\delta_{AB} \right) \sqrt{6} \left(z_C z_D^* - \frac{1}{3}\delta_{AB} \right)$$

$$= \frac{1}{2} \left(\delta_{AD}\delta_{CB} - \frac{1}{3}\delta_{AB}\delta_{CD} \right)$$

Multi-jet processes

Beyond any colour treatment a summation over different flavours is also needed.

Up to now the most straightforward way was to count the distinct processes and then multiply with a multiplicity factor, i.e.

process	flavour
$gg \to ggg$	1
$q\bar{q} \to ggg$	8
$qg \to qgg$	8
$qg \to qgg$	8
$gg \to q\bar{q}g$	5
$q\bar{q} \to q\bar{q}g$	8
$q\bar{q} \to r\bar{r}g$	32
$qq \to qqg$	8
$q\bar{r} \to q\bar{r}g$	24
$qr \to qrg$	24
$qg \to qq\bar{q}$	8
$qg \to qr\bar{r}$	32
$gq \to qq\bar{q}$	8
$gq \to qr\bar{r}$	32

initial-state type		distinct processes	multiplicity factor
A	(gg)	$C_1(n)$	$\chi(n_0, n_1, \ldots, n_f; f)$
B	$(q\bar{q})$	$C_2(n)$	$\chi(n_0, n_2, \ldots, n_f; f-1)$
C	$(gq$ and $qg)$	$C_2(n-1)$	$\chi(n_0, n_2, \ldots, n_f; f-1)$
D	(qq)	$C_2(n-2)$	$\chi(n_0, n_2, \ldots, n_f; f-1)$
E	$(qq'$ and $q\bar{q}')$	$C_3(n-2)$	$\chi(n_0, n_3, \ldots, n_f; f-2)$

In order to clarify what we mean we consider the example of the type A initial state. Each distinct process is defined by an array $(n_0, n_1, \ldots, n_f)$. For instance, in the case of four-jet production we have

$$
\begin{array}{ll}
(4,0,0,0,0,0) & gg \to gggg \\
(2,1,0,0,0,0) & gg \to ggq\bar{q} \\
(0,2,0,0,0,0) & gg \to q\bar{q}q\bar{q} \\
(0,1,1,0,0,0) & gg \to q\bar{q}r\bar{r}
\end{array}
$$

$$C_1(n) = \sum_{n_0+2n_1+\ldots+2n_f=n} \Theta(n_1 \geq n_2 \geq \ldots \geq n_f)$$

$$C_2(n) = \sum_{n_0+2n_1+\ldots+2n_f=n} \Theta(n_2 \geq n_3 \geq \ldots \geq n_f)$$

and

$$C_3(n) = \sum_{n_0+2n_1+\ldots+2n_f=n} \Theta(n_3 \geq n_4 \geq \ldots \geq n_f)$$

A distinct process, given by the array $(n_0, n_1, \ldots, n_f)$ has a multiplicity factor :

$$\chi(n_0, n_1, \ldots, n_f; f) = n_f(n_f - 1)\ldots(n_f - j + 1)/j!$$

$$\begin{array}{rcl} j = f & \text{if} & \prod_{i=1}^{f} n_i \neq 0 \\ j = f - 1 & \text{if} & \prod_{i=1}^{f-1} n_i \neq 0 \\ \ldots & & \\ j = 1 & \text{if} & n_1 \neq 0 \\ j = 0 & \text{otherwise} & \end{array}$$

Now we can think of a flavour-MC, so the wave function is multiplied by an N_f-dimensional array representing flavour , $\vec{f} = \sqrt{N_f}(f_1, f_2, \ldots)$ such that $\langle f_i f_j \rangle = \delta_{ij}$ with a weight proportional to the relevant pdf for initial state flavours

In that case a process like

$$gg \to ggq\bar{q}q\bar{q}$$

will actually represent a plethora of processes.

The number of distinct processes is now given by

$$9k + 3 \ \text{ if } \ n = 2k \quad \text{and} \quad 9k + 7 \ \text{ if } \ n = 2k + 1$$

# of jets	2	3	4	5	6	7	8	9	10
# of D-processes	12	16	21	24	30	34	39	43	48
# of dist.processes	10	14	28	36	64	78	130	154	241
total # of processes	126	206	621	861	1862	2326	4342	5142	8641

Multi-jet rates

$$p_{T\ i} > 60\ GeV, \quad \theta_{ij} > 30^o \quad |\eta_i| < 3$$

# jets	3	4	5	6	7	8
$\sigma(nb)$	91.41	6.54	0.458	2.97×10^{-2}	2.21×10^{-3}	2.12×10^{-4}
% Gluon	45.7	39.2	35.7	35.1	33.8	26.6

A new code $\Rightarrow$ `JetI`

- anybody to tell us how many Feynman graphs in $gg \to 8g$?
- or $gg \to 2g3u3\bar{u}$?

- **Feynman graphs in $gg \to 8g$** **10,525,900 !!**

- **or $gg \to 2g3u3\bar{u}$** **946,050!**

Higher-order corrections

- Fermion-loop corrections have been implemented and studied for up to 3-point vertices.
- We have started the computation of 4-point contributions.
  - `FORM` has been used to reduce the expressions to Passarino-Veltman coefficient functions
  - `FF` has been updated to include level-4 tensor coefficient functions for 4-point integrals.
  - Implementation and checking in `HELAC` is in progress.
- This implementation will allow to study 4 fermion+γ and 6 fermion production including the running of electroweak couplings.

Outlook

PHEGAS/HELAC: a framework for high-energy phenomenology

- **Standard Model fully included**

★ **High color charge processes: multijet production**
P.Draggiotis, R.H.Kleiss and C.G.Papadopoulos, Phys. Lett. B439 (1998) 157; Eur. Phys. J. C 24 (2002) 447 hep-ph/0202201

★ **Higher order corrections**

– **Direct approach. Ongoing work to better understand Dyson-Schwinger equations and loop calculations: stepping equations, recursive actions, etc.**

– **Running couplings and masses: 4-point FL contributions and BBC non-local approch to go beyond 4-fermion final states.**