# Matching Matrix Elements and Parton Showers with HERWIG

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- Introduction
- Approach
- CKKW Algorithm
- $e^+e^-$ collisions
- Hadron collisions
- Conclusions

# Introduction

- Parton Shower (PS) simulations use the soft/collinear approximation:
  - Good for simulating the internal structure of a jet;
  - Can't produce high $p_T$ jets.
- Matrix Elements (ME) compute the exact result at fixed order:
  - Good for simulating many high $p_T$ jets;
  - Can't give the structure of a jet.
- We want to use both in a consistent way, *i.e.*
  - ME gives hard emission,
  - PS gives soft/collinear emission,
  - Smooth matching between the two,
  - No double counting of radiation.

# Introduction

- There have been a number of attempts to do this.
- The first attempts got the hardest emission correct for simple processes, for example
  - $e^+e^- \to q\bar{q}$
  - DIS
  - Drell-Yan
  - Top Decay
- There were problems with this
  - Only the hardest emission was correctly described.
  - The leading-order normalization was retained.

# Introduction

- Recently there has been renewed interest.

- Two types of approach have emerged.

1. NLO Simulation
   - NLO normalization of the cross section.
   - Gets the hardest emission correct.
   - Only programs by Frixione/Webber and Dobbs for gauge boson pairs.

2. Multi-jet Leading Order
   - Still leading order.
   - Gets many hard emissions correct.
   - Programs by Krauss et. al. and Lonnblad for $e^+e^-$.

# Approach

- I will be looking at the second type of approach.
- The idea is to implement the algorithm of CKKW to produce a multi-jet simulation using existing tools
  - Use the Les accord compliant M.E. generators to calculate matrix elements (MADGRAPH, ALPGEN)
  - Perform the reweighting etc. specified by the algorithm before passing the events to HERWIG.
  - Use the Les accord interface to read the events into HERWIG.
  - Make only minimal changes to the event generator itself.
- Produce a better simulation using existing well tested tools.
- I will first review the algorithm of CKKW and then present some preliminary results.

# Procedure

- In order to match the ME and PS we need to separate the phase space:
  - One region should contain the soft/collinear region and be filled by the PS;
  - The other is filled by the matrix element.

- In Catani-Krauss-Kuhn-Webber (CKKW) prescription this is done via the Durham jet algorithm.
  - For all final-state particles compute the resolution variables
    - $d_{kB} \simeq E_k^2\theta_{kB}^2 \simeq k_{\perp kB}^2, \quad \theta_{kB} \to 0.$
    - $d_{kl} \simeq \min(E_k^2, E_l^2)\theta_{kl}^2, \quad \theta_{kl} \to 0$
  - The smallest of these $d_{kB}, d_{kl}$ is then selected. If $d_{ij}$ is the smallest the two particles are merged, if $d_{iB}$ is the smallest the particle is merged with the beam.
  - This procedure is repeated until the minimum value is above some stopping parameter $d_{\rm cut}$.
  - The remaining particles and pseudo-particles are then the hard jets.

# Procedure

1. Select the jet multiplicity with probability

$$P_n = \frac{\sigma_n}{\sum_{k=0}^{k=N} \sigma_k}$$

where $\sigma_n$ is the $n$-jet matrix element evaluated at resolution $d_{\mathrm{ini}}$ using $d_{\mathrm{ini}}$ as the scale for the P.D.F's and $\alpha_S$, $n$ is the number of additional jets.

2. Distribute the jet momenta according to the ME.

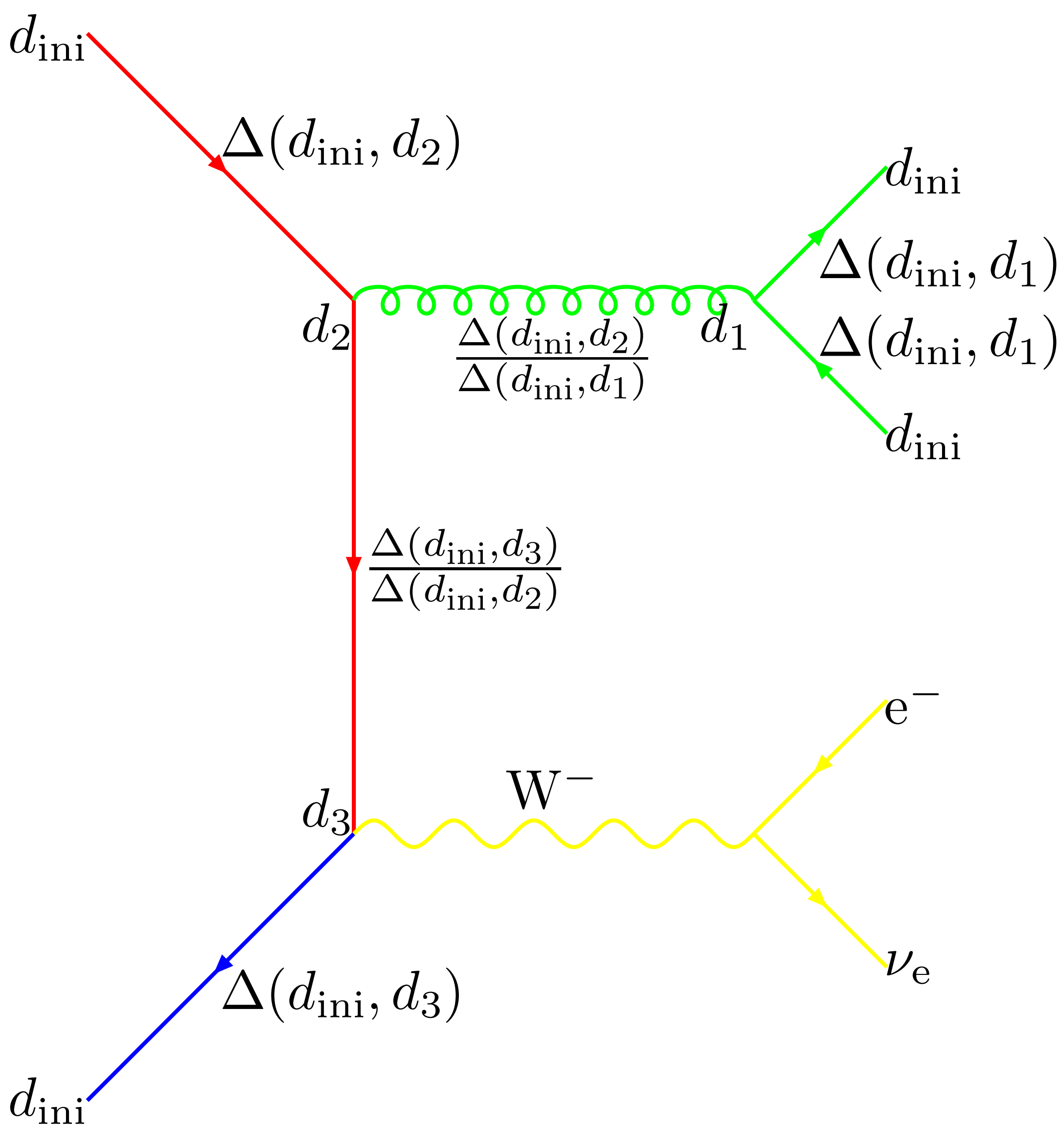
3. Cluster the partons to determine the values at which 1,2,..$n$-jets are resolved. These give the nodal scales for a tree diagram.

4. Apply a coupling-constant reweighting $\alpha_S(d_1)\alpha_S(d_2)..\alpha_S(d_n)/(\alpha_S(d_{\mathrm{ini}})^n \leq 1$.

5. Reweight the lines by a Sudakov factor $\Delta(d_{\mathrm{ini}}, d_j)/\Delta(d_{\mathrm{ini}}, d_k)$.

6. Accept the configuration if the product of the $\alpha_S$ and Sudakov weight is less the $\mathcal{R} \in [0,1]$., otherwise return to step 1.

# Procedure




7. Generate the parton shower from the event starting the evolution of each parton at the scale it was created and vetoing emission above the scale $d_{\rm ini}$

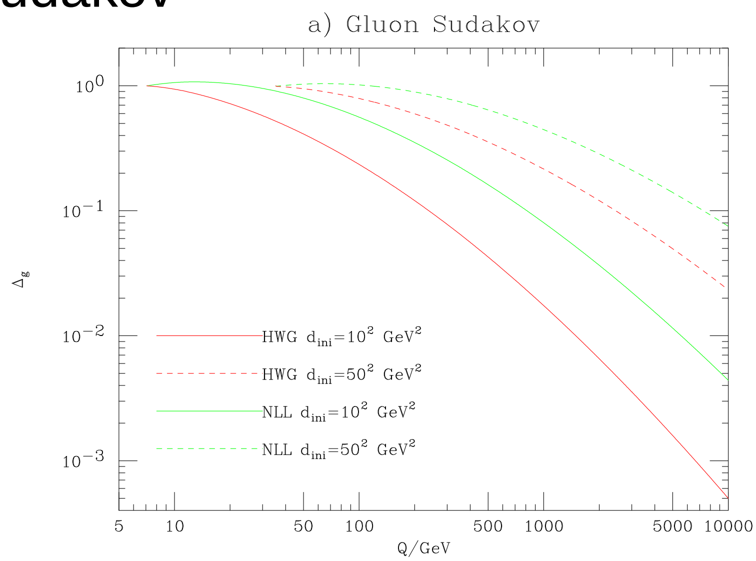
# Free Parameters

- Although the algorithm above ensures smooth matching at NLL log level there are a number of free choices
  - Sudakov Form Factors
    - HERWIG or NLL Sudakovs
    - Definition of the scale
    - Prefactor for the scale
  - $\alpha_S$
    - LO or NLO definition
    - Definition of the scale
    - Prefactor for the scale
  - Treatment of Highest Multiplicity Matrix Element
    - Same as other multiplicities?
    - Different treatment to describe more emissions?
  - Definition of $k_T$-algorithm away from soft/collinear limit
    - Definition of the $k_T$-measure.
    - Recombination scheme.

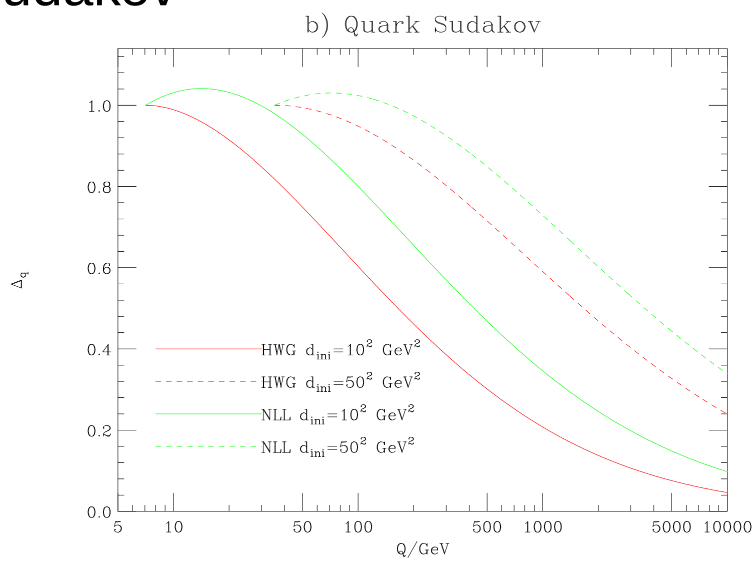
# Sudakov Form Factors

- Due to the choice of scale in $\alpha_S$ and regularization the HERWIG Sudakov is always smaller than the NLL one.

- Gluon Sudakov




- Quark Sudakov



# Sudakov Form Factors

- To look at the effects of the different Sudakov form factors study $e^+e^-$ collisions at $M_Z$.

- Look at the differential distributions with respect to $y_n$ where
$$y_n = \frac{d_n}{s},$$
and $d_n$ is the $k_T^2$ scale where the event changes from $n$ to $n-1$ jets.

- Start with a matching scale of $y = 0.001 \Rightarrow d = 8.3\mathrm{GeV}^2$.

- This is a very low scale which enhances the differences of different choices.

- Start by studying different Sudakovs, different scales in the Sudakov, and different scales in $\alpha_S$.

# Choice of Scale

- There are a number of choices to make.
- First for the Sudakovs

$$d_{\rm SUD} = \texttt{QFACT}(1) \begin{cases} k_T^2 & \texttt{ISCALE} = 1, \\ 2p_i \cdot p_j & \texttt{ISCALE} = 2, \\ Q^2 & \texttt{ISCALE} = 3, \end{cases}$$

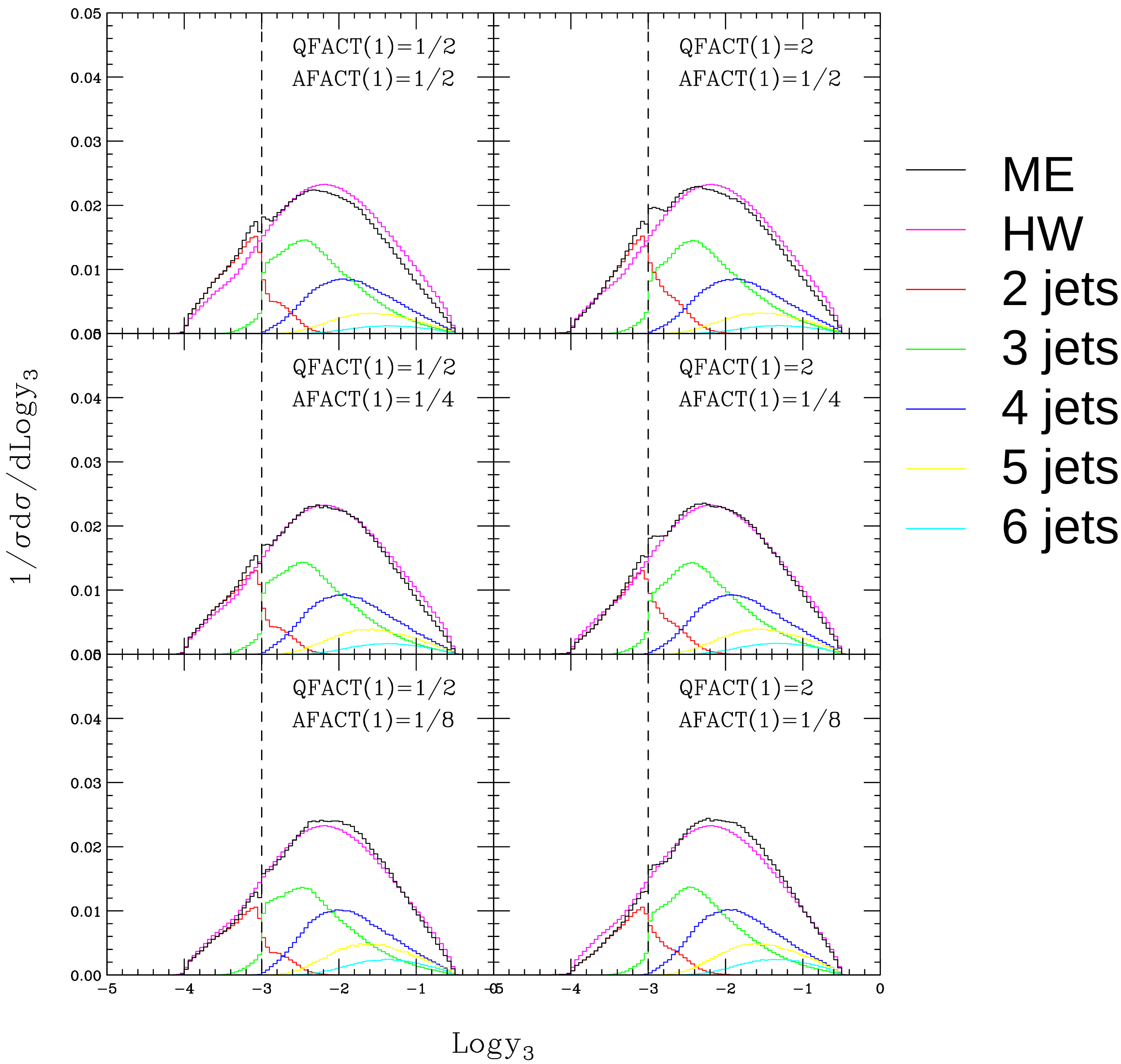
- and for $\alpha_S$.

$$d_{\alpha_S} = \texttt{AFACT}(1) \begin{cases} k_T^2 & \texttt{ISCALE} = 1, \\ 2p_i \cdot p_j & \texttt{ISCALE} = 2, \\ Q^2 & \texttt{ISCALE} = 3, \end{cases}$$

- HERWIG would use `QFACT(1)=1/2` for $e^+e^- \to \rm jets$ or Drell-Yan.
- However no radiation in shower above $\frac{1}{2}d_{\rm shower} \Rightarrow$ `QFACT(1)=2`
- HERWIG always has a smaller scale for $\alpha_S$ than in the Sudakov form factor.

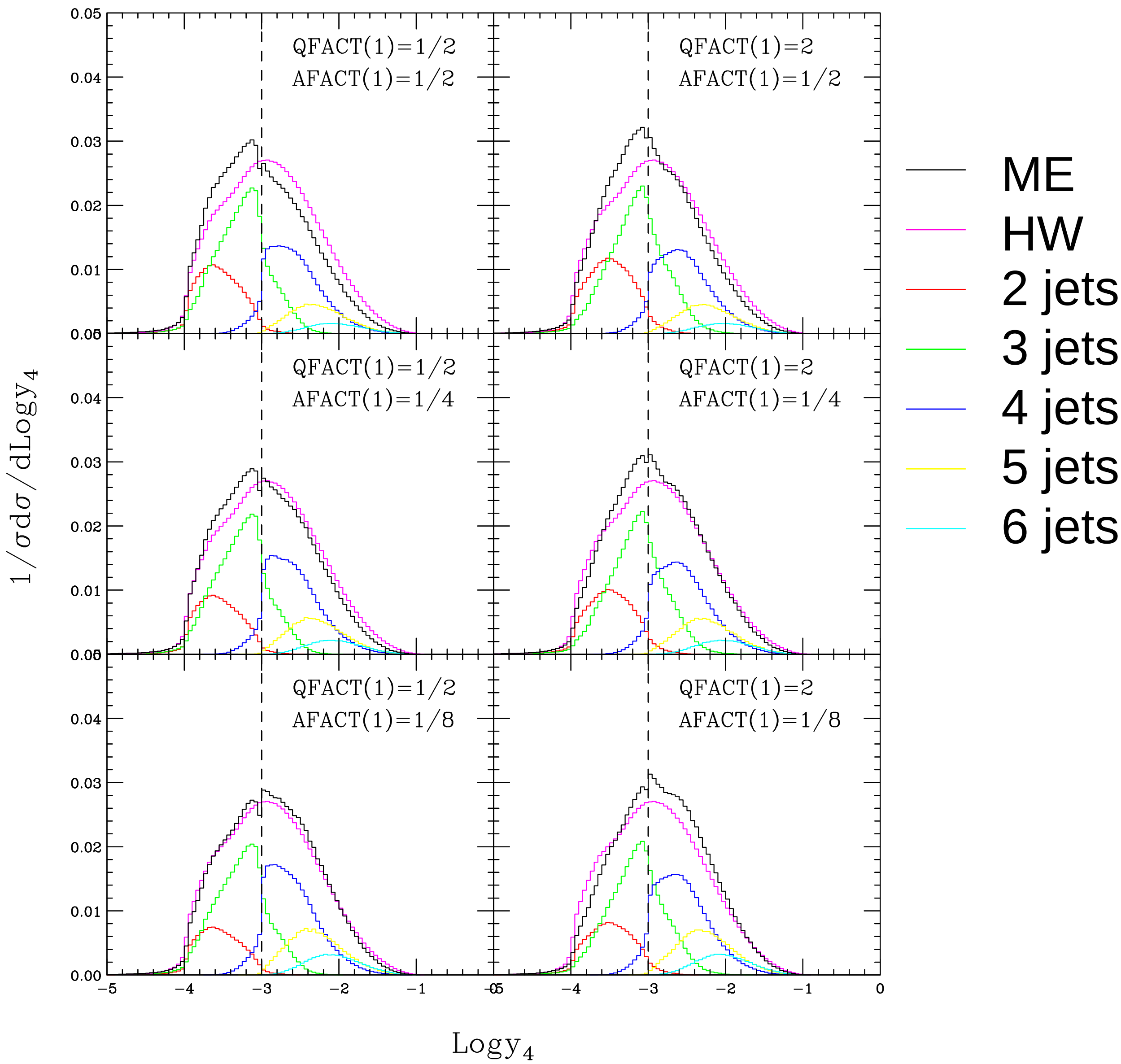
# NLL Sudakovs

- NLL Sudakovs with `ISCALE=1`.
- $y_3$ distribution.



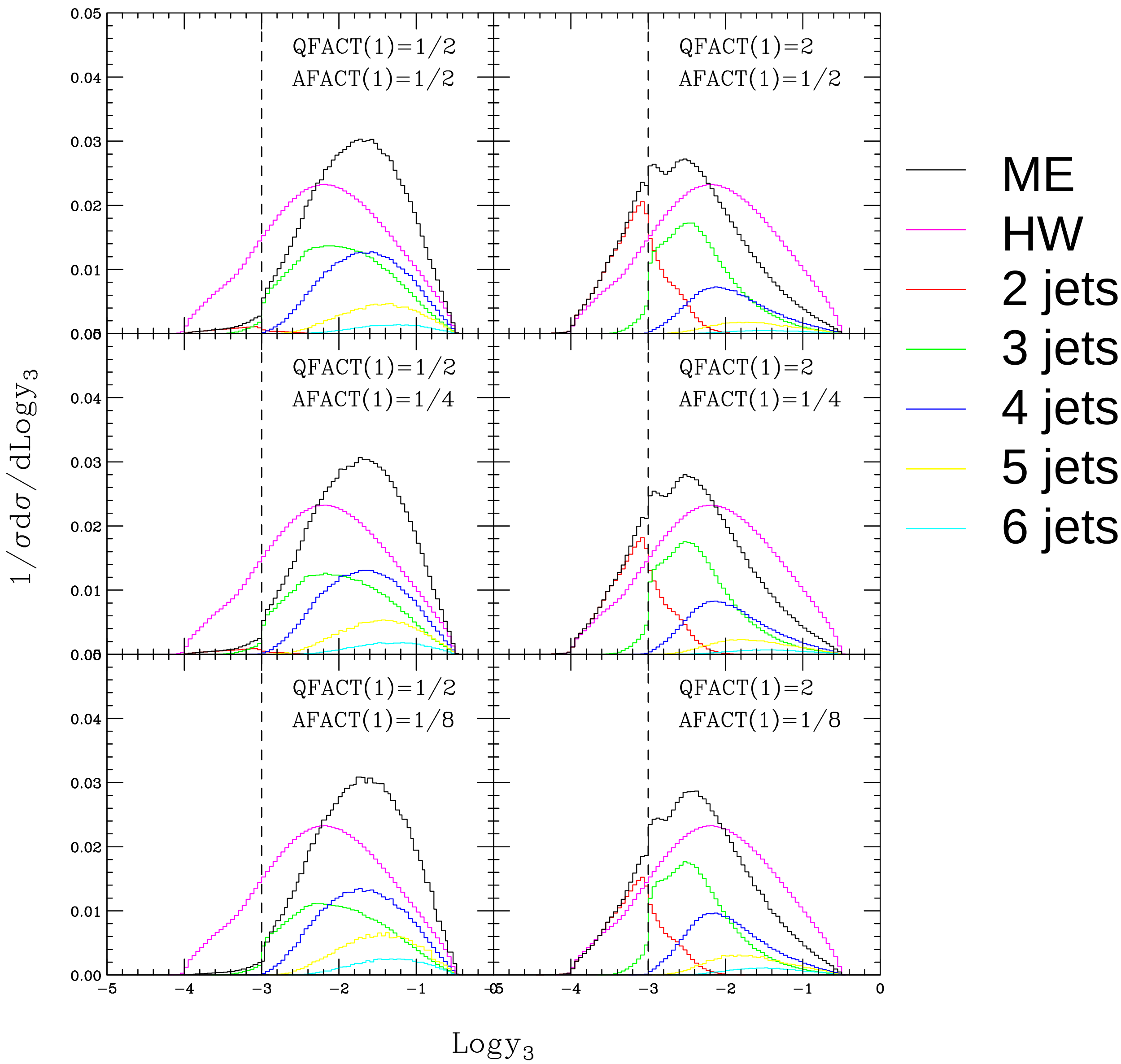
# NLL Sudakovs

- NLL Sudakovs with `ISCALE=1`.
- $y_4$ distribution



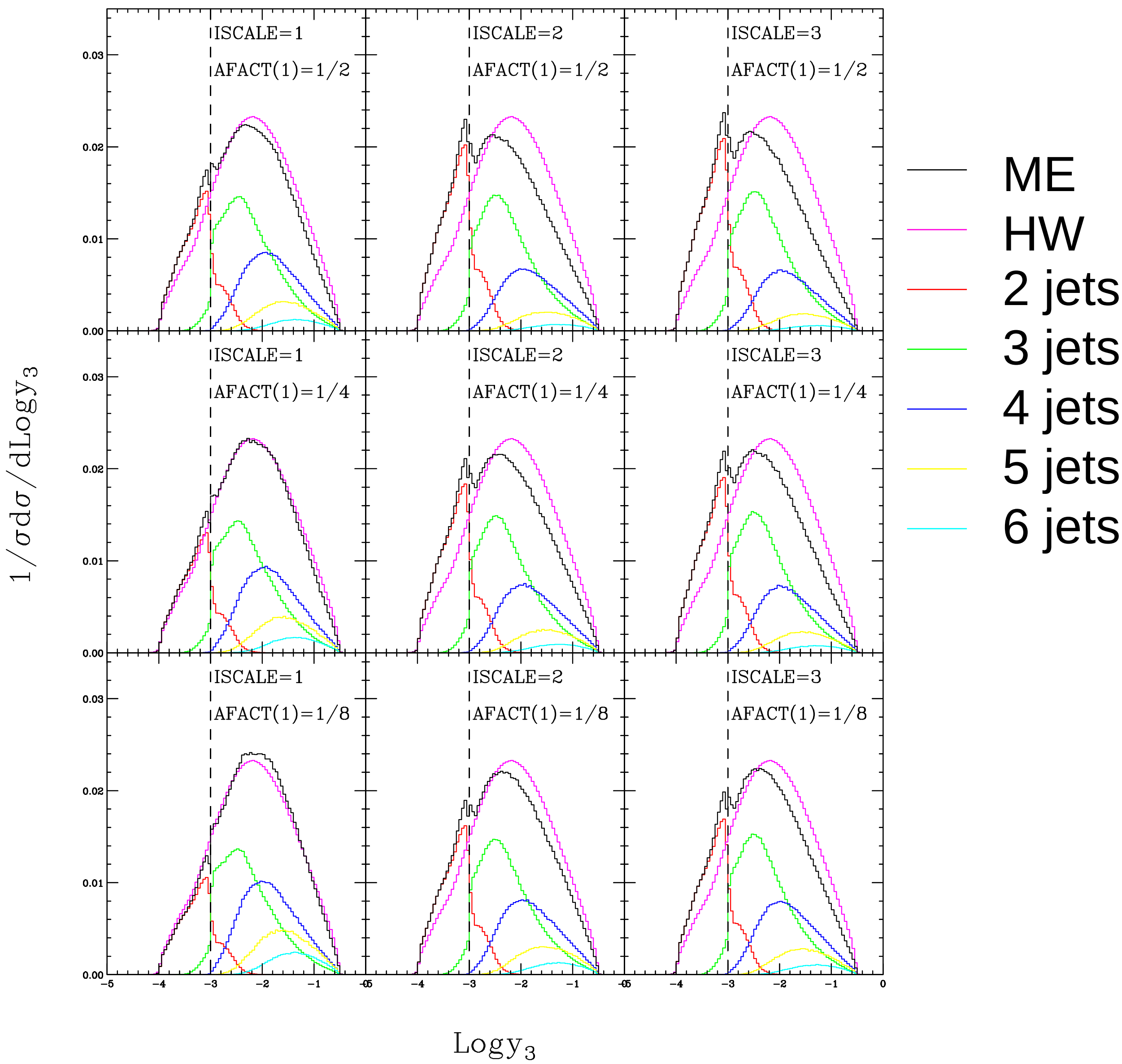
# HERWIG Sudakovs

- HERWIG Sudakovs with `ISCALE=1`.
- $y_3$ distribution.



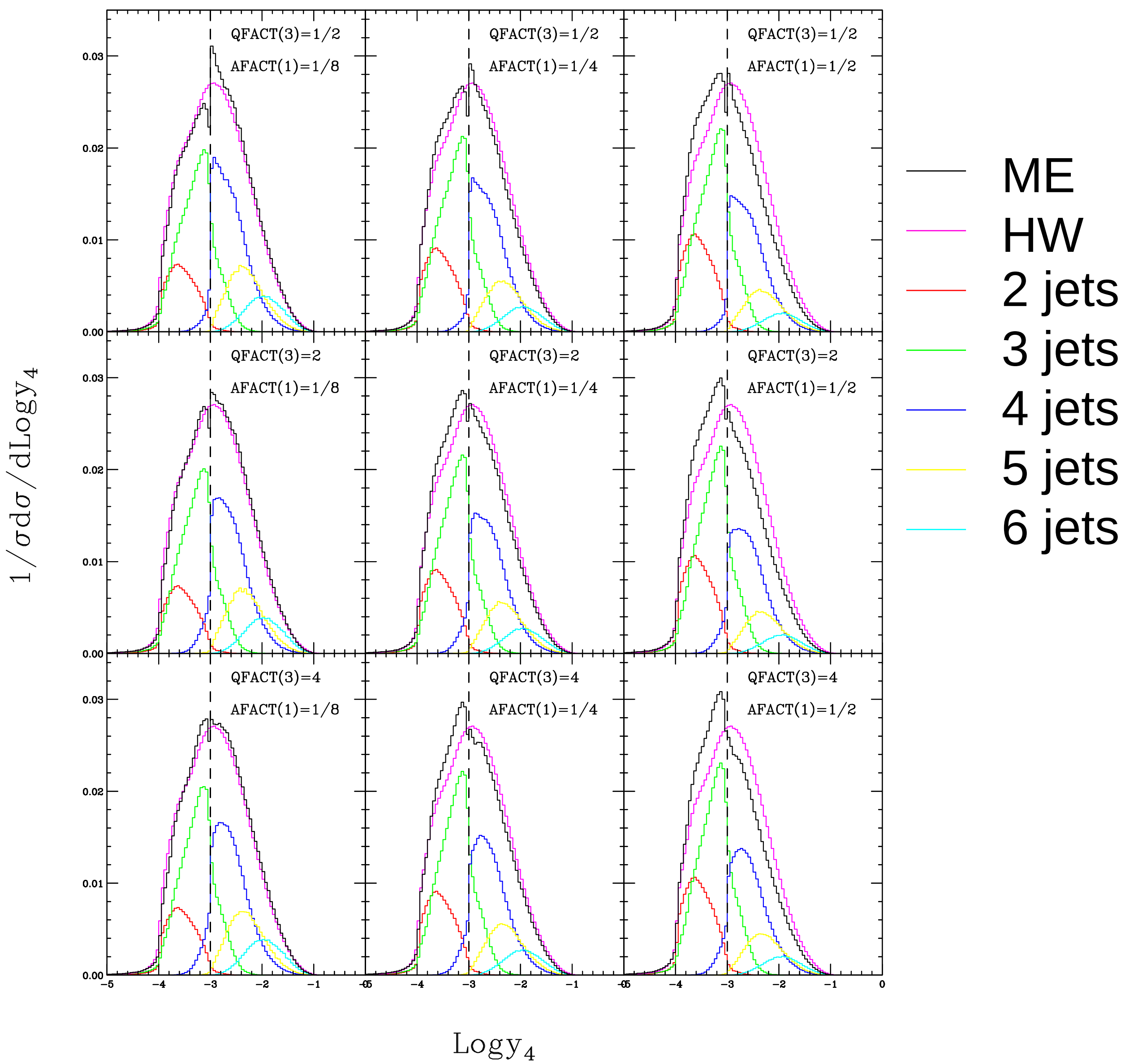
# NLL Sudakovs

- NLL Sudakovs with `QFACT(1)=1/2`.
- $y_3$ distribution.



# NLL Sudakovs

- NLL Sudakovs with `QFACT(1)=1/2`.
- Variation of `QFACT(3)` which is the minimum scale starting scale for the HERWIG shower.
- $y_4$ distribution.



# Treatment of Highest Multiplicity

- In the original CKKW algorithm all jet multiplicities were treated in the same way, `IFINAL=1`.

- Not appropriate for highest multiplicity.

- Instead only apply $\alpha_S$ reweighting and Sudakovs for internal lines.

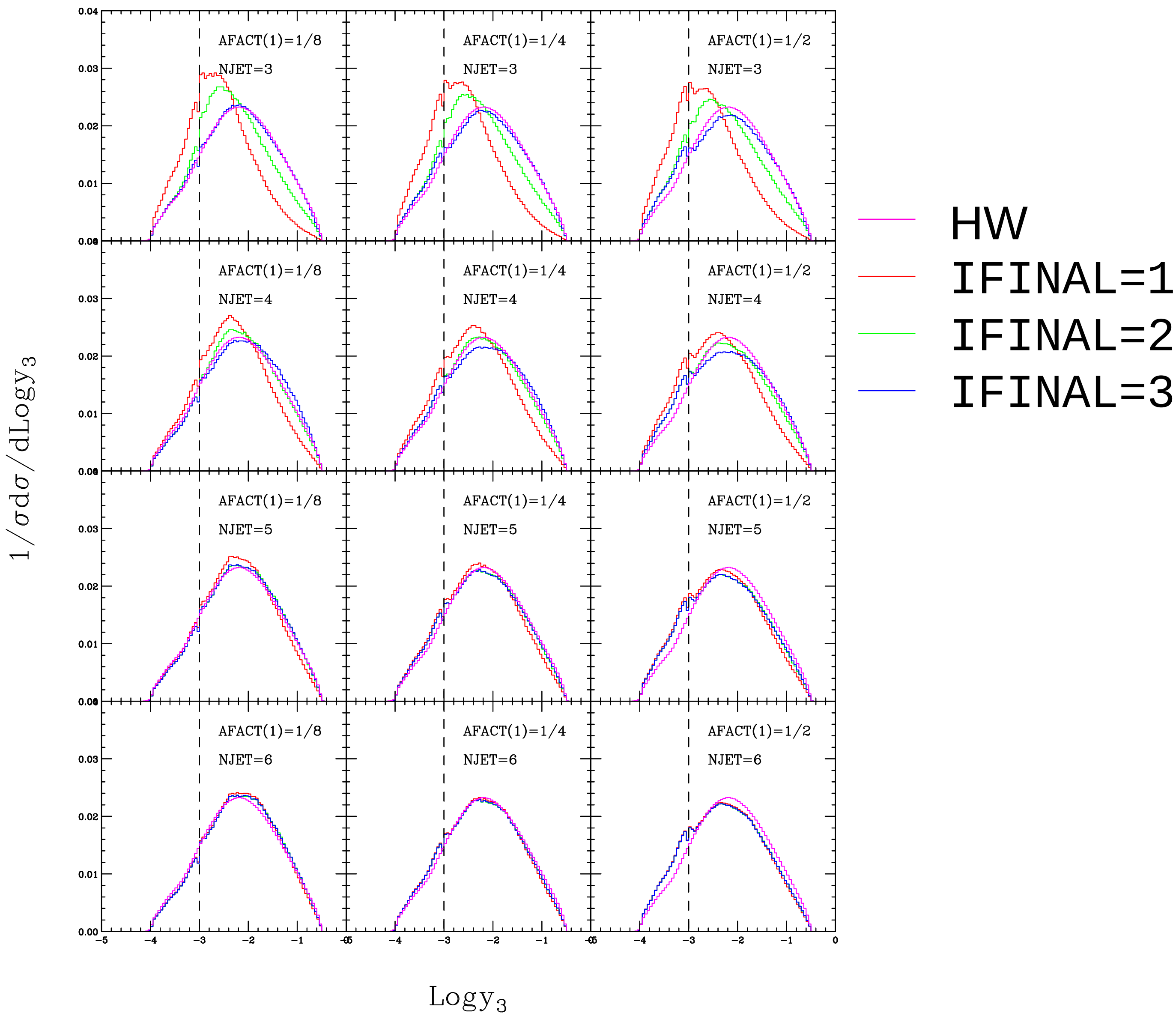
- Two options for the shower
  - Apply no veto and start at normal HERWIG scale, `IFINAL=2`
  - Start at same scale as other multiplicities but veto above scale of last emission in ME. `IFINAL=3`

- `IFINAL=2` is easier to implement but has some double counting problems

- `IFINAL=3` is the best theoretical choice.

# Treatment of Highest Multiplicity

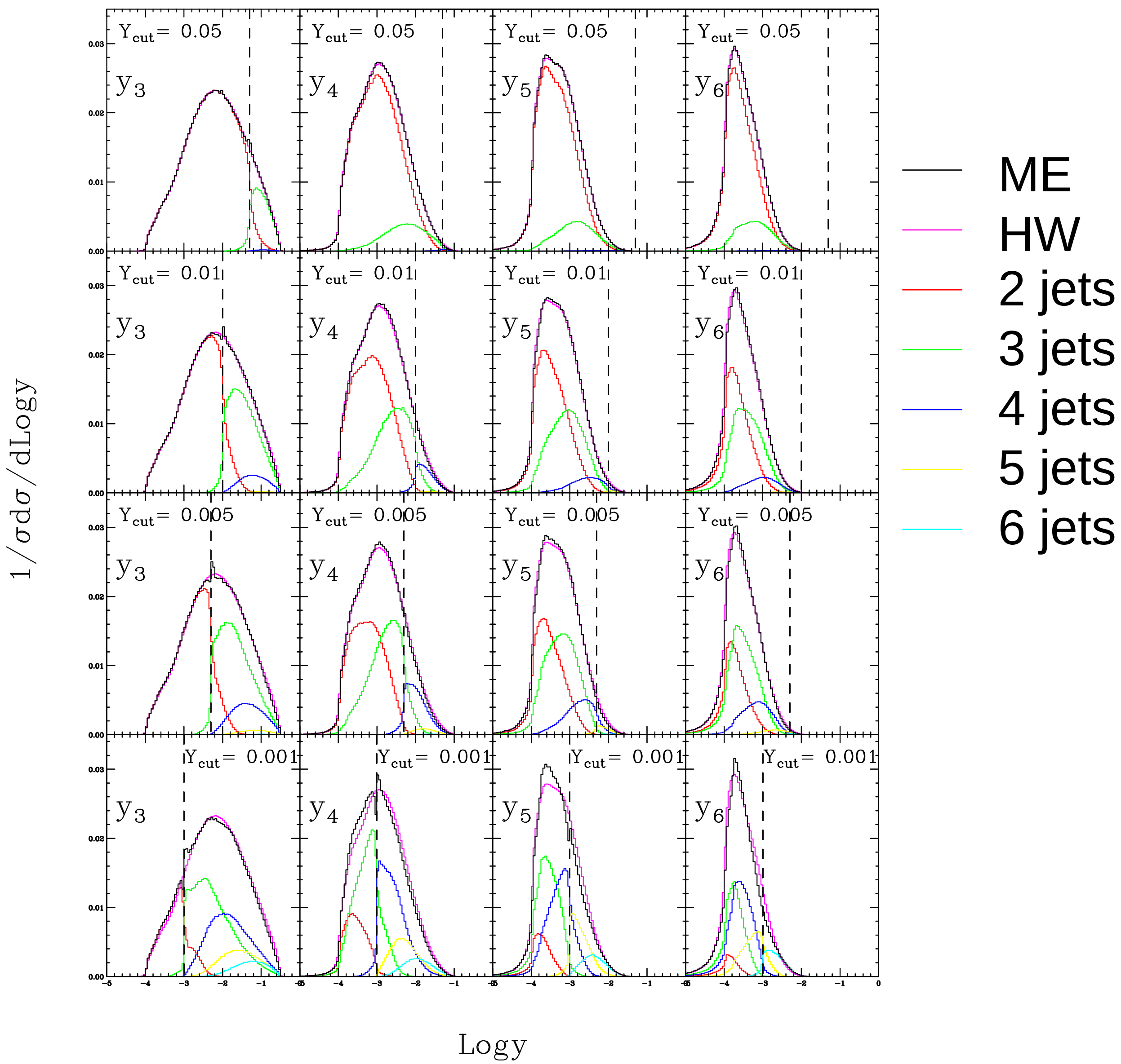
- NLL Sudakovs with `QFACT(1)=1/2`.
- $y_3$ distribution.



# Effect of Varying the cut-off at $M_Z$

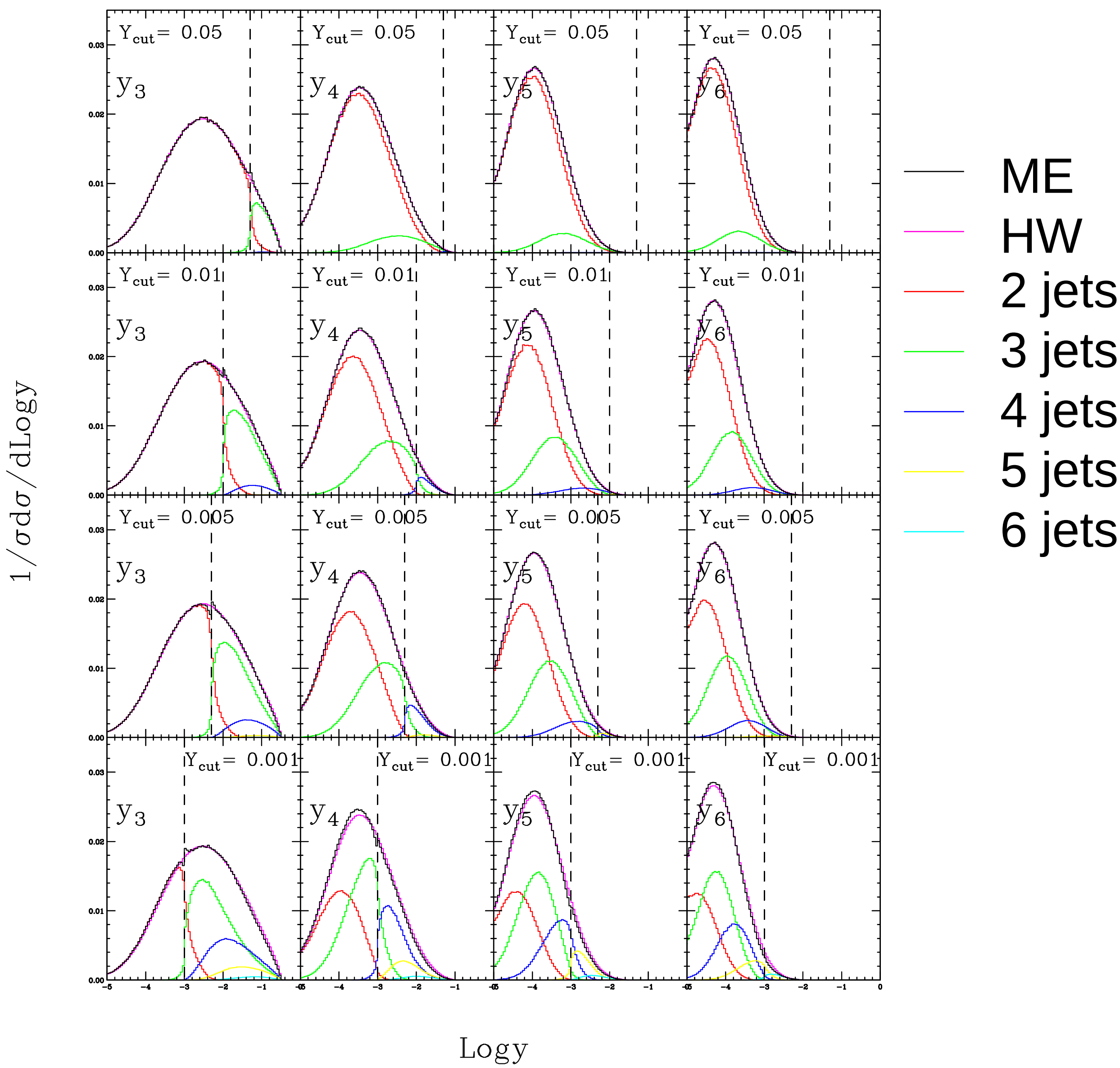
- NLL Sudakovs with `QFACT(1,2,3)=1/2`, `AFACT(1,2,3` `IFINAL=3`.

- Parton-level distributions.



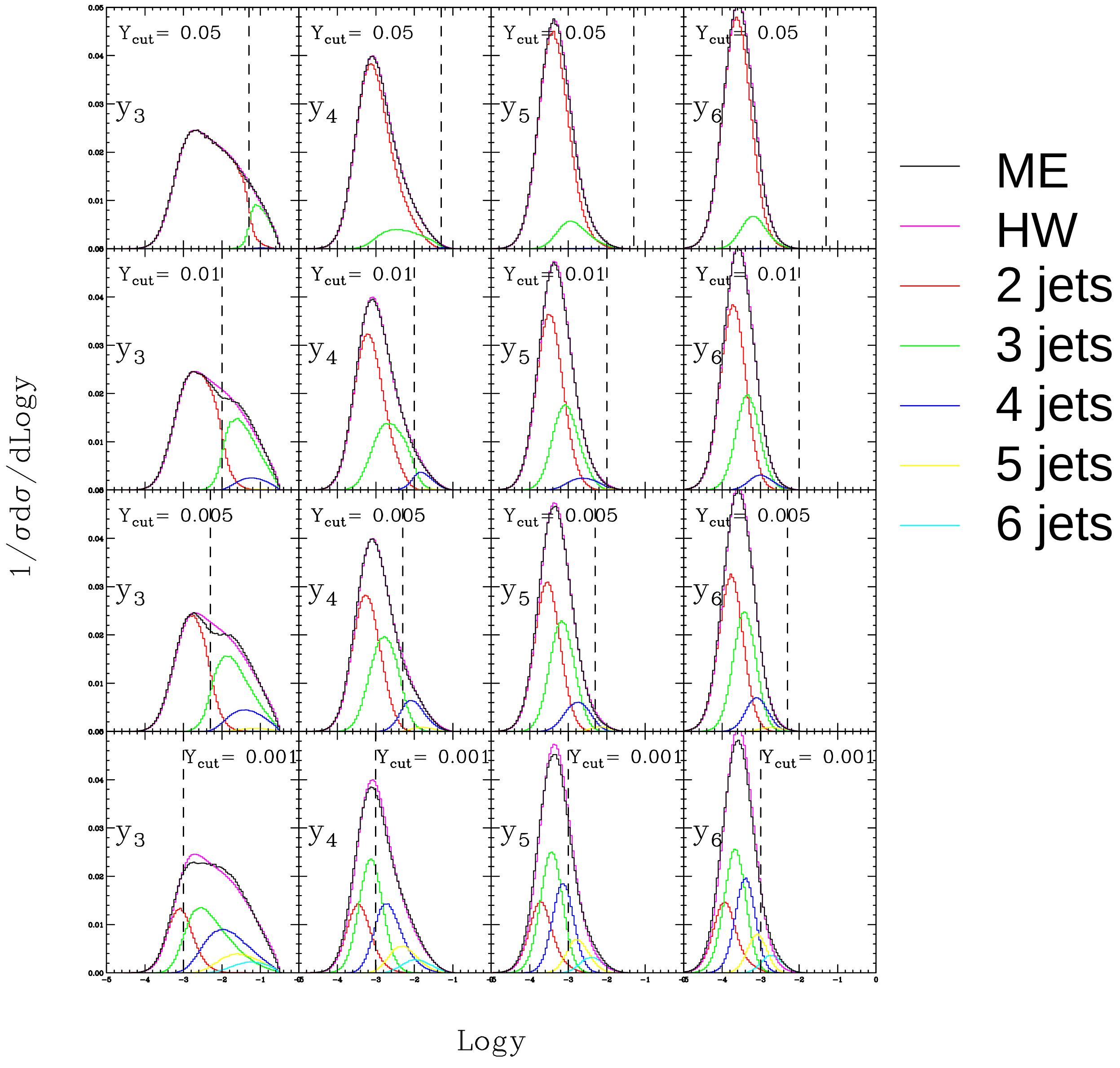
# Effect of Varying the cut-off at $\sqrt{s} = 500$ GeV

- NLL Sudakovs with `QFACT(1)=1/2`, `IFINAL=3`.
- Parton-level distributions.



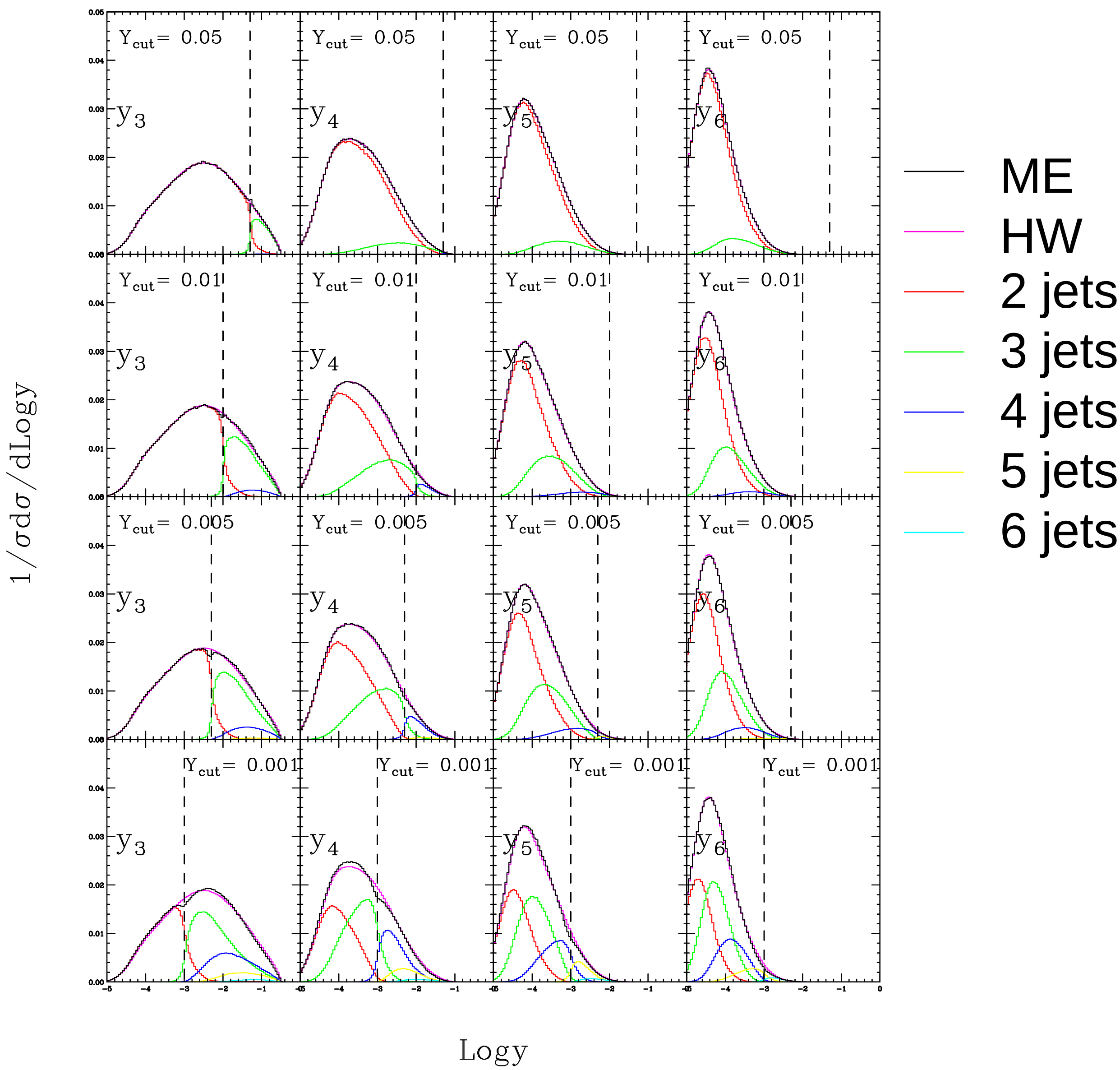
# Effect of Varying the cut-off at $M_Z$

- NLL Sudakovs with `QFACT(1,2,3)=1/2`, `AFACT(1,2,`
  `IFINAL=3`.
- Hadron-level distributions.
- Problems with hadronization model which remain to be solved.



# Effect of Varying the cut-off at $\sqrt{s} = 500$ GeV

- NLL Sudakovs with `QFACT(1)=1/2`, `IFINAL=3`.
- Hadron-level $y_3$ distribution.
- Problems with hadronization model reduced.



# Hadron Collisions

- In hadron collisions in addition to final-state radiation we have the problems of initial-state radiation.

- I will consider $W$ and $Z$ production in hadron collisions

- All the events were generated with MADGRAPH including the leptonic decays of the gauge bosons.

- For the $W$ samples no cuts were applied on the leptons

- For $Z$ production the photon diagrams were included and a cut on the invariant mass of the lepton pair imposed, $m_{\ell\ell} > 20$ GeV.

# Hadron Collisions

- We also have a greater choice of the clustering algorithm.

- I will consider three choices.

  - E-scheme

$$
\begin{aligned}
d_{iB} &= 2E_i^2(1-\cos\theta_{iB}), \\
d_{ij} &= 2\min(E_i^2, E_j^2)(1-\cos\theta_{ij}).
\end{aligned}
$$

    Combine 4-momenta to get pseudoparticles

  - Covariant E-scheme

$$
\begin{aligned}
d_{iB} &= p_{ti}^2, \\
d_{ij} &= \min(p_{ti}^2, p_{tj}^2)R_{ij}^2,
\end{aligned}
$$

    Combine 4-momenta to get pseudoparticles

  - Covariant monotonic $p_T^2$ scheme
    Same definition of $d$ as Covariant E-scheme but define $p_t$ and $R$ for pseudoparticles as

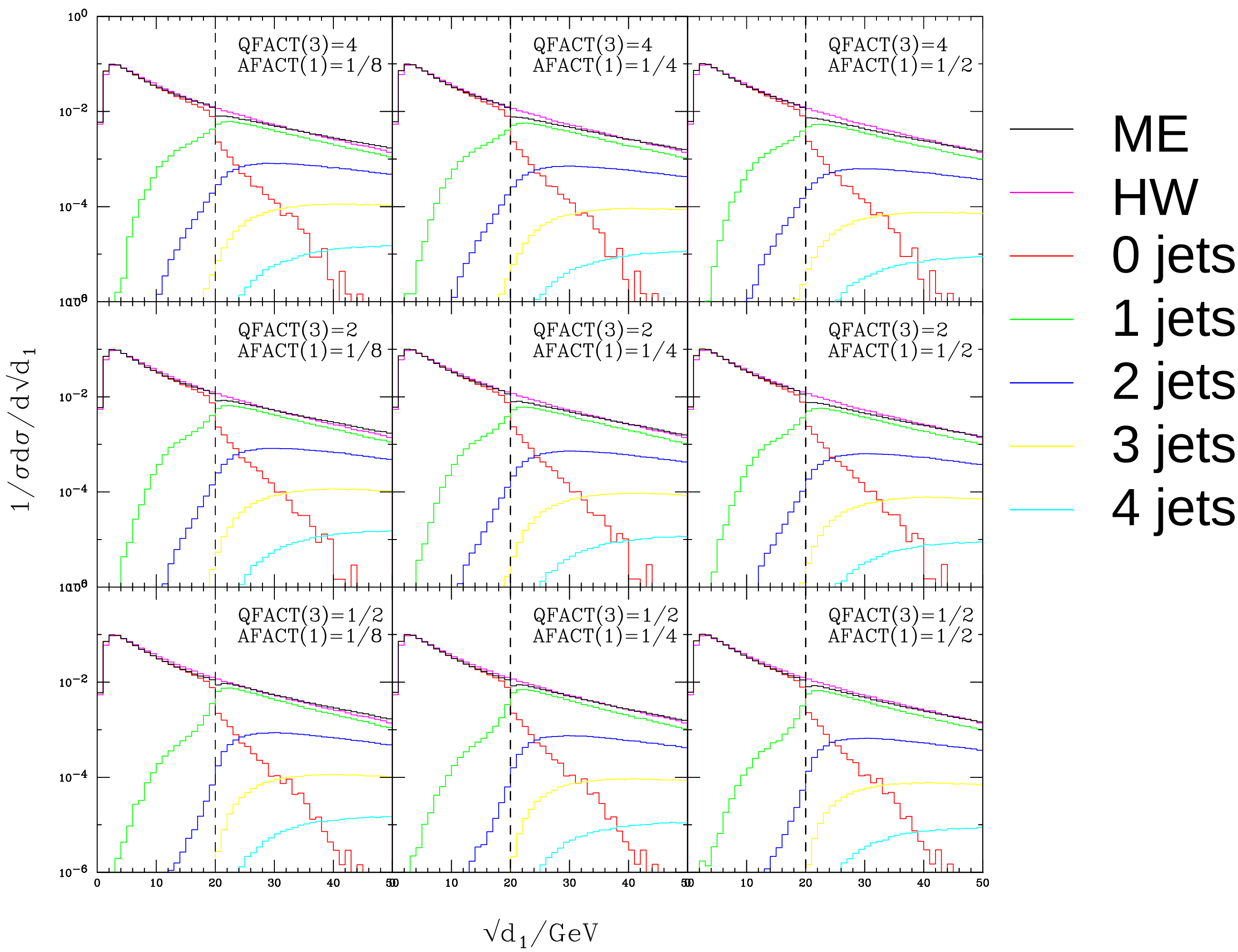
$$
\begin{aligned}
p_{t(ij)} &= p_{ti} + p_{tj}, \\
R_{(ij)k}^2 &= \frac{p_{ti}^2 R_{ik}^2 + p_{tj}^2 R_{jk}^2}{p_{ti}^2 + p_{tj}^2}.
\end{aligned}
$$

# Hadron Collisions

- Start by considering $W$ and $Z$ production at the Tevatron.
- Use the Covariant monotonic $p_T^2$ scheme.
- and a matching scale of $400\ \mathrm{GeV}^2$.
- Look at the differential cross section with respect to $\sqrt{d_n}$ where $d_n$ is the scale at which $n$ jets are resolved.
- This is like the $E_T$ in a cone algorithm.

# W Production

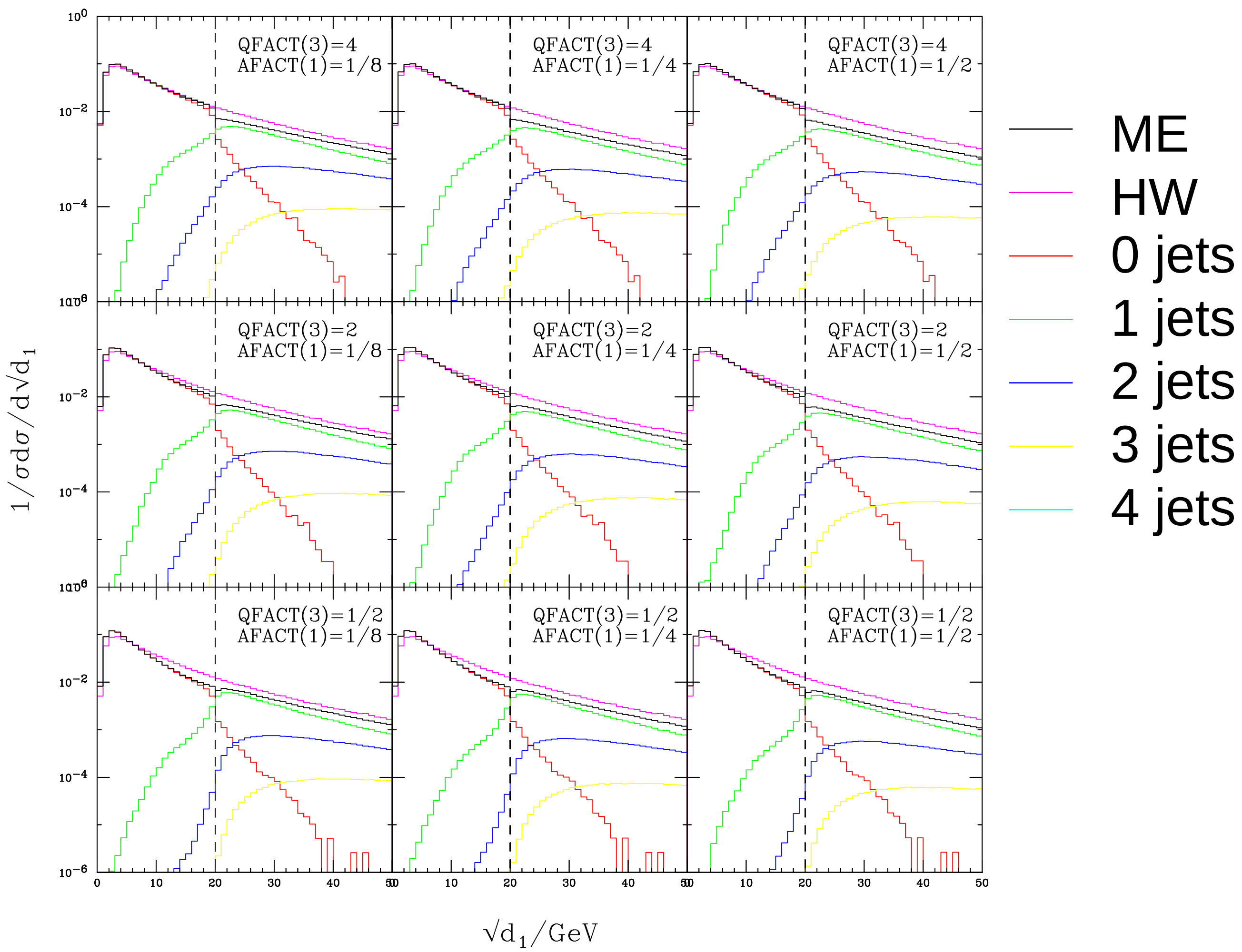
- Variation of results with scale of $\alpha_S$ and starting scale of shower.
- $d_1$ distribution at the Tevatron



# Z Production

- Variation of results with scale of $\alpha_S$ and starting scale of shower.

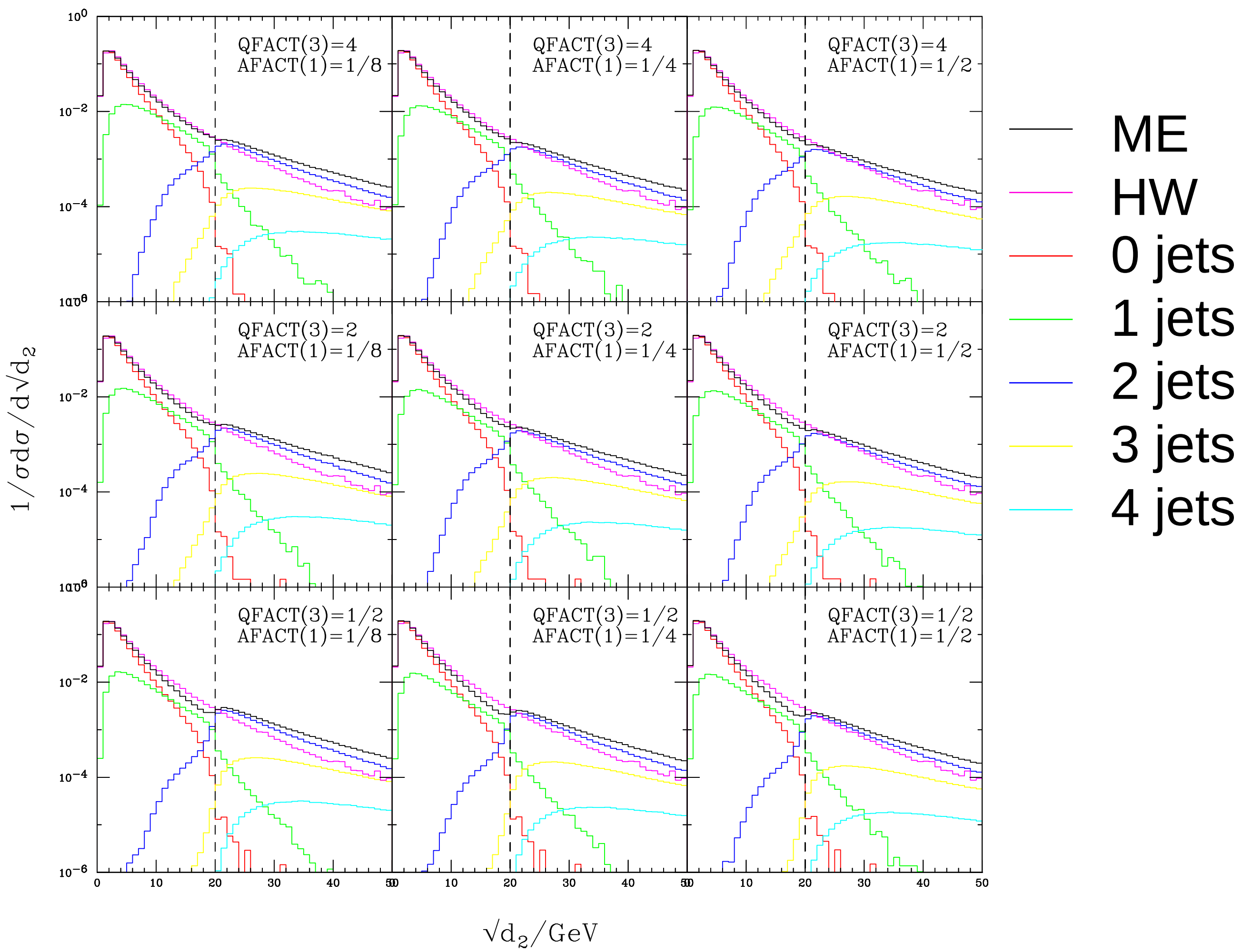
- $d_1$ distribution at the Tevatron



# W Production

- Variation of results with scale of $\alpha_S$ and starting scale of shower.

- $d_2$ distribution at the Tevatron



# Hadron Collisions

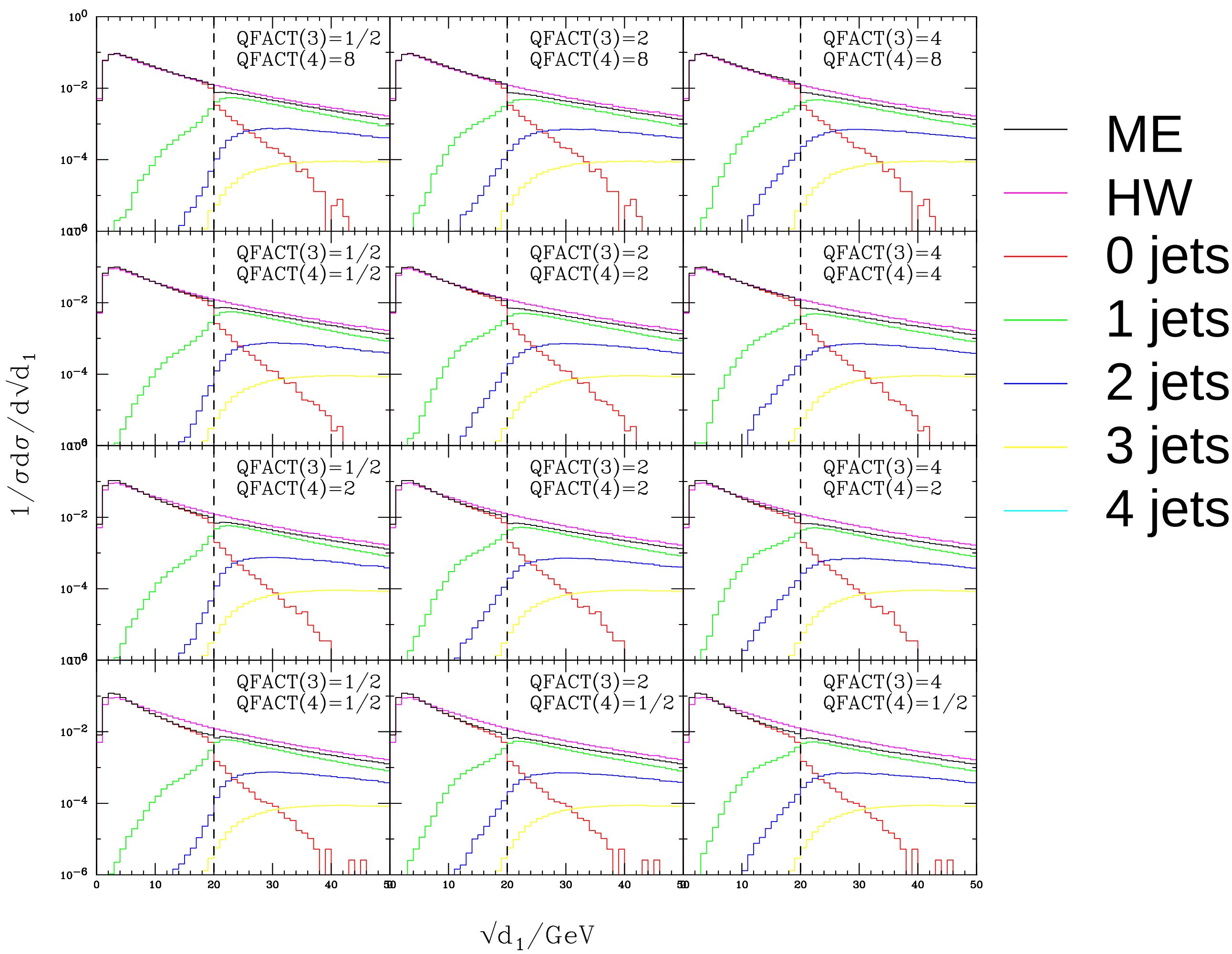
- There is a radiation dip at the matching scale $\Rightarrow$ high `QFACT(3)`.

- However there is also a problem with events migrating below the matching scale $\Rightarrow$ low `QFACT(3)`.

- Solution is to decouple the scale for the initial-state shower `QFACT(4)` from the scale for the final-state shower `QFACT(3)`?

# Z Production

- Variation of results with `QFACT(3,4)` from `AFACT(1,2)=1/8`.
- $d_1$ distribution at the Tevatron



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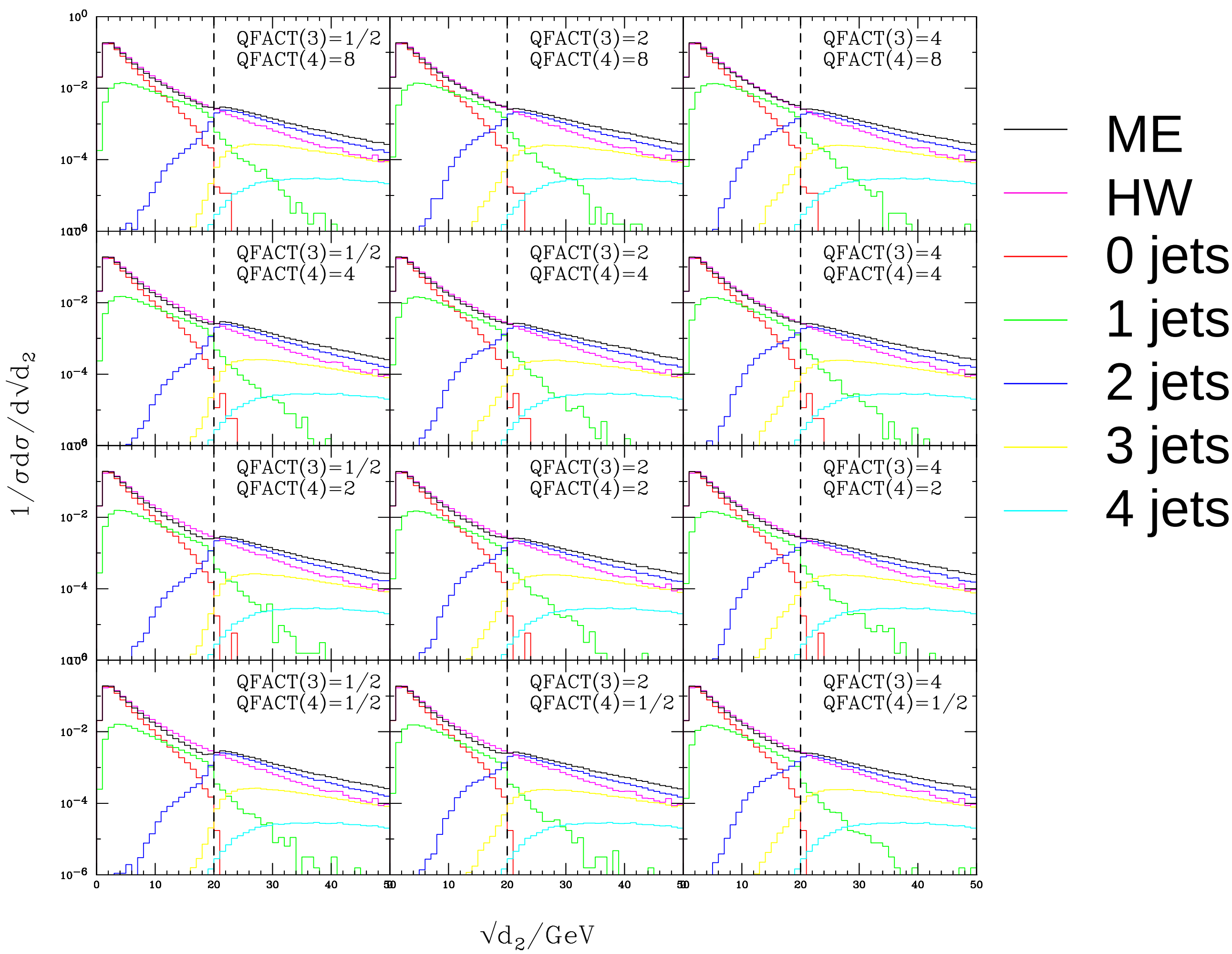

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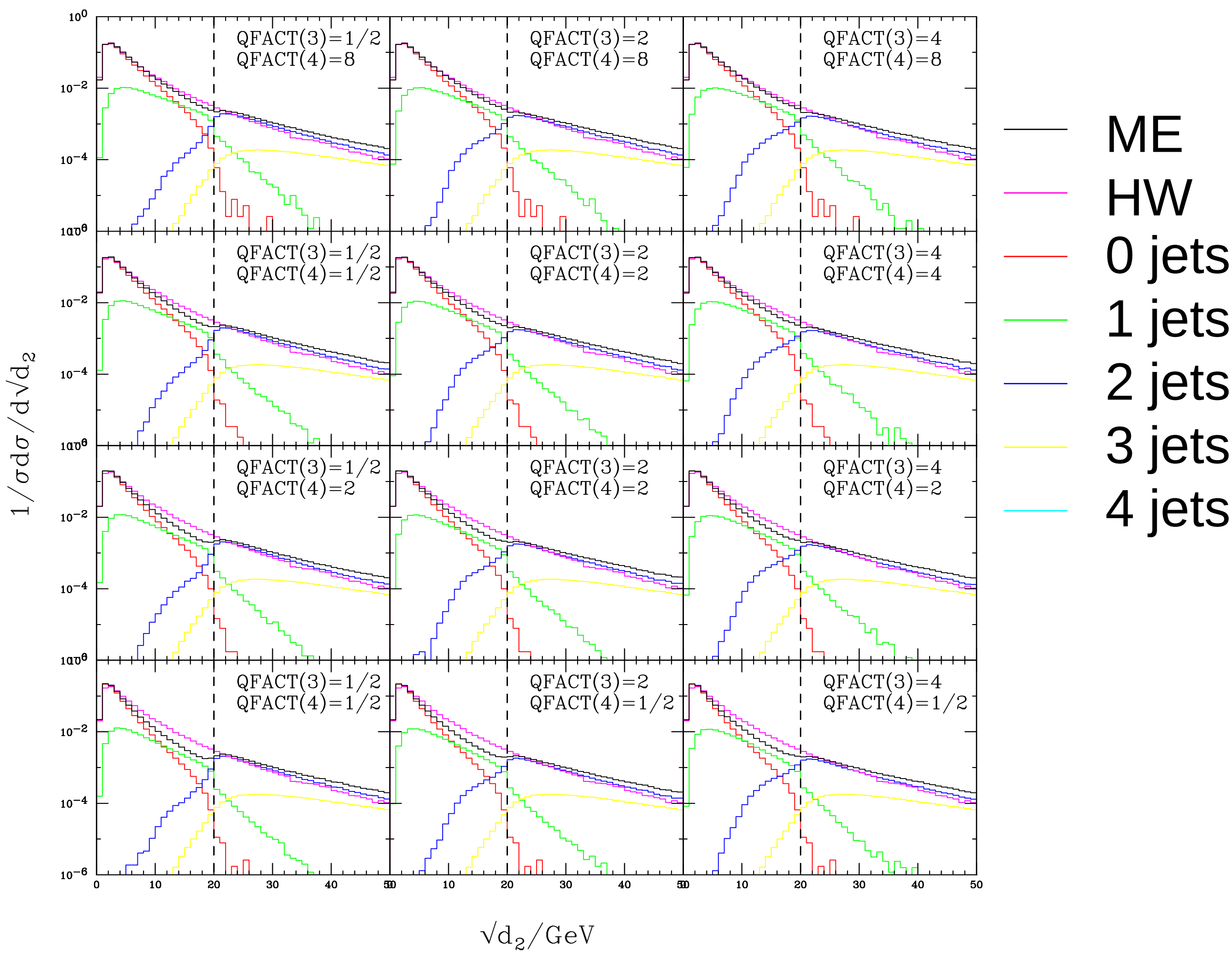
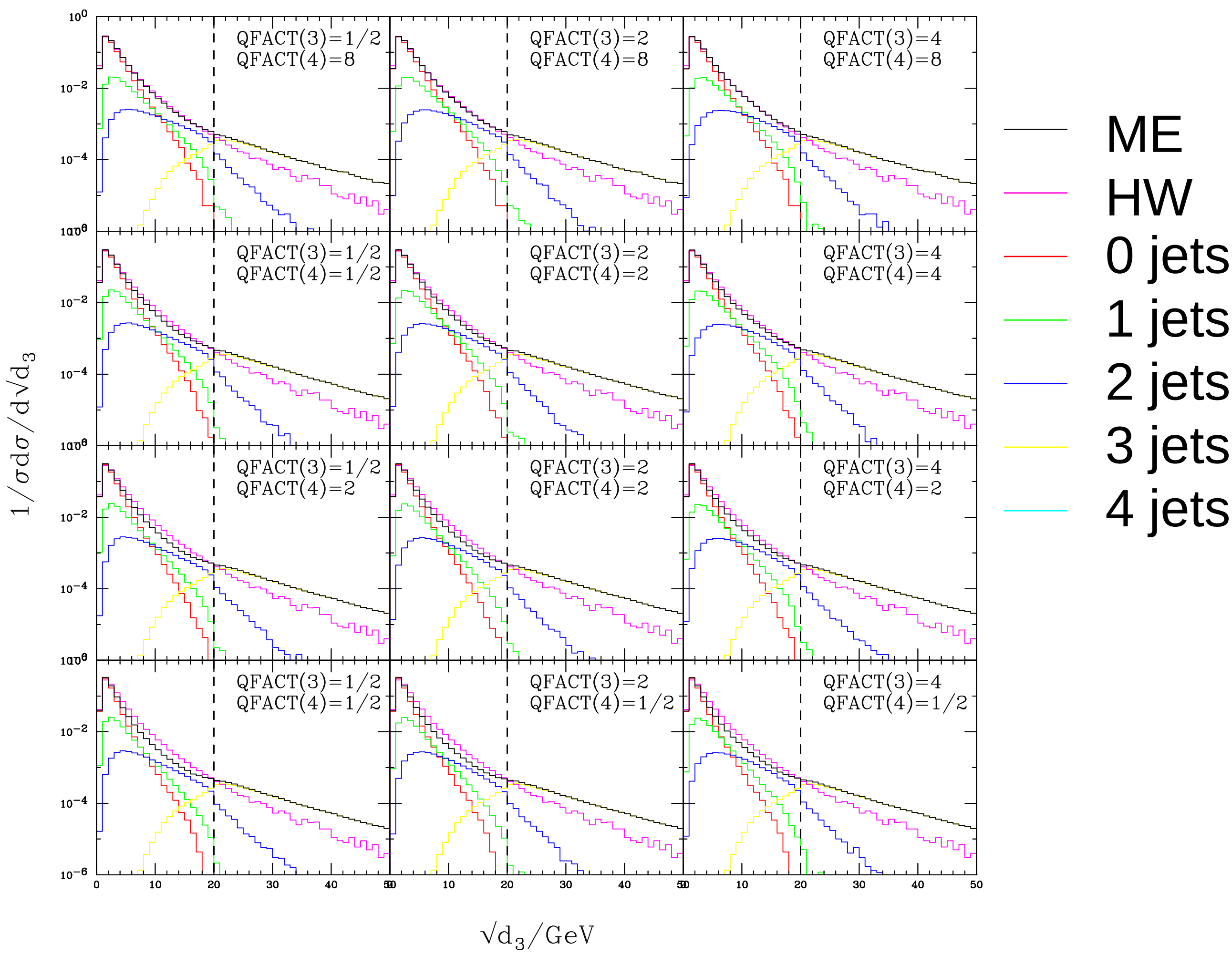


# Z Production

- Variation of results with `QFACT(3,4)` from `AFACT(1,2)=1/8`.

- $d_3$ distribution at the Tevatron



# Hadron Collisions

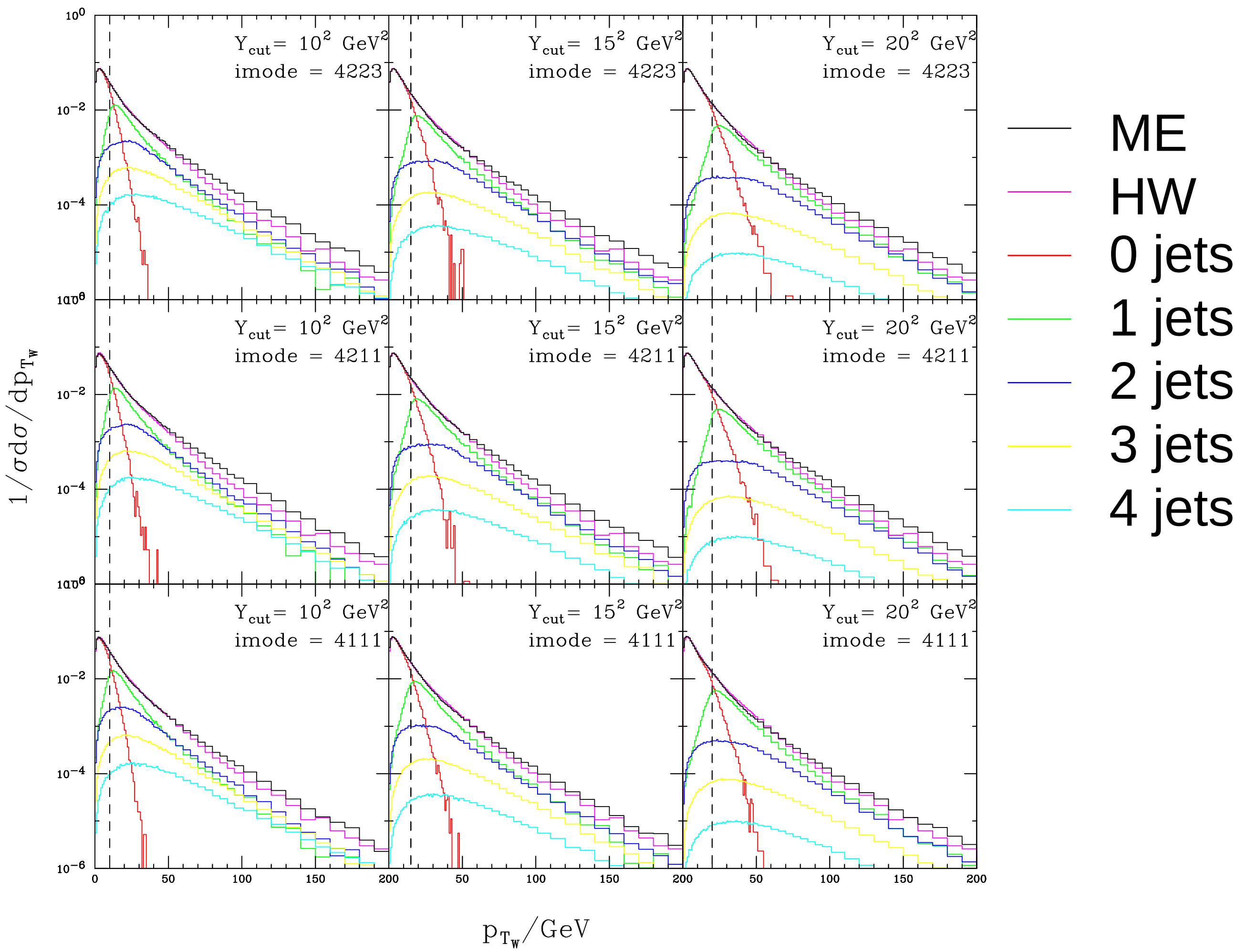
- Hard to get the HERWIG shower to produce more radiation than before below the matching scale.
- Hopefully there is some solution to this.
- Otherwise we will have to use low values of the matching scale.
- *i.e.* a region where the PS is already giving the right answer.
- One other option which helps is to always start the initial state shower at the highest scale in the process.

# Hadron Collisions

- Look at the variation with matching scale and algorithm
- Look at three scales
  - $10^2\ \mathrm{GeV}^2$
  - $15^2\ \mathrm{GeV}^2$
  - $20^2\ \mathrm{GeV}^2$
- and three algorithms
  - E-scheme, `IMODE=4111`
  - Covariant E-scheme, `IMODE=4211`
  - Covariant monotonic $p_T^2$ scheme, `IMODE=4223`.

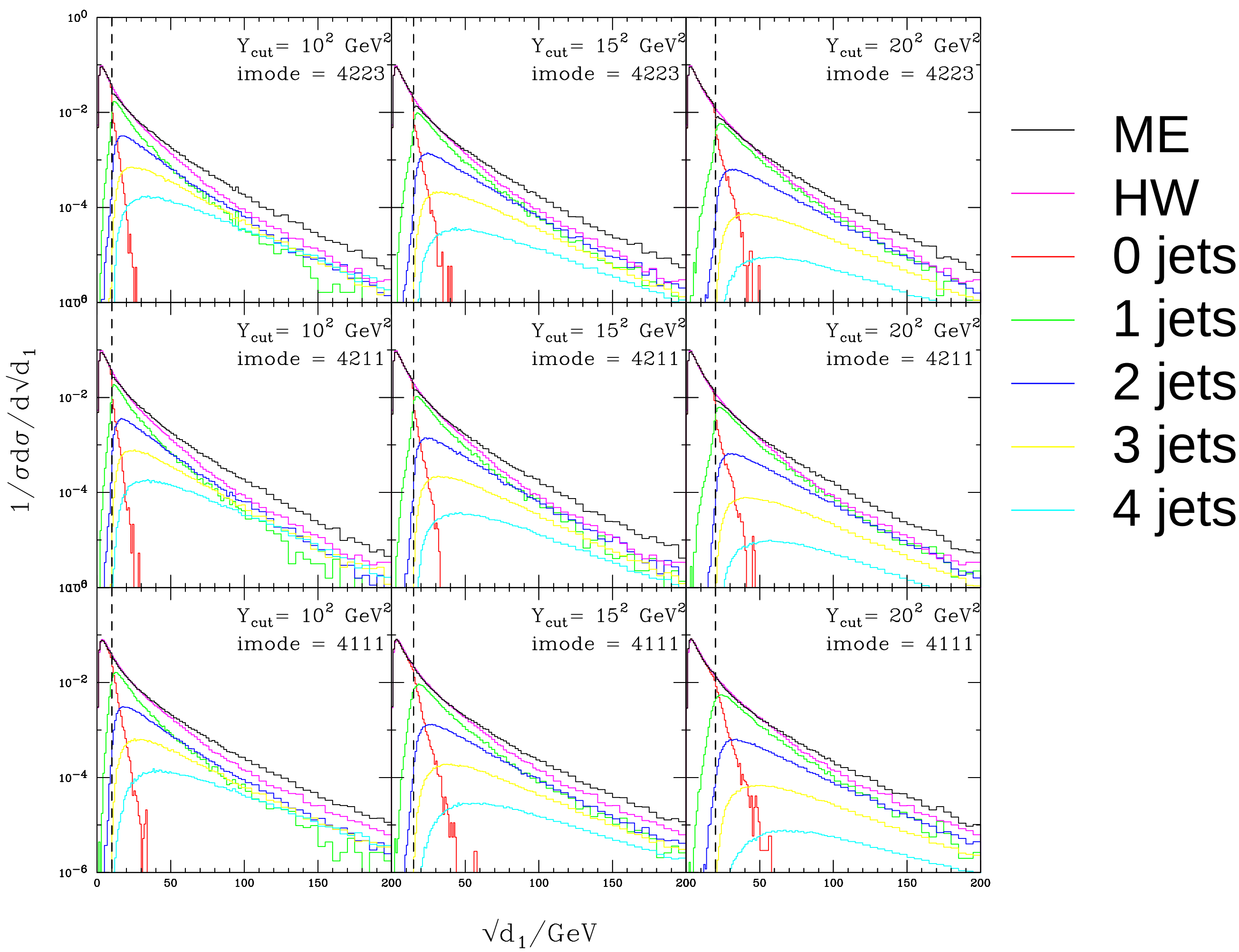
# W Production

- `AFACT(1,2)=1/2`.
- $p_T$ of the W at the Tevatron



# W Production

- `AFACT(1,2)=1/2`.
- $d_1$ distribution at the Tevatron



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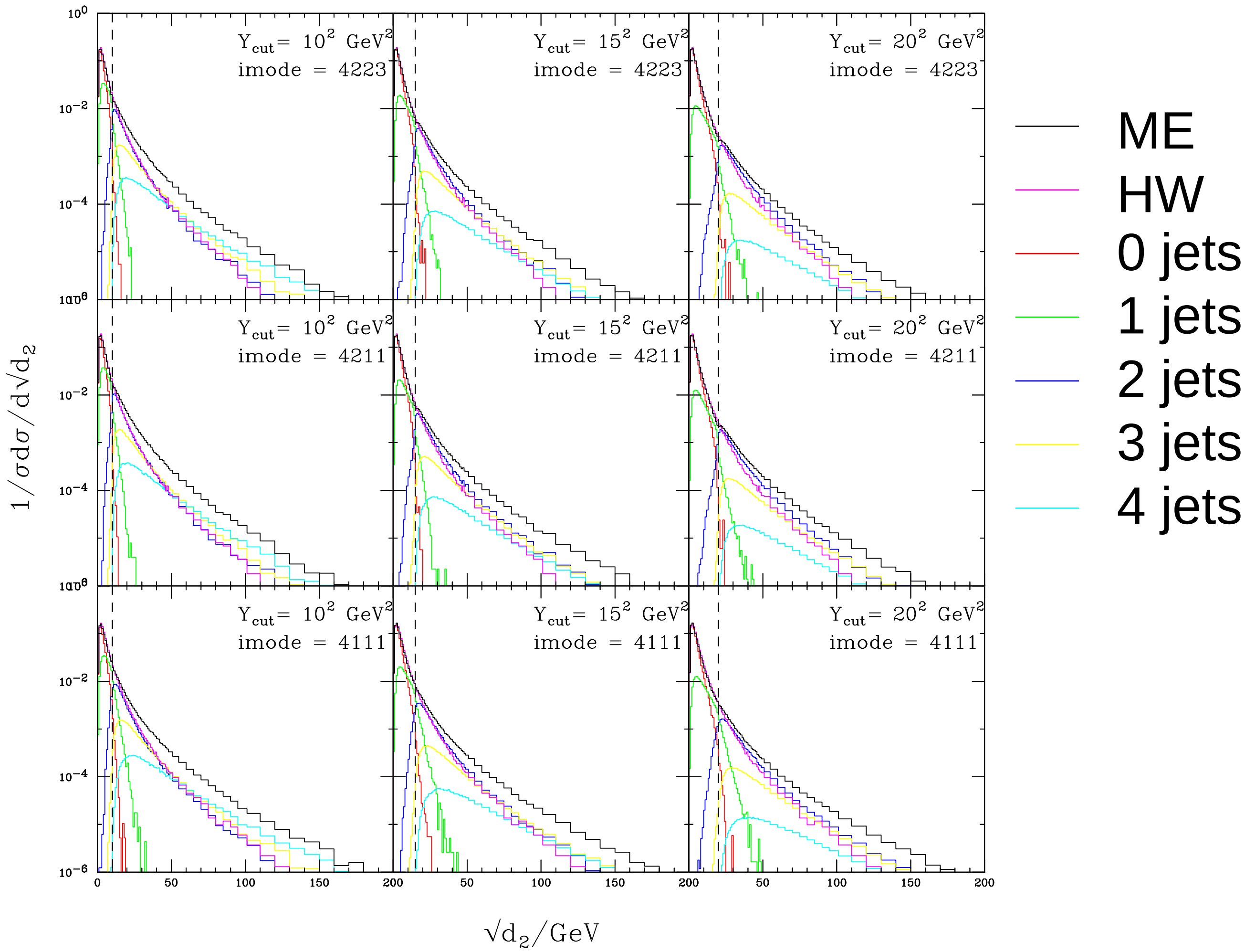

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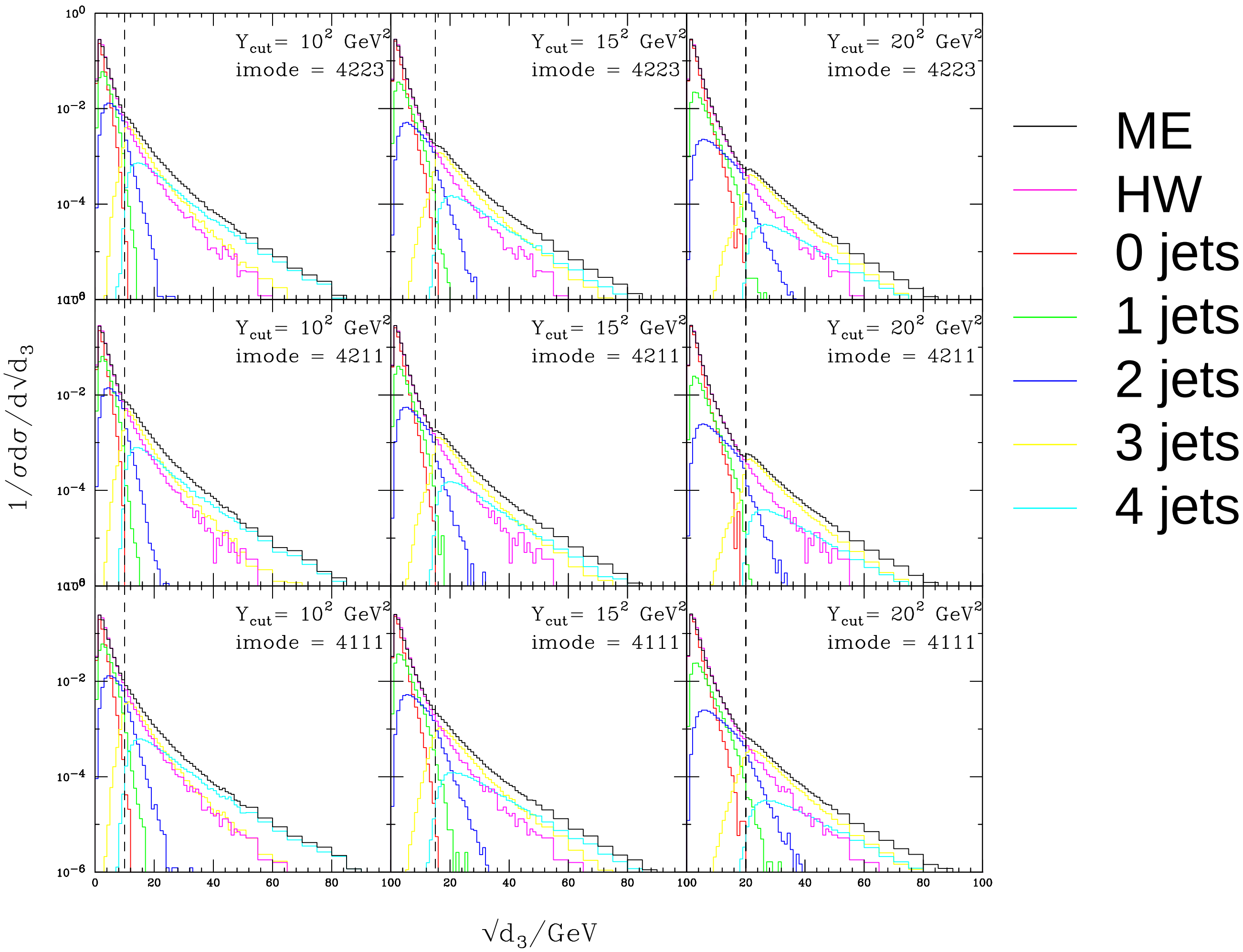
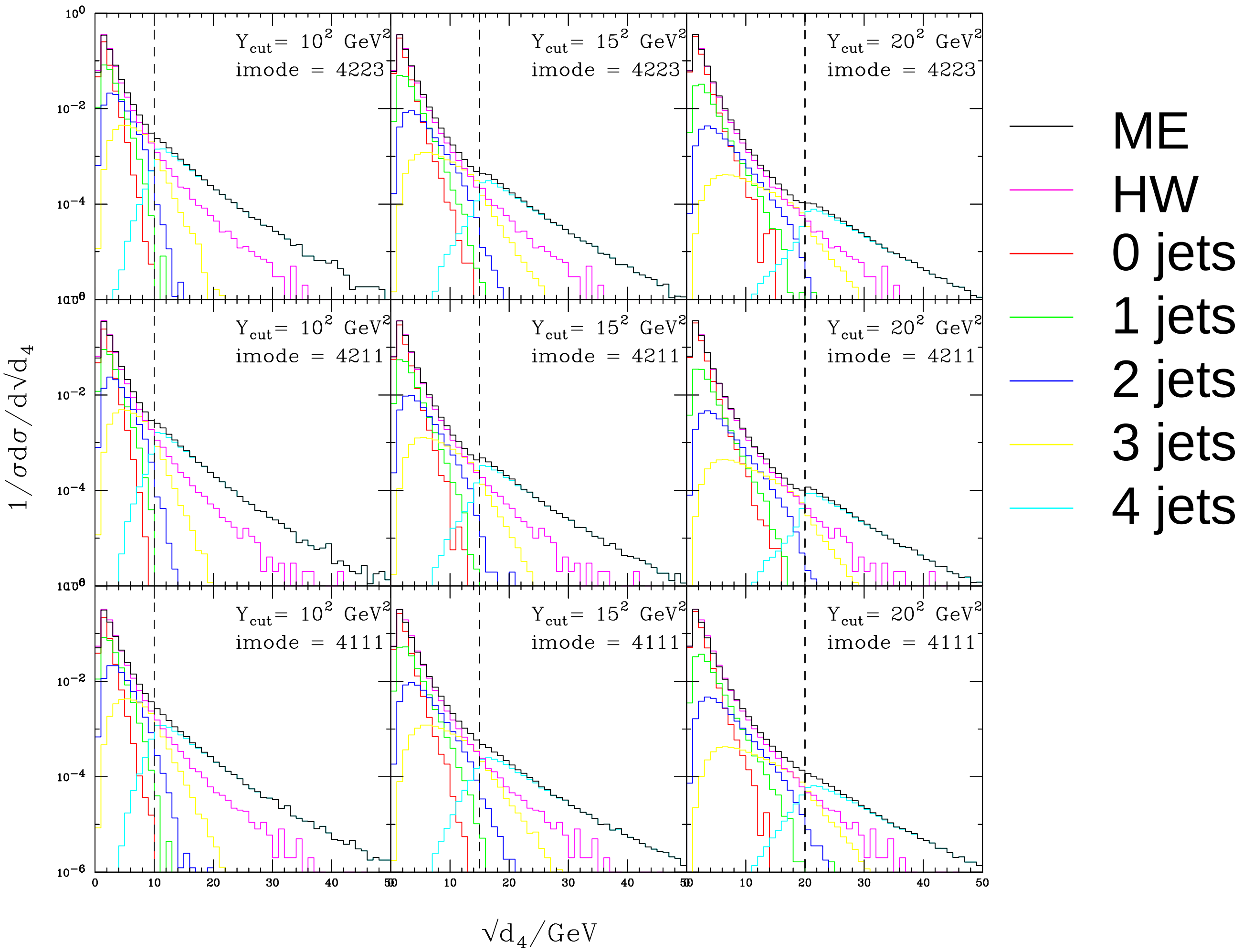


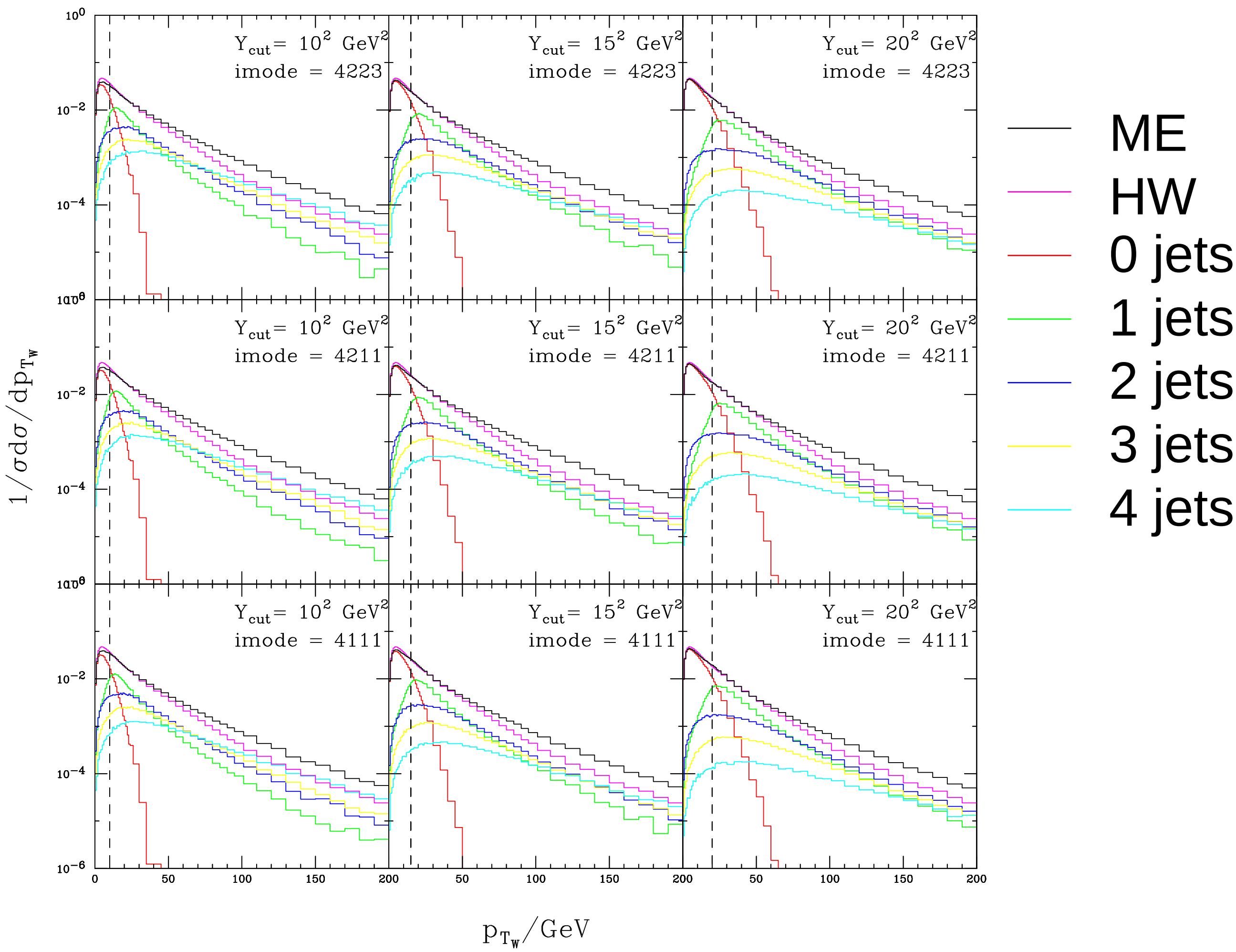
# W Production

- `AFACT(1,2)=1/2`.
- $d_4$ distribution at the Tevatron



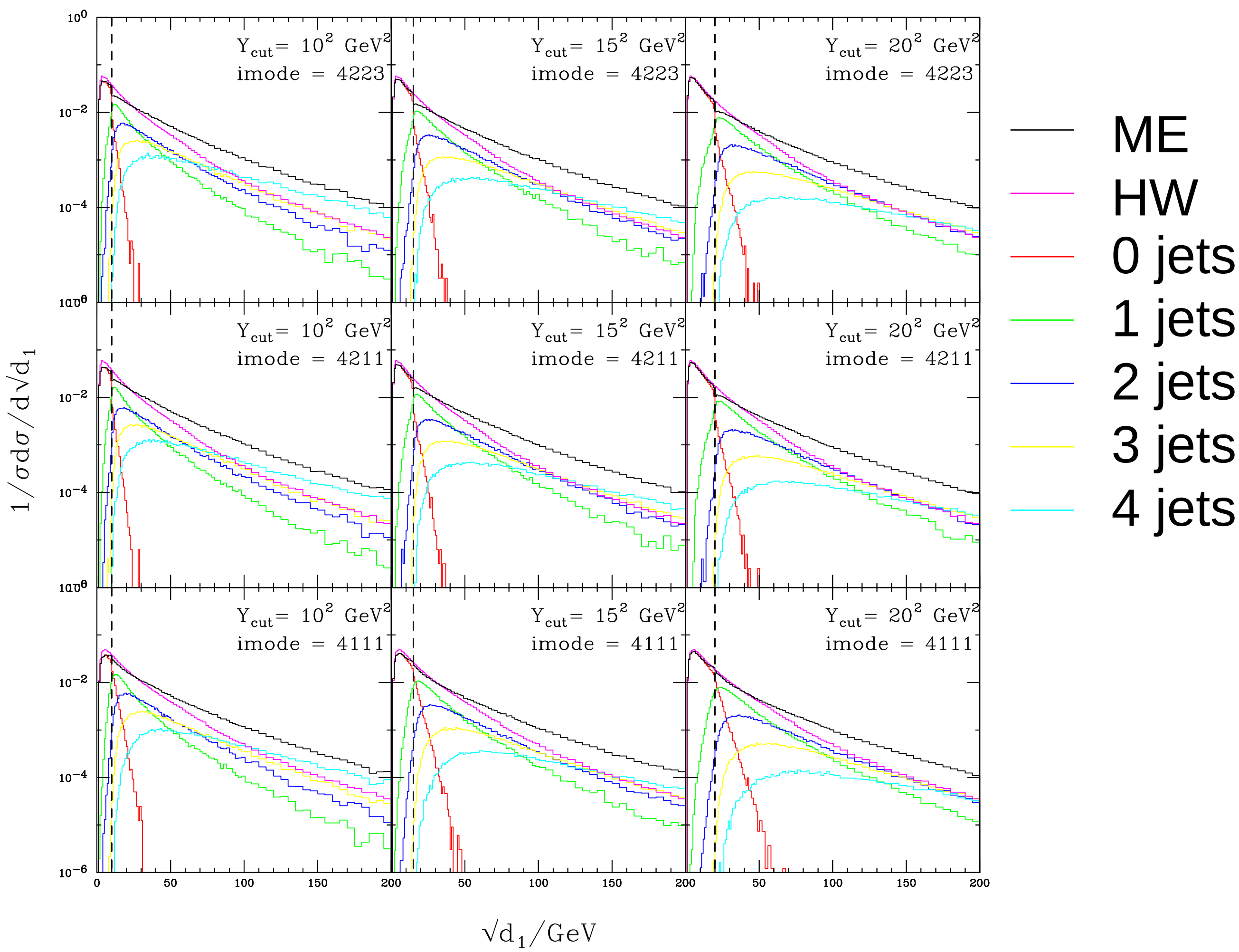
# W Production

- AFACT(1,2)=1/2.
- $p_T$ of the W at the LHC



# W Production

- `AFACT(1,2)=1/2`.
- $d_1$ distribution at the LHC



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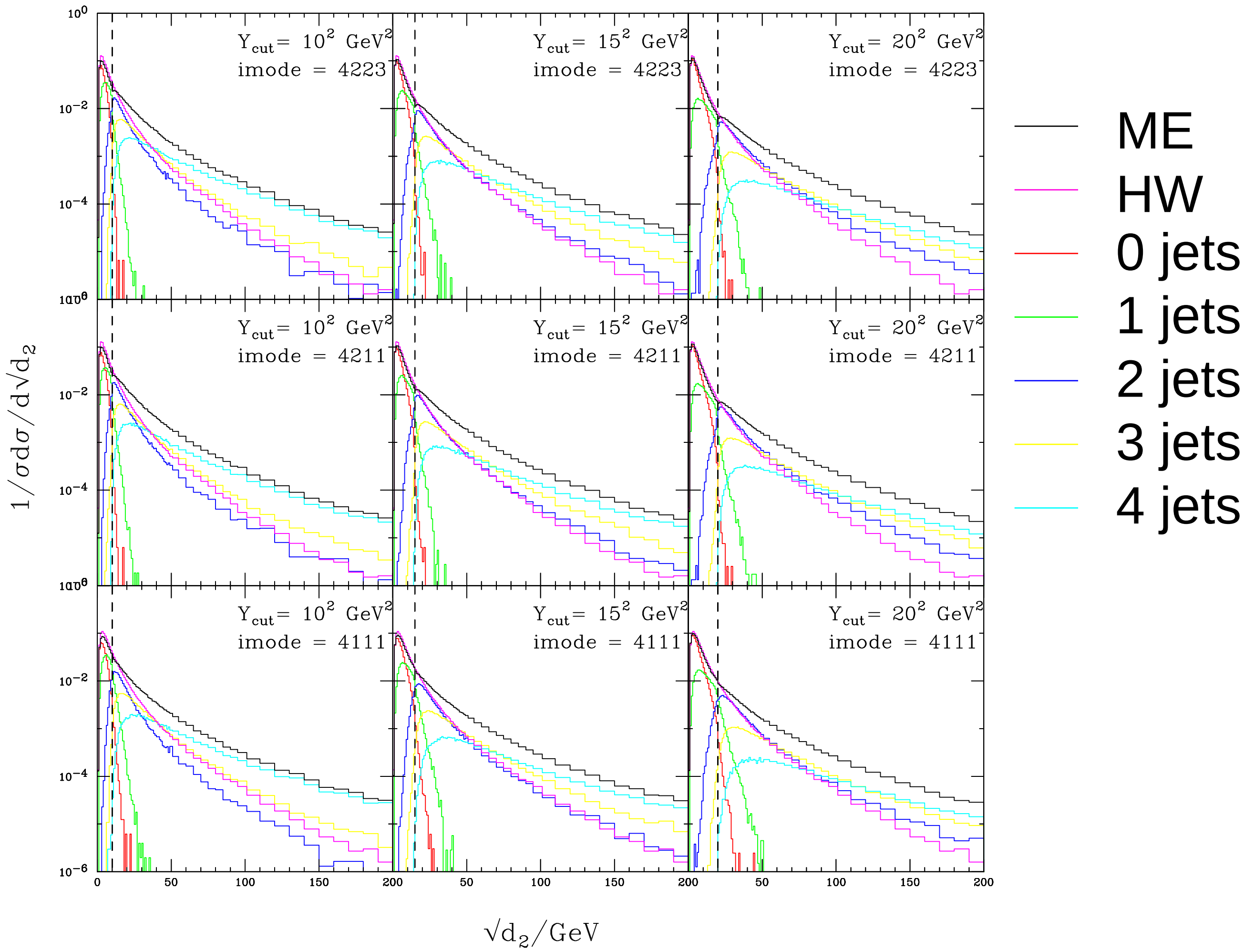

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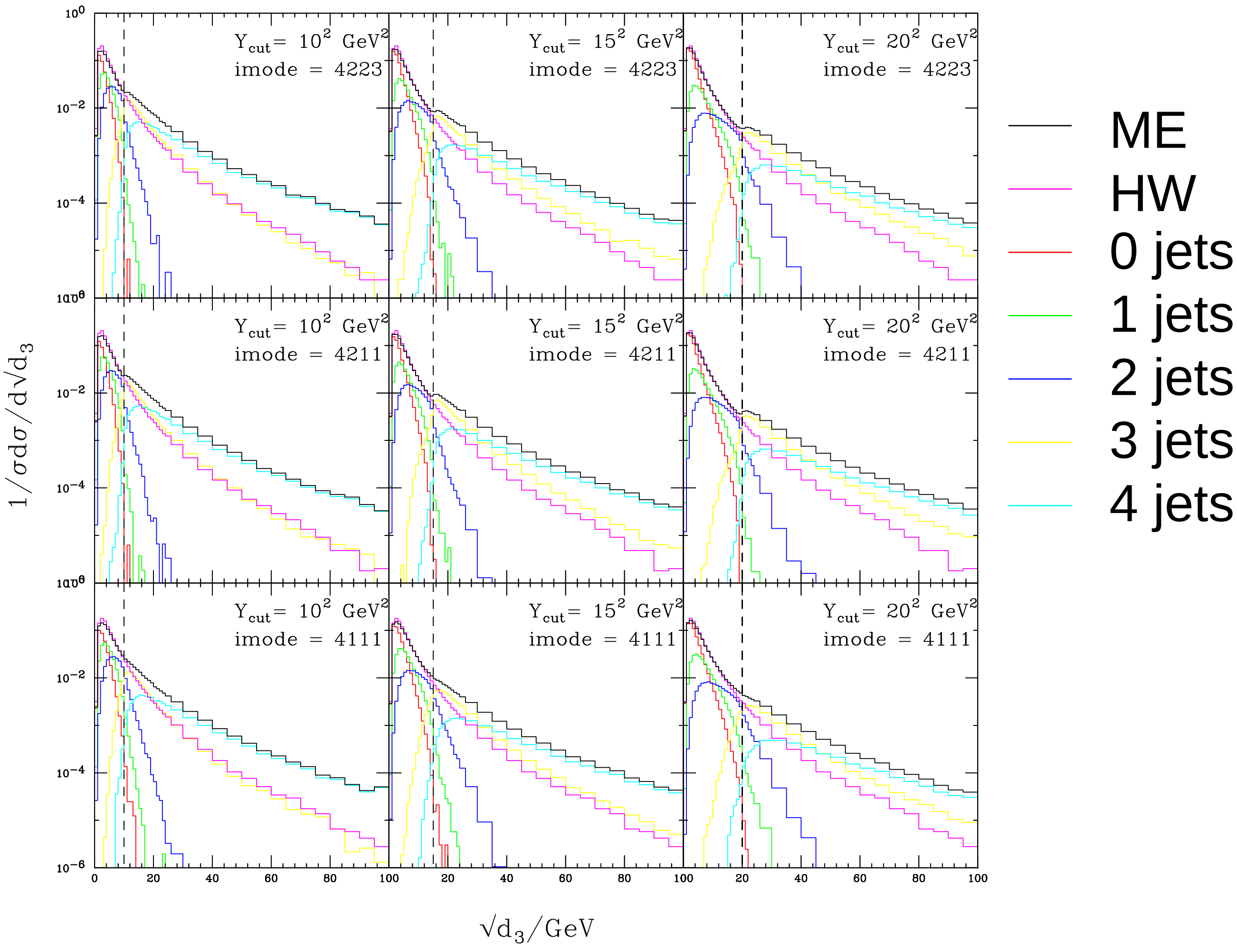

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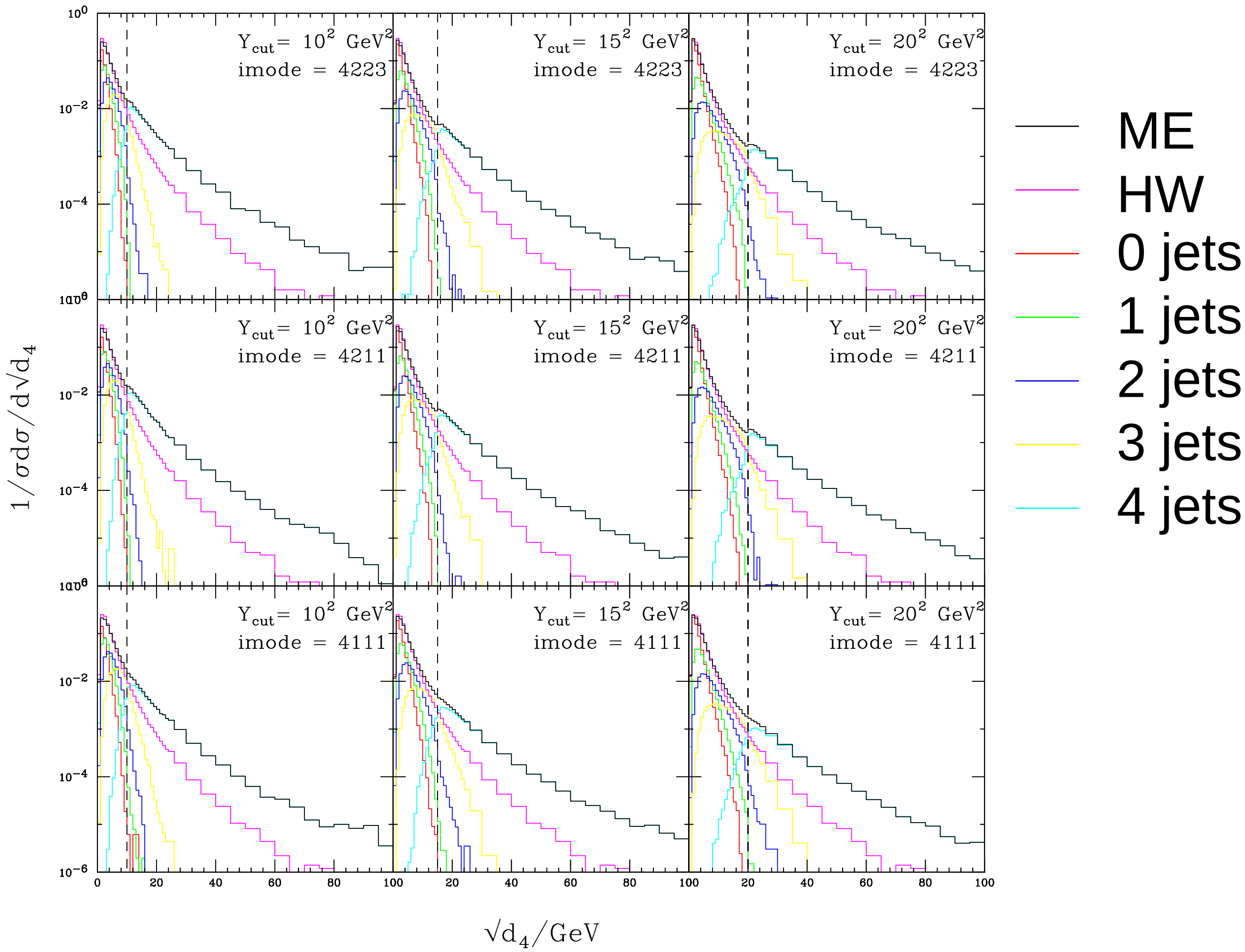

# Program

- I have written a program to implement the CKKW algorithm with HERWIG.
- Uses the Les Houches interface to HERWIG to pass events from MADGRAPH/ALPGEN.
- Performs the clustering, reweighting and works out stsrt scale for the shower.
- Should be available soon

# Conclusions

- I have presented results for the CKKW algorithm with HERWIG.
- We now have good results for $e^+e^-$ collisions
- Still some problems to iron-out in hadron collisions.
- Hopefully the code will be available soon.