CKKW Anatomy

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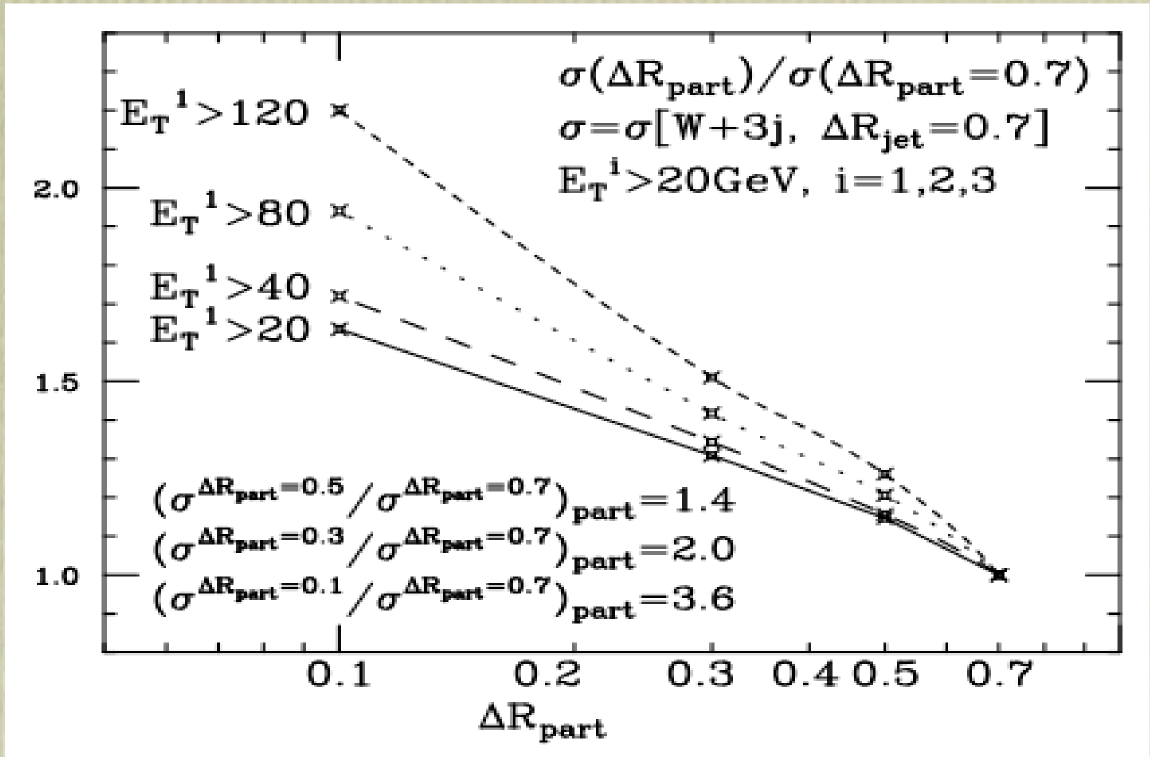
Overwiew

- CKKW as a scale-setting prescription for parton level calculations
- kT vs cone-jet final states
- inclusive vs exclusive parton-level final states
- CKKW implementation details for hadronic collisions

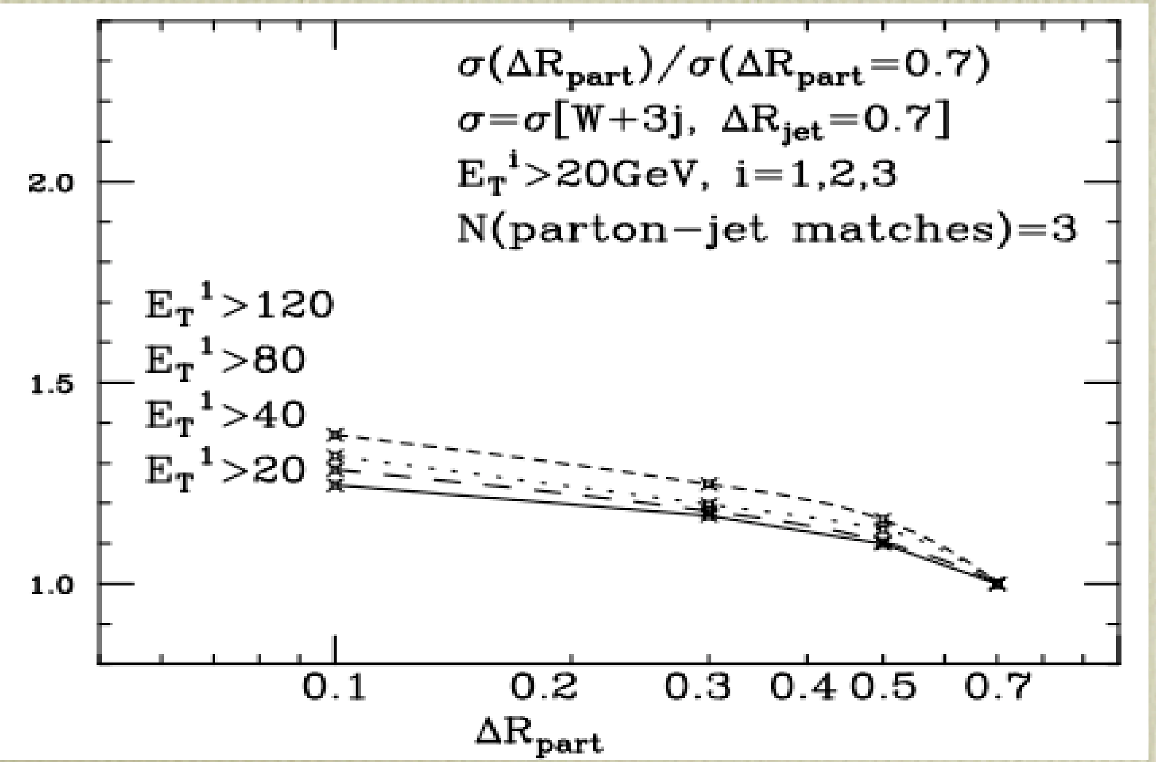
LO ME generators in a nutshell

- ME calculations provide rates for inclusive final states:
  - I.S. radiation is "included" in the PDF's, which are inclusive objects: f(i,x,Q) is the density of parton "i" with momentum fraction "x" at scale "Q", summed over all possible histories between Q_0 and Q.
  - F.S. radiation is included in the LO definition of a parton, which represents all its possible future histories
- The IS and FS histories may include the emission of jets
- The only possible use of a standard N-jet ME calculation with a shower MC generator is to generate N-jet inclusive distributions. Addition of ME samples of different multiplicity leads to double counting. Therefore the accuracy for the N+1 jet component of the sample is limited to the accuracy of the shower hard emission

Cut-generation dependence, example

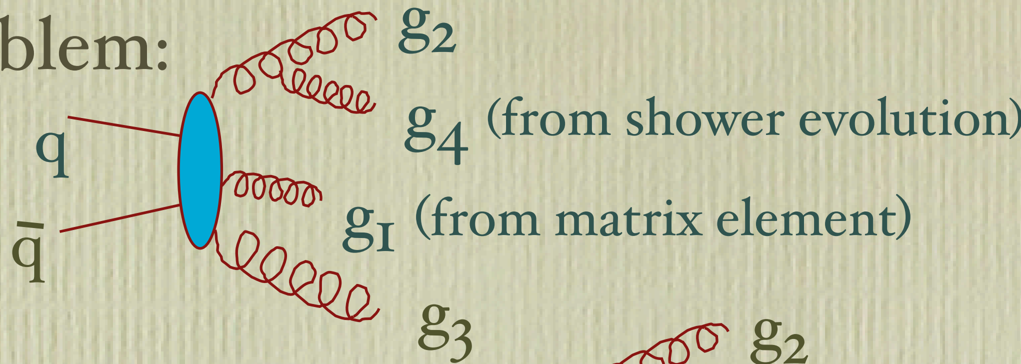


Dependence on generation ΔR in the spectrum of $\Delta R = 0.7$ jets



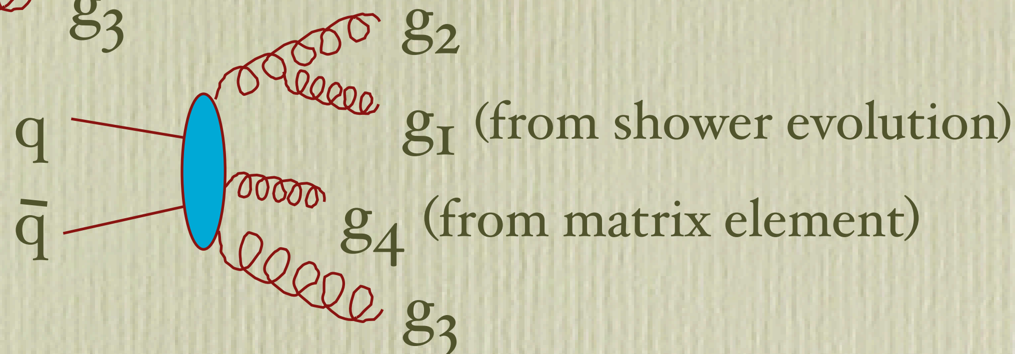
Same, after applying a (rather arbitrary and perhaps ill-defined) parton-jet matching requirement

Problem:



with $pT_1 << pT_4 << pT_2, pT_3$

versus

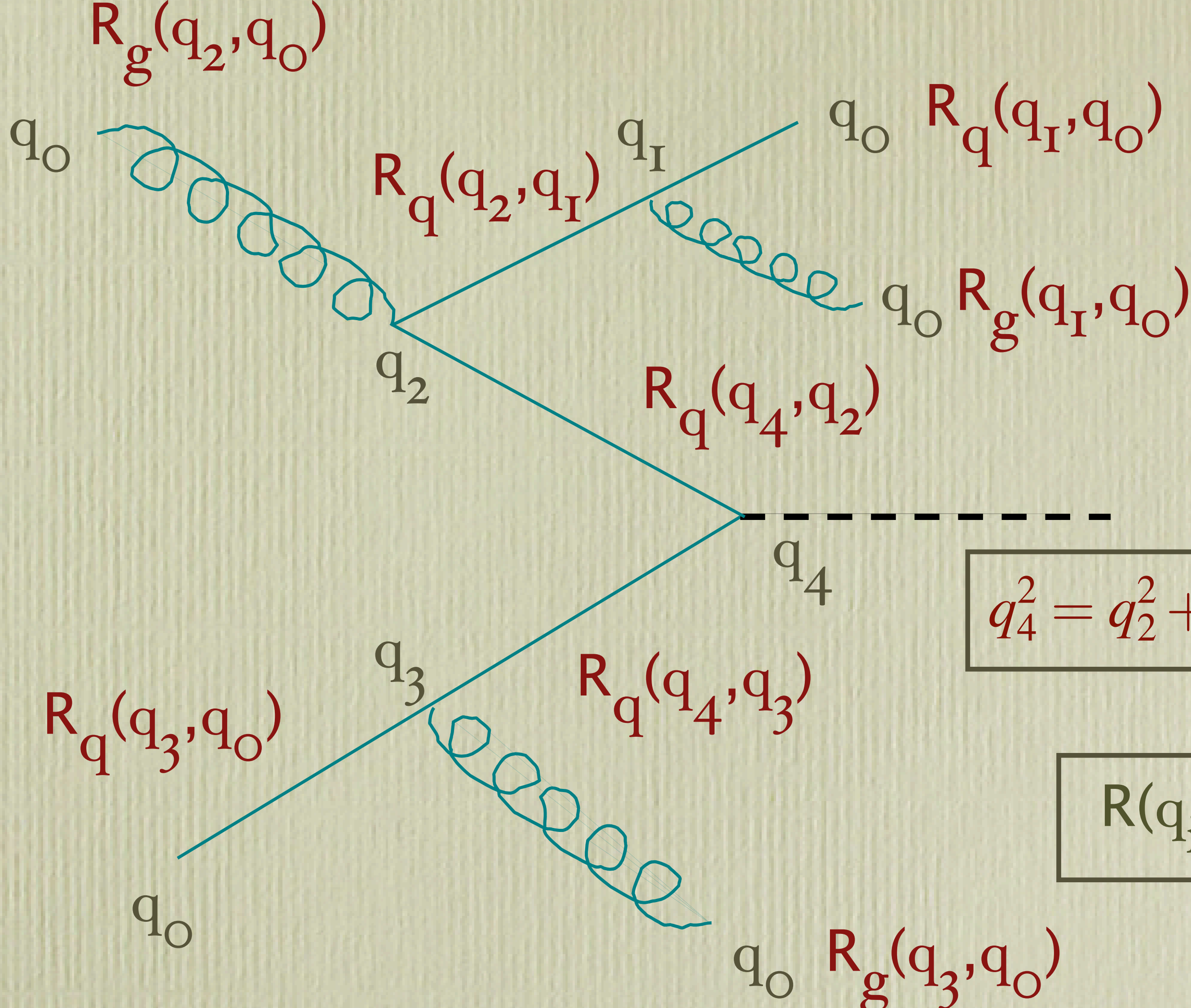


Solution:

- Separate multi-jet phase-space into
  - domains covered by the ME calculation
  - domains covered by the shower evolution
- Reweight the ME weight to reproduce the probability of an exclusive N-jet final state form the inclusive parton-level N-jet rate. This allows to add parton-level event samples of different jet multiplicity
- Veto showers with hard emissions which are supposedly already included in the higher-order ME phase-space

CKKW prescription in a nutshell

- Generate samples of N-jet configurations, defined by the $k_\perp$ algorithm, with a resolution parameter k_0
- Since all N-jets have to be resolved w.r.t. the beam, $k_0 = p_T^{min}$. No cut on η can be set, however
- Cluster the partons using the $k_\perp$ algorithm, allowing only for physical branchings in the tree
- Reevaluate α_s at each vertex of the tree, using $k_\perp$ as a scale
- For each line in the tree, associate a Sudakov weight giving the probability that no emission takes place along this line
- Samples of different N-jet multiplicity can now be put together, and evolved through the vetoed shower



$$q_4^2 = q_2^2 + q_3^2 + m_W^2 - min(q_2^2, q_3^2, m_W^2)$$

$$R(q_i,q_j)=\Delta(q_i,q_o)/\Delta(q_j,q_o)$$

$$w = \prod_1^3 \frac{\alpha_s(q_i)}{\alpha_s(q_0)} \times \prod R(q_i,q_j) = w_\alpha \times R_g(q_1,q_0)R_g(q_2,q_0)R_g(q_3,q_0)R_q^2(q_4,q_0)$$

In this presentation, I would like to concentrate on the features of the parton-level events produced after implementation of the reweighing procedure, as a first step towards understanding the results of full PL+shower evolution in the CKKW framework

Comments, part I:

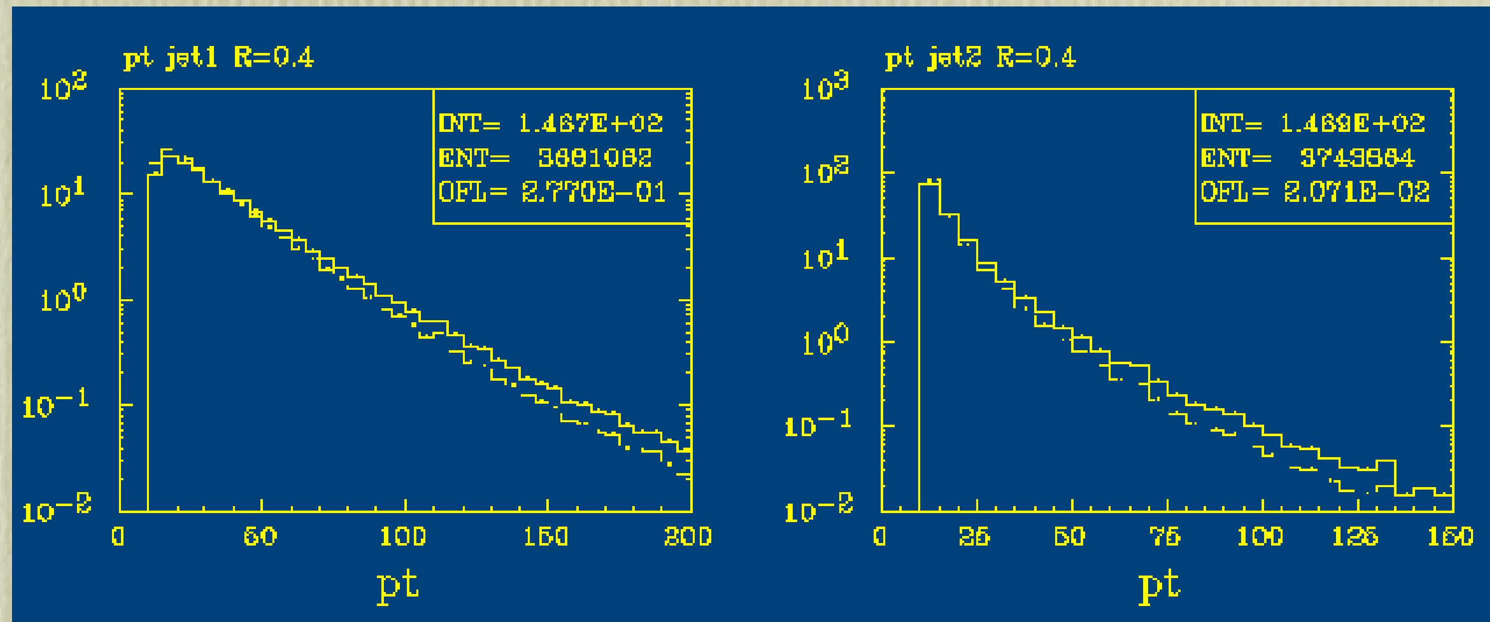
- α_s reweighting could be considered as a prescription per se.
- It is gauge invariant, contrary to other ways of implementing different scales in different vertices of a Feynman diagram
- It embodies the correct UV dynamics of each vertex (so it should better reproduce the effect of NLO corrections)
- NLO corrections DO lead to different scales at different vertices. This is given by left-over logarithms which compensate for the use of a single α_s for all vertices
- I would therefore propose that α_s reweighting be used as a scale-setting prescription for all inclusive ME calculations, independently of the issue of shower merging.
- Implications, for example, for gg->Hbb vs bb->H comparisons

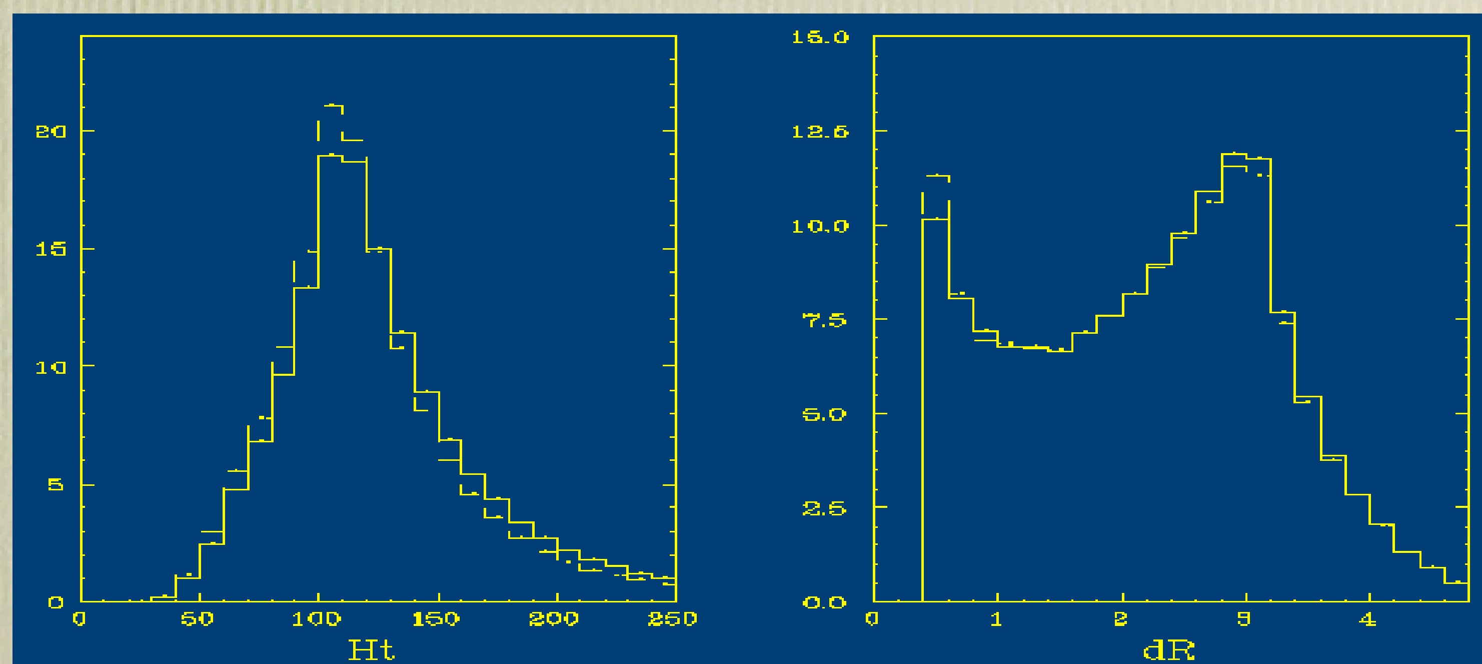
Impact of α_s rescaling on distributions: W+2 jets

$\sigma[\alpha_s(Mw)] = 0.62*\sigma[\alpha_s(k\perp)]$

In the plots I rescaled the $\alpha_s(k\perp)$ curves (dashes) to the rate of the $\alpha_s(Mw)$ ones (solid).

The change in rate, and the relative slopes, are similar in the case of R=0.7 jets



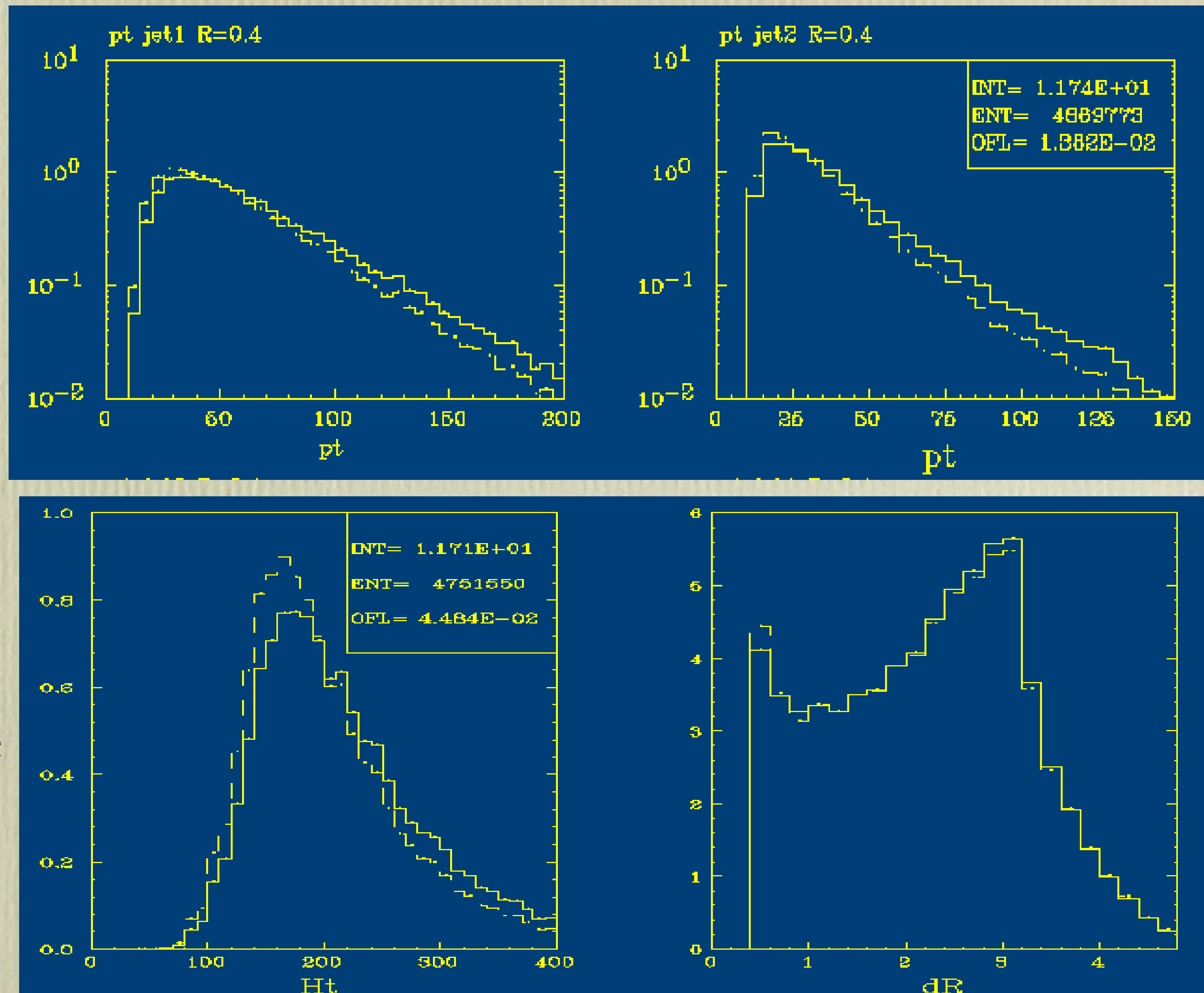


Impact of α_s rescaling on distributions: W+4 jets

$\sigma[\alpha_s(M_W)] = 0.42 * \sigma[\alpha_s(k_\perp)]$, K=2.4

In the plots I rescaled the $\alpha_s(k_\perp)$ curves (dashes) to the rate of the $\alpha_s(M_W)$ ones (solid).

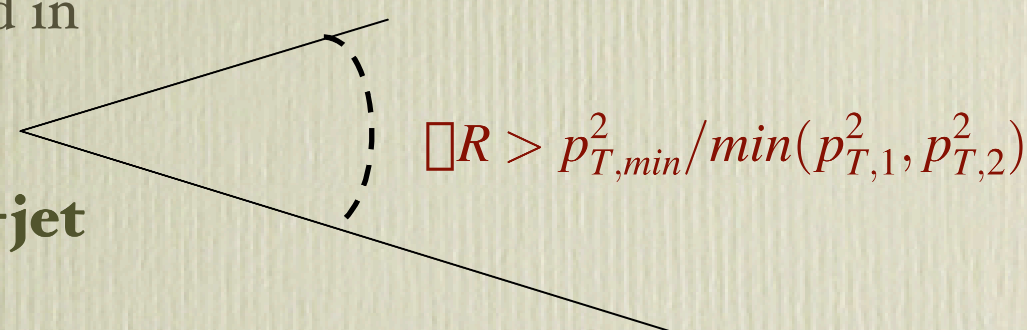
The change in rate, and the relative slopes, are similar in the case of R=0.7 jets, and for jet=3,4



Comments, II: kT vs cone event definition

Cone jets:	$p_T > p_{T,min}\ , \quad \Delta R > R_{min}$
kT jets:	$k^2_{ij} = p^2_{T,i}$ if i final state and j incoming parton $k^2_{ij} = \min(p^2_{T,i}, p^2_{T,j}) \times \Delta R(i,j)$ if i, j outgoing partons If a pair (i,j) has k(i,j) below a given resolution scale kres, cluster (i,j). The jet multiplicity is given by the number of clusters for which k(i,j) (for all i,j pairings) is above kres. To establish a link to experimental jets, we typically choose $k_{res} = p_{T,min}$

The typical dR separation between resolved kT jets is larger than the Rmin separation used in standard experimental analysis (0.4 or 0.7). **Therefore kT-N-jet final states only populate a fraction of the full cone-N-jet phase space.** If this fraction is small, the shower carries the burden of properly describing hard-jet emission



Example

R=0.4, Fixed Q for both kt and R samples

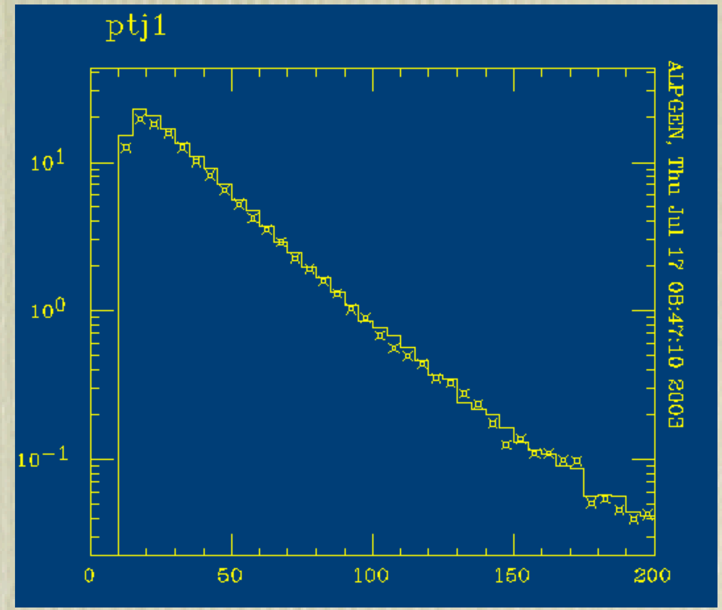
Consider 2 event samples:

a) N jets resolved by the kT algorithm (points)

b) N jets defined by the cone algorithm (histo)

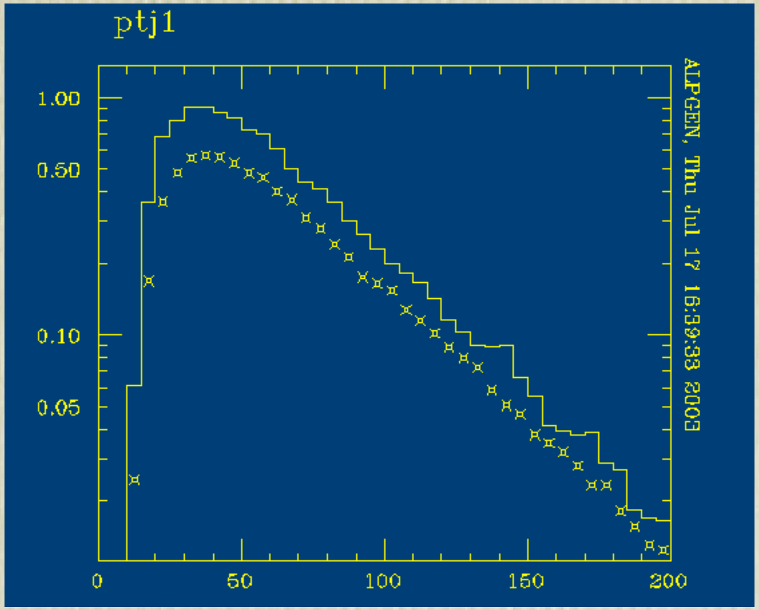
Then reconstruct jets in the two samples using the cone algorithm, and plot jet distributions.

W+2 jets,
σ(kt) = 132
σ(R) = 147



W+4 jets,
σ(kt) = 7.5
σ(R) = 11.8

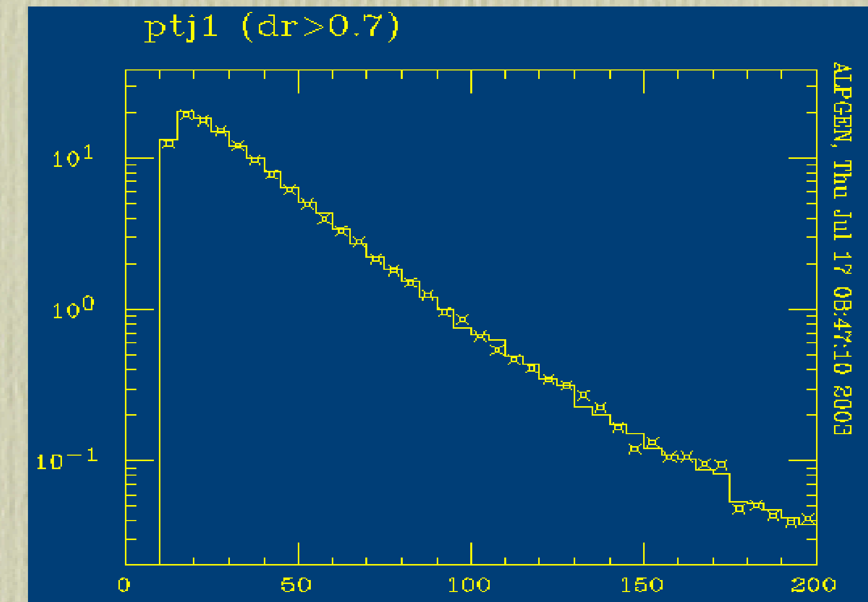
The kT 4-jet rate from the matrix element accounts only for 60% of the 4-jet rate, the rest is to be provided by showering 3-jet events

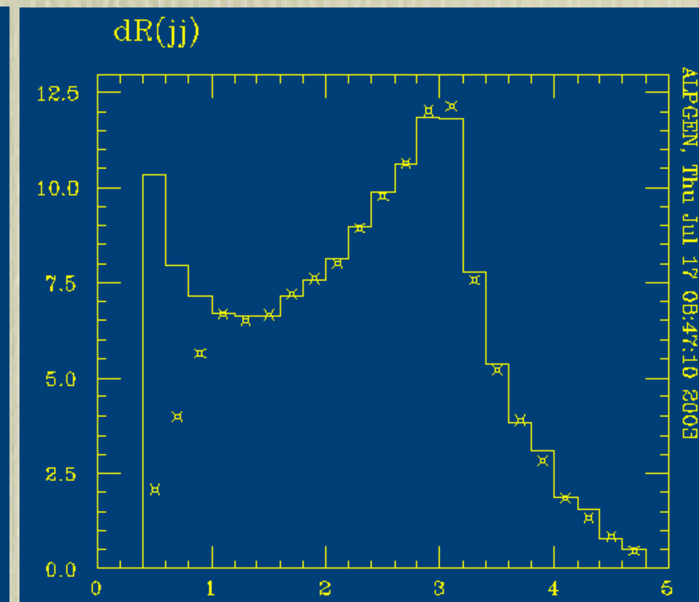


things improve of course for larger separation:

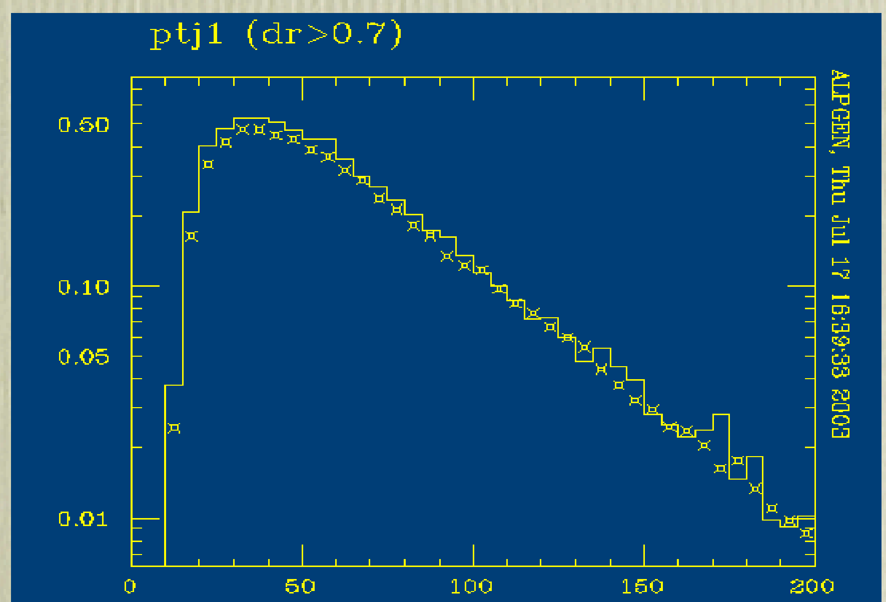
R=0.7, Fixed Q for both kt and R samples

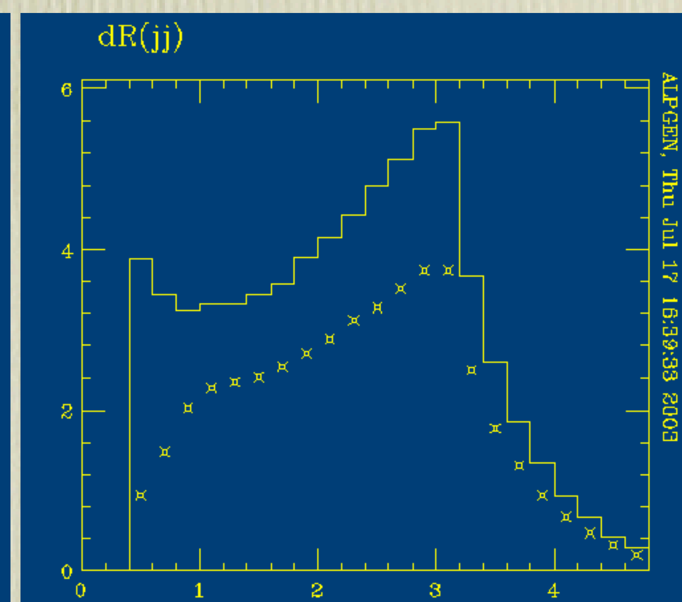
W+2 jets,
σ(kt) = 128
σ(R) = 132





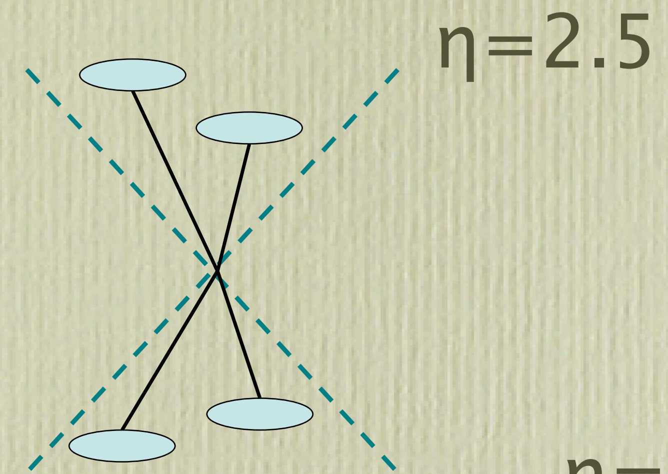
W+4 jets,
σ(kt) = 6.0
σ(R) = 6.7



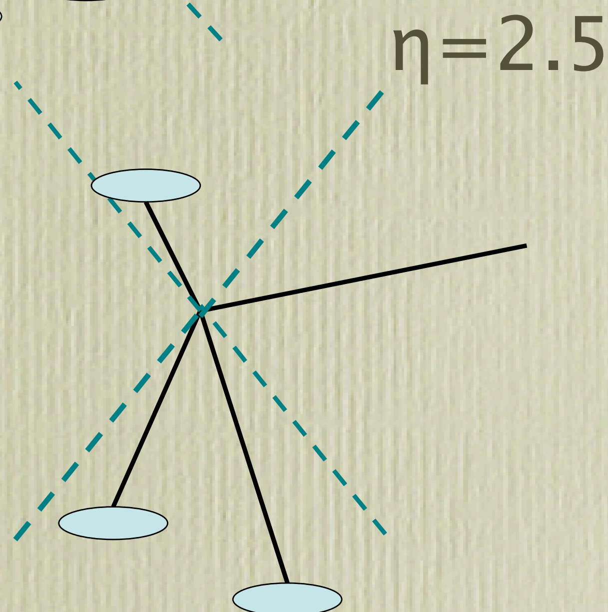


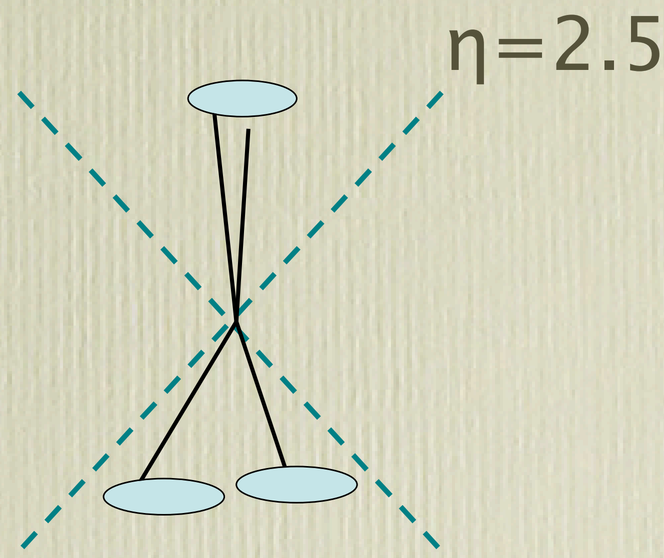
Comments III: reconstructed-jet composition of k⊥-defined jet sample. E.g.: N(k⊥ jets)=4

N(R jets)=4



N(R jets)=3

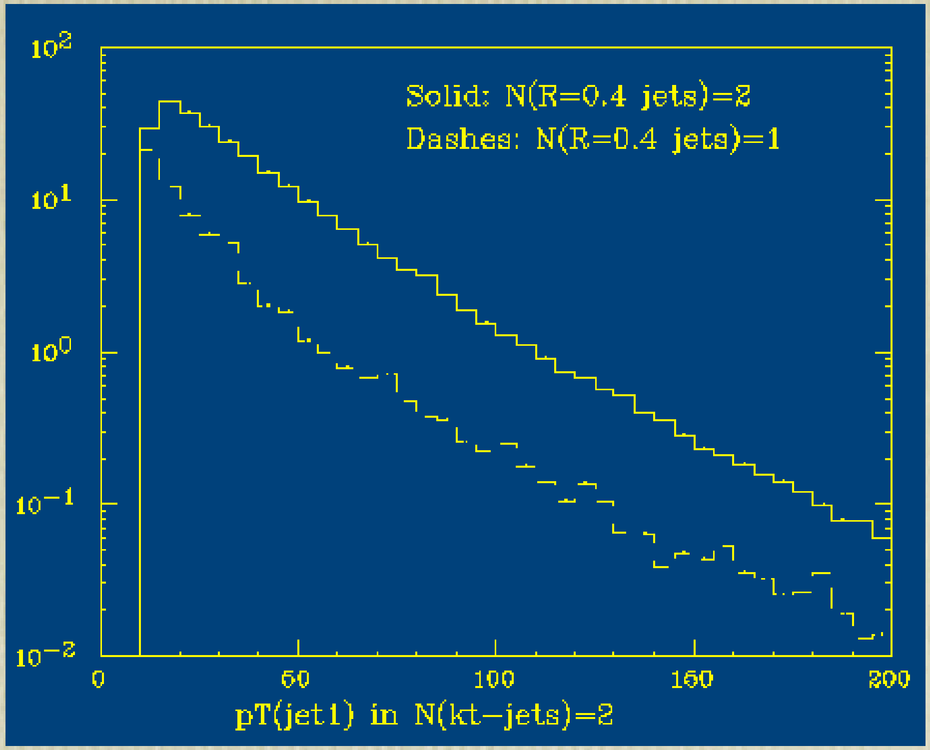


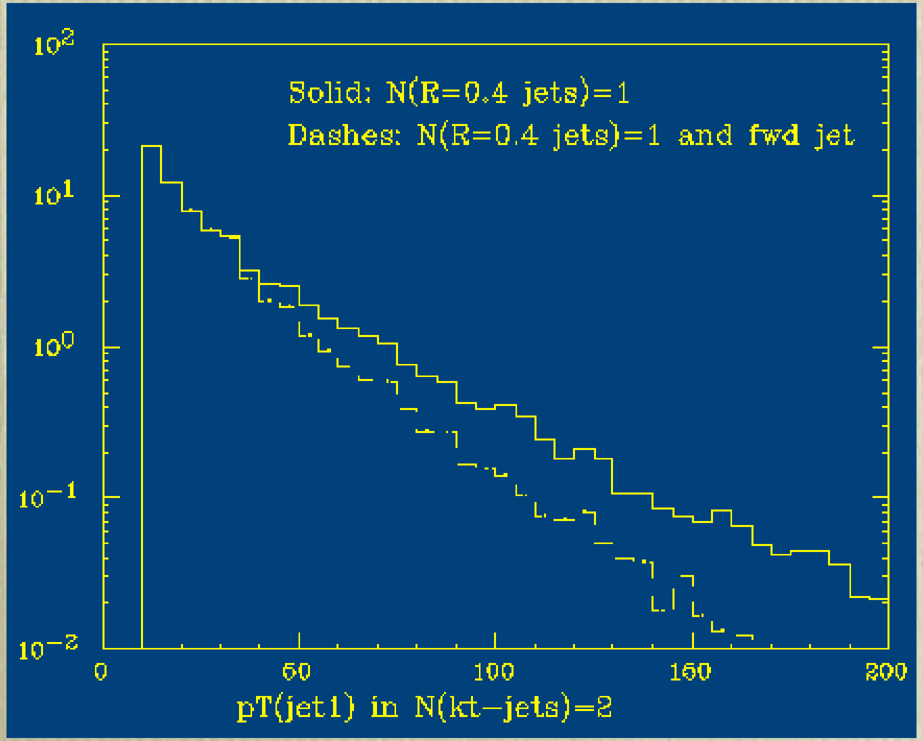


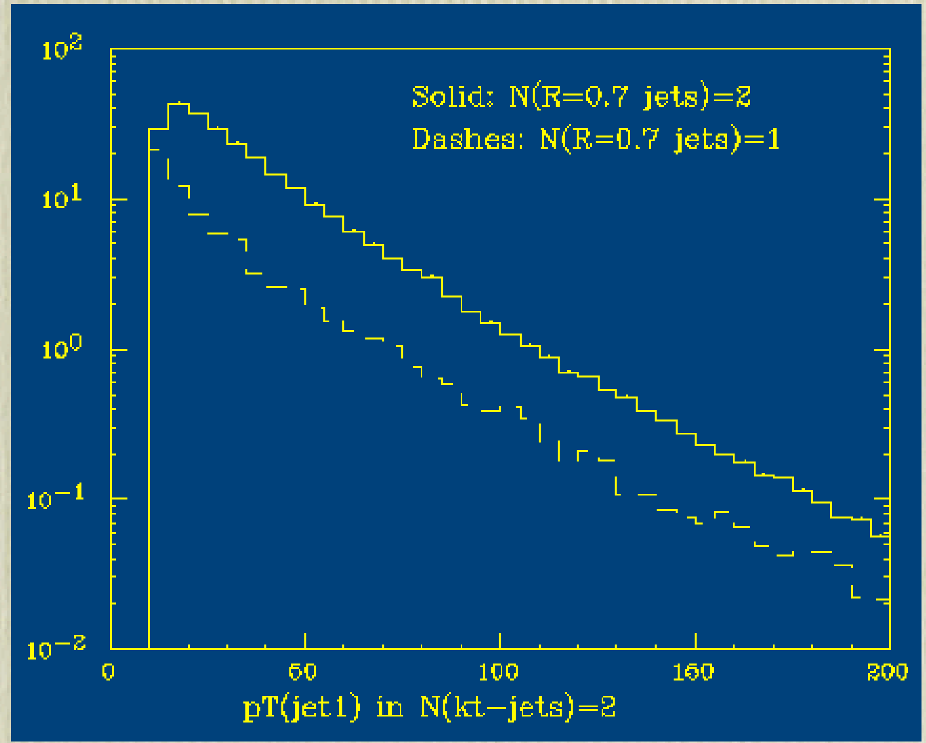
N(R jets)=2 ...etc...

Example: W+2 jets

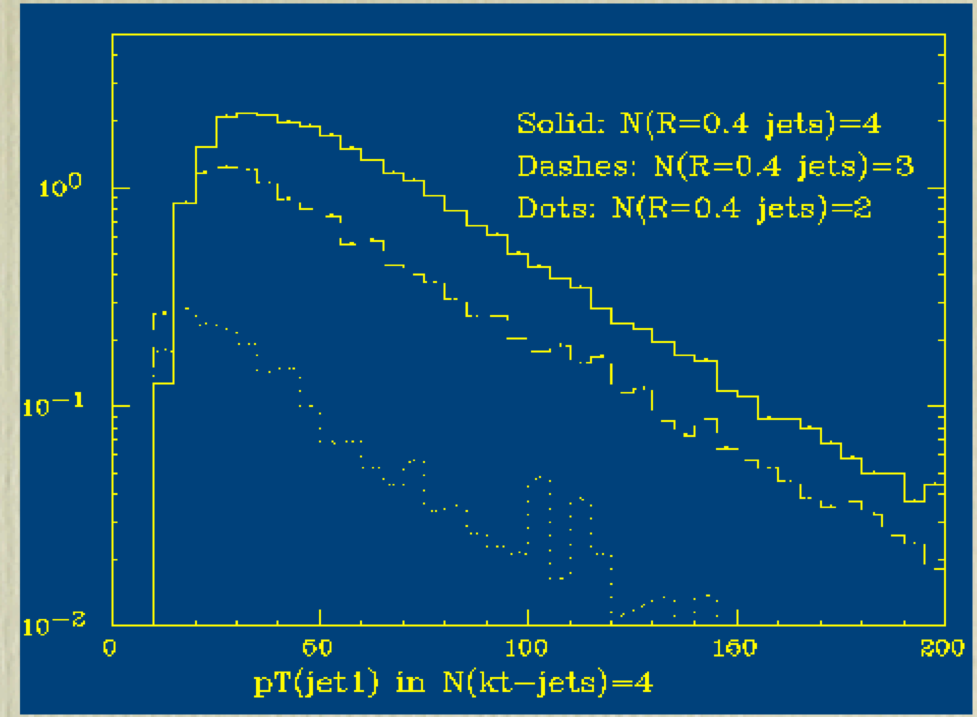
For pt>40 most of the 1-jet final states are due to merging of the two partons into a single jet.

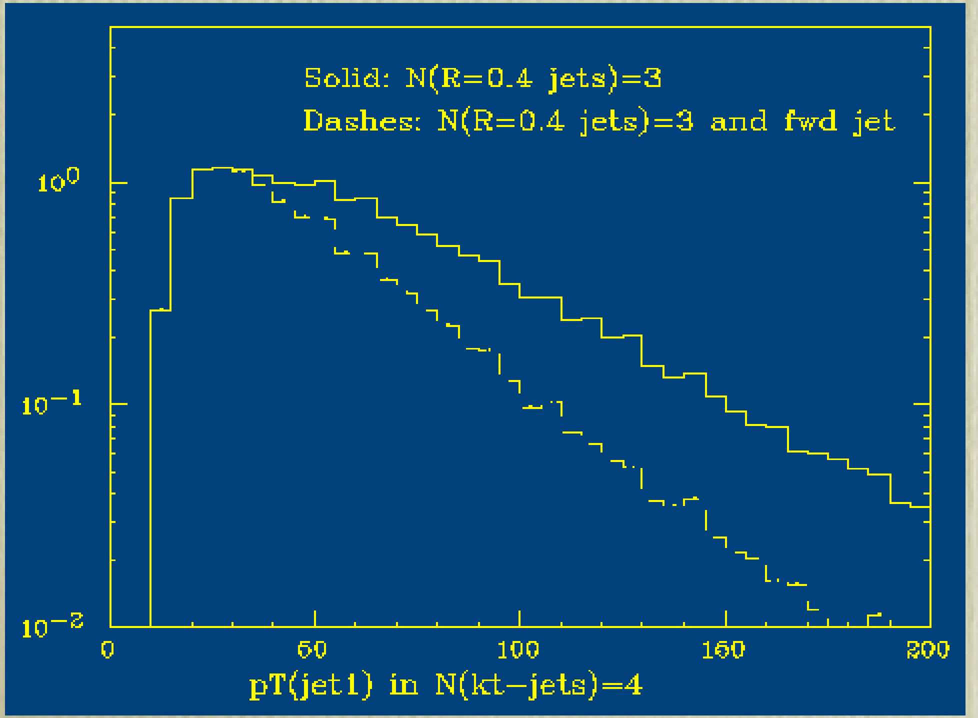


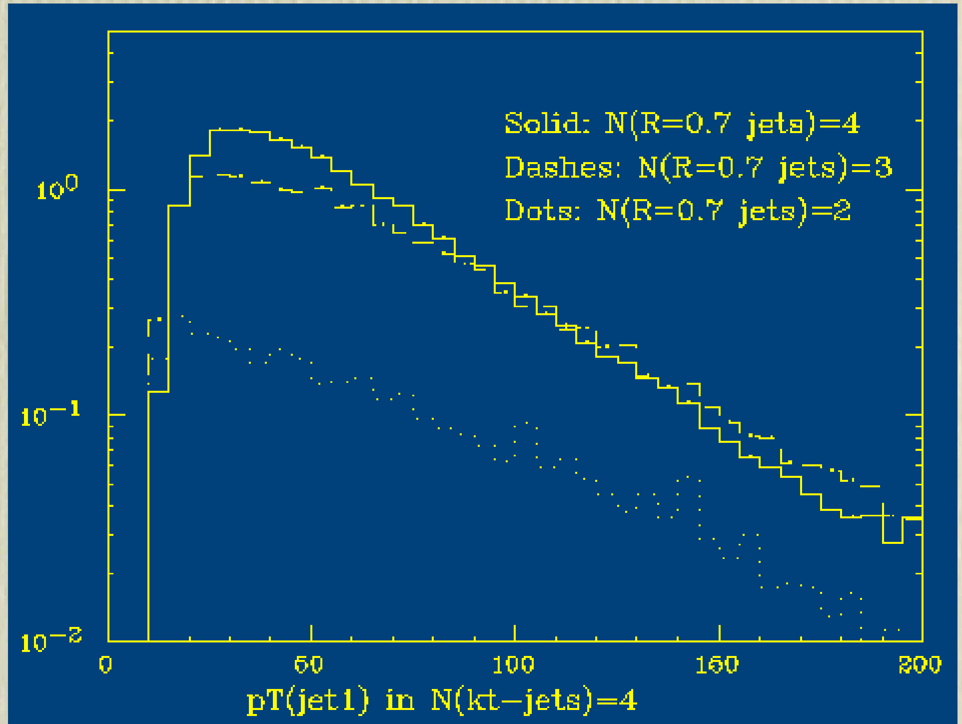




Same as before, for W+4 jets







Inclusive vs Exclusive

In principle, a consistent prescription should lead to:

$$\sigma_{inc} = \sum_{excl} \sigma_{excl}$$

For example, all of these should then be equivalent:

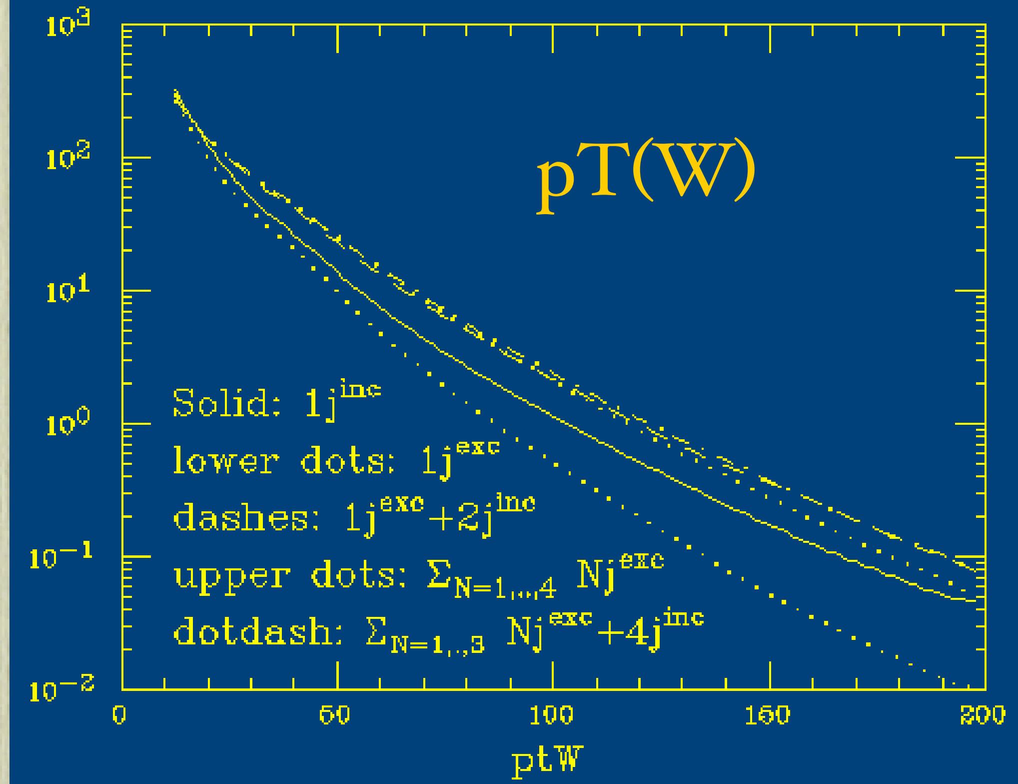
$$\sigma_{inc}^{W+1\,jet} = \sigma_{exc}^{W+1} + \sigma_{inc}^{W+2} = \sum_{i \geq 1} \sigma_{exc}^{W+i}$$

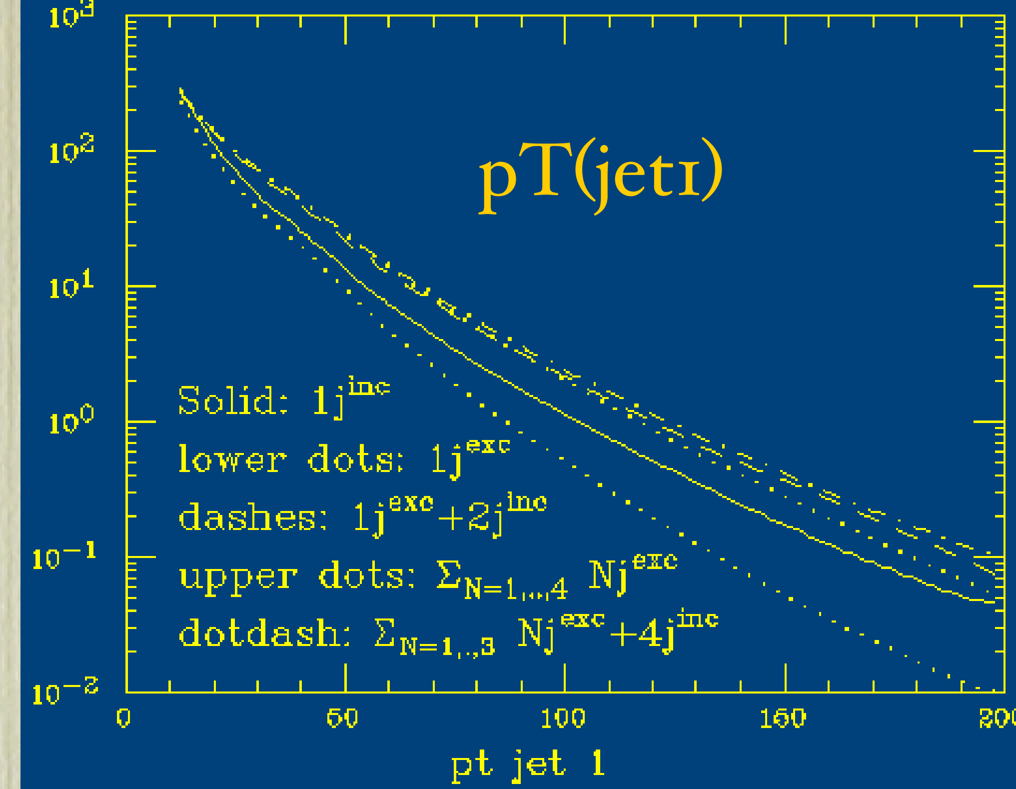
In practice, to which extent this is true needs to be tested.

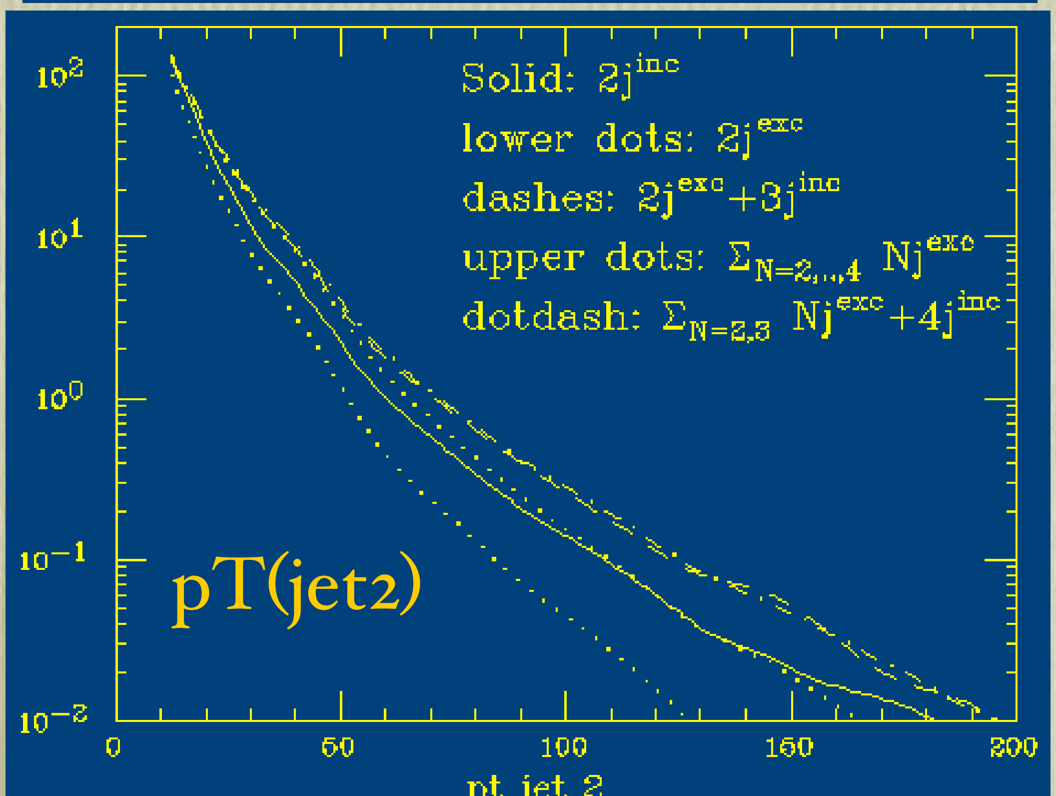
Strong deviations from the above relation can be expected, for example, when new processes open up at higher orders. E.g.:

$\sigma(pp \rightarrow W b\bar{b}\, jet) > \sigma(pp \rightarrow W b\bar{b})$ since $qg \rightarrow W b\bar{b} q > q\bar{q} \rightarrow W b\bar{b}$

In the following slides we compare distributions obtained with **inclusive weights** (ME+α_s reweighting) and with **exclusive weights** (ME+α_s+ Sudakov reweighting). Jet 1 (2) refers to the leading pt (2nd leading) R=0.4 jet.







Bottom line:

$$O[\Box_s^{Nj}]_{inc} < O[\Box_s^{Nj}]_{exc} + O[\Box_s^{Nj+1}]_{inc}$$

but:

$$O[\Box_s^{Nj+1}]_{inc} \sim \Box_{Nj+1}^{3} O[\Box_s^{n}]_{exc} + O[\Box_s^{4}]_{inc}$$

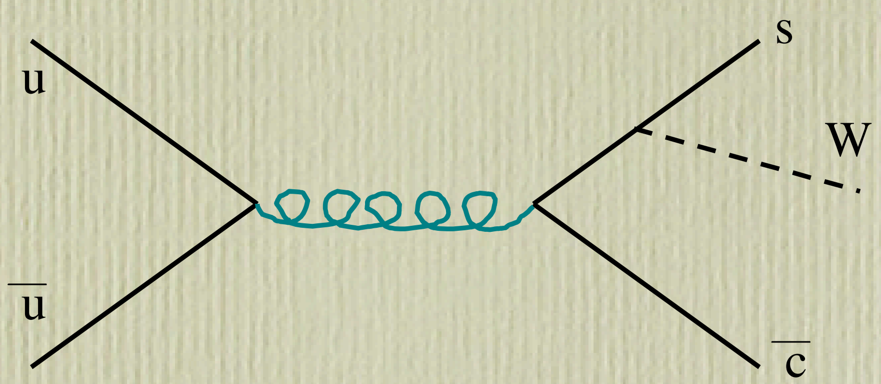
What do we expect the relation between inclusive and Σ over exclusive channels to be??

Implementation issues

- Choice of scales: $k_\perp$ or $k_\perp/2$?
- Scale of last node in the branching
- Same scales in IS and FS branchings?
- Definition of N-jet resolved flavour blind or driven by allowed branchings?
- Use of colour information, instead of flavour, to define the tree and the respective reweightings?
- Different resolution scales for different components of the tree (for example, in the case of b-bbar quark pairs, which experimentally might be subject to different jet-selection criteria than light jets)?
- Use pT or transverse mass in the clustering?
- Treatment of gauge bosons
- etc. etc.

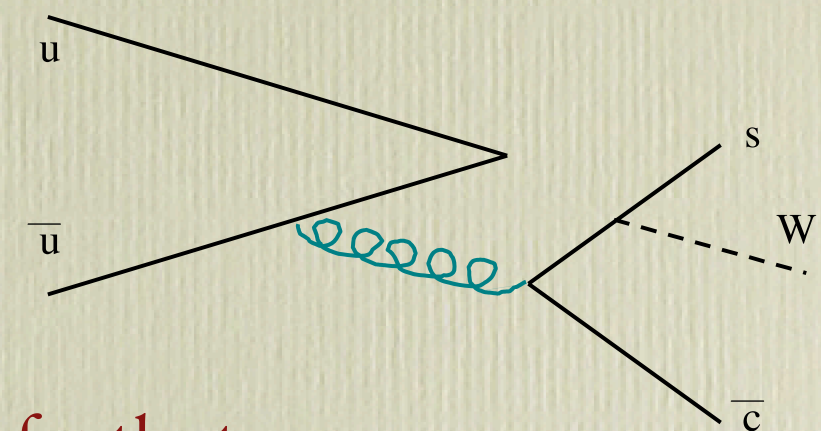
Implementation issues, cont.

Not all flavour configurations are clusterable to a 2->1 process:



Possible prescription: cluster all the possible partons, down to a 2->3 process, then set $k_\perp=\sqrt{s}$ at the remaining vertices (default in Alpgen)

Alternative: allow for W to cluster with quarks, then continue towards a 2->0 process:



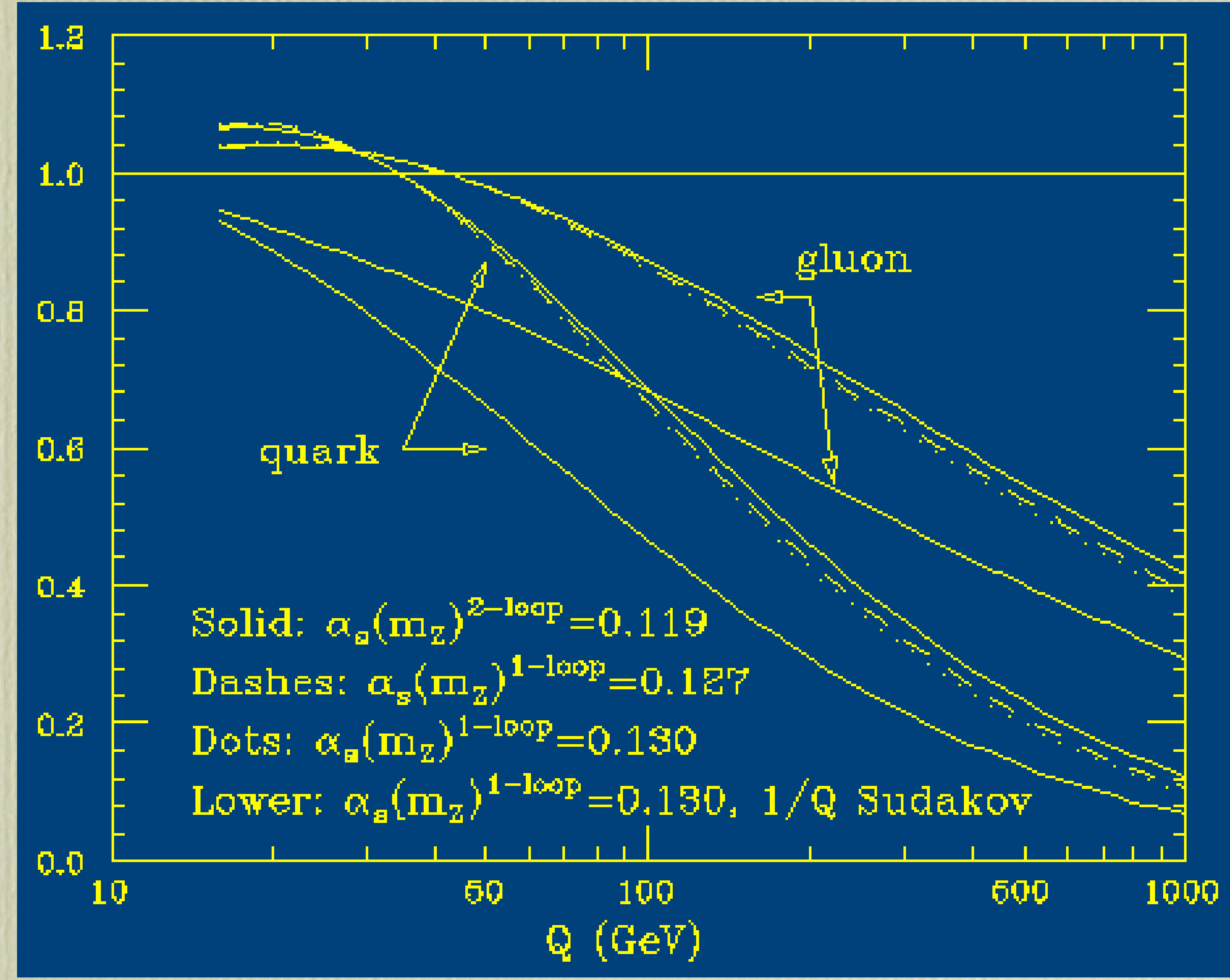
The resulting Sudakovs for the two approaches are approx. the same

The clustering of weak bosons allows flexibility to the code to deal with multi-boson final states. However I don't know what to do with Z's: include the weak charges to weight differently clusterings with u and d quarks?

Sudakov form factors

No significant sensitivity to the value of α_s, but large sensitivity to subleading 1/Q corrections:

In this plot 1/Q refers to the correct treatment of the z-range in the integration of the splitting functions:



$$\int_{q/Q}^{1-q/Q} dz \left[\frac{2}{1-z} - (1+z)\right] = 2\log\frac{Q}{q} - \frac{3}{4}(1-2\frac{q}{Q})$$

Conclusions

- There are several steps involved in setting up a merging programme for multi-jet final states in hadronic collisions.
- At each step, some arbitrariness is involved
- Already at the parton-level, the behaviour of distributions before and after α_s and Sudakov reweighing is significantly different
- The possibility that $\alpha_s(k\perp)$ reweighing provides the "right" scale-setting prescription for LO ME calculations is very interesting, and should be further explored in cases where multi-jet NLO results are available
- Until formal proofs are found indicating what the correct prescriptions are, multi-jet calculations won't have much predictive power, and at best we should aim at thorough, and possibly process-dependent, tuning on data.
- Given the large uncertainties intrinsic in these high-order calculations, tuning will likely remain the only valid approach even once these proofs are found.