Direct CP Violation in B decays with $\rho^0 - \omega$ Mixing

Phenomenolgy and Simulations

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- **Physical Motivations for the Vector Decay Channels : $B \to V_1 V_2$**
- **Helicity Formalism and its Applications**
- **F.S.I, $\rho^0 - \omega$ Mixing and Factorization Hypothesis**
- **Simulations and General Results**
- **Perspectives and Conclusion**

Physical Motivations for $B \to V_1 V_2$

$\star$ $V_{1,2}$ Vector Meson 1^{--} which decays into Pseudoscalar Mesons 0^{-+}

$\star$ Weak Decay of the B meson $\Longrightarrow$ $V_{1,2}$ are polarized

$\Rightarrow$ Specific Angular Distributions and Correlations among final particles

$\Downarrow$

Cross-checks of the Standard Model predictions

$\Downarrow$

Test of Models Beyond the SM (BSM)

(Dunietz, Lipkin et al Phys.Rev. D34 (1991), 2134)

• Special case of two Neutral Vector Mesons with $\bar{V}^0 = V^0$

$$s_B = 0 \Rightarrow \ell = S = 0, 1, 2 \Longrightarrow CP = (-1)^{\ell}$$

$\Longrightarrow$ Mixing of Different CP Eigenstates $\equiv$ CP non-conservation

$\Longrightarrow$ Test of CP violation in a Model Independent way.

$\Longrightarrow$ Constraint on any model BSM

Helicity Formalism and its Application

$\star$ Helicity defined by $\lambda \;=\; \vec{s}\cdot\vec{p}/p$ (s = spin of the particle, $\vec{p}$ its momentum)
$\star$ $B \rightarrow X\ Y$ (s_1 and s_2 are Spin of ANY particles X and Y)

$$\star\ \vec{s_B} = \vec{\ell} + \vec{s_1} + \vec{s_2} \;=\; \vec{0}$$

$\star$ Projection on the momentum-axis (Δ_p) in the B rest-frame

$$\Downarrow$$

$$\lambda_1 \;=\; \lambda_2$$

So, Corresponding Number of helicity states :

Decay Channel	Helicity values	Nb of amplitudes
P_1P_2	0	1
VP	0	1
V_1V_2	-1, 0, +1	3
γV	-1, +1	2
γP	FORBIDDEN	0

N.B : In any Radiative Decay, Photon Helicity : $\lambda_\gamma = \pm 1$

- Formulation of the Decay Amplitude, $B \to V_1 V_2 \ \to Hadrons$

$$V_i(\lambda) \to a_i + b_i \ \ \text{where} \ \ a_i, b_i \ = \ 0^{-+}$$

Partial Amplitude $H_\lambda \ = < V_1(\lambda) V_2(\lambda) | \mathcal{H}_W | B >$
($\mathcal{H}_W \ = \ Weak \ Hamiltonian$)

⋆ For a given Helicity State, the total amplitude factorizes according to :

$$M_\lambda \Big(B \to \sum_{i=1}^{2} (a_i + b_i) \Big) = H_\lambda (B \to V_1 + V_2) \times \prod_{i=1}^{2} A_i (V_i \to a_i + b_i)$$

(Kramer-Palmer, Phys.Rev.D45 (1992), 193)
where :
⋆ $A_i(V_i \to a_i + b_i) \ \equiv$ Decay Amplitude of V_i due to Strong or E.M. interaction

$$A_1(V_1 \to a_1 + b_1) = \sum_{m_1=-1}^{1} c_1 D^1_{\lambda, m_1}(0, \theta_1, 0)$$
$$A_2(V_2 \to a_2 + b_2) = \sum_{m_2=-1}^{1} c_2 D^1_{\lambda, m_2}(\phi, \theta_2, -\phi)$$

(Wigner-Eckart Theorem)

- c_1 , c_2 $=$ Dynamical Decay Parameters related to resonances V_1 and V_2
- $D^1_{\lambda,m_i}(\phi_i,\theta_i,-\phi_i)$ $=$ Wigner rotation matrix element for a spin-1 particle
- $\lambda(a_i)$, $\lambda(b_i)$ $=$ Helicities of the final particles a_i and b_i in the V_i rest frame
- θ_1 $=$ Polar Angle of meson a_1 in the V_1 helicity frame
- Decay Plane of V_1 $\equiv$ (X-Z) Plane $\Rightarrow$ Azimuthal Angle $\phi_1 = 0$
- θ_2 , ϕ $\equiv$ Polar and Azimuthal angles of meson a_2 in the V_2 helicity frame
- $m_i = \lambda(a_i) - \lambda(b_i)$

$\star$ Convention for the $D^1_{\lambda,m_i}(\alpha,\beta,\gamma)$ (Rose, Dover 1957)

$$D^1_{\lambda,m_i}(\alpha,\beta,\gamma) = \exp[-i(\lambda\alpha + m_i\gamma)]\; d^1_{\lambda,m_i}(\beta) \ .$$

⋆ General form of the Decay Amplitude $\mathcal{M}\big(B \to \sum_{i=1}^{2}(a_i + b_i)\big)$:

$\Longrightarrow$ Linear Superposition of the previous amplitudes $M_\lambda\big(B \to \sum_{i=1}^{2}(a_i + b_i)\big)$

$$\Longrightarrow \mathcal{M}\big(B \to \sum_{i=1}^{2}(a_i + b_i)\big) = \sum_{\lambda} M_\lambda\big(B \to \sum_{i=1}^{2}(a_i + b_i)\big) \ .$$

⋆ Decay Width $\Gamma(B \to V_1 V_2)$

$$d^3\Gamma(B \to V_1 V_2) \propto \Big| \sum_{\lambda} M_\lambda\big(B \to \sum_{i=1}^{2}(a_i + b_i)\big) \Big|^2 = \sum_{\lambda,\lambda'} h_{\lambda,\lambda'} F_{\lambda,\lambda'}(\theta_1) G_{\lambda,\lambda'}(\theta_2, \phi) \ ,$$

$$\Downarrow$$

3 Kinematic Parameters : θ_1, θ_2, ϕ

3 Density-Matrices : $h_{\lambda,\lambda'},\ F_{\lambda,\lambda'}(\theta_1),\ G_{\lambda,\lambda'}(\theta_2, \phi)$

$\star$ $h_{\lambda,\lambda'} = H_\lambda H^*_{\lambda'} \Longrightarrow$ Density-Matrix related to the B decay

$\star$ $F_{\lambda,\lambda'}(\theta_1)$ $\Longrightarrow$ Density-Matrix of the decay $V_1 \longrightarrow a_1 + b_1$

$\star$ $G_{\lambda,\lambda'}(\theta_2, \phi)$ $\Longrightarrow$ Density-Matrix of the decay $V_2 \longrightarrow a_2 + b_2$.

$$\Downarrow$$

Very General Analytic Formulation

$\star$ Independent of the Nature of the intermediate Resonances

$\star$ Independent of their Decay Modes
(except for the Spin of the final particles)

Physical Informations which can be inferred

⋆ Angular Distribution of each final particle $a_i(b_i)$ in the V_i rest frame

⋆ Correlations among the final B products

- "Practical" Applications

$$B^0 \to K^{*0}\rho^0(\omega)\ ,\quad K^{*0} \to K^+\pi^-\ ,\ \rho^0 \to \pi^+\pi^-$$

$$B^\pm \to K^{*(\pm)}\rho^0(\omega)\ ,\quad K^{*(\pm)} \to K^\pm\pi^0\ ,\ \rho^0 \to \pi^+\pi^-$$

$$B^\pm \to \rho^\pm\rho^0(\omega)\ ,\quad \rho^\pm \to \pi^+\pi^-\ ,\ \rho^0 \to \pi^+\pi^-$$

$$\frac{d^3\Gamma(B \to V_1 V_2)}{d(\cos\theta_1)d(\cos\theta_2)d\phi} \propto \big(h_{++} + h_{--}\big)\sin^2\theta_1 \sin^2\theta_2/4 + h_{00}\cos^2\theta_1\cos^2\theta_2$$
$$+\Big\{ Re(h_{+0})\cos\phi - Im(h_{+0})\sin\phi + Re(h_{0-})\cos\phi - Im(h_{0-})\sin\phi \Big\}\sin 2\theta_1 \sin 2\theta_2/4$$
$$+\Big\{ Re(h_{+-})\cos 2\phi - Im(h_{+-})\sin 2\phi \Big\}\sin^2\theta_1\sin^2\theta_2/2$$

Expression Completely Symmetric in θ_1 and θ_2

$\Longrightarrow$ Angular Distribution of a_1 in the V_1 frame is identical to that of a_2 in the V_2 frame.

- Probabilty Distribution Functions (PDF) of the Angles :

$$\begin{aligned} f(\cos\theta_{1,2}) &= (3h_{00} - 1)\cos^2\theta_{1,2} + (1 - h_{00}) \\ g(\phi) &= 1 + 2\ Re(h_{+-})\cos 2\phi - 2\ Im(h_{+-})\sin 2\phi\ . \end{aligned}$$

Final State Interactions and $\rho^0 - \omega$ Mixing

Why FSI are so Important

$\star$ Hadrons produced from B decays are scattered again by their mutual Strong Interactions

$\Longrightarrow$ Modification of the final state Wave-Function

$$\Downarrow$$

Computations of the Branching Ratios $B \to X$ must take account of the FSI
(H.Lipkin, 1980)

- Two regimes in the FSI :

$\star$ Perturbative regime

$\star$ Non-perturbative one

$\star$ How to deal with FSI in a simple and practical way ??

$$\Downarrow$$

Factorization Method

$\star$ In the Feynman Diagrams, Tree and/or Penguin, Soft Gluons exchanged between quarks are neglected.

$\star$ Effective Lagrangean approach :

i) Perturbative Calculations performed at NLO for an Energy Scale $\geq m_B$ via the Wilson Coefficients
ii) Non-perturbative effects estimated for the Energy Scale $\leq m_B$ $\Rightarrow$ Form Factors

Computation Details

$\star$ Two Matrix Elements :

$$\langle \rho^0(\omega)|(\bar{u}u)|0\rangle\langle K^*|(\bar{q}_i q_j)|B^{\pm,0}\rangle \text{ and } \langle K^*|(\bar{q}_i q_j)|0\rangle\langle \rho^0(\omega)|(\bar{u}b)|B^{\pm,0}\rangle$$

$\star$ q_i , q_j could be either u, s or d.

$\star$BSW decomposition of the Matrix Elements for $B \rightarrow X^*$

$$\langle X^*|J_\mu|B\rangle = \frac{2}{m_B + m_{X^*}}\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}p_B^\rho p_{X^*}^\sigma V(k^2) + i\left\{\epsilon_\mu^*(m_B + m_{X^*})A_1(k^2)\right.$$
$$\left. -\frac{\epsilon^* \cdot k}{m_B + m_{X^*}}(P_B + P_{X^*})_\mu A_2(k^2) - \frac{\epsilon^* \cdot k}{k^2}2m_{X^*} \cdot k_\mu A_3(k^2)\right\}$$
$$+i\frac{\epsilon^* \cdot k}{k^2}2m_{X^*} \cdot k_\mu A_0(k^2)$$

where :

- $J_\mu = \bar{q}\gamma^\mu(1-\gamma_5)b$ with $q = u, d, s$
- $k = p_B - p_{X^*}$
- ϵ_μ is the polarization vector of X^*
- A_0, A_1, A_2, A_3, V $\equiv$ Form Factors describing the transition $0^- \to 1^-$.

$\star$ Modification of the Effective Number of Color, ${N_c}^{eff}$:

$$\frac{1}{(N_c^{eff})_i} = \frac{1}{3} + \xi_i \quad \xi_\mathrm{i}\ \mathrm{describesNon-Factorizable\ effects}$$

Naive Factorization

$\Downarrow$

Four Fermion Operator replaced by the Product of Two Currents

Illustrating the FSI by Phase-Shift Angles

- The Quark Contribution

⋆ Absorptive Part of the Scattering Amplitude due to the Penguin Diagram

$\Longrightarrow$ BSS Mechanism (M.Bander, Phs.Rev.Lett.43 (19179), 242)

$\Longrightarrow$ Generation of a Phase Shift by comparison with the Tree Diagram

$$\mathcal{A} = \mathcal{A}^T \; + \; \mathcal{A}^P \;\; = \;\; \langle K^* \pi^- \pi^+ | H^T | B \rangle [1 + r e^{i\delta} e^{i\phi}]$$

⋆ ϕ , Weak Phase arising from the CKM Matrix Elements $\;\; = \;\; \arg[(V_{tb}V_{ts}^*)/(V_{ub}V_{us}^*)]$

⋆ $\delta =$ Strong Phase from Penguin diagram "a la BSS"

$$r \equiv \left| \frac{\langle \rho^0(\omega) K^* | H^P | B \rangle}{\langle \rho^0(\omega) K^* | H^T | B \rangle} \right| .$$

- The $\rho^0 - \omega$ Mixing Contribution

$$\rho^0 \longrightarrow \pi^+\pi^- \quad \approx \ 100\%$$

$$\omega \longrightarrow \pi^+\pi^- \quad \approx \ 2\%$$

$$\Downarrow$$

$\rho^0 - \omega$ Mixing is unavoidable in the study of $B \longrightarrow \pi\pi\ V$

$$\Downarrow$$

Generation of an Additionnal Phase-Shift δ_R
when $\pi^+\pi^-$ mass is in the vicinity of the Resonance ω

Final Expression of the Amplitude

$$\begin{aligned}
\langle K^*\pi^-\pi^+|H^T|B\rangle &= \frac{g_\rho}{s_\rho s_\omega}\tilde{\Pi}_{\rho\omega}t_\omega + \frac{g_\rho}{s_\rho}t_\rho \ , \\
\langle K^*\pi^-\pi^+|H^P|B\rangle &= \frac{g_\rho}{s_\rho s_\omega}\tilde{\Pi}_{\rho\omega}p_\omega + \frac{g_\rho}{s_\rho}p_\rho \ .
\end{aligned}$$

Here

- t_V $(V = \rho \text{ or } \omega)$ is the Tree Amplitude for producing a Vector Meson V

- p_V , Penguin Amplitude for producing V
- g_ρ is the coupling for $\rho^0 \to \pi^+\pi^-$
- $\tilde{\Pi}_{\rho\omega}$ is the effective $\rho^0 - \omega$ Mixing Amplitude
- s_V is the inverse propagator of the vector meson V

- $s_V = s - m_V^2 + i m_V \Gamma_V$ with $\sqrt{s}$ $\equiv$ Invariant Mass of the $\pi^+\pi^-$ pair.

- $\tilde{\Pi}_{\rho\omega}(s) = \tilde{\Pi}_{\rho\omega}(m_\omega^2) + (s - m_w^2)\tilde{\Pi}'_{\rho\omega}(m_\omega^2)$

where :

$\Re e\, \tilde{\Pi}_{\rho\omega}(m_\omega^2) = -3500 \pm 300\ \mathrm{MeV}^2, \quad \Im m\, \tilde{\Pi}_{\rho\omega}(m_\omega^2) = -300 \pm 300\ \mathrm{MeV}^2$, and
$\tilde{\Pi}'_{\rho\omega}(m_\omega^2) = 0.03 \pm 0.04.$

$\Longrightarrow \Longrightarrow \Longrightarrow$

$$r e^{i\delta} e^{i\phi} = \frac{\tilde{\Pi}_{\rho\omega} p_\omega + s_\omega p_\rho}{\tilde{\Pi}_{\rho\omega} t_\omega + s_\omega t_\rho}$$

- Final Phase-Shift δ depends both on the Quark Loop and the Resonance Mixing

Explicit Calculations, Simulations and General Results

$$B \to \text{(Weak Decays)} \Longrightarrow \text{Quarks} \quad \Longrightarrow \text{Hadrons}$$

$$\Downarrow$$

Wilson Operator Product Expansion (OPE)

Effective Hamiltonian $\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \sum_i V_{CKM} C_i(\mu) O_i(\mu)$

where

- G_F = Fermi Constant
- V_{CKM} = CKM Matrix Elements
- $C_i(\mu)$ are the Wilson Coefficients
- $O_i(\mu)$ are the Operators from the OPE
- $\mu \equiv$ Renormalization Scale

⋆ OPE represents physics processes for an Energy Scale lower than μ.

⋆ $C_i(\mu)$ represent Perturbative parts computed at Energy Scale greater than μ.

General Form of a Wilson Operator

$$O_i = (\bar{q}_i \Gamma_{n1} q_j)(\bar{q}_k \Gamma_{n2} q_l)$$

$\star$ Γ_{n1} , Γ_{n2} $\equiv$ Combination of Dirac Gamma Matrices
$\star$ q_i , Quark Flavor.

$\star$ Three Categories of Operators :

Tree Operators (O_1, O_2)

QCD Penguin Operators, (O_3 to O_6)

EW Penguin Operators (O_7 to O_{10})

Wilson Coefficients

$\star$ W.C are calculated perturbatively at NLO by Renormalization Group techniques (Buras, Fleischer hep-ph/9806471)
$\star$ Energy Scale $\mu = m_B$.

Combining Wilson OPE and BSW Helicity Formalism

$$
\begin{aligned}
&H_\lambda\big(B\to\rho^0(\omega)V_2\big)=\Big(V_{ub}V_{us}^{*}c_{t_1}^{\rho}-V_{tb}V_{ts}^{*}c_{p_1}^{\rho}\Big)\Big\{\beta_1^{\rho}\varepsilon_{\alpha\beta\gamma\delta}\epsilon_{V_2}^{*\alpha}(\lambda)\epsilon_{\rho}^{*\beta}(\lambda)P_B^{\gamma}P_{V_2}^{\delta}\\
&+i\Big(\beta_2^{\rho}\epsilon_{V_2}^{*}(\lambda)\epsilon_{\rho}^{*}(\lambda)-\beta_3^{\rho}(\epsilon_{V_2}^{*}(\lambda)P_B)(\epsilon_{\rho}^{*}(\lambda)P_B)\Big)\Big\}+\Big(V_{ub}V_{us}^{*}c_{t_2}^{\rho}-V_{tb}V_{ts}^{*}c_{p_2}^{\rho}\Big)\\
&\Big\{\beta_4^{\rho}\varepsilon_{\alpha\beta\gamma\delta}\epsilon_{\rho}^{*\alpha}(\lambda)\epsilon_{V_2}^{*\beta}(\lambda)P_B^{\gamma}P_{\rho}^{\delta}+i\Big(\beta_5^{\rho}\epsilon_{\rho}^{*}(\lambda)\epsilon_{V_2}^{*}(\lambda)-\beta_6^{\rho}(\epsilon_{\rho}^{*}(\lambda)P_B)(\epsilon_{V_2}^{*}(\lambda)P_B)\Big)\Big\}\\
&+\frac{\tilde{\Pi}_{\rho\omega}}{(s_\rho-m_\omega^2)+im_\omega\Gamma_\omega}\Bigg[\Big(V_{ub}V_{us}^{*}c_{t_1}^{\omega}-V_{tb}V_{ts}^{*}c_{p_1}^{\omega}\Big)\Big\{\beta_1^{\omega}\varepsilon_{\alpha\beta\gamma\delta}\epsilon_{V_2}^{*\alpha}(\lambda)\epsilon_{\omega}^{*\beta}(\lambda)P_B^{\gamma}P_{V_2}^{\delta}\\
&+i\Big(\beta_2^{\omega}\epsilon_{V_2}^{*}(\lambda)\epsilon_{\omega}^{*}(\lambda)-\beta_3^{\omega}(\epsilon_{V_2}^{*}(\lambda)P_B)(\epsilon_{\omega}^{*}(\lambda)P_B)\Big)\Big\}+\Big(V_{ub}V_{us}^{*}c_{t_2}^{\omega}-V_{tb}V_{ts}^{*}c_{p_2}^{\omega}\Big)\\
&\Big\{\beta_4^{\omega}\varepsilon_{\alpha\beta\gamma\delta}\epsilon_{\omega}^{*\alpha}(\lambda)\epsilon_{V_2}^{*\beta}(\lambda)P_B^{\gamma}P_{\omega}^{\delta}+i\Big(\beta_5^{\omega}\epsilon_{\omega}^{*}(\lambda)\epsilon_{V_2}^{*}(\lambda)-\beta_6^{\omega}(\epsilon_{\omega}^{*}(\lambda)P_B)(\epsilon_{V_2}^{*}(\lambda)P_B)\Big)\Big\}\Bigg]
\end{aligned}
$$

with

- $\epsilon_{V_2,\rho,\omega}(\lambda)$ being the K^*, ρ^0 and ω polarization 4-vectors expressed in the B rest frame

- ❖ $\varepsilon_{\alpha\beta\gamma\delta}$ Antisymmetric Tensor in Minkowski space.
- ❖ β_i , Form Factors describing transitions between Vector Mesons.

⋆ Simplification of the Helicity Amplitude in the B rest-frame : $P_B = (m_B, \vec{0})$

$$H_\lambda\big(B \to \rho^0(\omega)V_2\big) = iB^\rho_\lambda(V_{ub}V^*_{us}c^\rho_{t_1} - V_{tb}V^*_{ts}c^\rho_{p_1}) + iC^\rho_\lambda(V_{ub}V^*_{us}c^\rho_{t_2} - V_{tb}V^*_{ts}c^\rho_{p_2}) + \frac{\tilde{\Pi}_{\rho\omega}}{(s_\rho - m^2_\omega) + im_\omega\Gamma_\omega}\Big[iB^\omega_\lambda(V_{ub}V^*_{us}c^\omega_{t_1} - V_{tb}V^*_{ts}c^\omega_{p_1}) + iC^\omega_\lambda(V_{ub}V^*_{us}c^\omega_{t_2} - V_{tb}V^*_{ts}c^\omega_{p_2})\Big]$$

$$\Downarrow$$

$$h_{\lambda,\lambda'} = H_\lambda\big(B \to \rho^0(\omega)V_2\big)H^*_{\lambda'}\big(B \to \rho^0(\omega)V_2\big)\ .$$

⋆ Hermiticity of the Density-Matrix ($h_{\lambda,\lambda'}$) $\Longrightarrow$ Only Six Elements have to be calculated.

Form Factors

- Two Models of FF have been used :

⋆ Pioneering Model of BSW (Bauer et al, Z.Phys.C34(1987),103)

$\star$ Modified BSW Model by Guo and Huang (GH, Phys.Rev.D43(1991),2931) which uses the light-cone approach.

$$V(k^2) = \frac{h_V}{\left(1 - \frac{k^2}{m_V^2}\right)} \ , \qquad A_j(k^2) = \frac{h_{A_j}}{\left(1 - \frac{k^2}{m_{A_j}^2}\right)} \ ,$$

where

- m_{A_j} , m_V are the Pole Masses associated with the transition current
- h_V , h_{A_j} are the values of the Form Factors at $k^2 = 0$.

Simulations and Main Results

Input Values

$\star$ Wolfenstein Parameters

$$\lambda_c = 0.2237 \ , \quad A = 0.8113 \ , \quad 0.190 < \rho < 0.268 \ , \quad 0.284 < \eta < 0.366 \ .$$

In the numerical simulations, the Average Values of ρ and η are used.

⋆ Decay Constant of each Vector Meson :

$$f_{K^*} = 214 \text{ MeV} \ , \ f_\rho = 221 \text{ MeV} \ , \ f_\omega = 195 \text{ MeV}$$

Free Parameters

⋆ Ratio $q^2/{m_b}^2$ related to the Penguin Diagram, where q^2 is the Gluon squared mass

$$\Rightarrow \quad 0.3 \leq q^2/{m_b}^2 \leq 0.5$$

(R.Fleischer, Int.J.Mod.Phys. A12(1997), 2459)

⋆ Form Factor Models : BSW or GH

⋆ Color Effective Number, N_c^{eff} $\Rightarrow$ It depends on the Neutral Transition $b \longrightarrow d$ $(N_c^{eff} = \ 2.01)$ or $b \longrightarrow s$ $(N_c^{eff} = \ 2.84)$

Determination of Density Matrix $h_{i,j}$

$\star$ $h_{i,j}$ elements depend essentially on the masses of the Resonances.

$\star$ Mass of each Vector Resonance generated according to a Relativistic Breit-Wigner distribution :

$$\frac{d\sigma}{dM^2} = C_N \frac{\Gamma_R M_R}{(M^2 - M_R^2)^2 + (\Gamma_R M_R)^2}$$

- C_N , Normalization Constant
- M_R and Γ_R , respectively the Mass and the Width of the Vector Meson

$\star$ Simplified Simulation of the $\rho^0 - \omega$ Mixing

The ρ^0-propagator is replaced by the following one :

$$\frac{1}{s_{\rho\omega}} = \frac{1}{s_\rho} + \frac{T_\omega}{T_\rho} \frac{\Pi_{\rho\omega}}{s_\rho s_\omega} \ ,$$

(P.Langacker, Phys.Rev. D20(1979), 2983)

$\star$ $\Pi_{\rho\omega}$, Mixing amplitude
$\star$ Natural Choice, $T_\omega / T_\rho = 1$

Main Results

1) Spectrum of $h_{i,j}$ too wide because of the Resonance Widths, especially the ρ^0 width, $\Gamma_\rho = 150\ \mathrm{MeV/c^2}$

2) The Longitudinal Polarization $h_{00} = |H_0|^2$ is largely Dominant

$B^0 \to \rho^0(\omega)K^{*0} \implies$ Mean Value of h_{00} is $\approx 87\%$

$B^+ \to \rho^0(\omega)\rho^+ \implies$ Mean Value of h_{00} is $\approx 90\%$

3) $h_{--} = |H_{-1}|^2$ very tiny, $\leq 0.5\%$

4) Non-Diagonal matrix elements $h_{i,j}$:

- Smallness of both Real and Imaginary parts
- $Im/Re \approx 0.001 \Rightarrow 0.1$
- Special case of $B^+ \to \rho^0(\omega)\rho^+$, $Im(h_{i,j}) \approx 0.0$

First Conclusion

"Universal behavior" of the Density-Matrix Elements, whatever the decay $B \to \pi^+\pi^- V$ is !! ($V = K^{*0}, K^{*\pm}, \rho^\pm$)

Consequences on Angular Distributions

$\star$ Spin $s_B = \ 0 \Longrightarrow$ Isotropic Angular Distribution of V_i in B rest-frame.

$\star$ Helicity Frame of each Vector Meson V_i :
Angular Distributions given by the DM elements $h_{i,j}$

$\star$ $g(\phi) = 1 + 2 \ \mathcal{R}e(h_{+-})\cos 2\phi - 2 \ \mathcal{I}m(h_{+-})\sin 2\phi$

Smallness value of $\langle h_{+-} \rangle \Rightarrow$ Azimuthal angle distribution $\approx$ FLAT

$\star$ $f(\cos\theta_{1,2}) = (3h_{00} - 1)\cos^2\theta_{1,2} + (1 - h_{00})$

Polar Angle distribution $\approx \cos^2\theta$ because of the dominant longitudinal part h_{00}.

Branching Ratios and Asymmetries

$\star$ Energy E_i and momentum p_i of each vector meson vary significantly according to the generated event

$\Longrightarrow$ Branching Ratio of each channel computed by Monte-Carlo methods from the relation :

$$d\Gamma(B \to V_1 V_2) \ = \ \frac{1}{8\pi^2 M} \, |\mathcal{M}(B \to V_1 V_2)|^2 \, \frac{d^3\vec{p}_1}{2E_1} \, \frac{d^3\vec{p}_2}{2E_2} \, \delta^4(P - p_1 - p_2) \ .$$

and

$$Br(B \to f) = \frac{\Gamma(B \to f)}{\Gamma(B \to All)}$$

$\star$ For a fixed value of $q^2/{m_b}^2$, BRs depend strongly on the Form Factor model
$\Longrightarrow$ Variation up to a factor 2

$\star$ Relative Difference between two conjugate branching ratios, $Br(B \to f)$ and $Br(\bar{B} \to \bar{f})$, is $\approx$ independent of the form-factor models.

Asymmetry according to $\pi\pi$ Invariant Mass

$\star$ New and unexpected effect "discovered" wth the Differential Asymmetry according to the $\pi\pi$ invariant mass.

$$a_{CP}(m) = \frac{\Gamma_m(B \to f) - \bar{\Gamma}_m(\bar{B} \to \bar{f})}{\Gamma_m(B \to f) + \bar{\Gamma}_m(\bar{B} \to \bar{f})}$$

$\Longrightarrow$ $a_{CP}(m)$ is amplified in the vicinity of the ω Resonance mass : $\pm 20\ \mathrm{MeV}/c^2$ around $M_\omega =\ 782\mathrm{MeV}/c^2$

$\star$ Differential Asymmetry is $\approx 15\%$ in the case of $B^0 \to K^{*0}\rho^0(\omega)$ and

$\star$ $a_{CP}(m) =\ 90\%$ in the channel $B^\pm \to \rho^\pm\rho^0(\omega)$

$\Downarrow$

Asymmetry almost independent of the Form Factor model

Only explanation of this effect : $\rho^0 - \omega$ MIXING

$\star$ Results predicted analytically in the channel $B \to VP$: $B \Rightarrow \pi^+\pi^-\pi$ (O.Leitner, X.Guo, A.W.Thomas, Phys.Rev. D66(2002), 096008)

Physical Consequences

$\star$ New Method to detect and to measure Direct CP Violation both in B^0 and $B^\pm$ decays.

$\star$ According to the analytical expression :

$$a_{CP}^{dir} = \frac{A^2 - \bar{A}^2}{A^2 + \bar{A}^2} = \frac{-2\ r\ \sin\delta\ \sin\phi}{1 + r^2 + 2\ r\ \cos\delta\ \cos\phi}\ .$$

$\Longrightarrow$ Possibility of measuring $\sin\Phi$, where Φ is a Weak Angle resulting from the CKM Matrix elements :

Channel $B \to \rho^0(\omega)K^*$

$\Longrightarrow \Phi = Arg[V_{tb}V_{ts}^*/V_{ub}V_{us}^*]\ =\ \gamma$

Channel $B \to \rho^0(\omega)\rho^\pm$

$\Longrightarrow \Phi = Arg[V_{tb}V_{td}^*/V_{ub}V_{ud}^*] =\ \beta + \gamma\ = \Pi - \alpha$

$\star$ Contribution of the Penguin Diagram to the amplitude :

Ratio $r = |P/T| \longleftrightarrow q^2/m_b^2$

$$q^2/m_b^2 = \ 0.3 \ \Rightarrow < r > = 0.31 \pm 0.03$$

$$q^2/m_b^2 = \ 0.5 \ \Rightarrow < r > = 0.27 \pm 0.03$$

$\star$ Standard Estimation of the ratio $P/T \ \approx \ 30\%$ (Buras et al..)

Final Results

channel	$\frac{q^2}{m_b^2}$	BSW	GH
$\bar{B}^0 \to \bar{K}^{*0}\rho^0(\omega)$	0.3	2.1	1.0
	0.5	1.5	0.73
$B^- \to K^{*-}\rho^0(\omega)$	0.3	6.6	3.9
	0.5	6.2	3.6
$B^- \to \rho^-\rho^0(\omega)$	0.3	24	13
	0.5	24	14

Table 1: $\bar{B}^0, B^-$ branching ratios (in units of 10^{-6}) using either the BSW or GH form factor models, for $q^2/m_b^2 = 0.3(0.5)$, with $N_{cmax}^{b\to s} = 2.84(2.82)$, $N_{cmax}^{b\to d} = 2.01(1.95)$, $\rho = 0.229$ and $\eta = 0.325$.

channel	$\frac{q^2}{m_b^2}$	BSW	GH
$B^0 \longrightarrow K^{*0}\rho^0(\omega)$	0.3	2.1	1.0
	0.5	1.7	0.83
$B^+ \longrightarrow K^{*+}\rho^0(\omega)$	0.3	5.8	3.4
	0.5	3.8	2.3
$B^+ \longrightarrow \rho^+\rho^0(\omega)$	0.3	20	11
	0.5	20	11

Table 2: B^0, B^+ branching ratios (in units of 10^{-6}) using either the BSW or GH form factor models, for $q^2/m_b^2 = 0.3(0.5)$, with $N_{cmax}^{b\to s} = 2.84(2.82)$, $N_{cmax}^{b\to d} = 2.01(1.95)$, $\rho = 0.229$ and $\eta = 0.325$.

Comparison with recent Experimental Results

First Results of CDF (Phys.Rev.Let.85, 4668)

$B \to J/\psi K^*$ $\Longrightarrow$ Longitudinal Decay Amplitude dominates :

$$|H_0|^2 = 0.59 \pm 0.06 \pm 0.01$$

Belle Collaboration

$$\mathrm{Br}(B^+ \to \rho^+ \rho^0) = \Big(31.7 \pm 7.1(stat.)^{+3.8}_{-6.7}(sys.)\Big) \times 10^{-6}$$

Longitudinal Polarization

$$\Gamma_L/\Gamma \ = \ (94.8 \pm 10.6(stat.) \pm 2.1(syst.))\%$$

$\star$ Computed Value of Longitudinal Polarization : $h_{00} \ \approx 90\%$

Babar Collaboration (hep-ex/0307026)

Channel	Br($\times 10^{-6}$)	$f_L = \lvert H_0 \rvert^2$	A_{CP}
$\rho^0 K^{*+}$	$10.6^{+3.0}_{-2.6} \pm 2.4$	$0.96^{+0.04}_{-0.15} \pm 0.04$	$0.20^{+0.32}_{-0.29} \pm 0.04$
Our results	$2.3 \rightarrow 5.8$	87%	$-6.4\% \rightarrow -22\%$
$\rho^0 \rho^+$	$22.5^{+5.7}_{-5.4} \pm 5.8$	$0.97^{+0.03}_{-0.07} \pm 0.04$	$-0.19 \pm 0.23 \pm 0.03$
Our results	$11.0 \rightarrow 20.0$	90%	$-8.5\% \rightarrow -10\%$

Perspectives and Conclusion

• Helicity Formalism : Very Powerful Tool

$\Longrightarrow$ Rigorous and Detailed Calculations carried out completely
$\Longrightarrow$ Applying the formalism to any $B \to VV$ channel
$\Longrightarrow$ Implemented in the LHCb generator code

• Naive Factorization always very useful in Weak Hadronic B decays.
BUT... It involves large Uncertainties.

• Longitudinal Polarization is largely dominating, whatever the form factor model is.

• Importance of the Form Factor Model, especially for the estimation of the different Branching Ratios.
$\Longrightarrow$ B.R. could vary up to a factor 2

• Importance of the $\rho^0(\omega)$ Mixing in the enhancement of Direct CPV with regard to $\pi^+\pi^-$ Invariant Mass.

$\Downarrow$

$\star$ New Way to look for CP Violation

$\star$ New Methods (in the future) for measuring Angles γ and α

$\star$ Performing Generators requires good agreement between Theorists and Experimentalists

$\Longrightarrow$ Very Good Success

$\star$ Waiting with patience for LHC and for Experimental Data

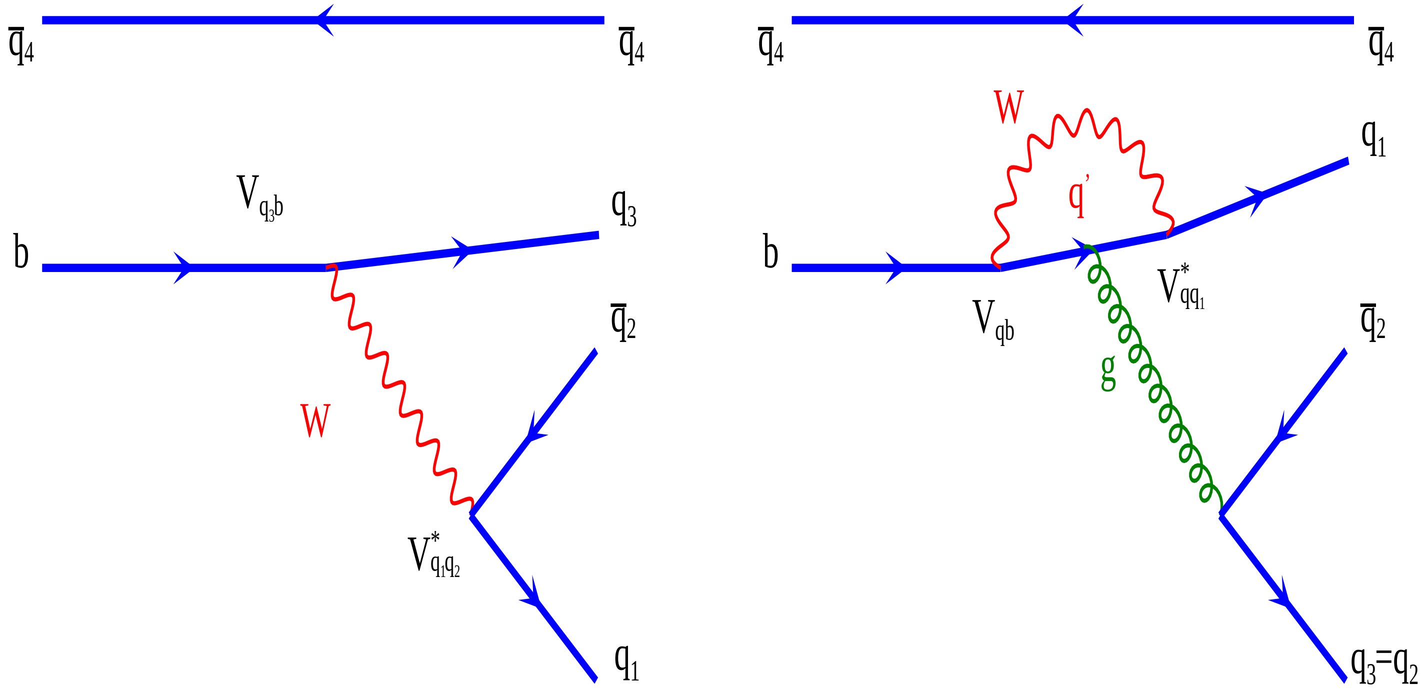


Figure 1: Tree diagram (left), and QCD-penguin diagram (right), for B decays.

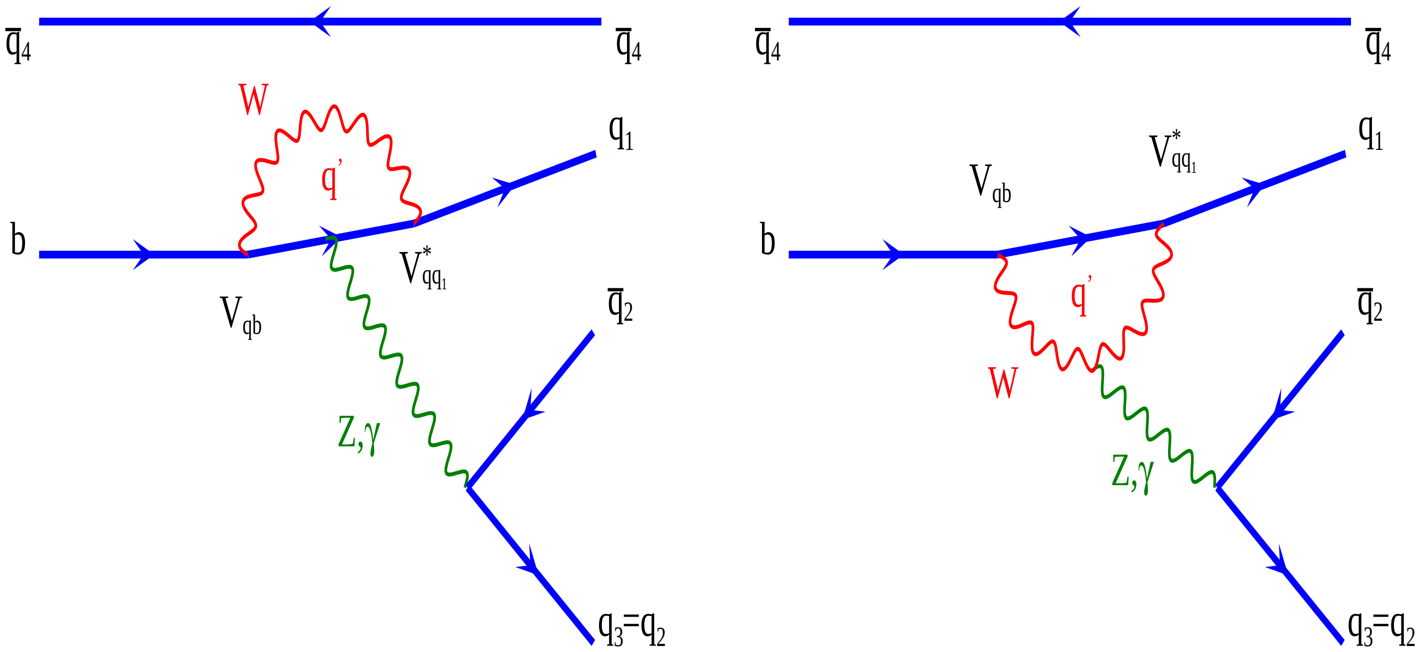
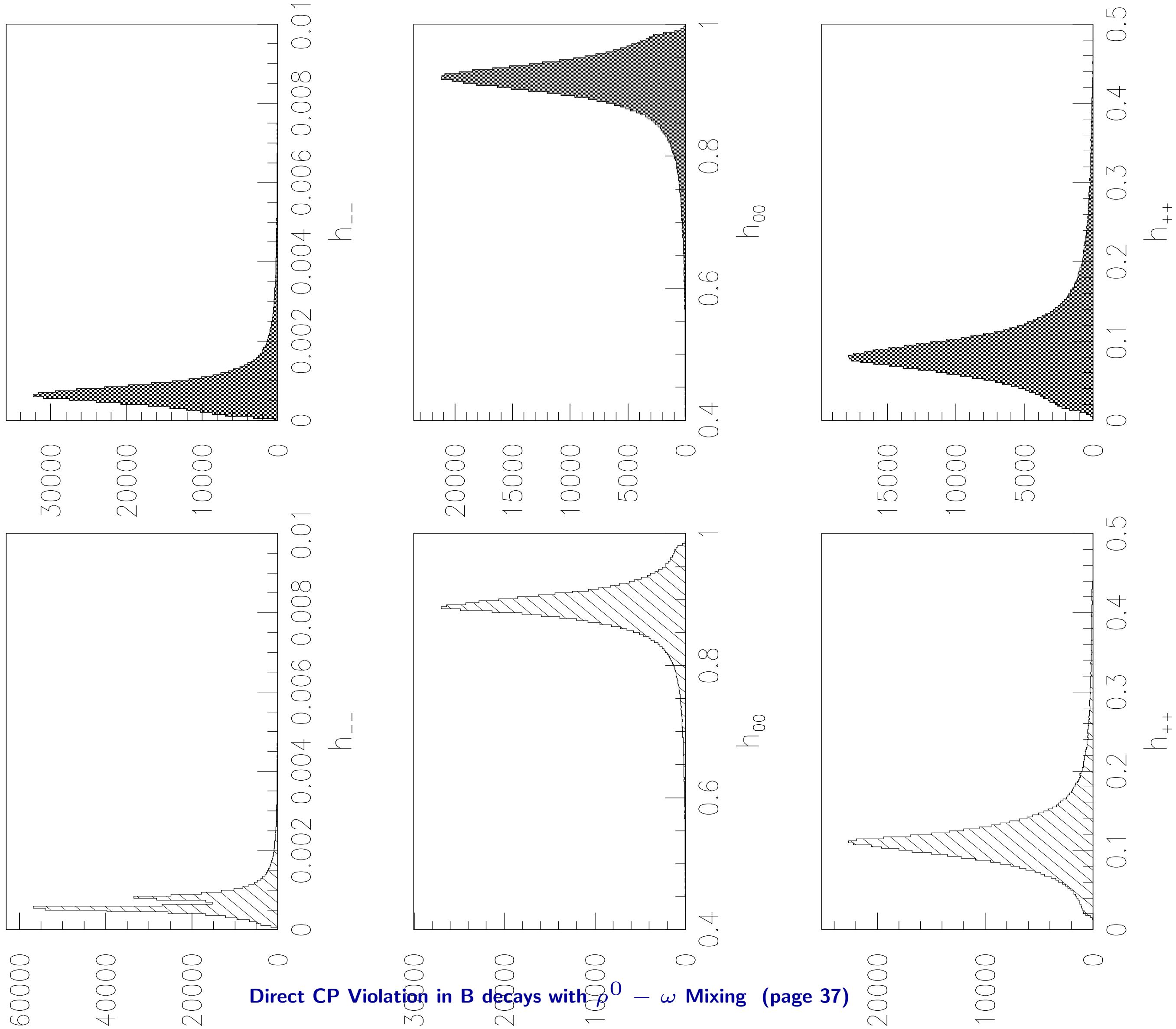
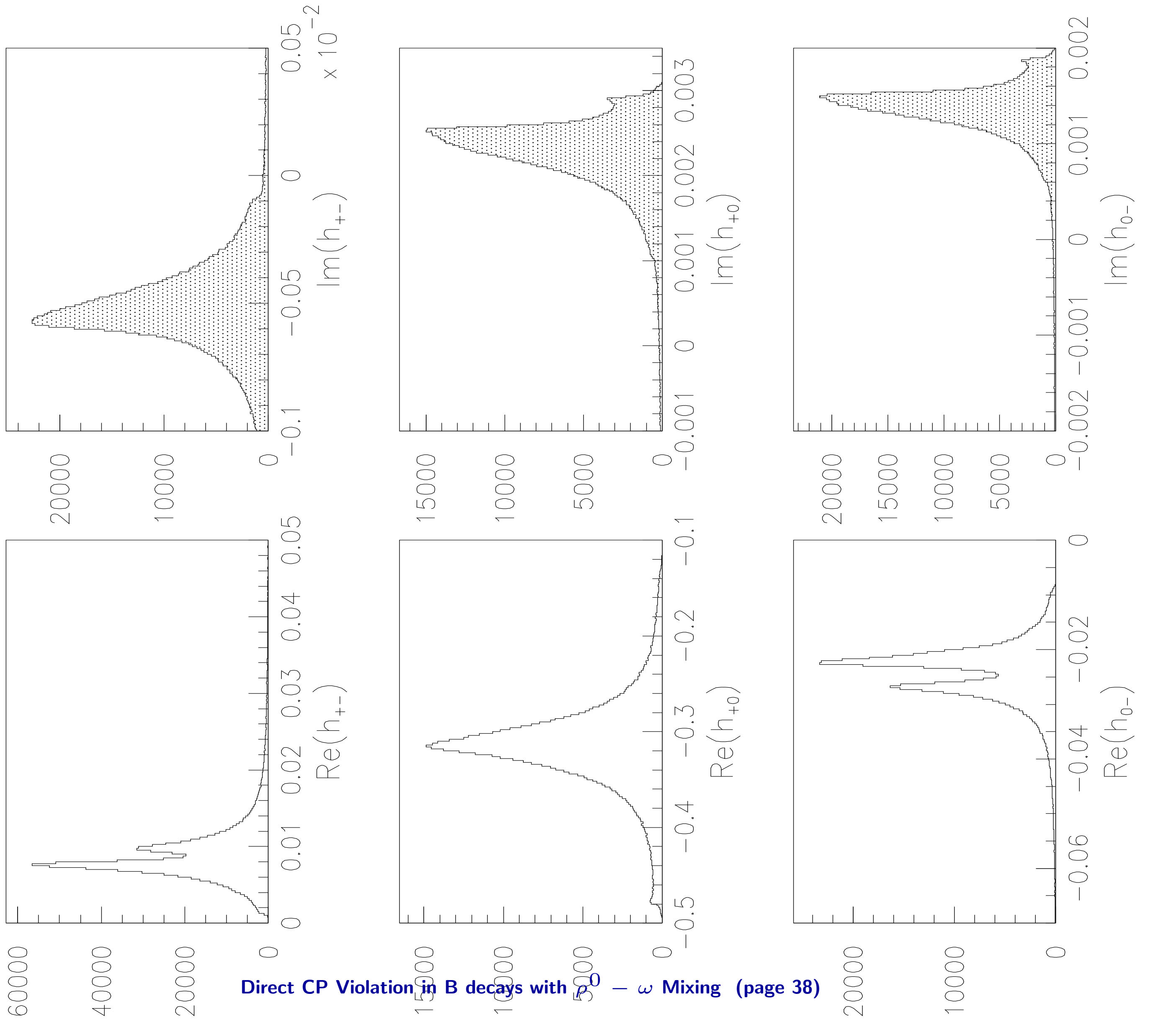
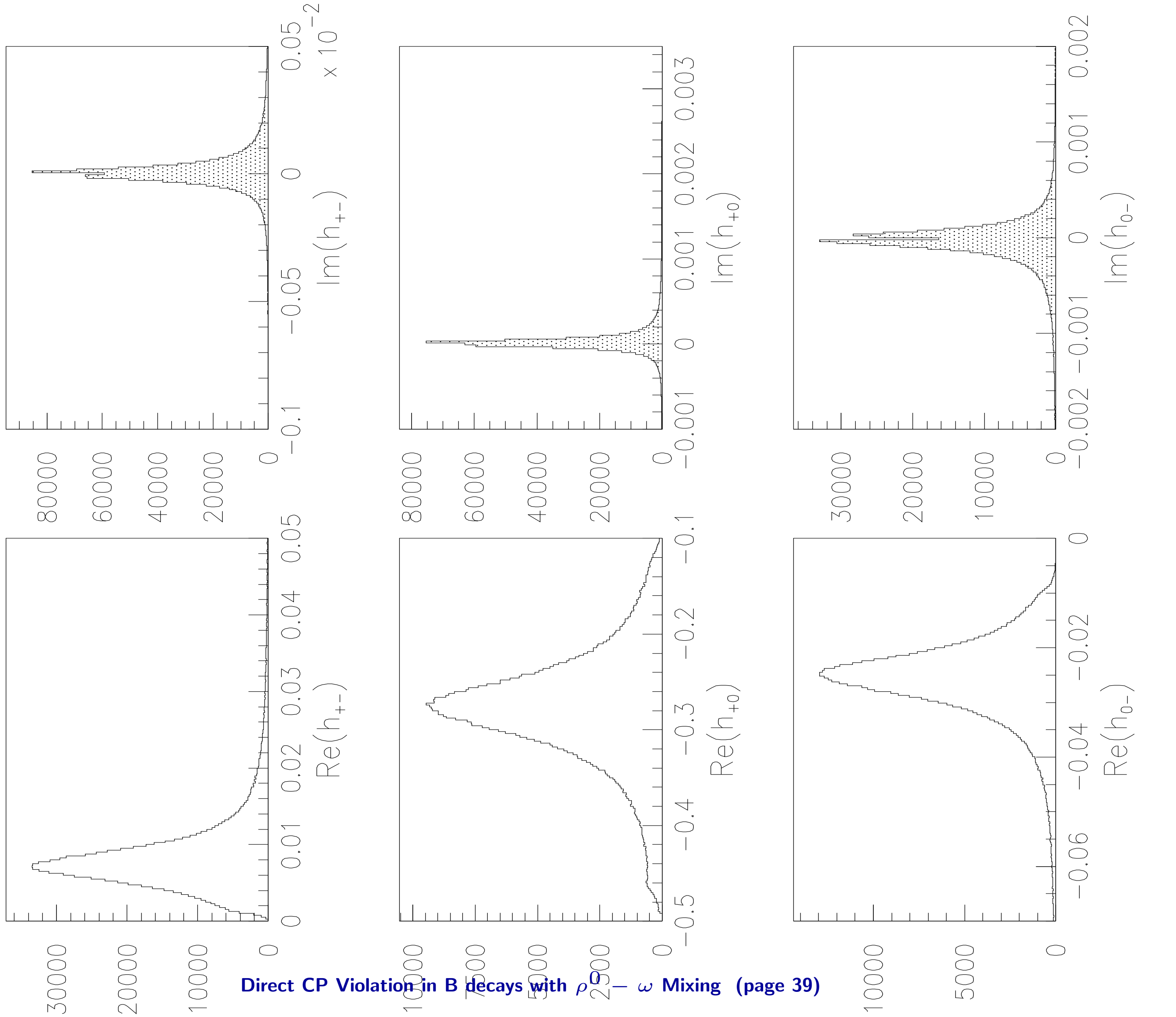
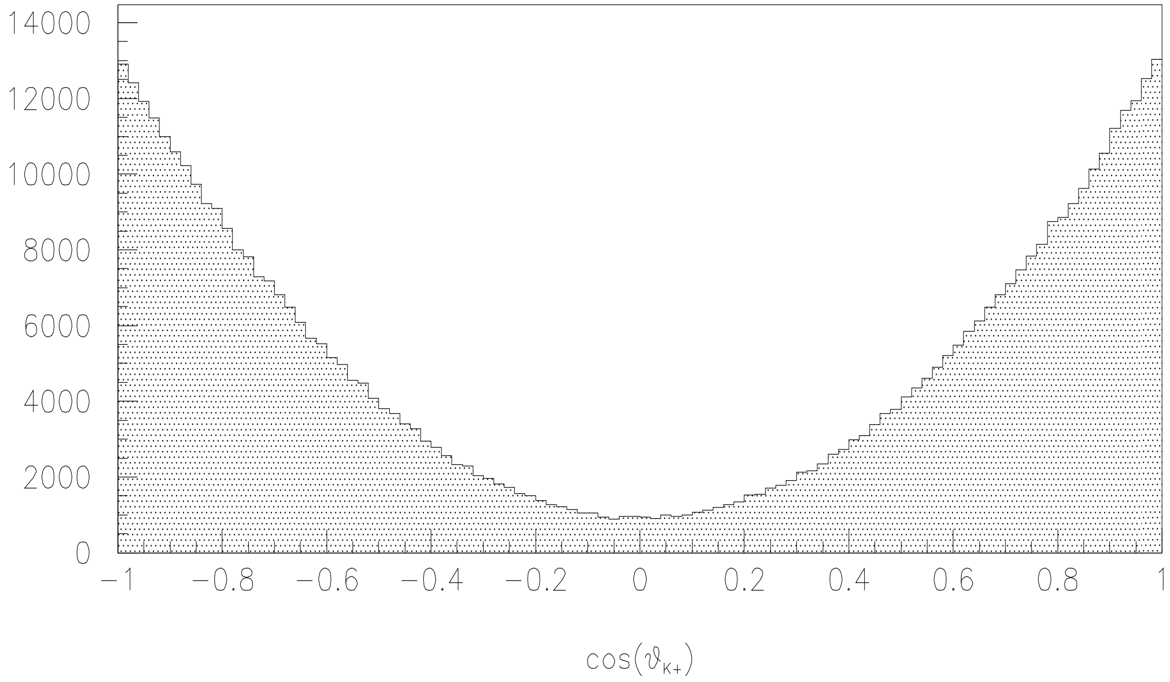


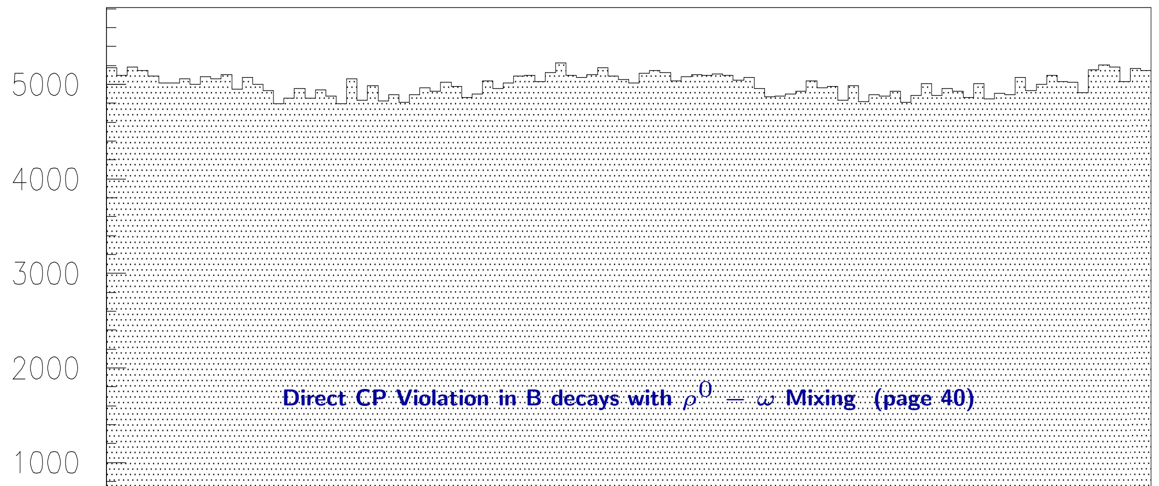
Figure 2: Electroweak-penguin diagram (left), and electroweak-penguin diagram with coupling between Z, γ and W (right), for B decays.

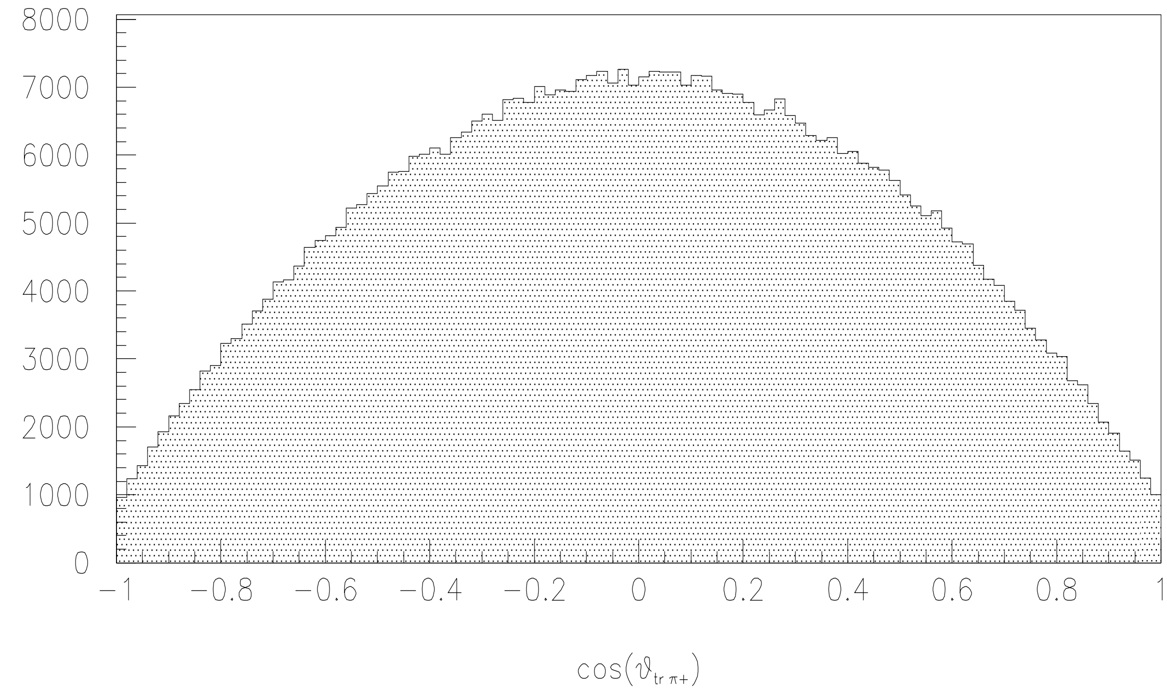

h_{--}
h_{00}
h_{++}

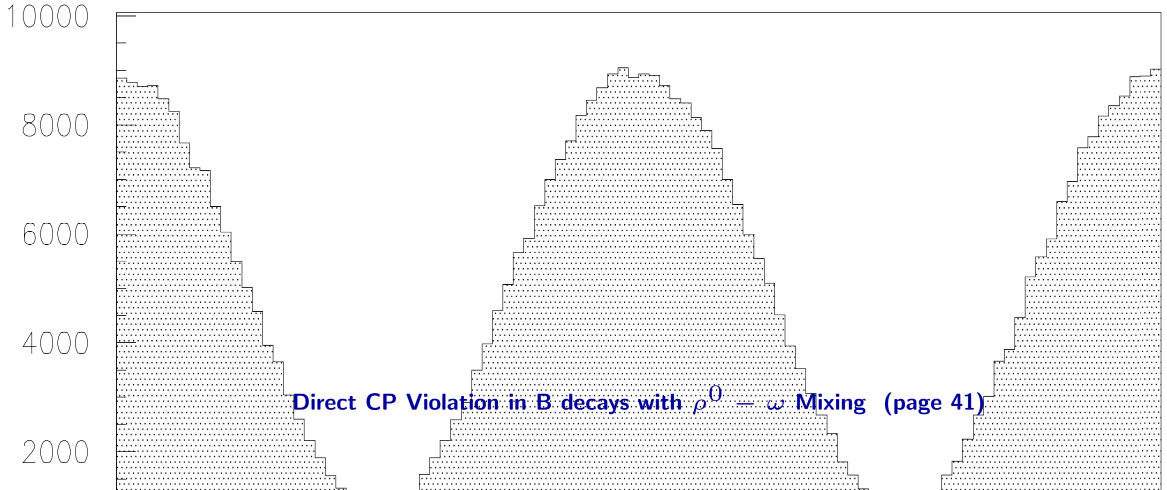

Re(h+-)
Im(h+-)
x 10-2
Re(h+0)
Im(h+0)
Re(h0-)
Im(h0-)

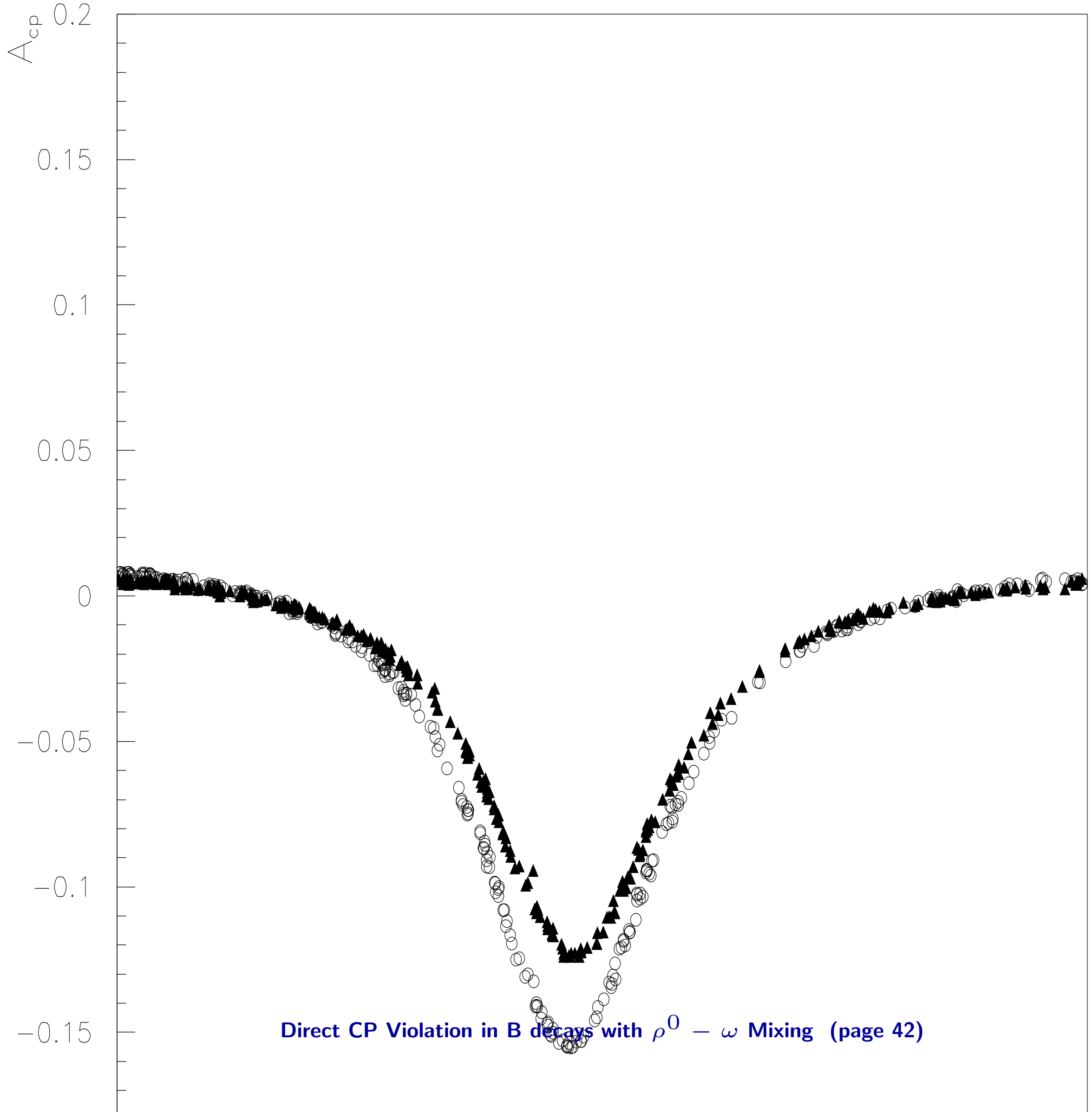

Re(h+-)
Im(h+-)
x 10^-2
Re(h+0)
Im(h+0)
Re(h0-)
Im(h0-)

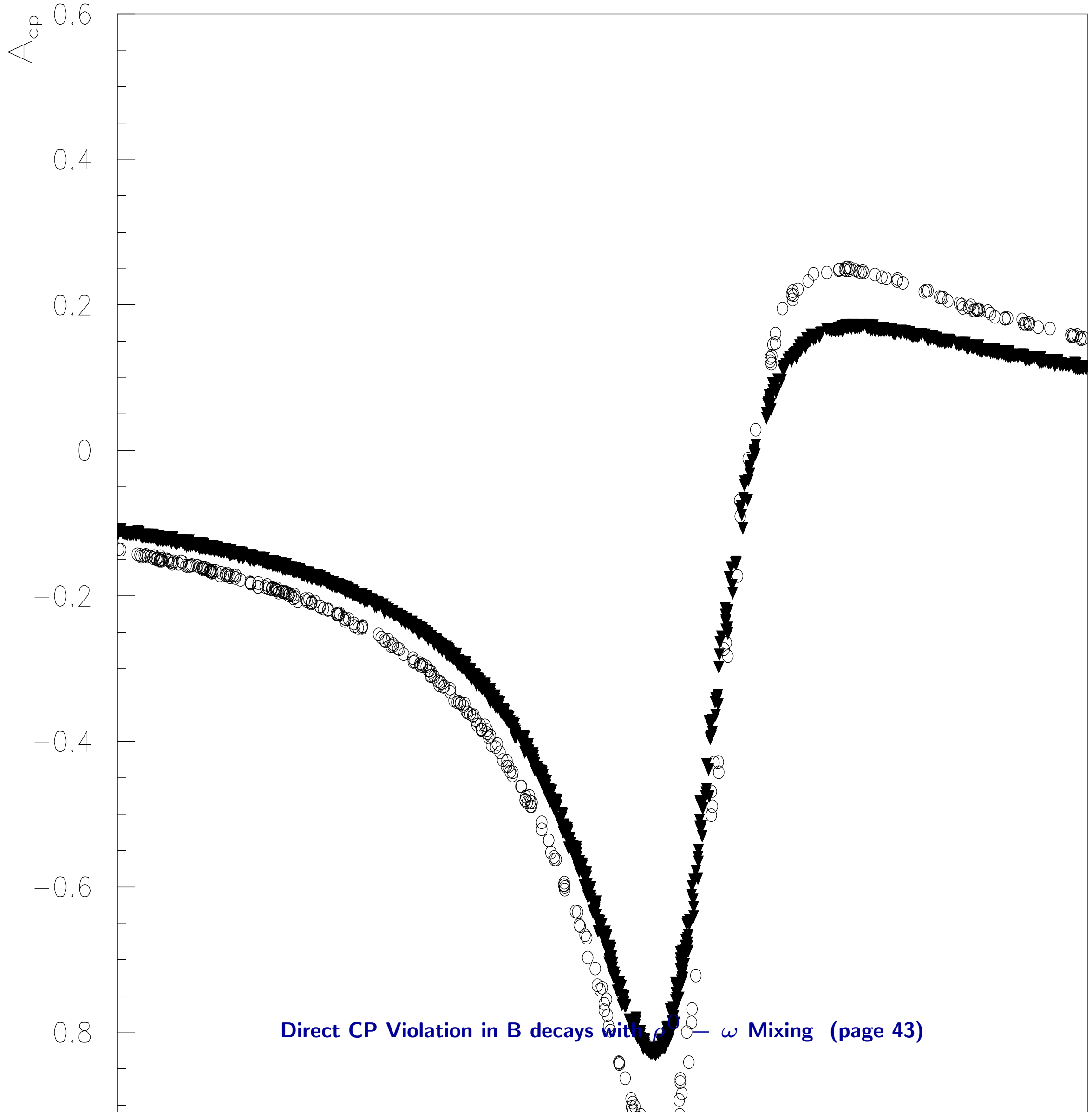

14000
12000
10000
8000
6000
4000
2000
0
−1
−0.8
−0.6
−0.4
−0.2
0
0.2
0.4
0.6
0.8
1
cos(ϑ_{K+})


5000
4000
3000
2000
1000


8000
7000
6000
5000
4000
3000
2000
1000
0
−1
−0.8
−0.6
−0.4
−0.2
0
0.2
0.4
0.6
0.8
1
cos($\vartheta_{tr\ \pi+}$)


10000
8000
6000
4000
2000


A_{cp}
0.2
0.15
0.1
0.05
0
−0.05
−0.1
−0.15



A_{cp}
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8