Determination of $\Delta\Gamma_s$ from Untagged Analysis of Decay $B_s^0 \to J/\psi\,\phi$ by Using the Method of Angular Moments

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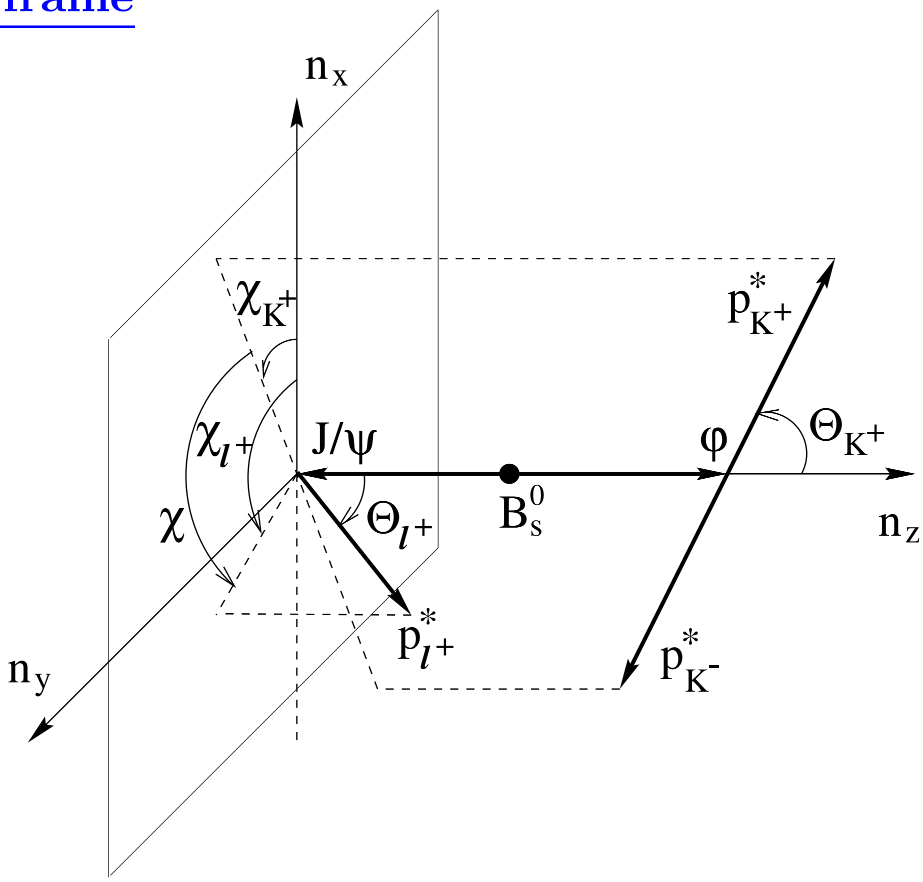
Decays $B_s^0, \overline{B}_s^0 \to J/\psi(\to l^+l^-)\,\phi(\to K^+K^-)$

- **Helicity frame**



- **Angular distribution**:

$$f(\Theta_{l^+},\Theta_{K^+},\chi;t) \equiv \frac{d^4N}{d\cos\Theta_{l^+}\,d\cos\Theta_{K^+}\,d\chi dt} \propto \sum_{i=1}^{6} b_i(t) g_i(\Theta_{l^+},\Theta_{K^+},\chi)$$

- **Observables** (for B_s^0 decay) in terms of transversity amplitudes:

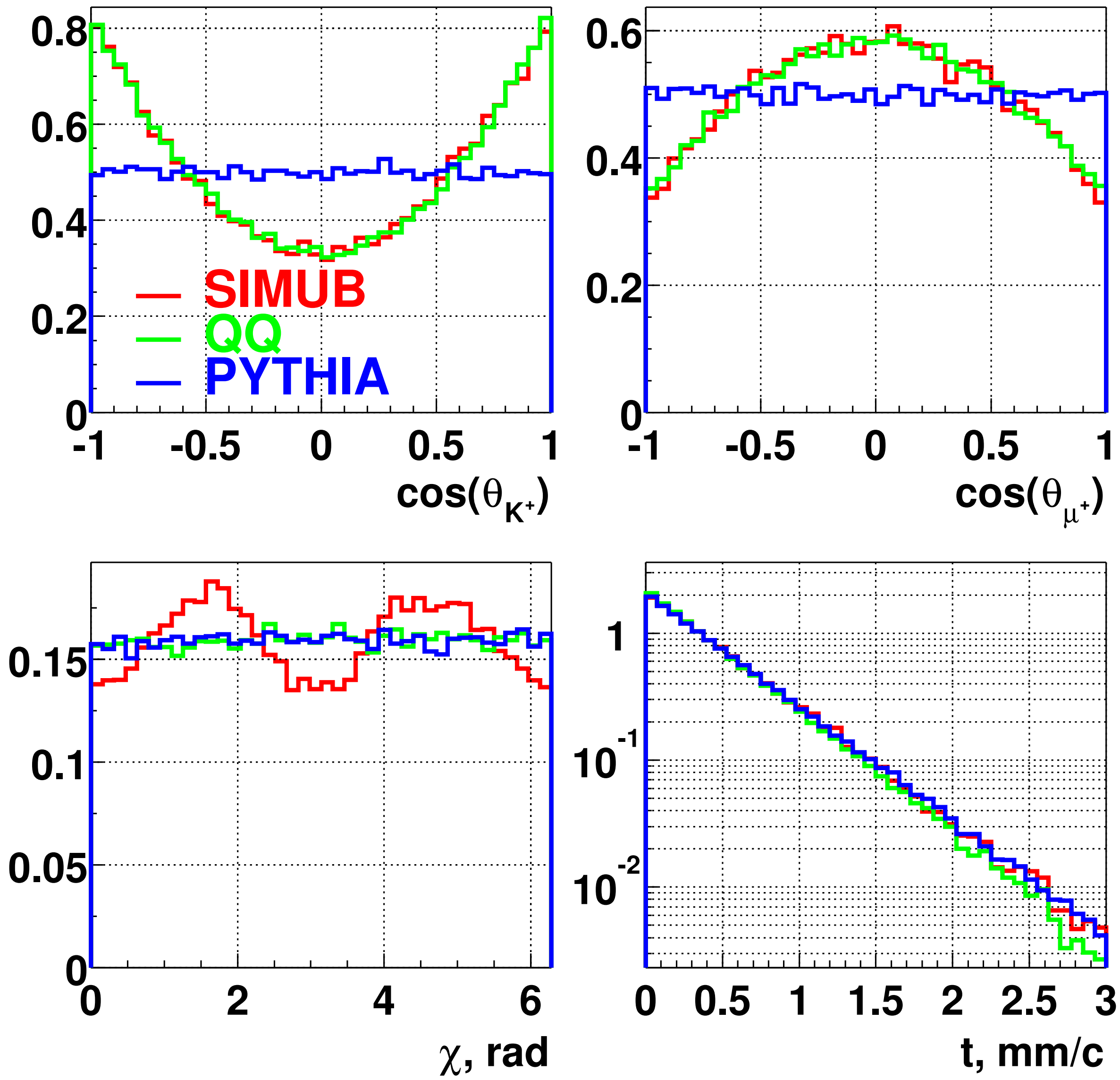
$$b_1 = |A_0(t)|^2\,, \quad b_2 = |A_{||}(t)|^2\,, \quad b_3 = |A_\perp(t)|^2\,,$$
$$b_4 = \mathrm{Im}(A_{||}^*(t)A_\perp(t))\,, \quad b_5 = \mathrm{Re}(A_0^*(t)A_{||}(t))\,, \quad b_6 = \mathrm{Im}(A_0^*(t)A_\perp(t))$$

- **Angular functions**:

$$g_1 = 2\cos^2\Theta_{K^+}\,\sin^2\Theta_{l^+}\,, \quad g_2 = \sin^2\Theta_{K^+}\,(1-\sin^2\Theta_{l^+}\,\cos^2\chi)\,,$$
$$g_3 = \sin^2\Theta_{K^+}\,(1-\sin^2\Theta_{l^+}\,\sin^2\chi)\,, \quad g_4 = -\sin^2\Theta_{K^+}\,\sin^2\Theta_{l^+}\,\sin 2\chi\,,$$
$$g_5 = \frac{1}{\sqrt{2}}\sin 2\Theta_{K^+}\,\sin 2\Theta_{l^+}\,\cos\chi\,, \quad g_6 = \frac{1}{\sqrt{2}}\sin 2\Theta_{K^+}\,\sin 2\Theta_{l^+}\,\sin\chi$$

Spin-angular correlations

Angular distributions in the helicity frame for decays
$B_s^0, \bar{B}_s^0 \to J/\psi(\to \mu^+\mu^-)\,\phi(\to K^+K^-)$



PYTHIA — no angular correlations

QQ — no azimuthal correlation

SIMUB — correct description of angular correlations

Documentation on the package SIMUB:
http://cmsdoc.cern.ch/~shulga/SIMUB/SIMUB.html

Physics parameters

- $\alpha = 1$ – no CP violation in $B_s^0 - \overline{B}_s^0$-mixing

- $M_s \equiv (M_L + M_H)/2 = 5.3696 \pm 0.0024$ GeV

- $\Delta M_s \equiv M_H - M_L$
  $\Delta M_s > 10.6$ ps^{-1} at 95% CL;
  all values $\Delta M_s < 14.3$ ps^{-1} are excluded at 95% CL
  [Particle Data Group, 9.11.2001]

- $\Gamma_s = (\Gamma_L + \Gamma_H)/2 = 1/\tau\,, \quad \tau = 1.493 \pm 0.062$ ps

- $\Delta\Gamma_s/\Gamma_s = (\Gamma_H - \Gamma_L)/\Gamma_s \leq 0.33$
  [PDG, Eur. Phys. J., C15 (2000)]

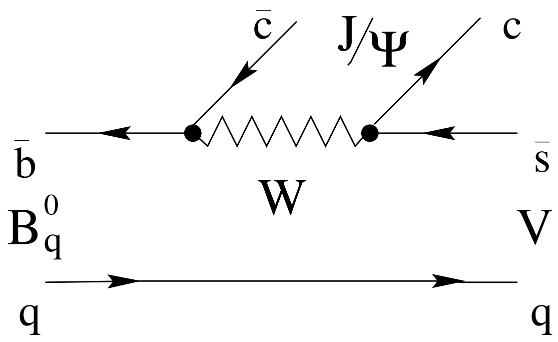
- $\phi_c^{(s)} \equiv 2[\arg(V_{ts}^* V_{tb}) - \arg(V_{cq}^* V_{cb})] = -2\delta\gamma$
  – CP-violating weak phase
  $\phi_c^{(s)} = -0.03 \pm 0.01$
  [Combined results based on the experimental constraints within Standard Model]

- $|A_0(0)|\,, \quad |A_{||}(0)|\,, \quad |A_\perp(0)|$
  – initial values of transversity amplitudes at $t = 0$

- $\delta_1 \equiv \arg[A_{||}^*(0) A_\perp(0)]\,, \quad \delta_2 \equiv \arg[A_0^*(0) A_\perp(0)]$
  – CP-conserving strong phases

Transversity amplitudes

- **Theory** (Effective Hamiltonians + factorization) for $B_s^0, \bar{B}_s^0 \to J/\psi\,\phi$ ($B_d^0, \bar{B}_d^0 \to J/\psi\, K^*$)



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Parameter	$\lvert A_0(0)\rvert^2$	$\lvert A_\perp(0)\rvert^2$	$\arg(A_\perp)$	$\arg(A_{\vert\vert})$
BSW	0.55(0.57)	0.09(0.09)	π	0
Soares	0.41(0.42)	0.32(0.32)	π	0
Cheng	0.54(0.56)	0.16(0.16)	π	0

Normalization: $|A_0|^2 + |A_{||}|^2 + |A_\perp|^2 = 1\,, \quad arg(A_0) = 0$

- **Summary of polarization measurements**

$B_d^0, \bar{B}_d^0 \to J/\psi(\to l^+l^-)\, K^*(\to K\pi)$

Exp	Stat	$\lvert A_0\rvert^2$	$\lvert A_\perp\rvert^2$	$\arg(A_\perp)$	$\arg(A_{\vert\vert})$
CLEO	80	0.52 ± 0.08	0.16 ± 0.09	3.00 ± 0.37	-0.11 ± 0.46
CDF	193	0.59 ± 0.06	$0.13^{+0.13}_{-0.11}$	2.2 ± 0.5	-0.6 ± 0.5
BaBar	940	0.60 ± 0.04	0.16 ± 0.03	2.50 ± 0.22	-0.17 ± 0.17
Belle	1471	0.62 ± 0.04	0.19 ± 0.04	2.83 ± 0.20	0.09 ± 0.14

$B_s^0, \bar{B}_s^0 \to J/\psi(\to l^+l^-)\, \phi(\to K^+K^-)$

Exp	Stat	$\lvert A_0\rvert^2$	$\lvert A_\perp\rvert^2$	$\arg(A_\perp)$	$\arg(A_{\vert\vert})$
CDF	40	0.61 ± 0.14	0.23 ± 0.19	1.1 ± 1.3	

Experimental data confirm the Cheng's model

Angular–moments method

- **Angular distribution:**

$$f(\Theta_{l^+}, \Theta_{K^+}, \chi; t) \propto \sum_{i=1}^{6} b_i(t)\, g_i(\Theta_{l^+}, \Theta_{K^+}, \chi)$$

- **Weighting functions:**

$$b_i(t) = \int f(\Theta_{l^+}, \Theta_{K^+}, \chi; t)\, w_i(\Theta_{l^+}, \Theta_{K^+}, \chi) [D\Theta],$$

where $[D\Theta] \equiv d\cos\Theta_{l^+} d\cos\Theta_{K^+} d\chi$

- **Standard method of the weighting functions calculation:**

$$w_i(\Theta_{l^+}, \Theta_{K^+}, \chi) = \sum_{j=1}^{6} \lambda_{ij}\, g_j(\Theta_{l^+}, \Theta_{K^+}, \chi),$$

where λ_{ij} are 36 coefficients defined as the solution of 36 linear equations:

$$\sum_{j=1}^{6} \lambda_{ij} \int g_j\, g_k [D\Theta] = \delta_{ik}$$

- **The experimentally measured observables:**

$$b_i^{(exp)}(t_0) = \frac{1}{N} \sum_{j=1}^{N} w_i(\Theta_{l^+}^j, \Theta_{K^+}^j, \chi^j)$$

$$\delta b_i^{(exp)}(t_0) = \frac{1}{N} \sqrt{\sum_{j=1}^{N} \left(b_i^{(exp)}(t_0) - w_i(\Theta_{l^+}^j, \Theta_{K^+}^j, \chi^j) \right)^2}$$

summation over events in the time bin arround $t = t_0$

Weighting function

- **Set A:**

$$w_1 = 2 - 5\cos^2\Theta_{l+}\,, \qquad w_2 = 2 - 5\sin^2\Theta_{l+}\cos^2\chi\,,$$

$$w_3 = 2 - 5\sin^2\Theta_{l+}\sin^2\chi\,, \qquad w_4 = -\frac{5}{2}\sin^2\Theta_{l+}\sin 2\chi\,,$$

$$w_5 = \frac{25}{4\sqrt{2}}\sin 2\Theta_{K+}\sin 2\Theta_{l+}\cos\chi\,,$$

$$w_6 = \frac{25}{4\sqrt{2}}\sin 2\Theta_{K+}\sin 2\Theta_{l+}\sin\chi$$

see [A.S. Dighe et al., Eur. Phys. J., **C6** (1999) p. 647]

Transversity Frame $\Longrightarrow$ Helicity Frame

- **Set B:**

$$\begin{aligned}
w_1 &= \frac{1}{12}[28\cos^2\Theta_{K+}\sin^2\Theta_{l+} - 3\sin^2\Theta_{K+}(1+\cos^2\Theta_{l+})] \\
w_2 &= -\frac{1}{8}[4\cos^2\Theta_{K+}\sin^2\Theta_{l+} - 29\sin^2\Theta_{K+}(1-\sin^2\Theta_{l+}\cos^2\chi) \\
&\quad +21\sin^2\Theta_{K+}(1-\sin^2\Theta_{l+}\sin^2\chi)] \\
w_3 &= -\frac{1}{8}[4\cos^2\Theta_{K+}\sin^2\Theta_{l+} + 21\sin^2\Theta_{K+}(1-\sin^2\Theta_{l+}\cos^2\chi) \\
&\quad -29\sin^2\Theta_{K+}(1-\sin^2\Theta_{l+}\sin^2\chi)] \\
w_4 &= -\frac{25}{8}\sin^2\Theta_{K+}\sin^2\Theta_{l+}\sin 2\chi \\
w_5 &= \frac{25}{4\sqrt{2}}\sin 2\Theta_{K+}\sin 2\Theta_{l+}\cos\chi \\
w_6 &= \frac{25}{4\sqrt{2}}\sin 2\Theta_{K+}\sin 2\Theta_{l+}\sin\chi
\end{aligned}$$

obtained from the system of linear equations

$$\sum_{j=1}^{6}\lambda_{ij}\int g_j\,g_k[D\Theta] = \delta_{ik}.$$

Time-integrated observables

- **Time dipendence of observables:**

$$b_1(t) = |A_0(0)|^2\, G_L(t)$$
$$b_2(t) = |A_{||}(0)|^2\, G_L(t)$$
$$b_3(t) = |A_\perp(0)|^2\, G_H(t)$$
$$b_4(t) = |A_{||}(0)|\,|A_\perp(0)|\cos\delta_1 \sin\phi_c^{(s)}\, Z(t)$$
$$b_5(t) = |A_0(0)|\,|A_{||}(0)|\cos(\delta_2 - \delta_1)\, G_L(t)$$
$$b_6(t) = |A_0(0)|\,|A_\perp(0)|\cos\delta_2 \sin\phi_c^{(s)}\, Z(t)$$

For **untagged sample** with equal initial numbers of B_s^0 and $\overline{B}_s^0$

$$G_{L/H}(t) = \frac{1}{2}\Big[(1 \pm \cos\phi_c^{(s)})\mathrm{e}^{-\Gamma_L t} + (1 \mp \cos\phi_c^{(s)})\mathrm{e}^{-\Gamma_H t}\Big]$$
$$Z(t) = \frac{1}{2}\Big(\mathrm{e}^{-\Gamma_H t} - \mathrm{e}^{-\Gamma_L t}\Big)$$

- **Time-integrated observables** $\tilde{b}_i(T_0)$ and $\hat{b}_i(T_0)$:

$$\tilde{b}_i(T_0) = \frac{1}{\widetilde{L}(T)} \int_0^{T_0} dt \int [D\Theta]\, w_i(\Theta_{l^+}, \Theta_{K^+}, \chi)\; f(\Theta_{l^+}, \Theta_{K^+}, \chi; t)$$

and

$$\hat{b}_i(T_0) = \frac{1}{\widetilde{L}(T)} \int_0^{T_0} dt \int [D\Theta]\, w_i(\Theta_{l^+}, \Theta_{K^+}, \chi)\; \mathrm{e}^{\Gamma' t}\, f(\Theta_{l^+}, \Theta_{K^+}, \chi; t)$$

where

$$\widetilde{L}(T) \equiv \int_0^T dt\, L(t)$$

$$L(T) \equiv \int [D\Theta]\, f(\cos\Theta_{l^+}, \cos\Theta_{K^+}, \chi; t) = b_1(t) + b_2(t) + b_3(t)$$

and Γ' is some **arbitrary initial approximation** of the B_s^0-meson total decay width

- **Analytical expressions** for $\hat{b}_i(T_0)$:

$$\hat{b}_1(T_0) = |A_0(0)|^2\, \hat{G}_L(T_0)/\widetilde{L}(T)$$
$$\hat{b}_2(T_0) = |A_{||}(0)|^2\, \hat{G}_L(T_0)/\widetilde{L}(T)$$
$$\hat{b}_3(T_0) = |A_\perp(0)|^2\, \hat{G}_H(T_0)/\widetilde{L}(T)$$
$$\hat{b}_4(T_0) = |A_{||}(0)|\,|A_\perp(0)|\cos\delta_1 \sin\phi_c^{(s)}\, \hat{Z}(T_0)/\widetilde{L}(T)$$
$$\hat{b}_5(T_0) = |A_0(0)|\,|A_{||}(0)|\cos(\delta_2-\delta_1)\, \hat{G}_L(T_0)/\widetilde{L}(T)$$
$$\hat{b}_6(T_0) = |A_0(0)|\,|A_\perp(0)|\cos\delta_2 \sin\phi_c^{(s)}\, \hat{Z}(T_0)/\widetilde{L}(T)$$

with

$$\widetilde{L}(T) = (|A_0(0)|^2 + |A_{||}(0)|^2)\hat{G}_L(T) + |A_\perp(0)|^2\hat{G}_H(T)$$

$$\hat{G}_{L/H}(T_0) = (1\pm\cos\phi_c^{(s)})\frac{\mathrm{e}^{\Delta\Gamma_L T_0/2}-1}{\Delta\Gamma_L} - (1\mp\cos\phi_c^{(s)})\frac{\mathrm{e}^{-\Delta\Gamma_H T_0/2}-1}{\Delta\Gamma_H}$$

$$\hat{Z}(T_0) = \frac{1-\mathrm{e}^{\Delta\Gamma_L T_0/2}}{\Delta\Gamma_L} + \frac{1-\mathrm{e}^{-\Delta\Gamma_H T_0/2}}{\Delta\Gamma_H}$$

where $\Delta\Gamma_{L/H} = \pm 2(\Gamma' - \Gamma_{L/H})$

- **Transversity amplitudes at $t=0$ and strong phases:**

$$|A_0(0)|^2 = \frac{\hat{b}_1(T)}{\hat{b}_1(T)+\hat{b}_2(T)+\hat{b}_3(T)/\hat{\gamma}(T)}$$

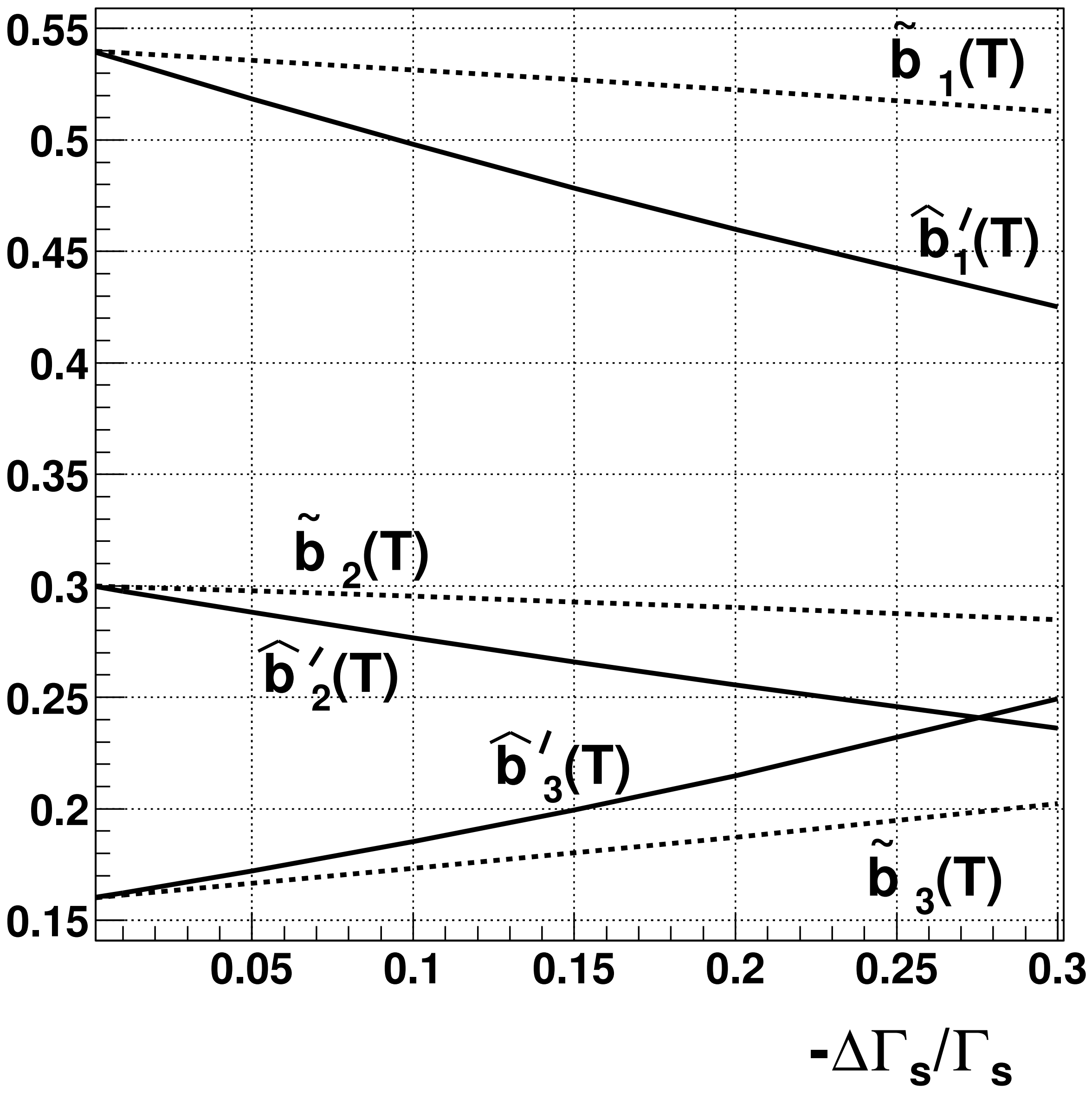
$$|A_{||}(0)|^2 = \frac{\hat{b}_2(T)}{\hat{b}_1(T)+\hat{b}_2(T)+\hat{b}_3(T)/\hat{\gamma}(T)}$$

$$|A_\perp(0)|^2 = \frac{\hat{b}_3(T)/\hat{\gamma}(T)}{\hat{b}_1(T)+\hat{b}_2(T)+\hat{b}_3(T)/\hat{\gamma}(T)}$$

$$\cos(\delta_2-\delta_1) = \frac{\hat{b}_5(T)}{\sqrt{\hat{b}_1(T)\,\hat{b}_2(T)}}$$

where $\tilde{\gamma}(T) \equiv \tilde{G}_H(T)/\tilde{G}_L(T)$

• $\Delta\Gamma_s$-dependence of the time-integrated observables:



Dependence of $\tilde{b}_i(T)$ and $\hat{b}'_i(T) \equiv \hat{b}_i(T)(1 - \mathrm{e}^{-\Gamma_s T})/\Gamma_s T$ on the values of $\Delta\Gamma_s/\Gamma_s$

$$\tilde{b}_{1,2,3}(T)|_{\Delta\Gamma_s=0} = \hat{b}'_{1,2,3}(T)|_{\Delta\Gamma_s=0} = |A_{0,||,\perp}(0)|^2$$

Monte Carlo studies

- **Physics parameters** are fixed in the SIMUB as follows:

$$\delta_1 = \pi\,,\ \ \delta_2 = 0\,,\quad \Gamma_s = 1/\tau_s = 2.278\ [\mathrm{mm/c}]^{-1}\,,\quad \phi_c^{(s)} = 0.04$$

$$\Delta\Gamma_s/\Gamma_s \in [-0.3, -0.01]\,,\quad t \in [0, T = 2\mathrm{mm}/c]$$

$$\text{Experiment:}\ \ |\Delta\Gamma_s|/\Gamma_s < 0.31\ \ \text{at 95\% CL}$$

- **Measured values**of the time integrated observables:

$$\hat{b}_i^{(exp)}(T_0) = \frac{1}{N(T)} \sum_{j=1}^{N(T_0)} W_i^j\,,\quad W_i^j \equiv \mathrm{e}^{\Gamma' t^j} w_i^j$$

- **Statistical error**:

$$(\delta\tilde{b}_i)^{(stat)} = \frac{1}{N(T)} \sqrt{\sum_{j=1}^{N(T_0)} (\tilde{b}_i^{(exp)} - W_i^j)^2}\,,$$

- **Random generator resolution**: For each randomly generated variable $V \in \{\cos\Theta_{l+}, \cos\Theta_{K+}, \chi, t\}$ the number of bins in the region $[V_{min}, V_{max}]$ was set as $N^{gen} = 50\,000$
$\Longrightarrow \Delta(V) = (V_{max} - V_{min})/N^{gen}$

- **Systematic error** coased by limited generator resolution:

$$(\delta\hat{b}_i)^{(sys)} = \sqrt{\frac{\sum_{j=1}^{N(T_0)} \Delta_i^j}{N(T)}}$$

$$\Delta_i^j = \left[\frac{\partial W_i^j}{\partial \cos\Theta_{l+}} \Delta(\cos\Theta_{l+})\right]^2 + \left[\frac{\partial W_i^j}{\partial \cos\Theta_{K+}} \Delta(\cos\Theta_{K+})\right]^2 + \left[\frac{\partial W_i^j}{\partial \chi} \Delta(\chi)\right]^2 + \left[\frac{\partial W_i^j}{\partial t} \Delta(t)\right]^2$$

- **Extraction of observables $\tilde{b}_i$:** from Monte Carlo sample of 100 000 decay events generated with $\Delta\Gamma_s/\Gamma_s = -0.15$. The first errors are statistical while the second errors correspond to the systematic uncertainties (due to limited generator resolution)

a) BSW model:

i	$\tilde{b}_i^{(theor)}(T)$	$\tilde{b}_i^{(exp)}(T)$ (set A)	$\tilde{b}_i^{(exp)}(T)$ (set B)
1	0.5425	$0.5409 \pm 0.0044 \pm 0.0003$	$0.5432 \pm 0.0024 \pm 0.0002$
2	0.3551	$0.3619 \pm 0.0047 \pm 0.0004$	$0.3579 \pm 0.0036 \pm 0.0004$
3	0.1024	$0.0972 \pm 0.0049 \pm 0.0004$	$0.0991 \pm 0.0034 \pm 0.0004$
5	-0.4389	$-0.4344 \pm 0.0050 \pm 0.0003$	$-0.4344 \pm 0.0050 \pm 0.0003$

b) Model by Soares:

i	$\tilde{b}_i^{(theor)}(T)$	$\tilde{b}_i^{(exp)}(T)$ (set A)	$\tilde{b}_i^{(exp)}(T)$ (set B)
1	0.3908	$0.3900 \pm 0.0046 \pm 0.0003$	$0.3955 \pm 0.0023 \pm 0.0002$
2	0.2574	$0.2617 \pm 0.0049 \pm 0.0004$	$0.2551 \pm 0.0037 \pm 0.0004$
3	0.3518	$0.3483 \pm 0.0047 \pm 0.0004$	$0.3509 \pm 0.0037 \pm 0.0004$
5	-0.3171	$-0.3156 \pm 0.0052 \pm 0.0003$	$-0.3156 \pm 0.0052 \pm 0.0003$

c) Model by Cheng:

i	$\tilde{b}_i^{(theor)}(T)$	$\tilde{b}_i^{(exp)}(T)$ (set A)	$\tilde{b}_i^{(exp)}(T)$ (set B)
1	0.5271	$0.5228 \pm 0.0045 \pm 0.0003$	$0.5267 \pm 0.0024 \pm 0.0001$
2	0.2928	$0.2980 \pm 0.0048 \pm 0.0004$	$0.2950 \pm 0.0036 \pm 0.0004$
3	0.1801	$0.1791 \pm 0.0048 \pm 0.0004$	$0.1778 \pm 0.0035 \pm 0.0004$
5	-0.3928	$-0.3927 \pm 0.0051 \pm 0.0003$	$-0.3927 \pm 0.0051 \pm 0.0003$

- **Determination of $\Delta\Gamma_s$:**

◇ Under assumption $\phi_c^{(s)} = 0$, we have

$$\hat{G}^{(0)}_{L/H}(T) = \pm 2\frac{e^{\pm\Delta\Gamma_{L/H}T/2} - 1}{\Delta\Gamma_{L/H}}, \qquad \Delta\Gamma_{L/H} = \pm 2(\Gamma' - \Gamma_{L/H})$$

$$\Downarrow$$

$$\boxed{\begin{array}{lll} \dfrac{\hat{b}_i(T)}{\hat{b}_i(T_0)} = \dfrac{\hat{G}_L(T)}{\hat{G}_L(T_0)} & (i = 1,2,5) & \text{for } \Delta\Gamma_L \text{ determination} \\ \dfrac{\hat{b}_3(T)}{\hat{b}_3(T_0)} = \dfrac{\hat{G}_H(T)}{\hat{G}_H(T_0)} & & \text{for } \Delta\Gamma_H \text{ determination} \end{array}}$$

◇ Then, the parameters Γ_s and $\Delta\Gamma_s$ can be determined via Γ' and $\Delta\Gamma_{L/H}$ as

$$\Gamma_s = \Gamma' - \frac{\Delta\Gamma_L - \Delta\Gamma_H}{4}, \qquad \Delta\Gamma_s = \frac{\Delta\Gamma_L + \Delta\Gamma_H}{2}$$

◇ Results of determination of the decay-width parameters (in units $(\mathrm{mm}/c)^{-1}$) from analysis of 100 000 Monte Carlo events. The input value of $\Delta\Gamma_s$ corresponds to $\Delta\Gamma_s/\Gamma_s = -0.15$, $\Gamma' = 1.05\Gamma_s$, $T_0 = 0.1\,T$.

Parameter	Input value	Measurement (set A)	Measurement (set B)
$\Delta\Gamma_L$	-0.1139	$-0.103 \pm 0.058 \pm 0.003$	$-0.110 \pm 0.034 \pm 0.002$
$\Delta\Gamma_H$	-0.5696	$-0.478 \pm 0.137 \pm 0.012$	$-0.554 \pm 0.101 \pm 0.012$
Γ_s	2.2784	$2.299 \pm 0.037 \pm 0.003$	$2.281 \pm 0.027 \pm 0.003$
$\Delta\Gamma_s$	-0.3418	$-0.290 \pm 0.074 \pm 0.006$	$-0.332 \pm 0.053 \pm 0.006$

◇ To improve the precision of $\Delta\Gamma_s$ determination, the same procedure should be repeated with Γ' fixed to be equal to the value of Γ_s determined at the first step. Because of $\Delta\Gamma_s = \Delta\Gamma_L = \Delta\Gamma_H$ in case of $\Gamma' = \Gamma_s$, the value of $\Delta\Gamma_s$ is defined at the second step as to be equal to the value of $\Delta\Gamma_L$ determined from the ratio $\hat{b}_1^{(exp)}(T)/\hat{b}_1^{(exp)}(T_0)$.

$$\Downarrow$$

$$\Delta\Gamma_s^{exp} = -0.330 \pm 0.057 \pm 0.004 \qquad \text{(set A)}$$
$$\Delta\Gamma_s^{exp} = -0.338 \pm 0.034 \pm 0.002 \qquad \text{(set B)}$$

to be compared with the input value $\Delta\Gamma_s = -0.3418$

◇ Statistical errors of $\Delta\Gamma_s/\Gamma_s$ determination by the angular-moments method with set-B (set-A) weighting functions from different statistics and for various values of $\Delta\Gamma_s$:

$\Delta\Gamma_s/\Gamma_s$	200 ev	500 ev	10^3 ev	10^4 ev	10^5 ev
-0.03	-	-	-	0.035 (-)	0.014 (0.023)
-0.05	-	-	-	0.046 (-)	0.014 (0.022)
-0.1	-	-	0.11 (-)	0.046 (0.079)	0.014 (0.024)
-0.15	-	-	0.13 (0.19)	0.045 (0.078)	0.014 (0.024)
-0.2	-	0.23 (-)	0.12 (0.18)	0.048 (0.072)	0.015 (0.026)
-0.3	0.21 (-)	0.23 (-)	0.18 (0.20)	0.050 (0.083)	0.016 (0.028)

The statistical error of the ration $\Delta\Gamma_s/\Gamma_s \in [-0.3, -0.03]$ is independent on the $\Delta\Gamma_s/\Gamma_s$ value and of about 0.015 for 10^5 events

Maximum likelihood fit gives the precision for $\Delta\Gamma_s = -0.15$ not better than 0.03 with statistics 10^5 events while with statistics 10^4 events the fit becomes unstable.

- **Determination of transversity amplitudes at $t = 0$:**
  - ◇ The total errors for parameters

$$|A_{f_i}(0)|^2 = \frac{\hat{b}_i(T)}{\hat{b}_1(T) + \hat{b}_2(T) + \hat{b}_3(T)/\hat{\gamma}(T)}\,, \quad f_i = 0, ||\,, \quad (i = 1, 2)$$

$$|A_\perp(0)|^2 = \frac{\hat{b}_3(T)/\hat{\gamma}(T)}{\hat{b}_1(T) + \hat{b}_2(T) + \hat{b}_3(T)/\hat{\gamma}(T)}$$

also should include the additional uncertainty related with the error of calculation of $\tilde{\gamma}(T)$ caused by the error of $\Delta\Gamma_s$ determination.

  - ◇ For $\Delta\Gamma_s = -0.15\,\Gamma_s$ we have:

$$\frac{\delta(\Delta\Gamma_s)}{|\Delta\Gamma_s|} = \begin{cases} 30\% & \text{for } 10^4 \text{ events}\,, \\ 10\% & \text{for } 10^5 \text{ events}\,. \end{cases}$$

  - ◇ Determination of initial transversity amplitudes and strong-phase difference by using the values of observables $\tilde{b}_i^{(exp)}(T)$ extracted from Monte Carlo data (Cheng's model, $\Delta\Gamma_s = -0.15\,\Gamma_s$, set-B weighting functions). The first errors are statistical while the second errors are caused by uncertainties of $\Delta\Gamma_s$ determination.

Parameter	Input	10^4 events	10^5 events
$\lvert A_0(0)\rvert^2$	0.54	$0.527 \pm 0.007 \pm 0.012$	$0.5398 \pm 0.0023 \pm 0.0011$
$\lvert A_{\lvert\lvert}\rvert^2$	0.30	$0.337 \pm 0.011 \pm 0.008$	$0.3023 \pm 0.0036 \pm 0.0006$
$\lvert A_\perp\rvert^2$	0.16	$0.136 \pm 0.010 \pm 0.020$	$0.1579 \pm 0.0032 \pm 0.0018$
$\cos(\delta_2 - \delta_1)$	-1	-1.021 ± 0.044	-0.9962 ± 0.015

  - ◇ The usage of the $\tilde{b}_i^{(exp)}(T)$ observables provides rather better precision of transversity amplitudes determination than in case of the $\hat{b}_i^{(exp)}(T)$ usage

CONCLUSION AND OUTLOOK

- Angular-moments method has been studied for the extraction of the initial transversity amplitudes and $\Delta\Gamma_s$ from the analysis of untagged events of the decay $B_s^0 \to J/\Psi\,\phi$
- The angular-moments method gives stable results for the estimate of $\Delta\Gamma_s$ and is found to be an efficient and flexible tool for the quantitative investigation of the $B_s^0 \to J/\Psi\,\phi$ decay
- A considerable dependence of statistical and sistematic errors on choice of weighting functions is shown
- The statistical error of the ration $\Delta\Gamma_s/\Gamma_s \in [-0.3, -0.03]$ is independent on the $\Delta\Gamma_s/\Gamma_s$ value and of about 0.015 for 10^5 events
- Full simulation of the decay $B_s^0 \to J/\Psi\,\phi$ in the CMS detector and reconstruction for further Monte Carlo studies of the untagged analysis are in progress
- Oscillation study of the decay $B_s^0 \to J/\Psi\,\phi$ for the determination of ΔM_s from analysis of the tagged sample by using the angular-moments method