



LUND UNIVERSITY

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Monte Carlo tools for the LHC
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Multiple Interactions An Introduction

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What is it and why care?

Models

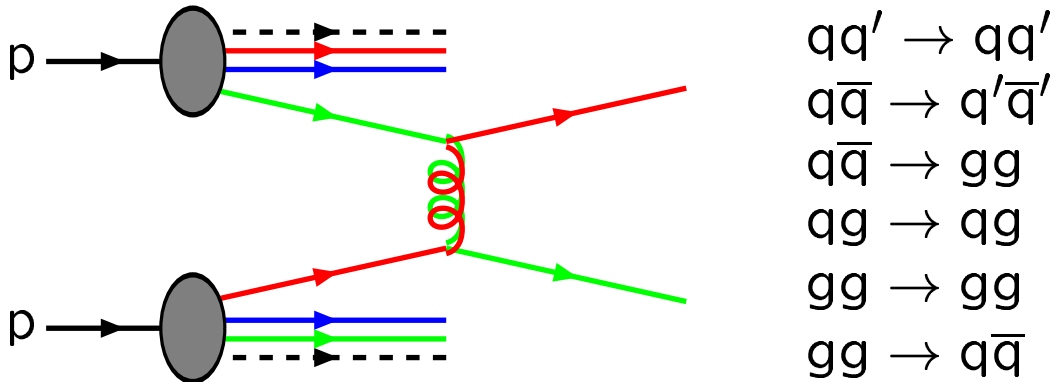
Data and comparisons

Outlook

What is multiple interactions?

Cross section for $2 \rightarrow 2$ interactions,

$$\sigma = \sum_{i,j} \iiint dx_1 dx_2 d\hat{t} f_i(x_1, Q^2) f_j(x_2, Q^2) \frac{d\hat{\sigma}_{ij}}{d\hat{t}}$$



is dominated by t -channel gluon exchange, so approximately (with $p_{\perp}^2 = \hat{t}\hat{u}/\hat{s} \approx |\hat{t}|$)

$$\frac{d\sigma}{dp_{\perp}^2} \approx \iint \frac{dx_1}{x_1} \frac{dx_2}{x_2} F(x_1, p_{\perp}^2) F(x_2, p_{\perp}^2) \frac{d\hat{\sigma}}{dp_{\perp}^2}$$

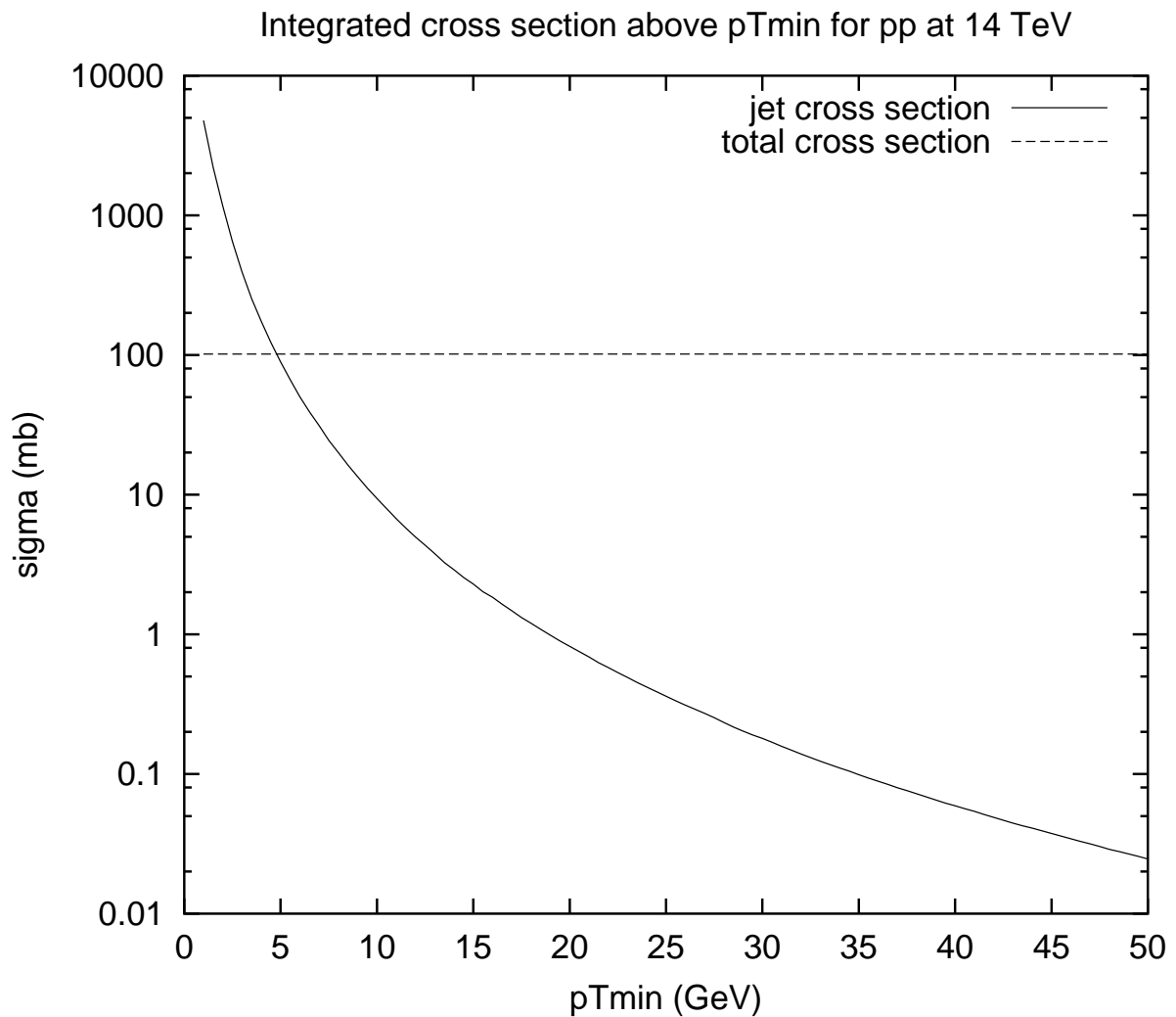
where
$$\frac{d\hat{\sigma}}{dp_{\perp}^2} = \frac{8\pi\alpha_s^2(p_{\perp}^2)}{9p_{\perp}^4}$$

and
$$F(x, Q^2) = \sum_q (x q(x, Q^2) + x \bar{q}(x, Q^2)) + \frac{9}{4}x g(x, Q^2)$$

Thus
$$\sigma_{\text{int}}(p_{\perp\text{min}}) = \int_{p_{\perp\text{min}}}^{\sqrt{s}/2} \frac{d\sigma}{dp_{\perp}} dp_{\perp} \propto \frac{1}{p_{\perp\text{min}}^2}$$

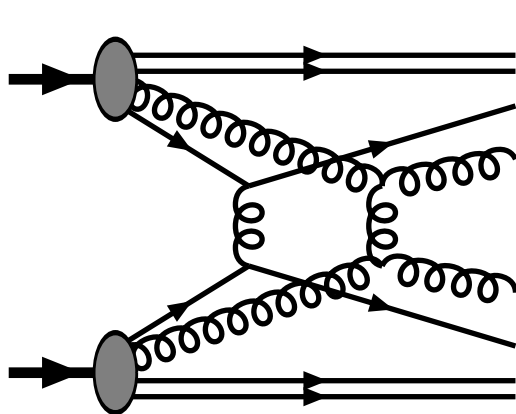
(for constant α_s and neglecting x integrals)

Numerically (for CTEQ 5L)



so $\sigma_{\text{int}}(p_{\perp\text{min}}) > \sigma_{\text{tot}}$ for $p_{\perp\text{min}} \lesssim 5 \text{ GeV}$

Half a solution: many interactions per event



$$\sigma_{\text{tot}} = \sum_{n=0}^{\infty} \sigma_n$$

$$\sigma_{\text{int}} = \sum_{n=0}^{\infty} n \sigma_n$$

$$\sigma_{\text{int}} > \sigma_{\text{tot}} \iff \langle n \rangle > 1$$

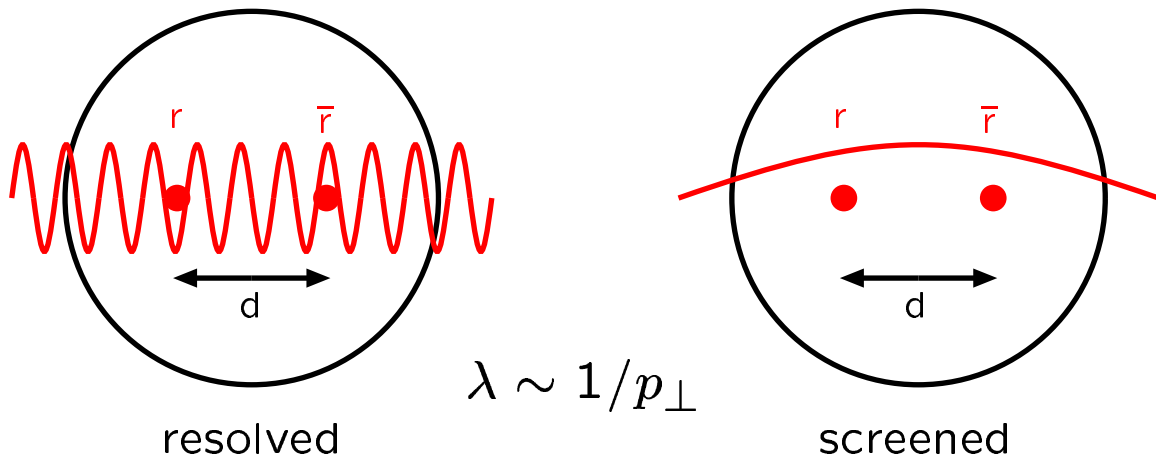
Other half of solution:

perturbative QCD not valid at small p_{\perp}
since q, g not asymptotic states.

Naively breakdown at

$$p_{\perp \min} \simeq \frac{\hbar}{r_p} \approx \frac{0.2 \text{ GeV} \cdot \text{fm}}{0.7 \text{ fm}} \approx 0.3 \text{ GeV} \simeq \Lambda_{\text{QCD}}$$

but better replace r_p by (unknown) colour screening length d in hadron



so modify

$$\frac{d\hat{\sigma}}{dp_{\perp}^2} \approx \frac{8\pi\alpha_S^2(p_{\perp}^2)}{9p_{\perp}^4}$$

$$\rightarrow \frac{8\pi\alpha_S^2(p_{\perp}^2)}{9p_{\perp}^4} \theta(p_{\perp} - p_{\perp \min}) \quad (\text{simpler})$$

$$\text{or } \rightarrow \frac{8\pi\alpha_S^2(p_{\perp 0}^2 + p_{\perp}^2)}{9(p_{\perp 0}^2 + p_{\perp}^2)^2} \quad (\text{more physical})$$

where $p_{\perp \min}$ or $p_{\perp 0}$ are free parameters,
empirically of order 2 GeV

Why care?

Energy	typical number of interactions per event
Tevatron	2 – 4
LHC	5 – 10

So multiple interactions are responsible for:

- large fraction of total multiplicity
- fluctuations to large multiplicities
- rapidity correlations in activity
- multiple (mini)jet production
- jet profile and jet pedestal
- shifts in jet energy scale

Precision physics involving jets or underlying events impossible without understanding of multiple interactions

Nontrivial example: Higgs $\rightarrow \gamma\gamma$

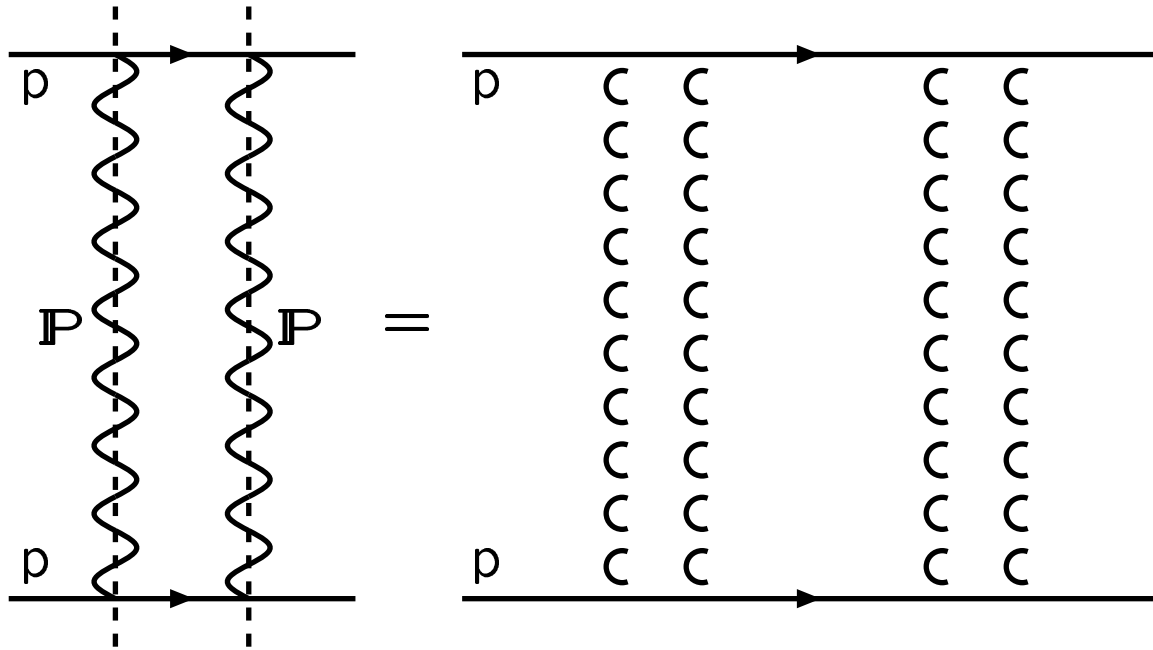
$$M_{\gamma\gamma}^2 = 2E_{\gamma 1}E_{\gamma 2}(1 - \cos\theta_{\gamma\gamma})$$

where $\theta_{\gamma\gamma}$ reconstructing requires to select one primary vertex among ~ 30 pp events.

Is the primary vertex of most or some special charged tracks also the $\gamma\gamma$ production vertex?

Models

Soft limit is cut pomeron language:



V.A. Abramovski, O.V. Kancheli, V.N. Gribov;
A. Capella, J. Tran Thanh Van (DTU);
P. Aurenche, F.W. Bopp, J. Ranft, R. Engel (DTUJET);
K. Fialkowski, A. Kotanski;
A.B. Kaidalov, K.A. Ter Martirosyan;
F. Paige (ISAJET); ...

Implicit in eikonal models for pp cross sections:

T.T. Chou, C.N. Yang;
C. Bourrely, J. Soffer, T.T. Wu;
P. L'Heureux, B. Margolis, P. Valin;
L. Durand, H. Pi; ...

Double parton scattering also studied:

P.V. Landshoff, J.C. Polkinghorne;
C. Goebel, D.M. Scott, F. Halzen;
N. Paver, D. Treleani;
B. Humpert, R. Odorico; ...

T. Sjöstrand, M. van Zijl, PRD36 (1987) 2019:
first model(s) for event properties based on
perturbative multiple interactions

(1) Simple scenario:

- Sharp cut-off at $p_{\perp\min}$ main free parameter
- Is only a model for inelastic nondiffractive events, representing $\sigma_{\text{nd}} \simeq (2/3)\sigma_{\text{tot}}$
- Average number of interactions per event is

$$\langle n \rangle = \frac{\sigma_{\text{int}}(p_{\perp\min})}{\sigma_{\text{nd}}} \quad (\text{naively; see further})$$

- Interactions occur independently

$$\Rightarrow \text{Poissonian statistics} \quad \mathcal{P}_n = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle}$$

with fraction $\mathcal{P}_0 = e^{-\langle n \rangle}$ pure low- p_{\perp}

- Interactions generated in ordered sequence
 $p_{\perp 1} > p_{\perp 2} > p_{\perp 3} > \dots$ by “Sudakov” trick

$$\frac{d\mathcal{P}}{dp_{\perp i}} = \frac{1}{\sigma_{\text{nd}}} \frac{d\sigma}{dp_{\perp}} \exp \left[- \int_{p_{\perp}}^{p_{\perp(i-1)}} \frac{1}{\sigma_{\text{nd}}} \frac{d\sigma}{dp'_{\perp}} dp'_{\perp} \right]$$

- Impose momentum conservation in PDF's
 $\Rightarrow \mathcal{P}_n$ narrower than Poissonian

- Need to simplify after first interaction:
only gg or $q\bar{q}$ final states, no showers, ...

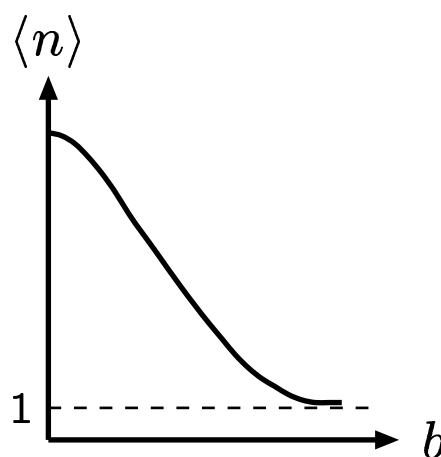
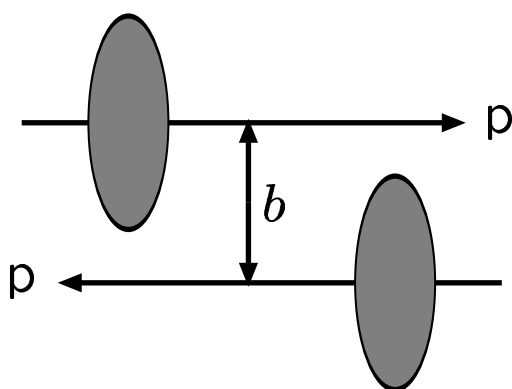
(2) More sophisticated scenario:

- Smooth turn-off at $p_{\perp 0}$ scale
- Require ≥ 1 interaction in an event
- Hadrons are extended:

$$\rho_{\text{matter}}(r) = N_1 \exp\left(-\frac{r^2}{r_1^2}\right) + N_2 \exp\left(-\frac{r^2}{r_2^2}\right)$$

where $r_2/r_1 \neq 1$ represents “hot spots”

- Events are distributed in impact parameter b



- Central collisions normally are more active
 $\Rightarrow \mathcal{P}_n$ broader than Poissonian
- More time-consuming (b, p_{\perp}) generation
- “Trigger bias”: hard jet \Rightarrow central collision
 \Rightarrow above-average active underlying event
- Need to simplify after first interaction, as before

The pedestal effect

Events with high- p_{\perp} jet or W/Z or ... have more underlying activity

“trigger bias”:

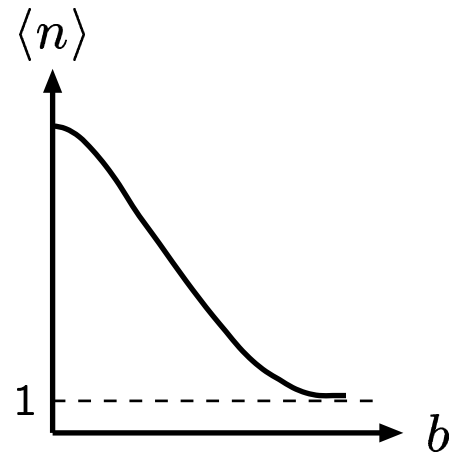
high- p_{\perp} jet

⇒ central collision

⇒ more interactions

⇒ larger activity

or



events with n interactions have n chances that one of them is hard

Define matter overlap of collision

$$\mathcal{O}(b) \propto \iint d^3x dt \rho \left(x - \frac{b}{2}, y, z - vt \right) \rho \left(x + \frac{b}{2}, y, z + vt \right)$$

with $\rho = \rho_{\text{matter,boosted}}$, such that $\langle \mathcal{O}(b) \rangle = 1$ (kind of). Then

$$\frac{d\mathcal{P}_{\text{hardest}}}{d^2b dp_{\perp}} = \mathcal{O}(b) \frac{1}{\sigma_{\text{nd}}} \frac{d\sigma}{dp_{\perp}} \exp \left[-\mathcal{O}(b) \int_{p_{\perp}}^{\sqrt{s}/2} \frac{1}{\sigma_{\text{nd}}} \frac{d\sigma}{dp'_{\perp}} dp'_{\perp} \right]$$

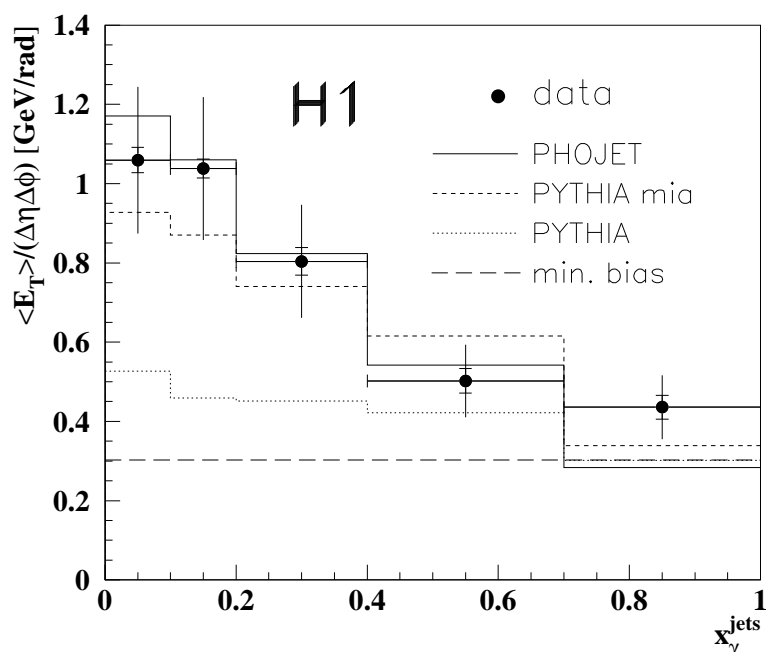
$$\rightarrow \mathcal{O}(b) \frac{1}{\sigma_{\text{nd}}} \frac{d\sigma}{dp_{\perp}} \quad (p_{\perp} \text{ and } b \text{ decouples})$$

$$\text{when } \int_{p_{\perp}}^{\sqrt{s}/2} \frac{d\sigma}{dp'_{\perp}} dp'_{\perp} \ll \sigma_{\text{nd}}$$

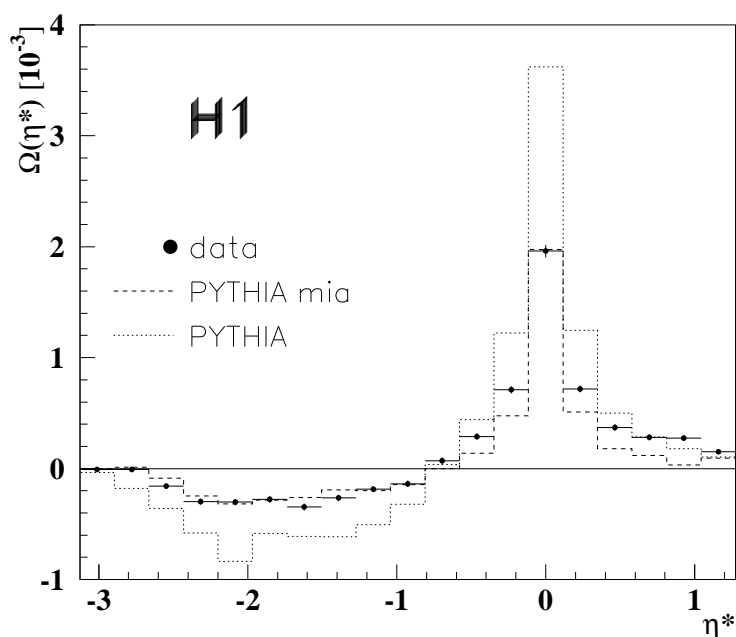
⇒ centrality effect saturates at $p_{\perp \text{hard}} \sim 10 \text{ GeV}$

Evidence for multiple interactions

- Width of multiplicity distribution: UA5, E735
- Forward–backward correlations: UA5
- Jet pedestal effect: UA1, **H1**, CDF



underlying activity in photoproduction vs. DIS



(anti)correlations in energy flow around jet

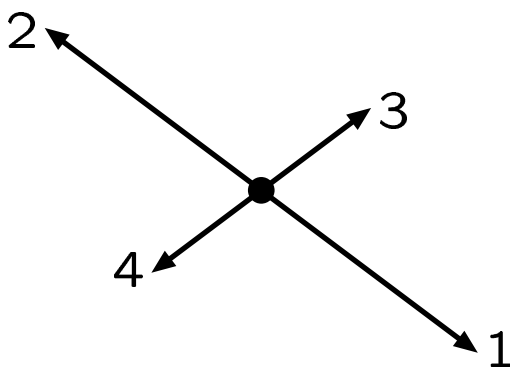
- Minijet rates: UA1

No. jets	UA1 (%)	no MI	simple	double Gaussian
1	9.96	14.30	11.51	8.88
2	3.45	2.45	2.45	2.67
3	1.12	0.22	0.32	0.74
4	0.22	0.01	0.04	0.25
5	0.05	0.00	0.00	0.07

- Direct observation: AFS, (UA2,) CDF

Order jets $p_{\perp 1} > p_{\perp 2} > p_{\perp 3} > p_{\perp 4}$ and define φ as angle between $p_{\perp 1} - p_{\perp 2}$ and $p_{\perp 3} - p_{\perp 4}$

Double Parton Scattering

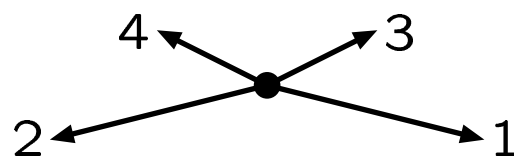


$$|p_{\perp 1} + p_{\perp 2}| \approx 0$$

$$|p_{\perp 3} + p_{\perp 4}| \approx 0$$

$d\sigma/d\varphi$ flat

Double BremsStrahlung



$$|p_{\perp 1} + p_{\perp 2}| \gg 0$$

$$|p_{\perp 3} + p_{\perp 4}| \gg 0$$

$d\sigma/d\varphi$ peaked at $\varphi \approx 0$

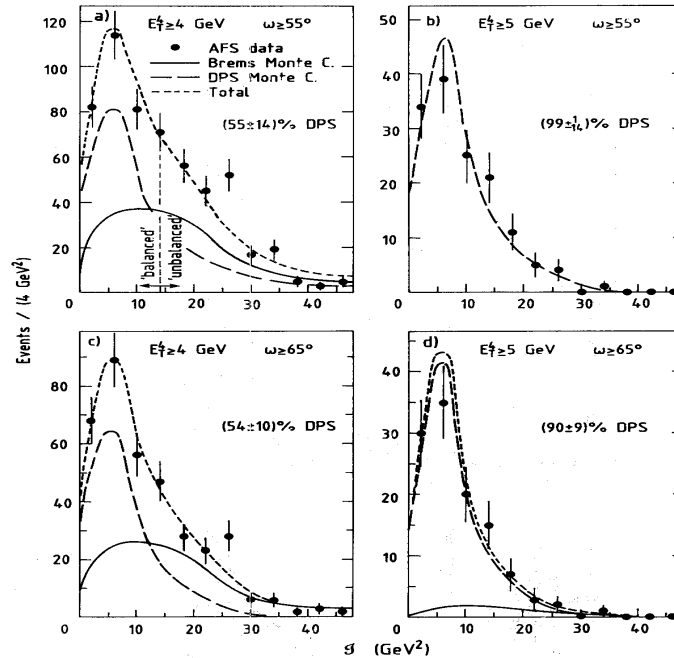


Fig. 3

AFS 4-jet analysis (pp at 63 GeV);
double bremsstrahlung subtracted:

observed	6	in arbitrary units
no MI	0	
simple MI	1	
double Gaussian	3.7	

UA2 4-jet analysis (at 630 GeV):

with ansatz $\sigma_{\text{DPS}} = \frac{1}{2} \frac{\sigma_{2\text{jet}}^2}{\sigma_{\text{eff}}}$

limit $\sigma_{\text{eff}} > 8.3 \text{ mb}$ at 95% CL

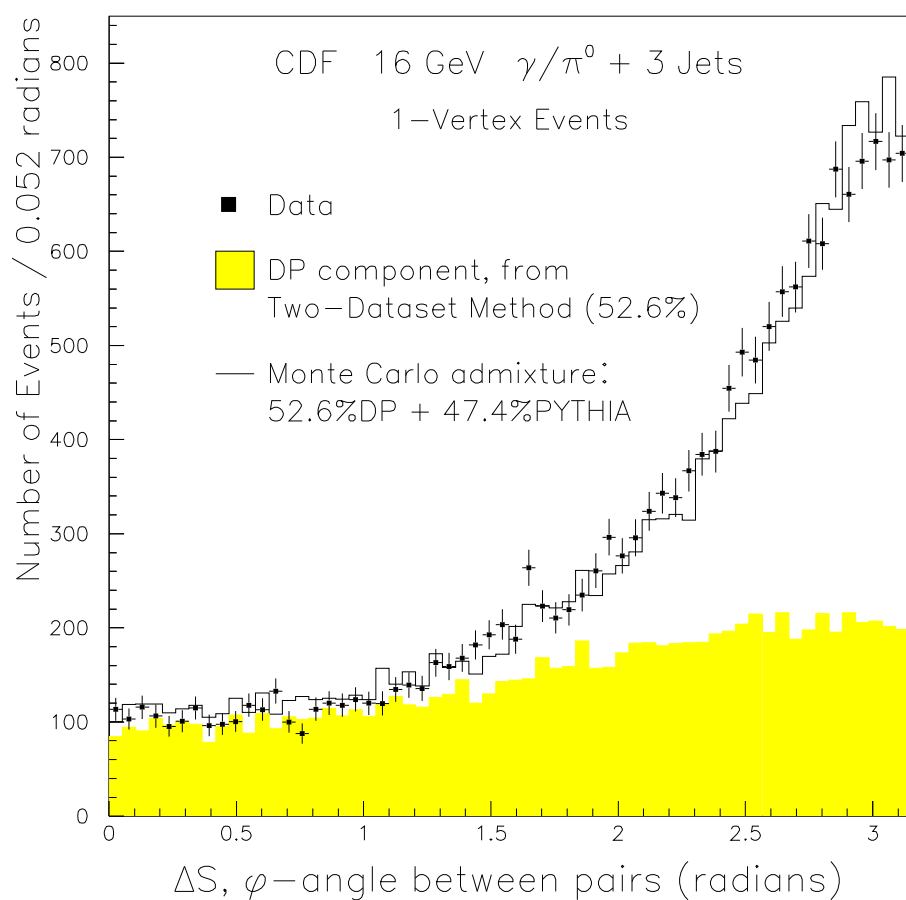
i.e. $\sigma_{\text{DPS}} < 4.5$ in 'AFS units'

... but best value 2.5 ± 1

CDF 4-jet analysis (at 1800 GeV):

$$\sigma_{\text{eff}} = 12.1^{+10.7}_{-5.4} \text{ mb}$$

CDF 3-jet + prompt photon analysis:

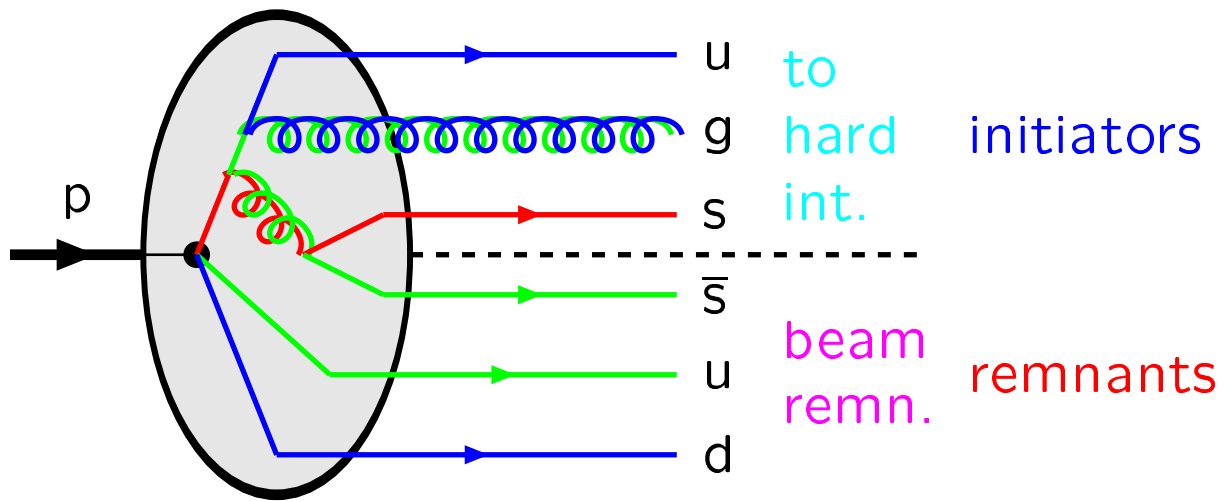


Warning!
PYTHIA
run with
multiple
interactions
off

$$\sigma_{\text{DPS}} = \frac{\sigma_A \sigma_B}{\sigma_{\text{eff}}} \quad \text{for } A \neq B$$

$$\sigma_{\text{eff}} = 14.5 \pm 1.7^{+1.7}_{-2.3} \text{ mb}$$

Initiators and Remnants

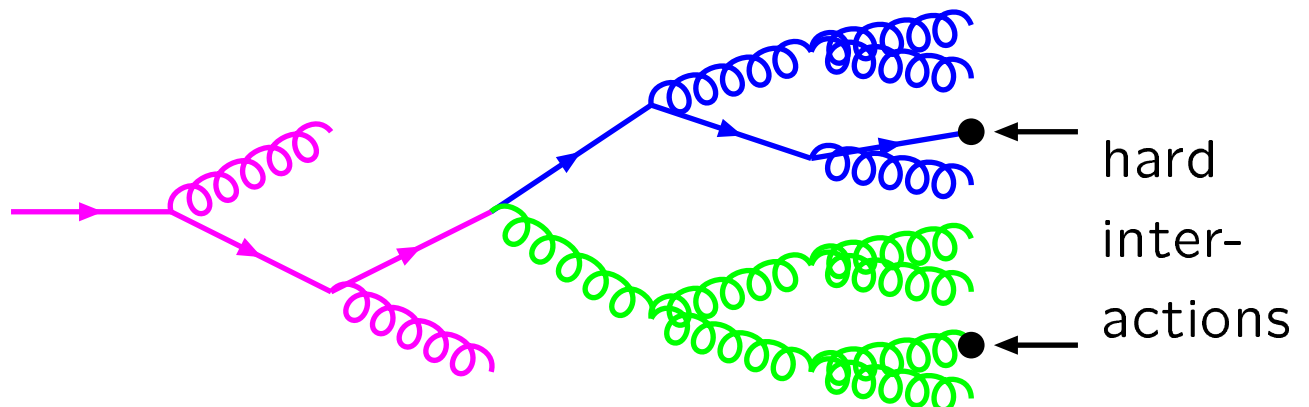


Need to assign:

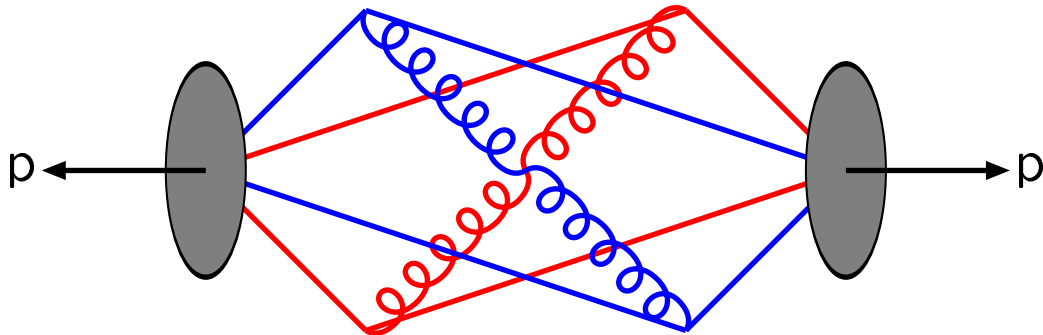
- correlated flavours

(Where does the baryon number go?)

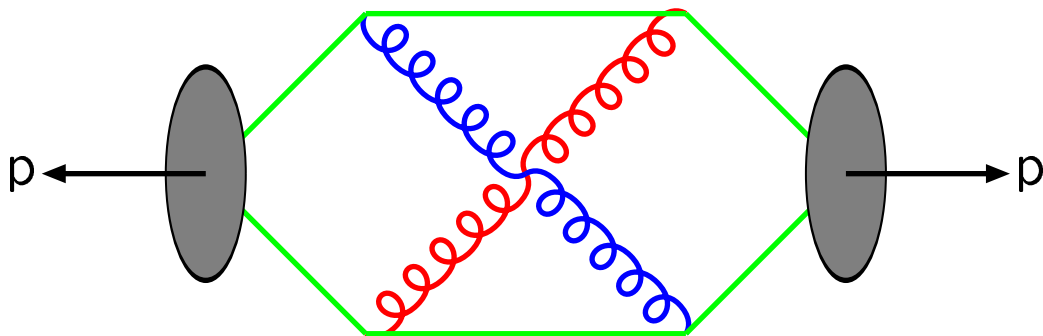
- correlated $x_i = p_{zi}/p_{ztot}$
- correlated primordial $k_{\perp i}$
- correlated colours
- correlated showers (?)



$\langle p_{\perp} \rangle(n_{\text{ch}})$ and colour correlations

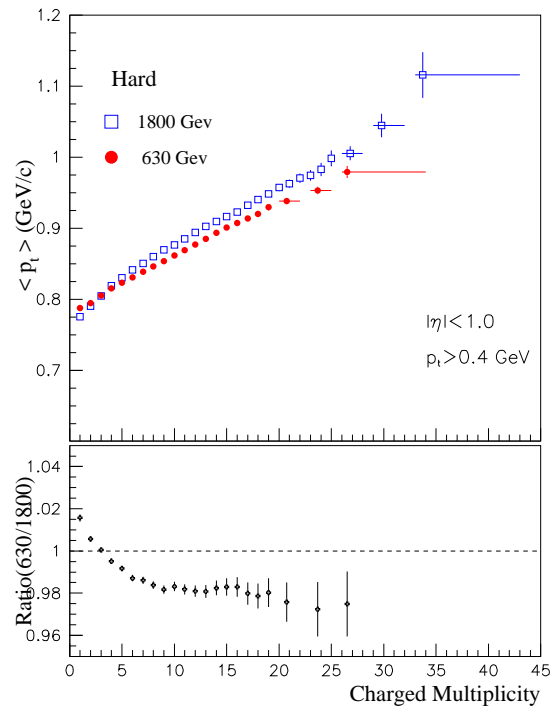
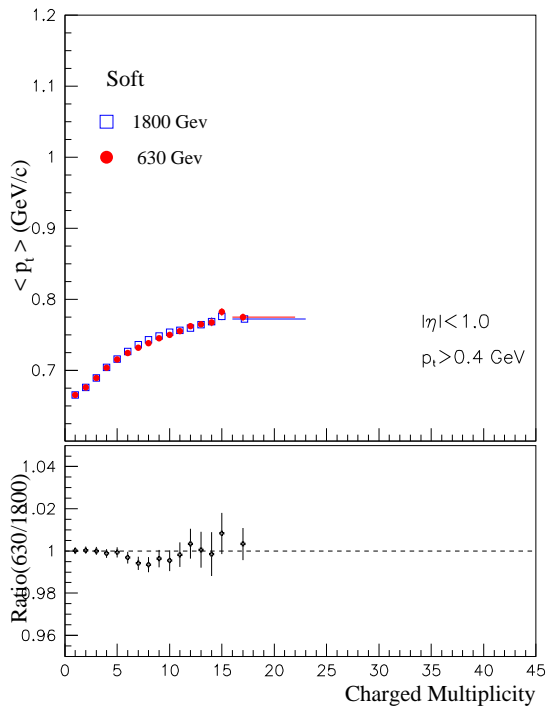
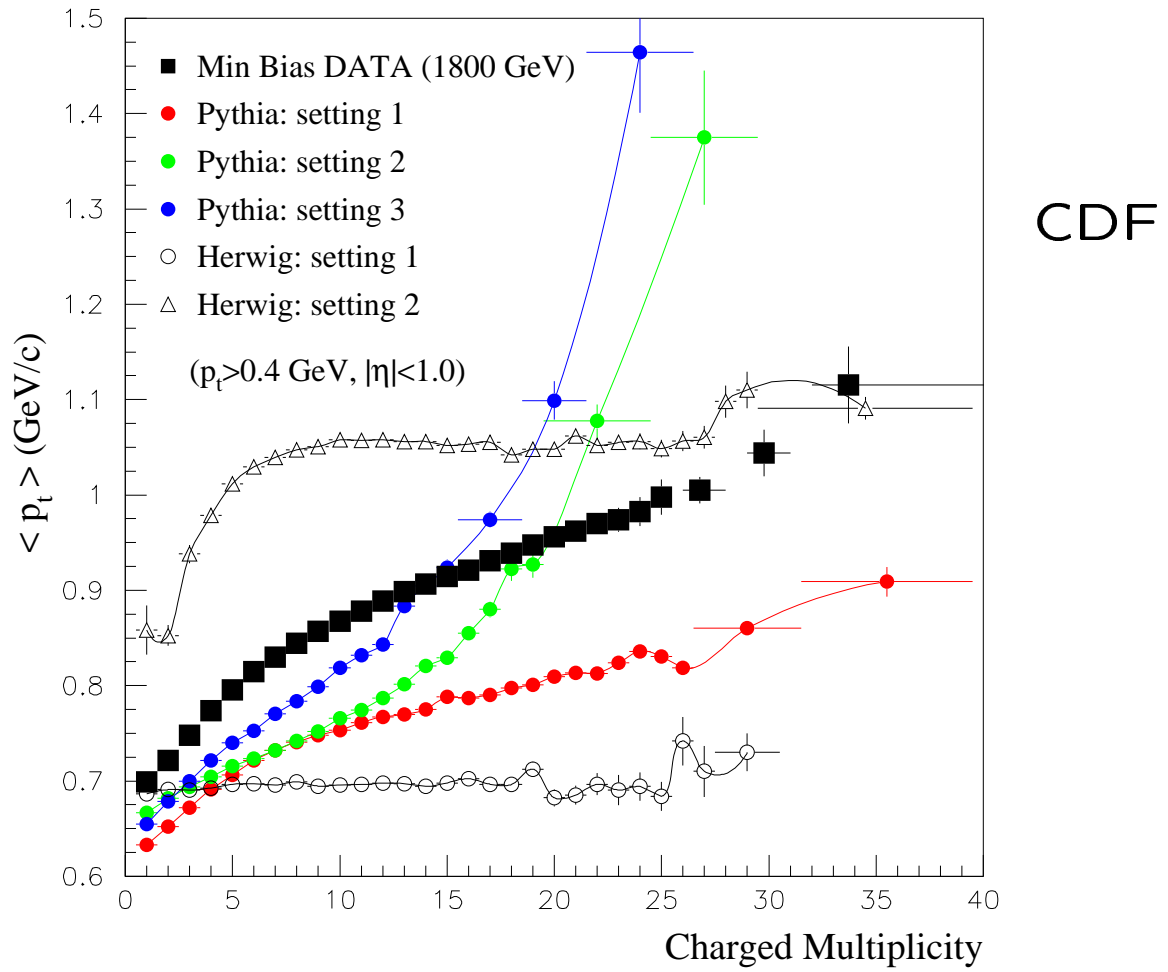


$$\begin{array}{ccccccc}
 l_{\text{string}} \nearrow & \Rightarrow & n_{\text{ch}}/n_{\text{int}} \nearrow & \Rightarrow & p_{\perp \text{min}} \nearrow & \Rightarrow & \\
 n_{\text{int}} \searrow & \Rightarrow & p_{\perp \text{pert}} \searrow & \Rightarrow & \langle p_{\perp} \rangle(n_{\text{ch}}) \sim \text{flat} & &
 \end{array}$$



$$\begin{array}{ccccccc}
 l_{\text{string}} \searrow & \Rightarrow & n_{\text{ch}}/n_{\text{int}} \searrow & \Rightarrow & p_{\perp \text{min}} \searrow & \Rightarrow & \\
 n_{\text{int}} \nearrow & \Rightarrow & p_{\perp \text{pert}} \nearrow & \Rightarrow & \langle p_{\perp} \rangle(n_{\text{ch}}) \nearrow & &
 \end{array}$$

... but need guiding principles
 or other mechanism to accomplish the same



Energy dependence of $p_{\perp\min}$ and $p_{\perp 0}$

Larger collision energy

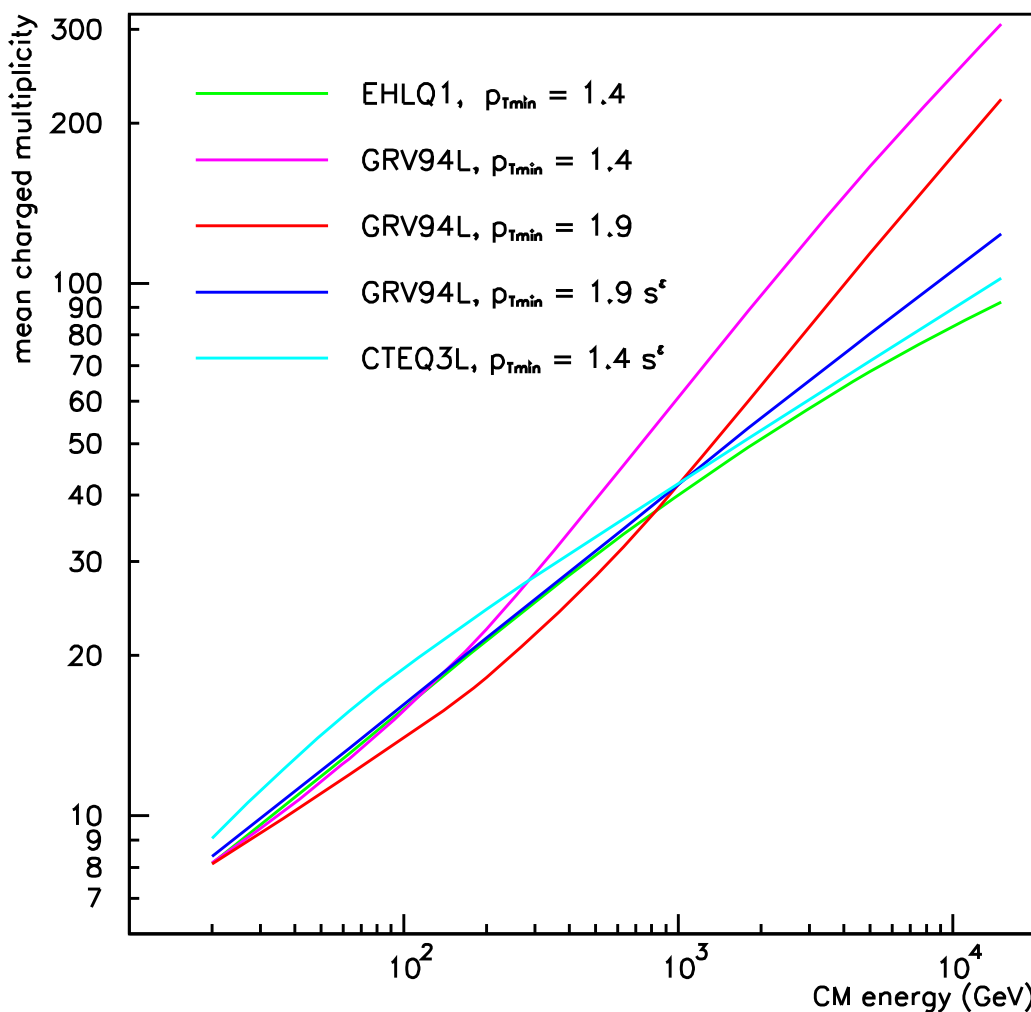
⇒ probe parton (\approx gluon) density at smaller x

⇒ smaller colour screening length d

⇒ larger $p_{\perp\min}$ or $p_{\perp 0}$

Post-HERA PDF fits steeper at small x

⇒ stronger energy dependence



Current PYTHIA default, tied to CTEQ 5L, is

$$p_{\perp\min}(s) = 1.9 \text{ GeV} \left(\frac{s}{1 \text{ TeV}^2} \right)^{0.08}$$

How to make progress?

Need model building \Leftrightarrow experimental tests

Reference samples over wide energy range:

- ~ 20 GeV: fixed target
- ~ 63 GeV: ISR
- ~ 200 GeV: Sp \bar{p} S, RHIC
- ~ 630 GeV: Sp \bar{p} S, Tevatron
- ~ 2 TeV: Tevatron

Corrected and reliable distributions
(over all or well-defined phase space) of:

- n_{ch}
- y and p_{\perp} single-particle spectra
- y correlations, $p_{\perp} \& \varphi$ correlations
- $\langle p_{\perp} \rangle(n_{\text{ch}})$
- n_{minijet} for different $E_{\perp\text{jet}}$
- jet profile and pedestal
- rapidity gap size and position
- other interesting properties?

in form usable to outsiders

\Rightarrow JetWeb?

Outlook

★ Multiple interactions concept compelling; it *has to* exist at some level

★ By now

- strong direct evidence
- overwhelming indirect evidence

★ Understanding of multiple interactions crucial for precision physics involving jets

★ Many details uncertain

- $p_{\perp\text{min}}/p_{\perp 0}$ cut-off
- impact parameter picture
- energy dependence
- multiparton densities in incoming hadron
- colour correlations between scatterings
- interferences between showers
- ...

★ Above physics aspects must be present

If a model is simple, it is wrong!

... so let's keep on complicating matters!