Modern Cosmology

HST 2003 lectures based on “An Introduction to Modern Cosmology” by Andrew Liddle

J.P. van der Schaar
CERN Theory Division

Outline

- The cosmological principle
- Hubble's law and Friedmann equation
- Geometry of the universe
- Weighing the universe

I

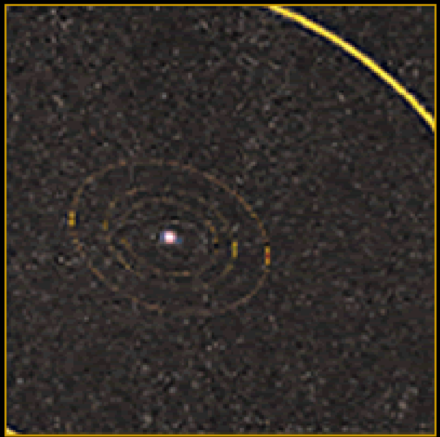
II

- The Cosmic Microwave Background
- Hot Big Bang problems
- Inflation
- CMB anisotropies

Cosmological distances

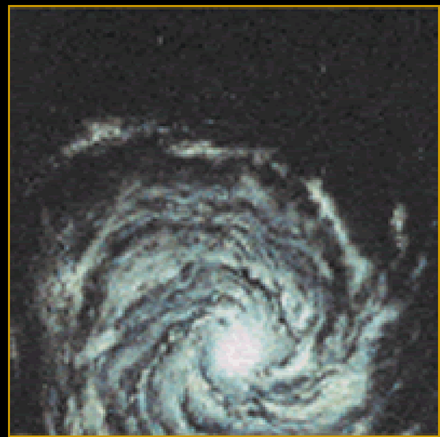
Alternative unit : 1 parsec ~ 3.261 lyr

Nearest similar galaxy : Andromeda ~ 770 kpc



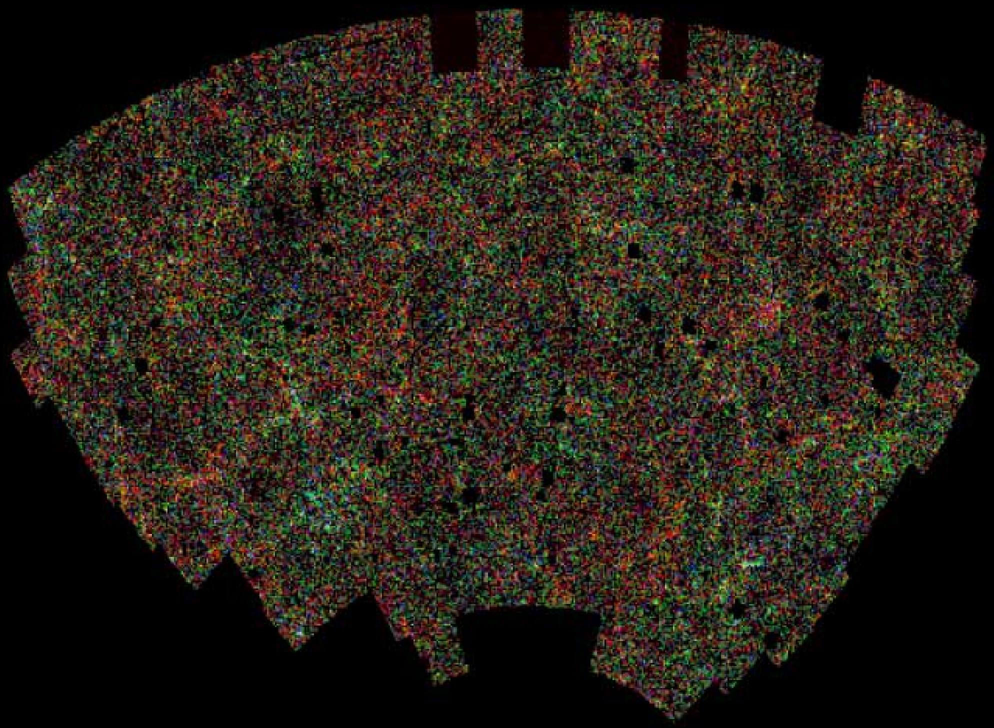
Mean distance Earth-Sun
1 Astronomical Unit (AU)
~ 10^{11} meters

Nearest stars
Few light years
~ 10^{16} meters



Milky Way galaxy
size ~ 13 kpc
~ 10^{20} meters

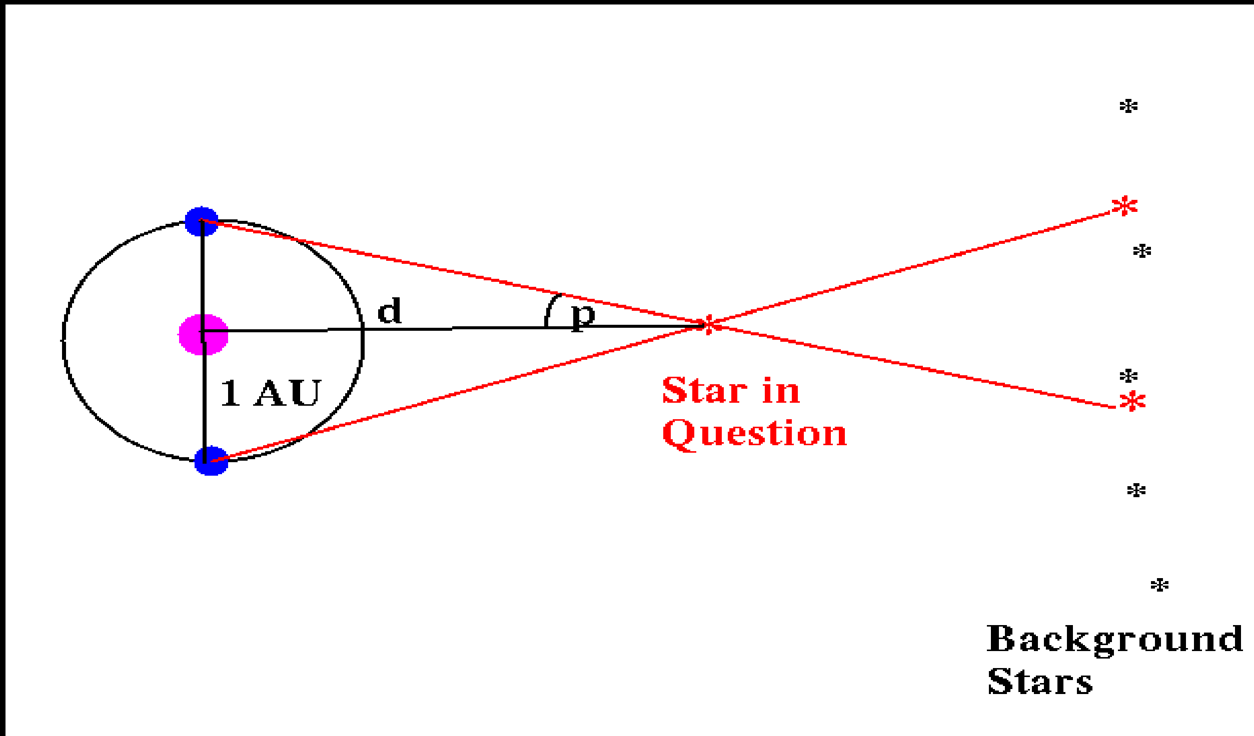
Typical galaxy group volume ~ cubic Million parsecs
Cosmologist's favorite unit: Megaparsec (Mpc) ~ 10^{22} meters



APM Survey picture of a large part of the sky, about 30 degrees across, showing almost a million galaxies out to a distance of about 2 billion light years.
MAP990047

Clusters, superclusters of galaxies and voids ~ 100 Mpc
Even larger scales > 100 Mpc: Universe appears smooth!

What is a parsec?



1 pc = 1 AU / tan(1 arcsecond)
1 arcsecond = 360 degrees / 3600

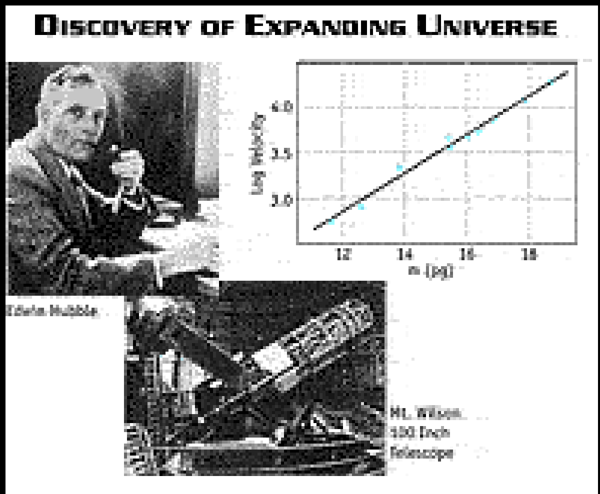
The cosmological principle

The belief that the universe is very smooth on extremely large scales (>100 Mpc)

- Underpinning of (theoretical) cosmology
- Only fairly recently it has been possible to provide convincing observational evidence supporting the smoothness of the matter distribution on large scales
    1. Cosmic Microwave Background
    2. Large scale galaxy (cluster) distribution
- Implies two important properties of our universe
    1. Homogeneity : Universe looks the same at each point
    2. Isotropy: Universe looks the same in all directions

Expansion of the Universe

"The further away galaxies are, the more rapid their recession appears to be"



- Velocities measured via redshift; basically the Doppler effect applied to light
- Distances typically determined via the use of standard candles (absolute vs. relative brightness)
- Leads to Hubble's law, whose form is consistent with the cosmological principle

$z = (\lambda_{obs}-\lambda_{em})/\lambda_{em} \sim v/c$ z : redshift

$v = H_0\, r$ H_0 : Hubble's constant

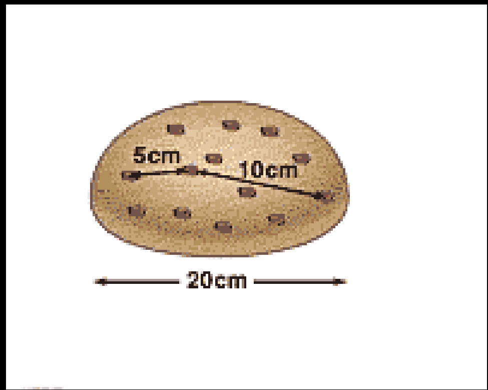
Big Bang Cosmology

Hubble's law is just what one would expect for a *homogeneous* expanding universe; each point in the universe sees all objects rushing away from them with a velocity proportional to distance

Useful analogy: raisin bread model

– Raisins correspond to (clusters of) galaxies

– Expanding bread corresponds to expanding space: More bread in between raisins implies larger relative velocity, exactly like Hubble's law would predict



We conclude that in the distant past everything in the Universe was much closer together:

Big Bang !

H_0 ~ 70 km/s/Mpc
Age ~ $1/H_0$ ~ 14 10^9 yrs
Size ?

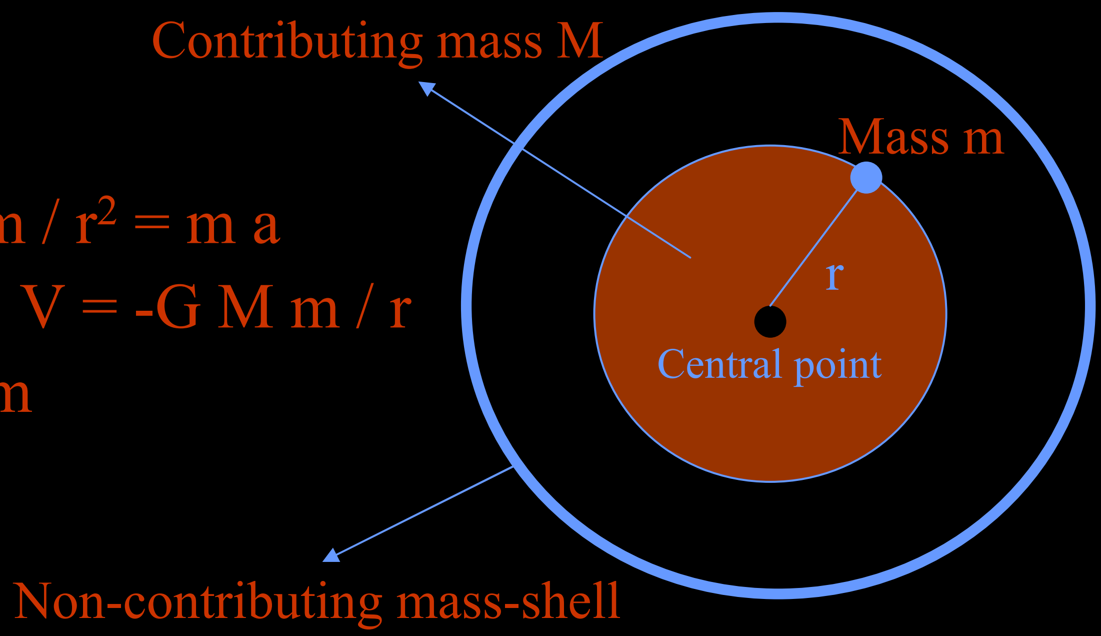
Newtonian gravity and cosmology

One can describe the expansion of the universe, without being completely rigorous, using simple Newtonian gravity, instead of using the equations of General Relativity

Force: $F = G\, M\, m / r^2 = m\, a$

Potential Energy: $V = -G\, M\, m / r$

Newton's theorem



Friedmann equation

Describes the expansion of the universe and can therefore be considered the most important equation in cosmology

To derive it we need to:

- Compute gravitational potential energy
- Compute kinetic energy of a test particle with mass m (e.g. a galaxy)
- According to the cosmological principle it does not matter which test particle we consider or which point we consider to be the center of the universe
- Use energy conservation!

$M = 4\pi \rho r^3 / 3$ ρ : mass density, mass per unit volume
Potential energy: $V=-G M m / r = -4\pi G \rho r^2 m / 3$
Kinetic energy: $T= \frac{1}{2} m v^2$ velocity test particle $v = dr / dt$
Energy conservation: U=T+V is constant (<u>but U will depend on initial separation</u>)

Friedmann equation continued…

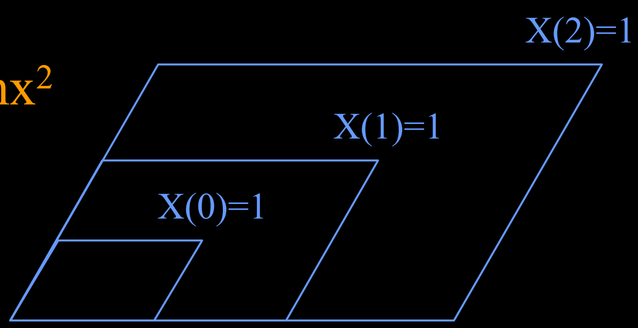
$$U = T + V = \frac{1}{2} m\,(dr/dt)^2 - 4\pi G \rho r^2 m / 3$$

Crucial step: the cosmological principle allows us to define so-called co-moving coordinates $r = a(t)\,x$, where only $a(t)$ is a function of time and x is a constant coordinate. The quantity $a(t)$ is known as the scale factor of the universe and it measures the universal expansion rate. If the scale factor doubles in value between time t and t', the universe has expanded by a factor of two.

Velocity $v = dr/dt = x\,da/dt$

Total energy $U = \frac{1}{2}(da/dt)^2 mx^2 - 4\pi G \rho a^2 mx^2 / 3$

Divide both sides by $\frac{1}{2} mx^2$ and define $kc^2 = -2U / mx^2$



$$[(da/dt)/a]^2 = (8\pi G/3)\,\rho - kc^2/a^2$$

Standard form of the Friedmann equation!

Friedmann equation continued…

$$[(da/dt) / a]^2 = (8\pi G / 3) \rho - kc^2 / a^2$$

- Constant k is independent of initial separation x to maintain homogeneity
- An expanding universe has a unique value of k, which it retains throughout its evolution
- To solve this equation we need to know how the mass density of the universe changes in time, standard examples are $\rho = \rho_0 / a^3$ for ordinary matter and $\rho = \rho_0 / a^4$ for radiation
- Hubble's law, $v = H r$, implies that the Hubble parameter $H = (da / dt) / a$

$$H^2 = (8\pi G / 3) \rho - kc^2 / a^2$$

Consider $k=0$, $\rho = \rho_0 / a^3$, $a(t) = c\, t^p$ and try to find a solution

So Hubble's constant is not really a constant and instead will typically decrease as a function of time. For ordinary forms of matter the expansion of the universe will be *decelerating*.

Time for a break…

Geometry of the universe

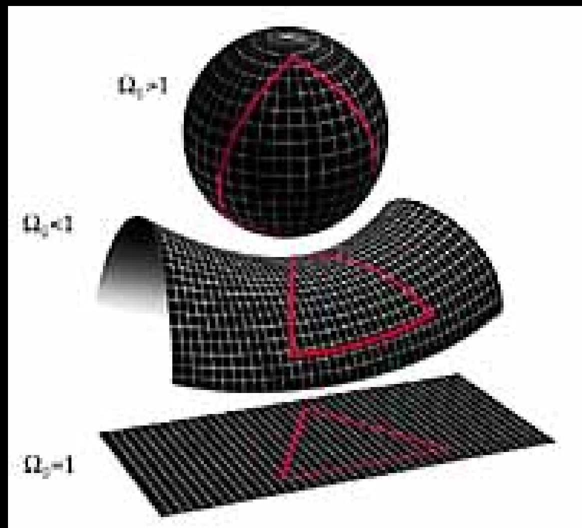
Meaning of constant k:

– Newtonian gravity: measure of the energy per test particle

– True interpretation from General Relativity: measure of the curvature of 3-space

3d generalization of intuitive and intrinsic notion of curvature for 2d surfaces

- Closed: k>0 (U<0), spherical geometry
  - Sum of triangle angles > 180º
  - Circle circumference < 2π r
  - Recollapse, Big Crunch
- Open: k<0 (U>0), hyperbolic geometry,
  - Sum of triangle angles < 180º
  - Circle circumference > 2π r
- Flat: k=0 (U=0), Euclidean geometry
  - Sum of triangle angles = 180º
  - Circle circumference = 2π r



Find a triangle on a 2d sphere with angles that add up to 270º and a circle with circumference 4r

Weighing the Universe

$$H^2 = (8\pi G / 3)\,\rho - kc^2 / a^2$$

Total density ρ includes particles, radiation and possibly even more exotic components. One defines ρ_{crit} as the density that would correspond to a flat universe k=0, giving $\rho_{crit} = 3H^2 / 8\pi G$ as a characteristic density scale.

$$\Omega - 1 = kc^2 / (a^2 H^2)\,,\ \Omega = \rho / \rho_{crit}$$

Present value $\rho_{crit} \sim 10^{11}\, M_{Sun} / (Mpc)^3$

How to estimate matter density components:

– Counting stars, $\Omega_{Stars} \sim 0.01$

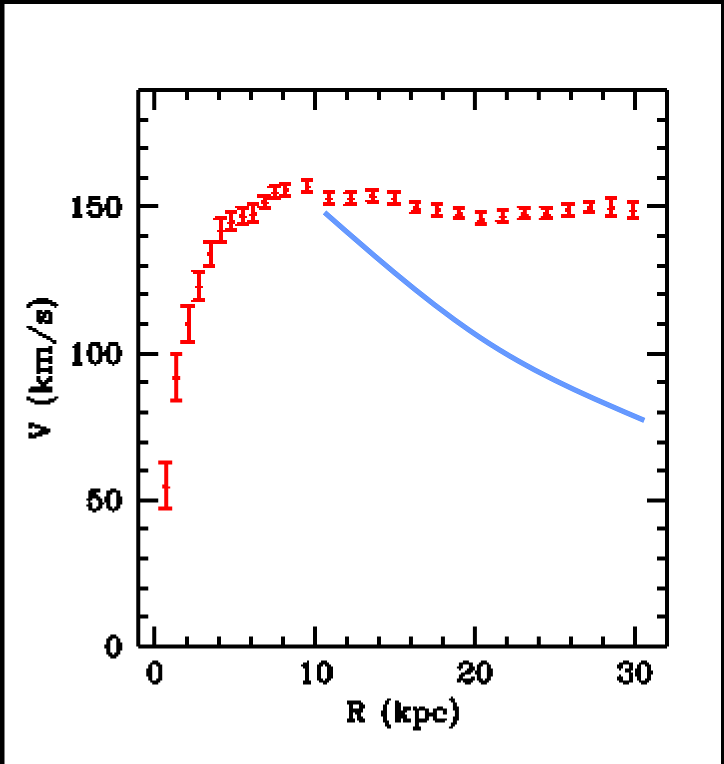
– Gravitational bulk motion in the universe; galaxy rotation curves, cluster composition, $\Omega \sim 0.1 – 0.3$

– Nucleosynthesis, $\Omega_{Baryons} \sim 0.03 – 0.05$

– Formation of structure, $\Omega > 0.2$

Dark Matter !

Galaxy rotation curves



Balance between gravitational pull and centrifugal force implies:

$$m\, v^2 / R = G\, m\, M(R) / R^2$$

$$\Rightarrow \quad v = [\, G\, M(R) \;/ R]^{1/2}$$

From the visible matter distribution in galaxies one expects that M(R) ~ constant for large R, leading to the $1/R^{1/2}$ behavior plotted as the blue curve in the diagram. This is clearly not in agreement with the observational curve. Apparently there is more matter than can be seen!

What kind of dark matter distribution does the experimental curve suggest?

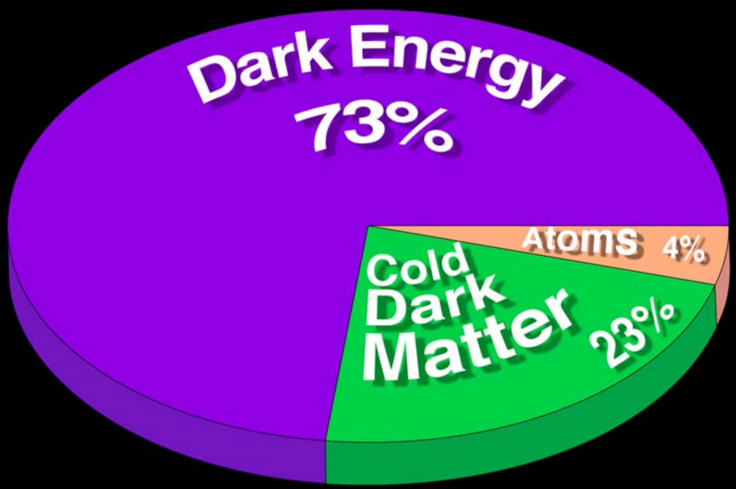
Other surprises

High redshift supernova project:

Determination of the time dependence of the Hubble parameter shows that the expansion of the universe is accelerating! No ordinary form of (dark) matter can lead to accelerating expansion, so one needs to introduce another form of energy, called **dark energy**, which does not cluster, one example being a **cosmological constant**

Cosmic Microwave Background anisotropies:

Geometry of the universe is flat! This implies that the total density equals the critical density and is consistent with the diagram below for the different component densities



Cosmological constant Λ:

- Constant energy density, requires negative effective pressure to do work as the universe expands

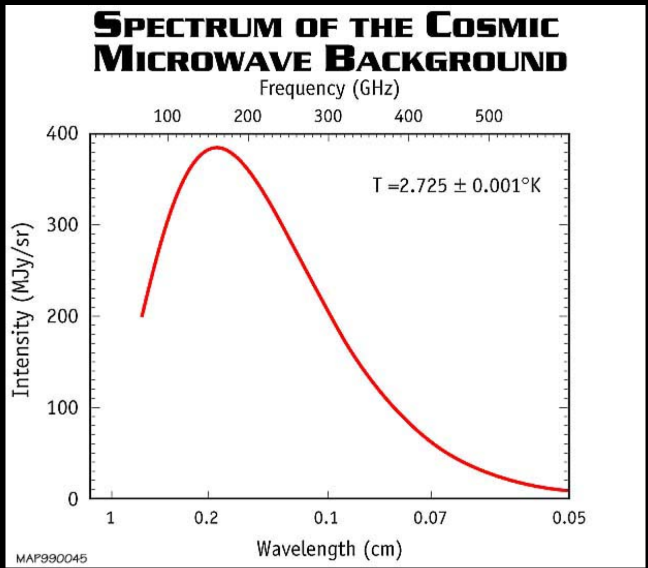
- Adds a $\rho_\Lambda = (\Lambda / 8\pi G)$ density component to Friedmann equation

The cosmic microwave background

- Thermal spectrum of microwave radiation that bathes the earth from all directions
- Discovered by accident in 1965 by Penzias and Wilson
- Temperature $T_0 = 2.725 \pm 0.001$ Kelvin, same "black body spectrum" in all directions
- Density radiation decreases as $1/a^4$, same is true for temperature

Origin of CMB:

– Energetic photons (13.6 eV) can ionize hydrogen
– Consider the universe at $1/10^6$ of present size
– $T \sim 3\ 10^6$ K, all hydrogen atoms ionized
– Cosmic ionized plasma, no free photons
– As universe expanded, temperature dropped
– Photons can no longer ionize hydrogen ; decoupling
– $T_{decoupling} \sim 5700$ K, free CMB photons!



CMB and the Hot Big Bang

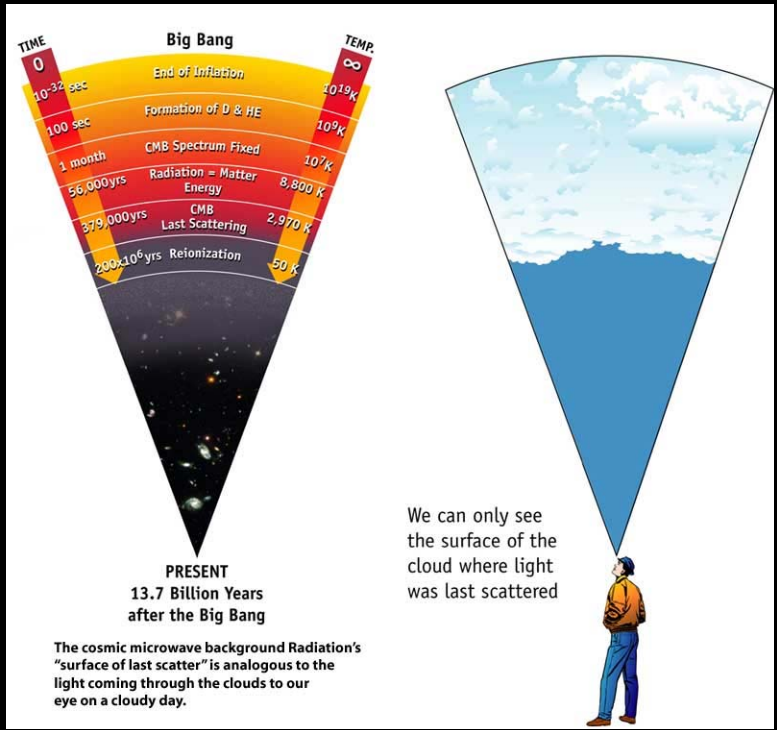
- CMB is our best confirmation that the cosmological principle applies
- Early universe was a very hot ionized plasma: Hot Big Bang
- Nucleosynthesis : Hot Big Bang predicts abundances of light elements

Surface of last scattering:

– Place where decoupled CMB photons first started traveling freely in our direction

– Roughly 6000 Mpc away, 300000 yrs

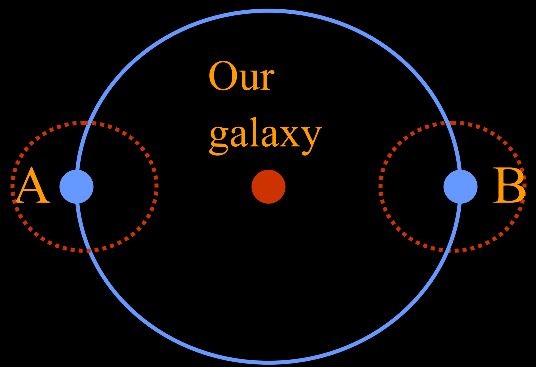
– With expansion temperature cooled to 3 K



Problems with the Hot Big Bang

- Flatness problem
- Horizon problem
- Origin of structure

Flatness problem:
$|\Omega - 1| = |k|\, c^2 / (a^2 H^2)$
For conventional matter content
$|\Omega - 1| \sim t^{|n|}$, which increases in time, so universe must have been extremely close to the flat geometry at very early times



Last scattering surface CMB

How can the CMB temperature from A and B be the same, since they could not have been in causal contact since the Big Bang?

Perfect Big Bang isotropy cannot lead to the structures we see in our universe. What are the seeds for structure formation?

Inflation

- Inflation ⇔ short period of accelerating, exponential expansion in the very early universe
- Extremely rapid expansion: by a factor of 10^{43} in less than 10^{-36} seconds
- Should connect cosmology and particle physics beyond the Standard Model

Solves the Big Bang problems:

– Flatness problem: inflation drives density towards critical density

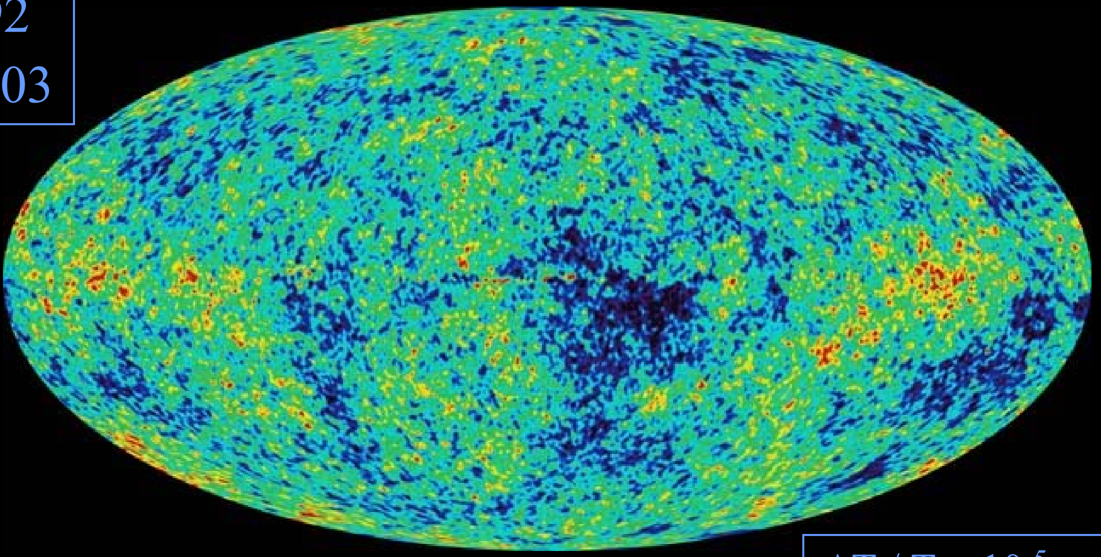
– Horizon problem: points A and B are causally connected through inflation

– Origin of structure : inflationary quantum fluctuations provide the seeds for structure formation

CMB anisotropies

CMB is not perfectly isotropic; extremely small temperature fluctuations that are due to tiny density variations in the ionized plasma. These small density variations are the seeds for all structure in our Universe.

COBE 1992
WMAP 2003



Average size of fluctuations ~ 1 degree in sky : flat universe

$\Delta T / T \sim 10^{-5}$, a swimming pool as smooth would contain ripples only 1/100 mm high!

CMB anisotropies continued…

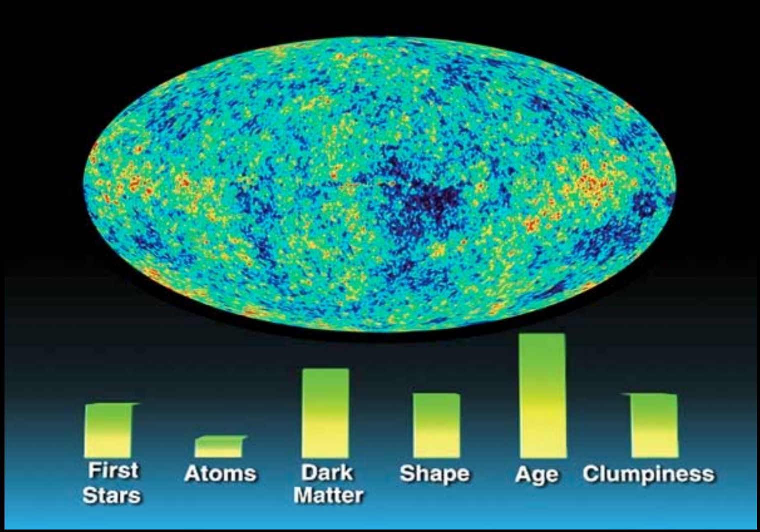
Anisotropy map is like a fingerprint, giving us a lot of information

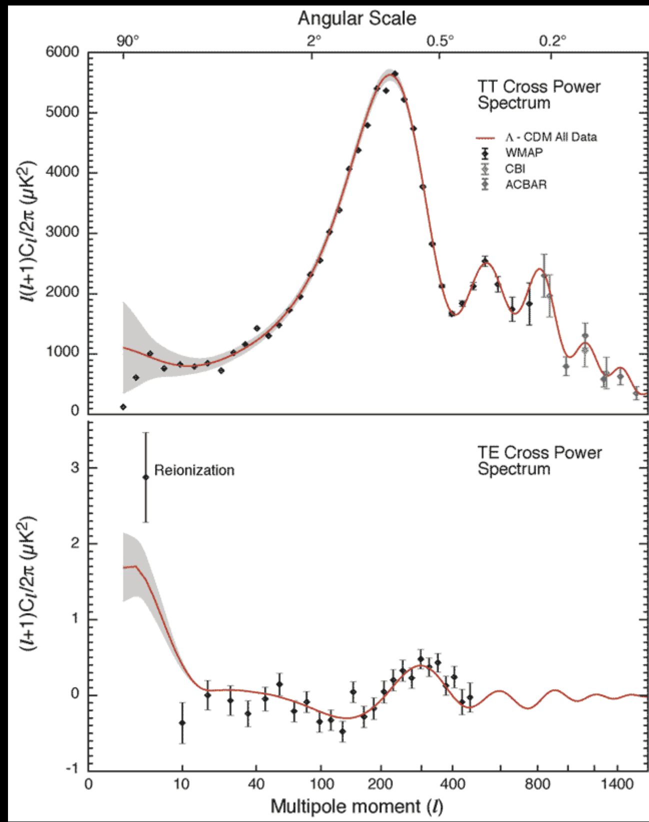
– geometry of the universe

– contents of the universe

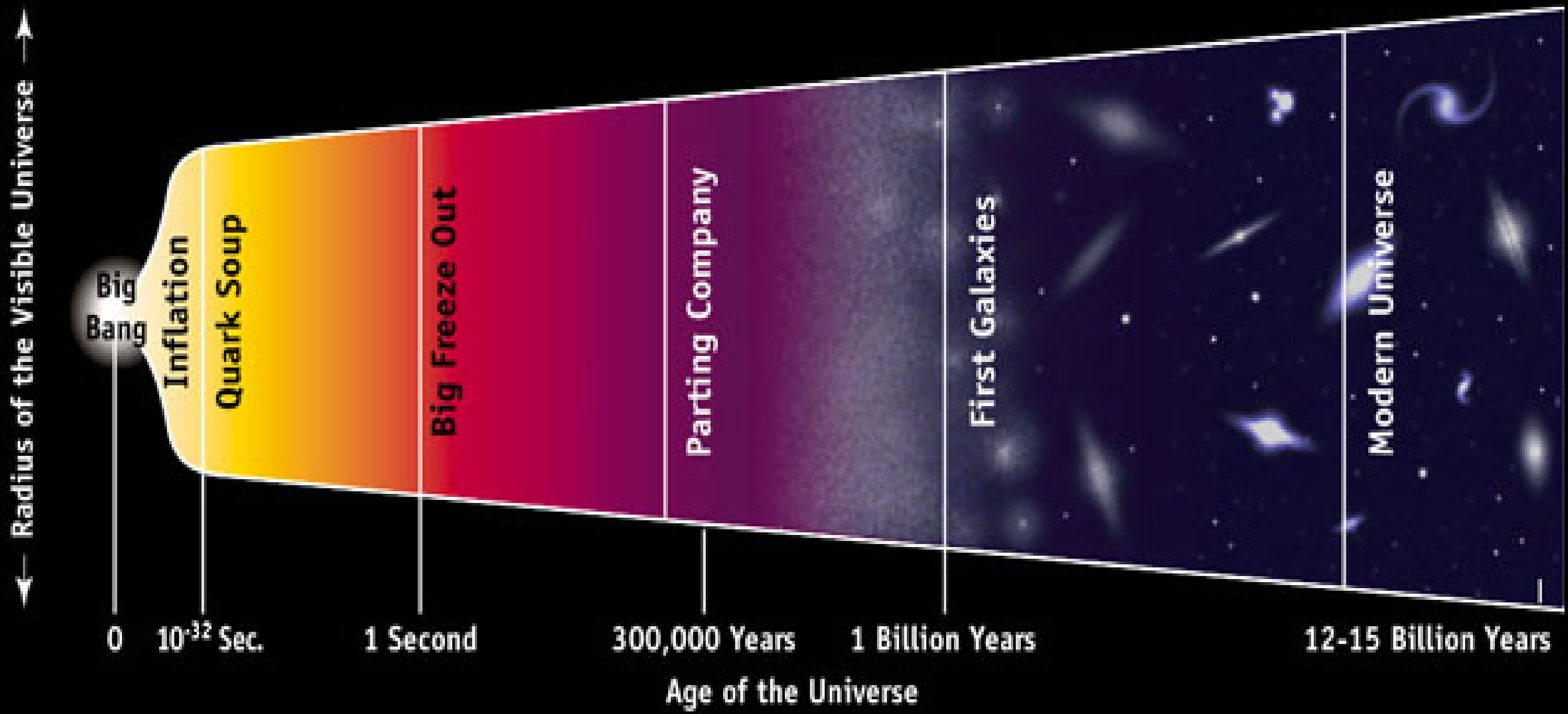
– physics of inflation

– and more…





History of the Universe



Credits

- "An introduction to Modern Cosmology" by Andrew Liddle
- Website http://astronomy.susx.ac.uk/~andrewl/cosbook.html
- The WMAP website:

  http://map.gsfc.nasa.gov/index.html

  including links to multimedia and teacher resources
- Many Figures are courtesy of NASA/WMAP Team