Overview

- **Basic Accelerator Principles :**
  - units and equations
  - acceleration concepts
  - storage rings
  - trajectory stability
  - collider concept
  - vacuum requirements
  - synchrotron radiation

→ design parameters for the LHC

Choices for the LHC

- super conducting RF
- R = 2784 meter

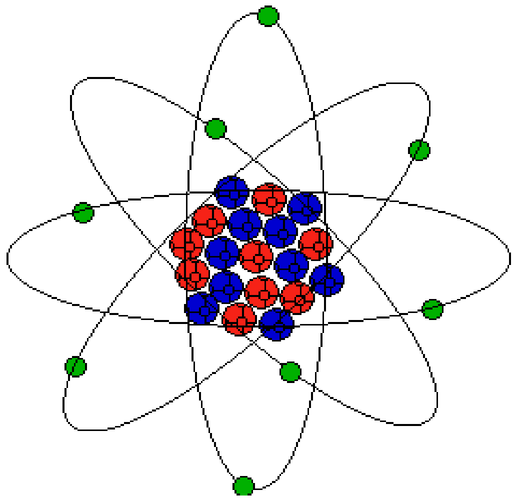
→ B_{max} = 8.38 T → *iron sarturation: 2 Tesla*
*earth: 0.3 * 10^{-4} Tesla*

- super conducting magnet technology
- FODO lattice
- proton collider
- 2 in 1 magnet design
- 2835 bunche with 10^{11} particles per bunch
- high luminosity insertions
- beam screen
- cryo pump at 2K

Search for Elementary Particles

Stage I:

Nuclear Physics



Chronology:

- **1803:** Dalton ⟶ Atom
- **1896:** M & P Currie ⟶ Atoms can decay
- **1896:** Thomson ⟶ Electron
- **1906:** Rutherford ⟶ Nucleus + Electron
- **1911:** Rutherford ⟶ $\alpha + N \longrightarrow O + H^{+}$

⟶ Disintegration of Nuclei!

⟶ Particle Accelerators

Stage II:

Particle Physics

Chronology (Theory):

- 1905: Einstein → $E = mc^2$
- 1930: Dirac → Antimatter
- 1935: Yukawa → π - Meson

Chronology (Experiments):

(Cosmic Rays)

- 1932: Anderson → e^+
- 1937: Anderson → μ

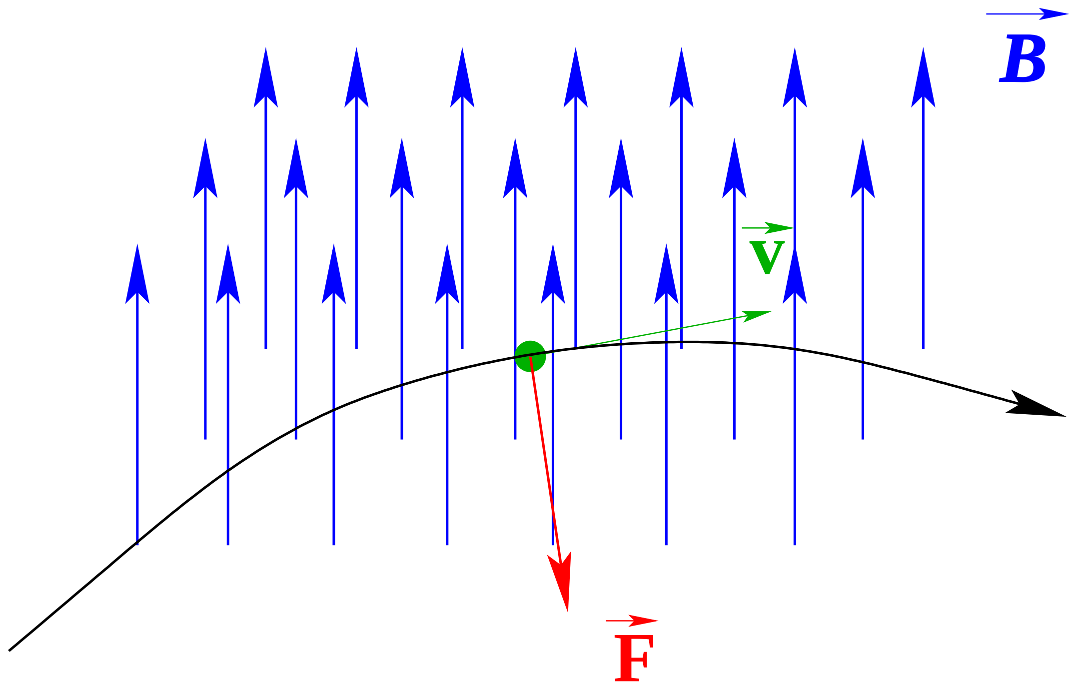
p^-, π } ? → Accelerators

Acceleration Concepts

- ***Lorentz Force:***

$$\frac{d\vec{p}}{dt} = Q * (\vec{E} + \vec{v} \times \vec{B})$$

- ***magnetic fields:***



→ ***Trajectory curvature due to B field!***

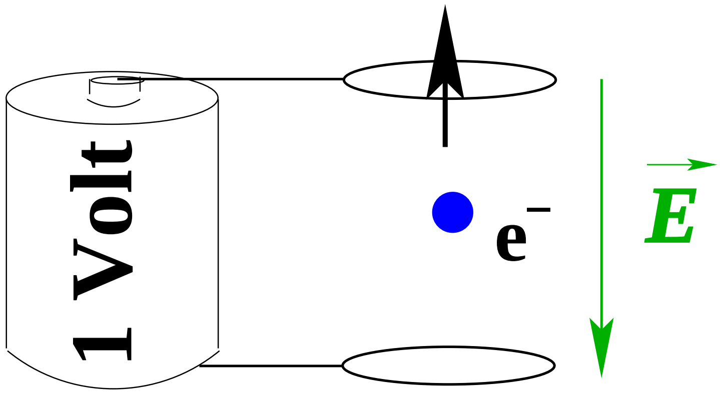
→ ***Energy gain only due to E field!***

Units

Energy Gain:

1 eV

→ $(1.6 * 10^{-19}\ J)$



Common Units:

keV, MeV, GeV, TeV

$(10^3,\ 10^6,\ 10^9,\ 10^{12})$

Total Particle Energy:

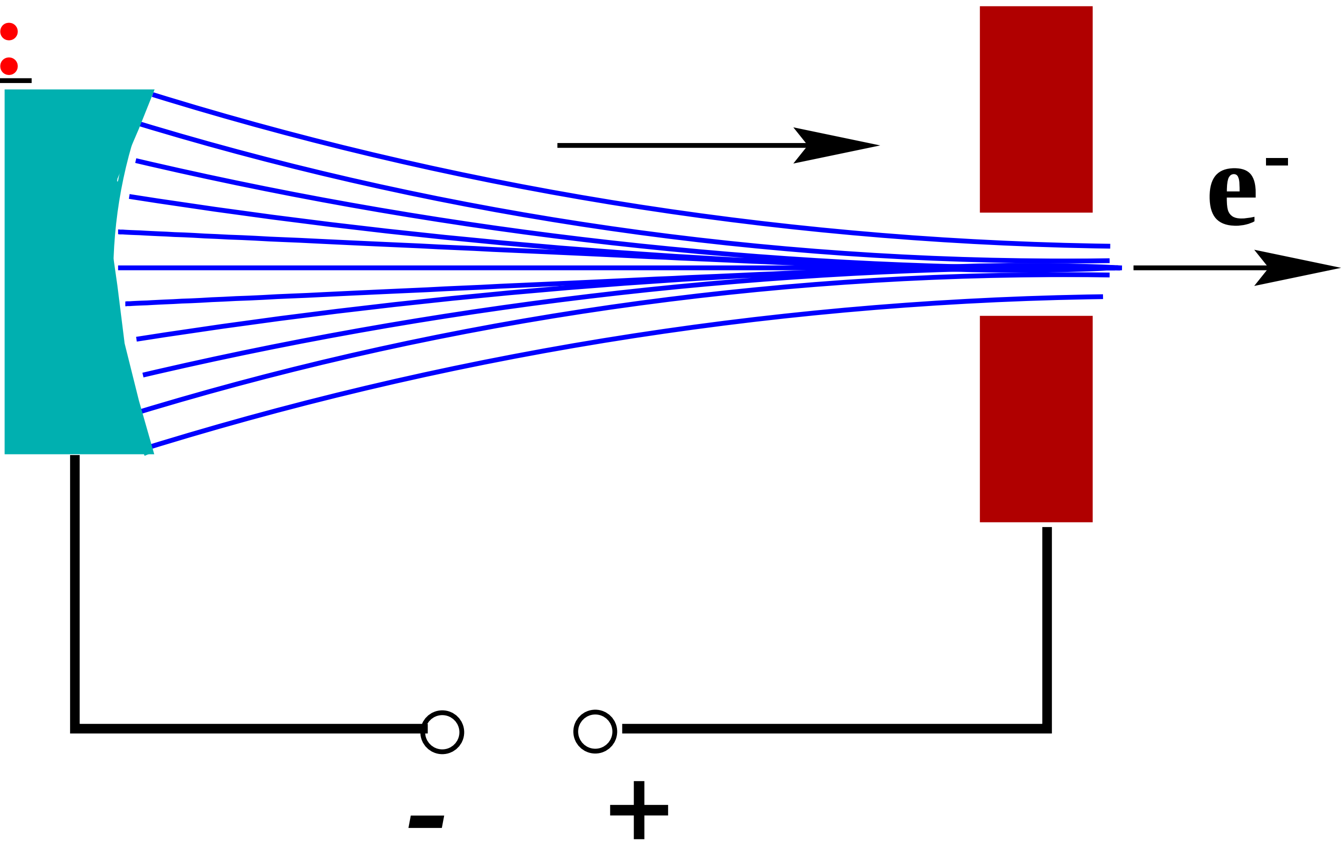
- **Relativity:** $E = mc^2 ;\ m = \gamma * m_0$

$$\gamma = 1/\sqrt{1-\beta^2};\quad \beta = v/c$$

electron: 0.51 MeV *proton:* 0.94 GeV

Particle Sources:

e^-:



Cathode Rays

p^+: *Cathode Tube with* **H**

$$H_2 + e^- \rightarrow H_2^+ + 2e^-$$

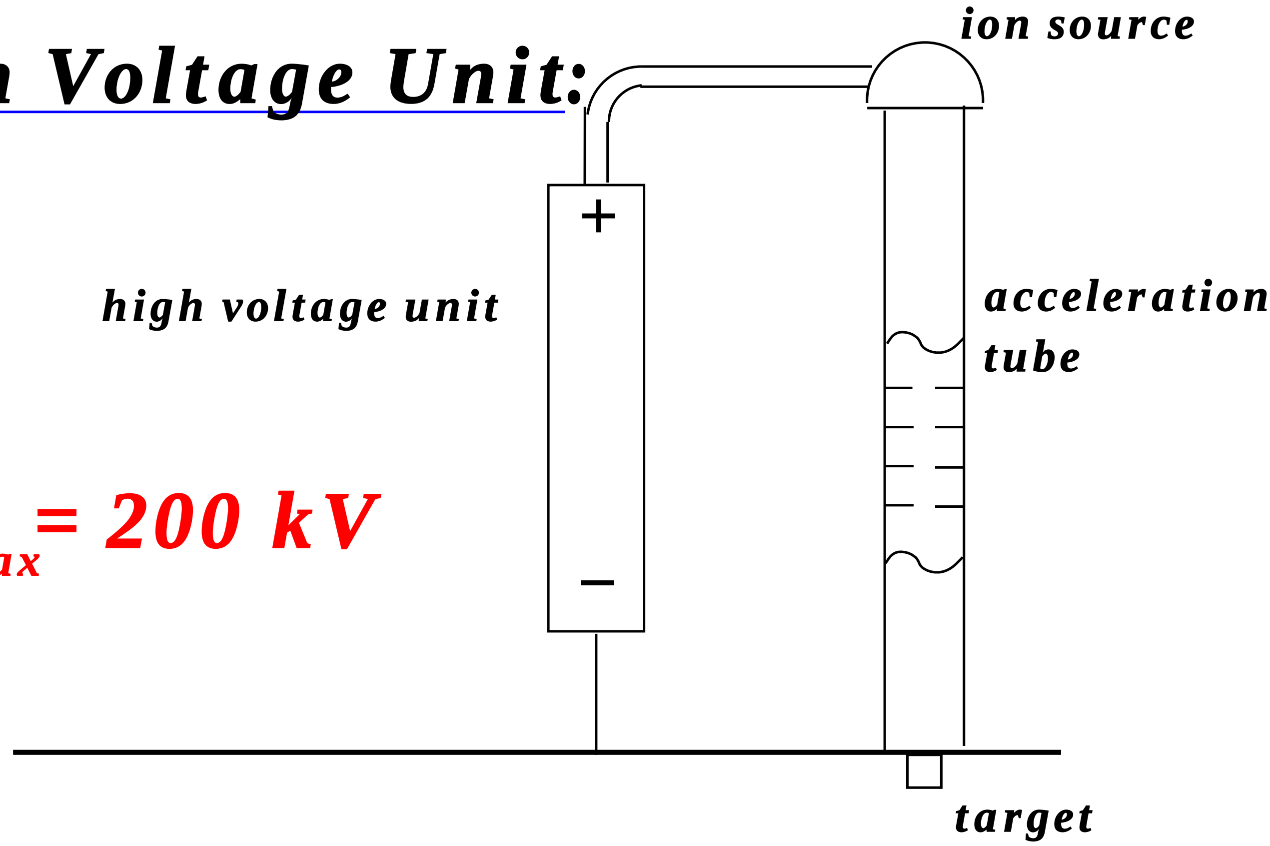
$$H_2^+ + e^- \rightarrow H^+ + H + e^-$$

$$H + e^- \rightarrow H^+ + 2e^-$$

Antimatter: ***Pair Production***

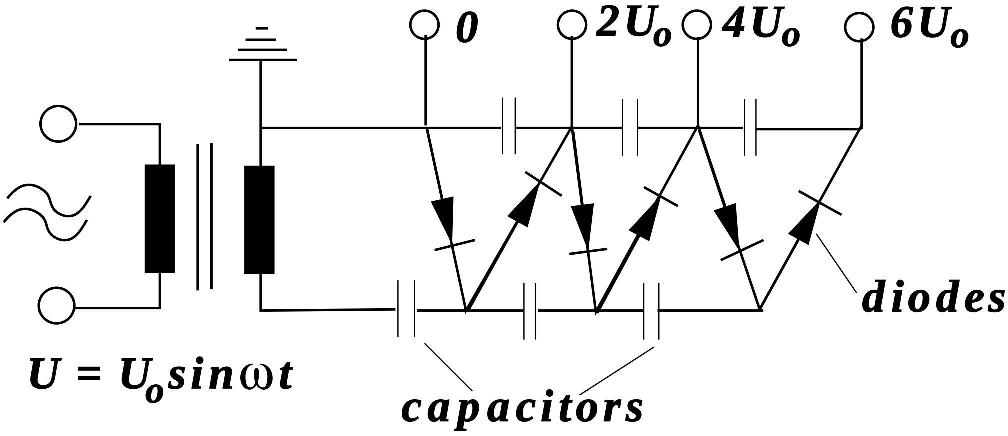
Electrostatic Fields

High Voltage Unit:



$V_{max} = 200\ kV$

Cascade Generator:

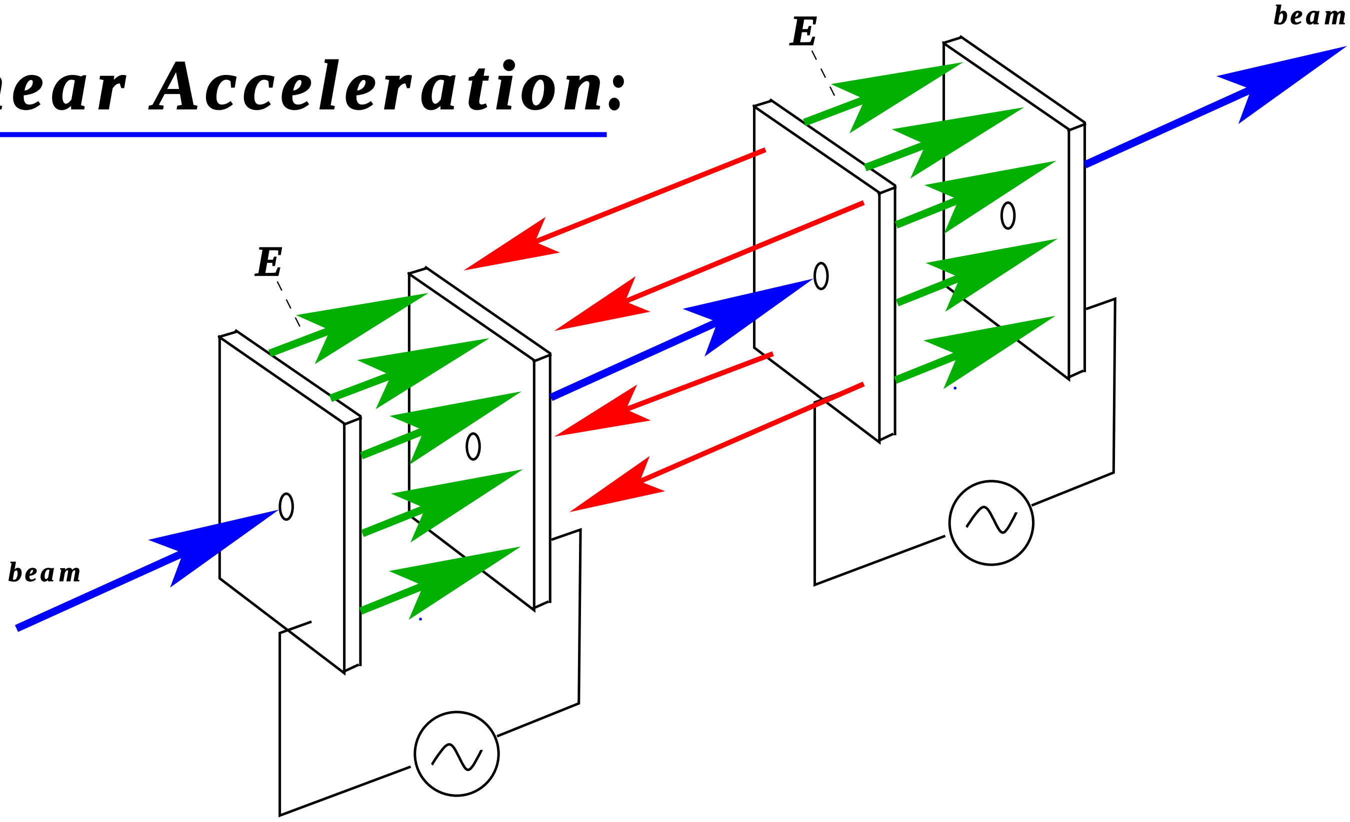


- **1928:** Cockroft + Walton 800kV
- **1932:** $p + Li \rightarrow 2\ He$ 700kV (p)

(Nobel Prize 1951)

Time Varying Fields

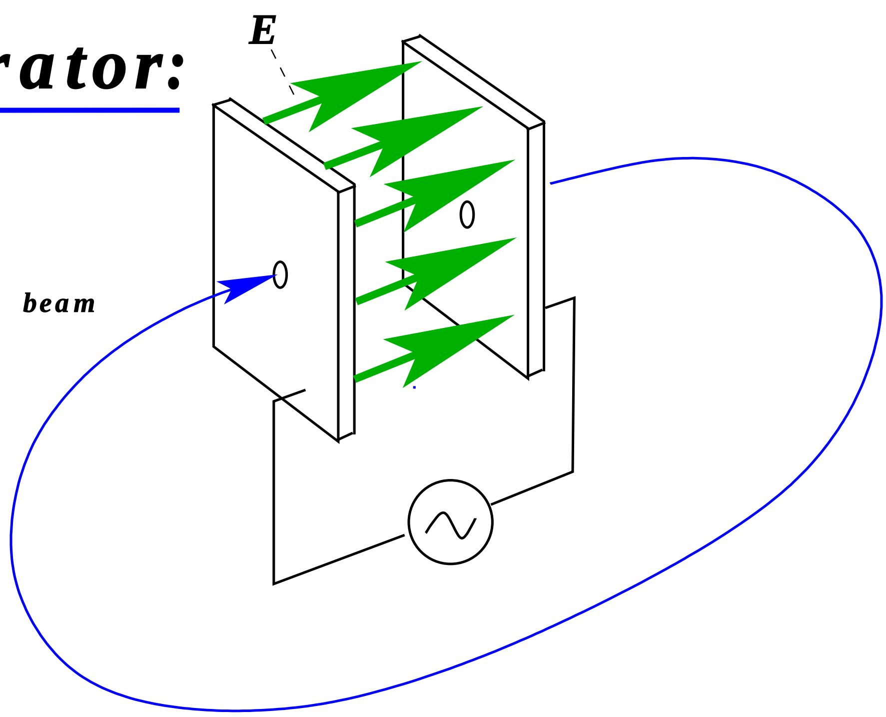
Linear Acceleration:



→ **bunched beam**

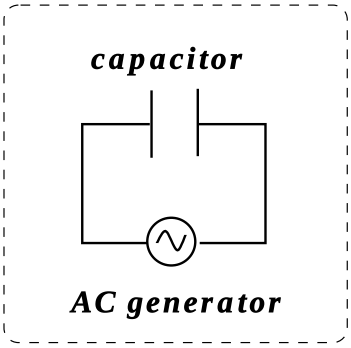
→ **long accelerator!**

Circular Accelerator:

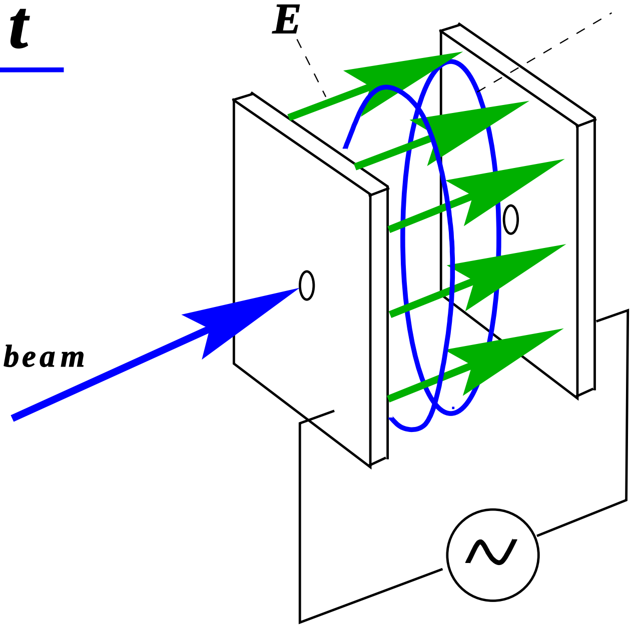


Time Varying Fields

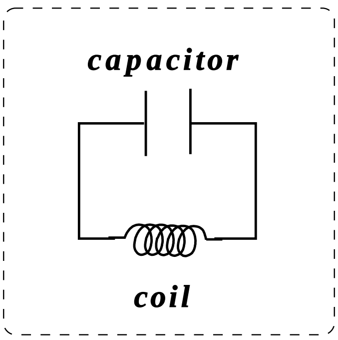
- $$E = -\frac{1}{c}\frac{\partial A}{\partial t}$$



$$\text{rot } B = \frac{\mu\varepsilon}{c}\frac{\partial E}{\partial t}$$

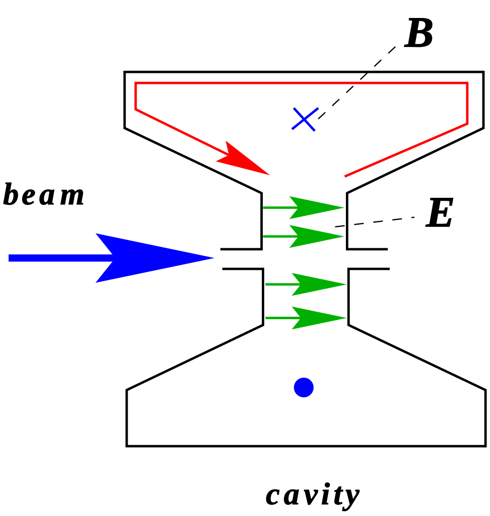


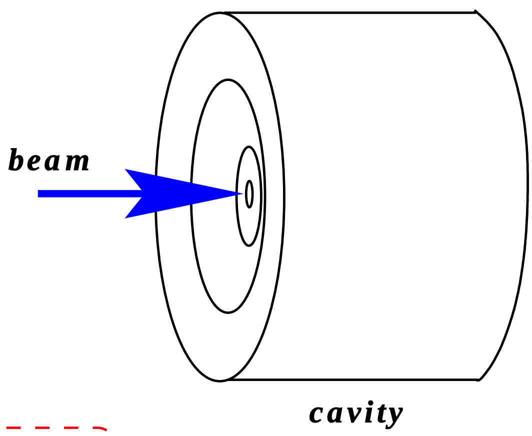
- **Resonator:**



$$L = \frac{\mu_0 \cdot N^2 \cdot A}{l}$$

$$C = \frac{\varepsilon_0 \cdot A}{d}$$





→ **f; Q; R**

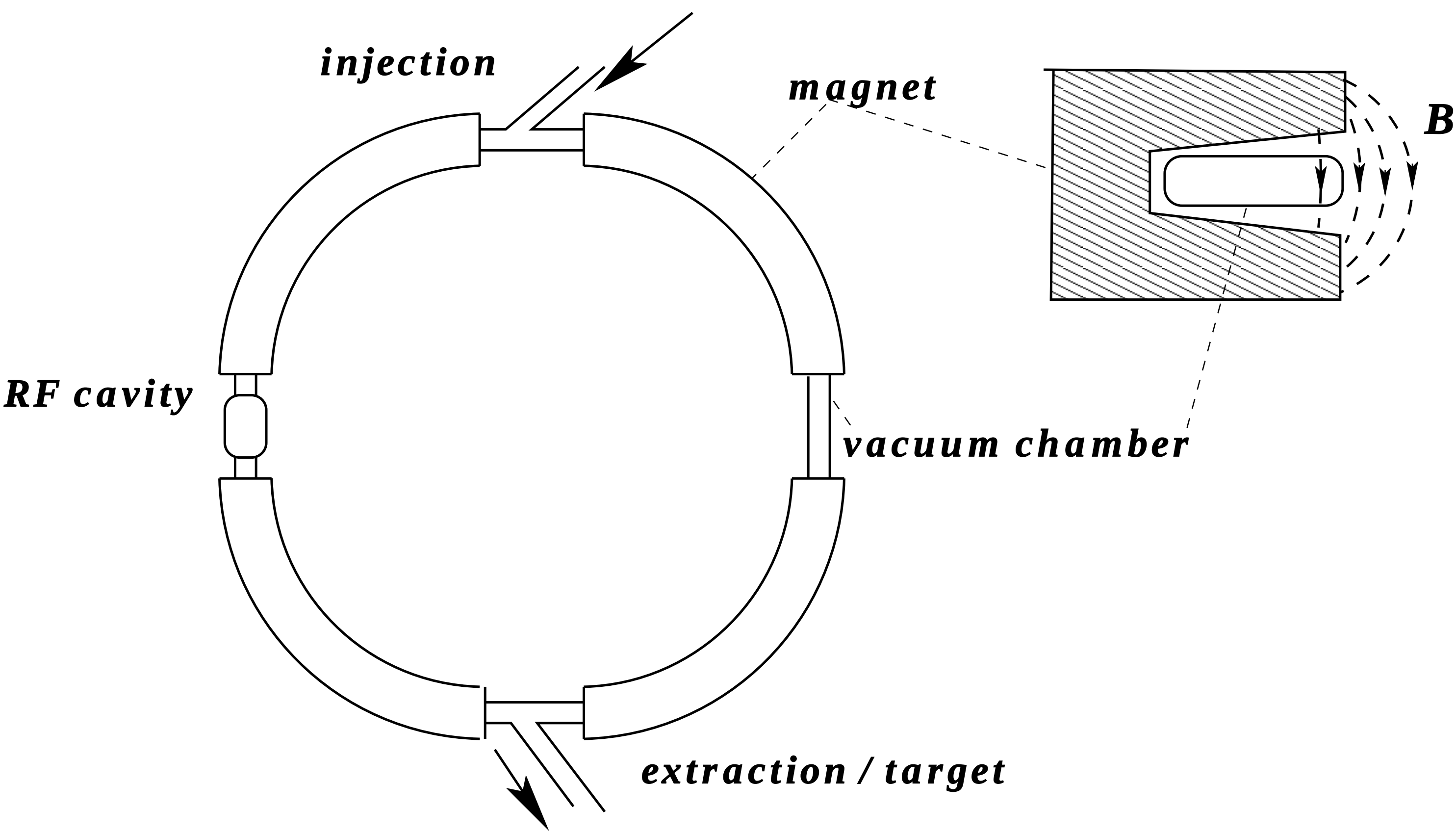
Circular Accelerators

Synchrotron: $R = const.$

$$\omega_0 = \frac{Q}{m_0} \cdot \frac{B}{\gamma}$$

(LHC/LEP: $\omega_0 = 11.3 kHz$)

$$r = \frac{m_0}{Q} \cdot \frac{\gamma}{B} \cdot v \longrightarrow B \neq const.$$



Why 8.4 Tesla?

- ***Synchrotron:*** $R = const.$

$$r = \frac{m_0}{Q} \cdot \frac{\gamma}{B} \cdot v \longrightarrow B \propto \gamma$$

$$\longrightarrow \quad B[T] = \frac{1}{0.3} \cdot \frac{p[GeV/c]}{R[meter]}$$

- ***Physics:*** → p = 7000 GeV/c
- ***LEP tunel:*** L = 27000 meter

→ arcs: L = 22200 meter

→ R = 3500 meter

- **Bending and Focusing:** → R = 2784 meter

→ $B_{max} = 8.38\ T$ → *iron sarturation: 2 Tesla*
*earth: 0.3 * 10^{-4} Tesla*

Power Consumption

- ***LEP:***

B = 0.135 Tesla

$$\boldsymbol{P = R \cdot I^2}$$

I = 4500A; R = 1mΩ → P = 20 kW / magnet

ca. 500 magnets → P = 10 MW

- ***LHC:***

$$\boldsymbol{B \propto I}$$

→ $\mathbf{B_{max} = 8.38\ T}$ → I = 280000 A

→ P = 78 MW / magnet

ca. 500 magnets → P > 39 GW

→ ***superconducting technology!***

8.4 T is at the limit of available technology!

Trajectory Stability

Vertical Plane:

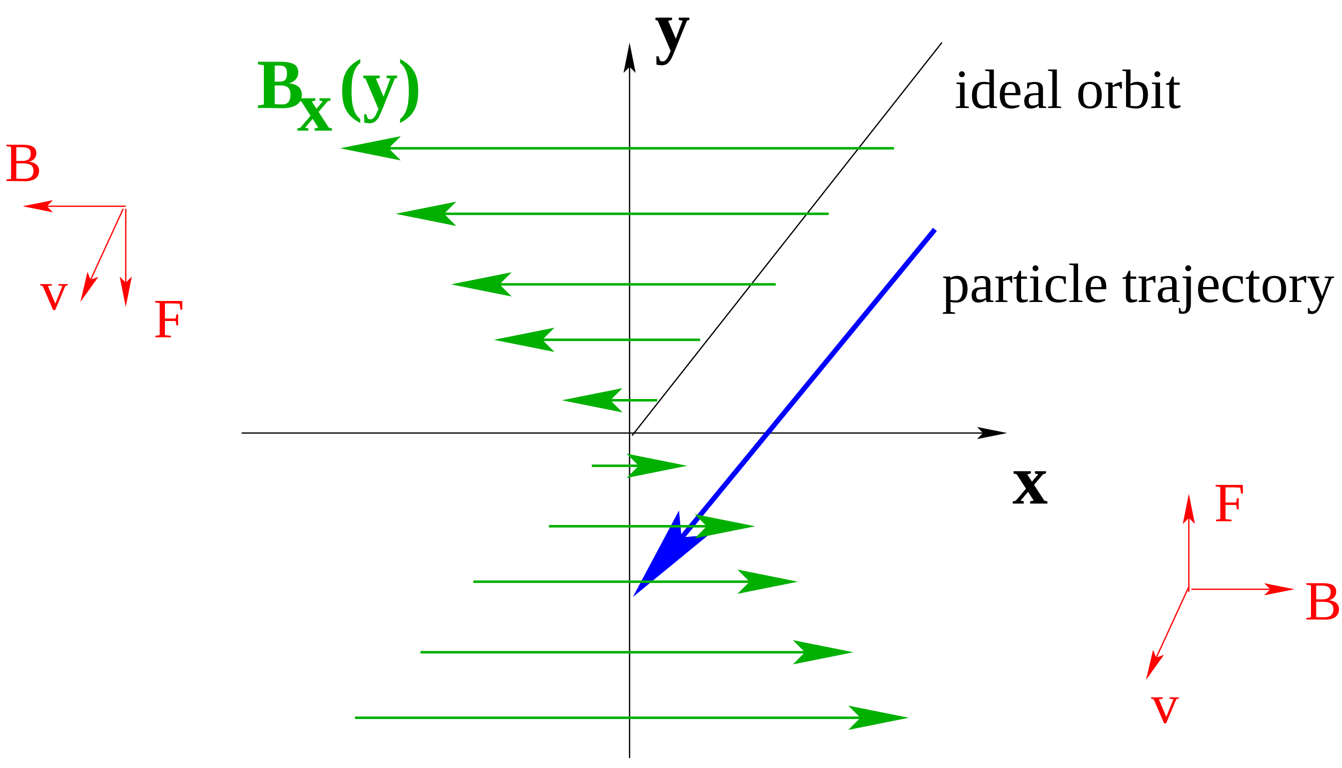
- gravitation: $\Delta s = \frac{1}{2} \cdot g \cdot \Delta t^2$

$g = 10 \cdot m \cdot s^{-2}$

$\Delta s = 18\ mm$ $\Delta t = 60\ msec$

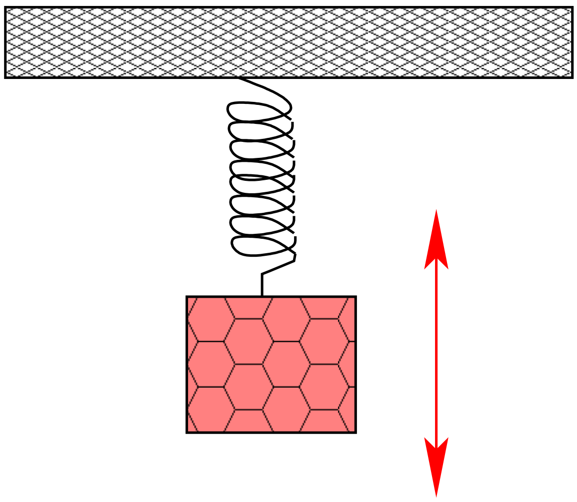
→ 660 Turns!

→ requires focusing!



Strong Focusing

oscillator (spring):



$$F = -g \cdot y$$

$$\Omega^2 \propto g$$

$$A \propto \frac{1}{g}$$

for a fixed energy

strong focusing:

→ **small amplitudes**

→ **small vacuum chamber**

→ **efficient magnets**

→ **high oscillation frequency**

Quadrupole Focusing

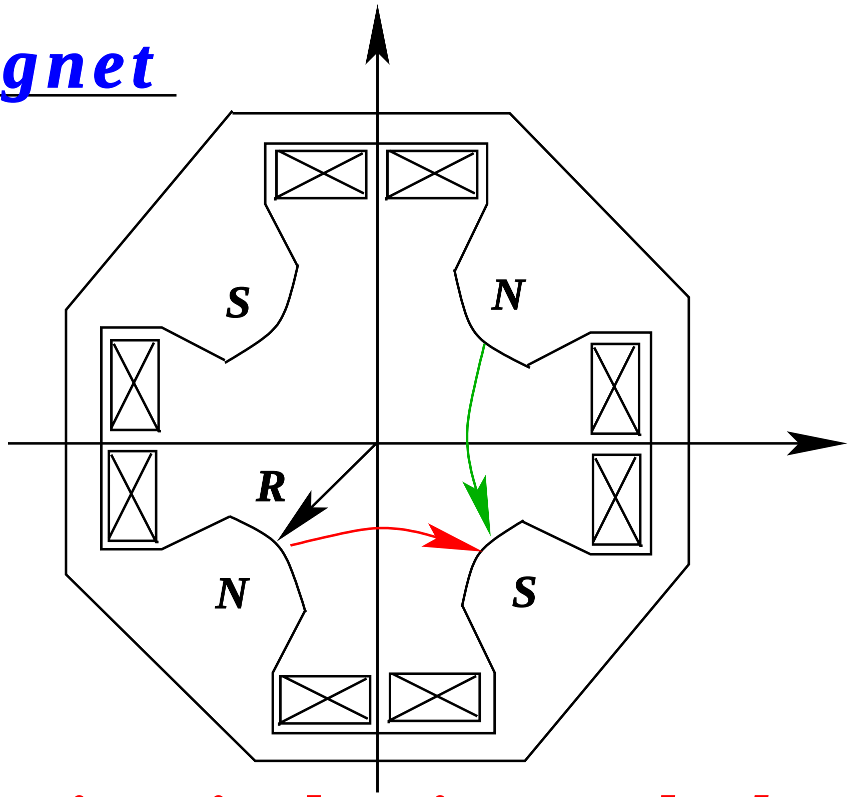
Quadrupole Magnet

$$B_x = -g \cdot y$$

$$B_y = -g \cdot x$$

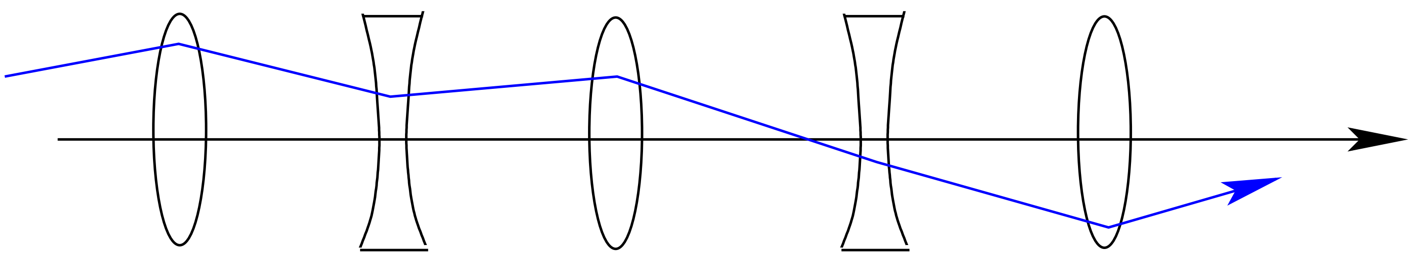
$$F_x = g \cdot x$$

$$F_y = -g \cdot y$$

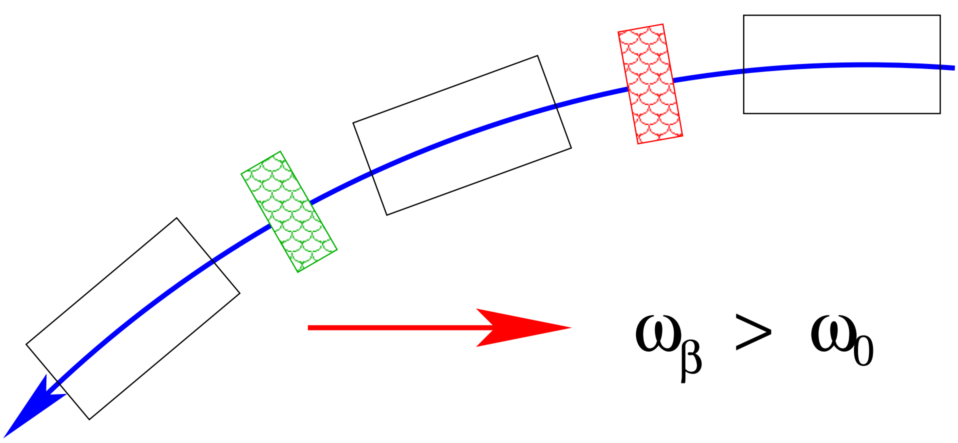


→ defocusing in horizontal plane!

Alternate Gradient Focusing



Idea: cut the arc sections in focusing and defocusing elements



→ $\omega_\beta > \omega_0$

Storage Ring

Tune:

$$Q = \frac{\text{number of oscillations}}{\text{turn}}$$

→ Q_x ; Q_y ; Q_s

Envelope Function:

- $$y(s) = \sqrt{A \cdot \beta} \cdot \sin\left(\frac{2\pi}{L} \cdot Q \cdot s + \phi_0 \right)$$

A: amplitude term due to injector

β: amplitude term due to focusing

L: sorage ring circumference

- $$\beta(s + L) = \beta(s)$$

- $$Q = \frac{1}{2\pi} \cdot \oint \frac{1}{\beta(s)} \, ds$$

Circular Accelerators

- ***uniform B−field:*** ***R = const.***

$$p = Q \bullet \frac{B \bullet L}{2\pi}$$

$$\approx E / c \qquad \text{for } E >> E_0$$

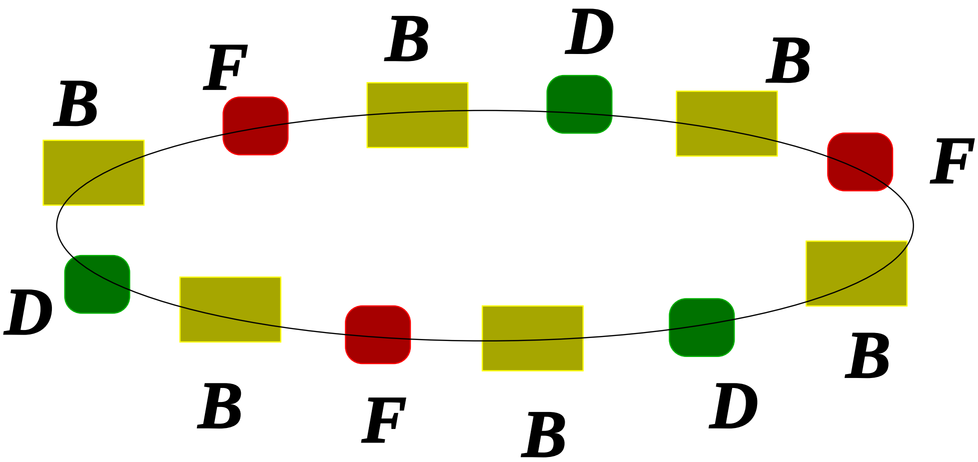
- ***realistic synchrotron:***

B−field is not uniform:
- −drift space for installation
- −different types of magnets
- −space for experiments etc

$$E = \frac{Q \bullet c}{2\pi} \bullet \oint \vec{B} \bullet d\vec{l}$$

→ high beam energy requires:
- −high magnetic field
- −large packing factor 'F'

Closed Orbit



$$B_x = -g \cdot y$$

$$B_y = -g \cdot x$$

Orbit Offset in Quadrupole:

$$x = x_0 + \tilde{x}$$

quadrupole

$$B_x = -g \cdot \tilde{y}$$

$$B_y = -g \cdot x_0 - g \cdot \tilde{x}$$

dipole component

→ orbit error

Sources for Orbit Errors

- Alignment: +/− 0.1 mm

- Ground motion
  - slow drift
  - civilisation
  - moon
  - seasons
  - civil engineering

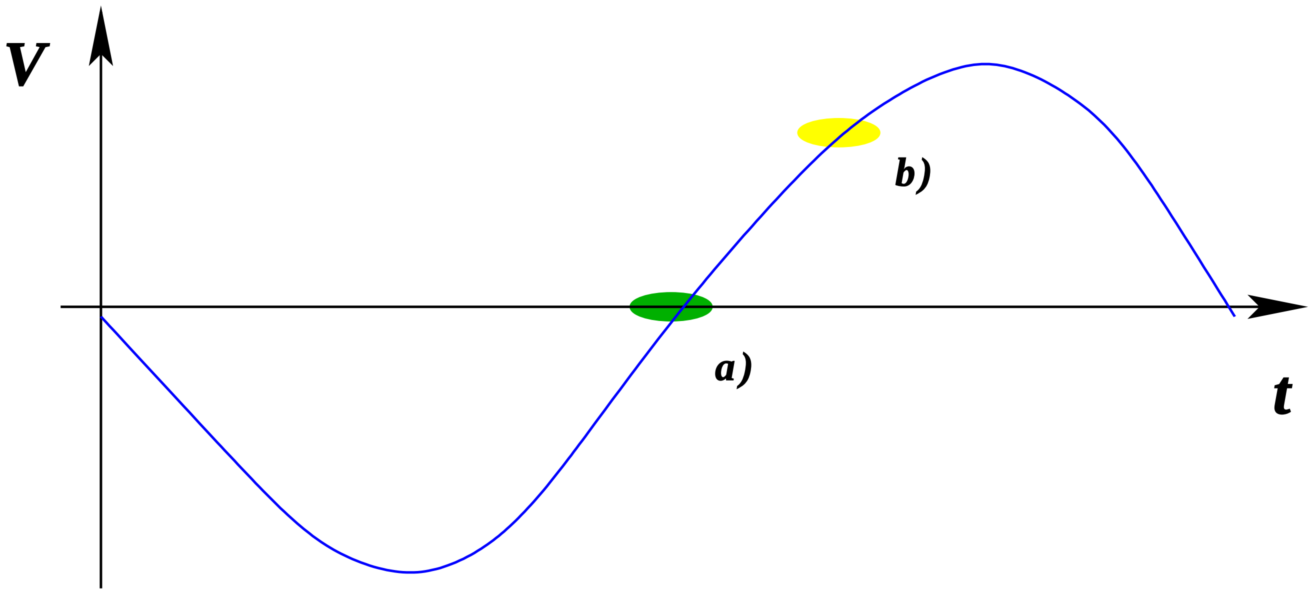
- Error in dipole strength
  - power supplies
  - calibration

- Energy error of particles

Synchrotron:

→ the orbit determines the particle energy!

assume: L > design orbit



→ energy increase

Equilibrium:

$$f_{RF} = h \cdot f_{rev}$$

$$f_{rev} = \frac{1}{2\cdot\pi} \cdot \frac{q}{m\cdot\gamma} \cdot B$$

→ E depends on orbit and magnetic field!

momentum compaction factor:

- ***increase particle energy***

→ *velocity increase*

shorter revolution time

→ *momentum increase*

longer revolution time

- ***transition energy***

$$\frac{\Delta R}{R} = \alpha \cdot \frac{\Delta p}{p}$$

$$\alpha = \frac{1}{\gamma_t^2} \qquad \alpha \approx \frac{1}{Q^2}$$

→ ***E error depends on transition energy!***

Collider Rings

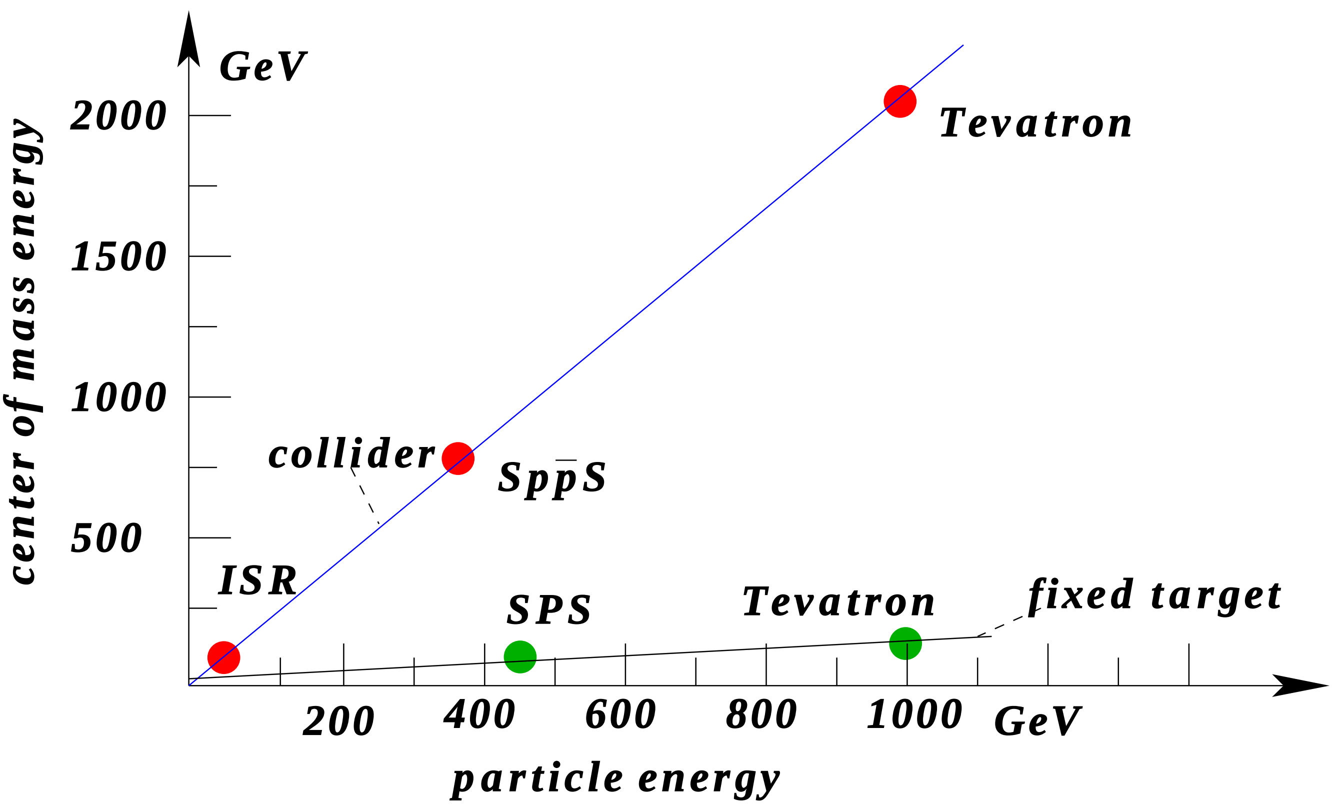
→ **1960:** *fixed target physics* *(bubble chamber)*

- **But:**

$$E_{cm} = 2 \cdot m_0 \cdot c^2 \left(\sqrt{1 + \frac{E}{2 \cdot m_0 \cdot c^2}} - 1 \right)$$

- **Collider:**

$$E_{CM} = 2 \cdot E_p$$



1960 ↗: e^+ / e^- collider

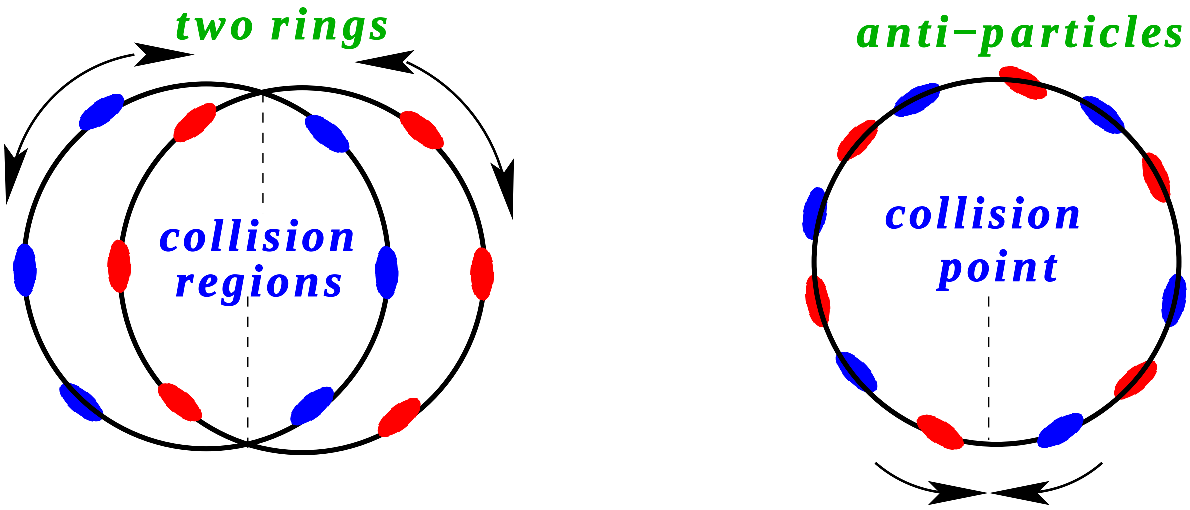
1970 ↗: p^+ / p^- collider

Features (+/−)

- not all particles collide in one crossing

→ long storage times

- requires 2 beams:



→ anti−particles hard to produc

- beam−beam interaction

→ requires beam separation

Lepton versus Hadron Collider

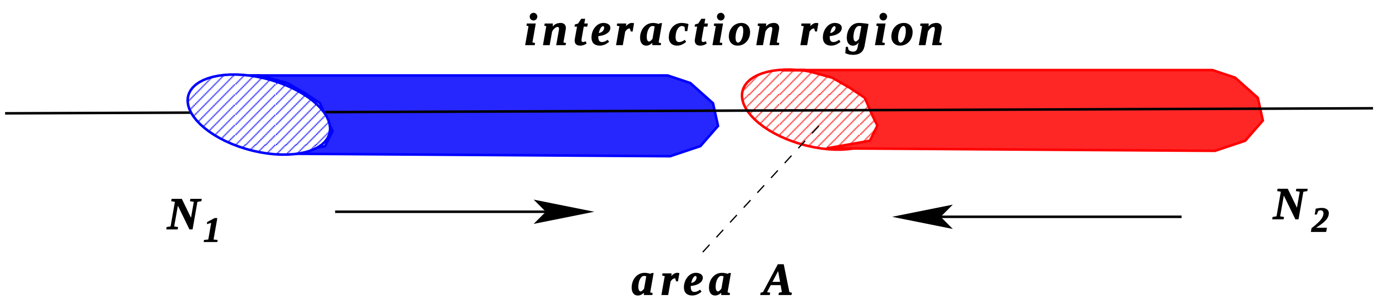
- **Leptons:** (e^+ / $\bar{e}$)
  - **elementary particles**
    - → well defined energy
    - → precision experiments
- **Hadrons:** (p^+ / $\bar{p}$)
  - **multi particle collisions**
    - → energy spread
    - → discovery potential
- **Example:**

Z_0

1985	$Sp\bar{p}S$	p^+p^-
1990	LEP	e^+e^-

Luminosity

- $N_{ev} / sec = \sigma \cdot L \qquad [L] = cm^{-2} \cdot s^{-1}$



$$L = \frac{n_b \cdot N_1 \cdot N_2 \cdot f_{rev}}{A}$$

- **high bunch current**

  beam−beam; collective effects

- **many bunches**

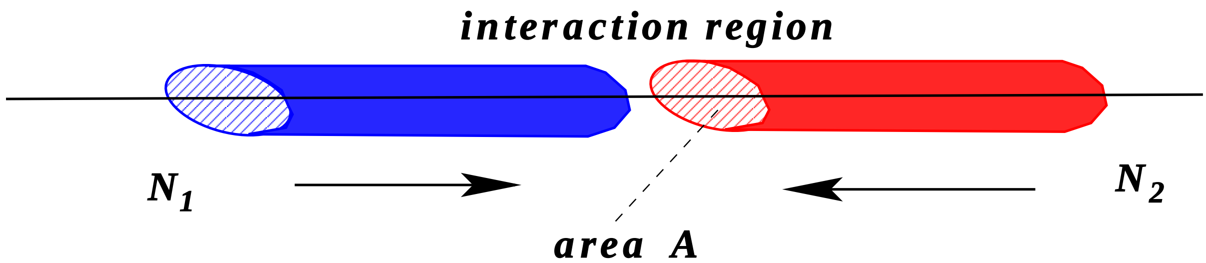
  total current (RF); collective effects

- **small beam size**

  coupling; dispersion; hardware

Beam Size

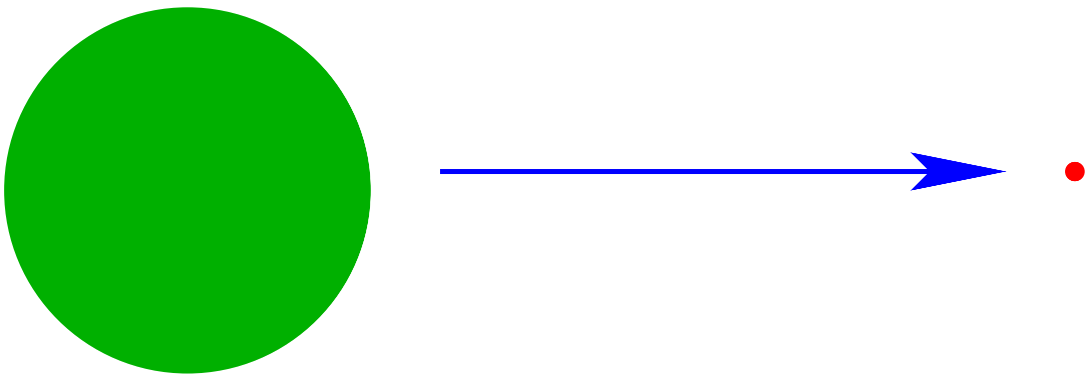
Luminosity:



$$L = \frac{n_b \cdot N_1 \cdot N_2 \cdot f_{rev}}{A}$$

$$A = \pi \cdot \beta \cdot \varepsilon$$

LHC:



$\langle\beta\rangle_{arc} = 80$ meter

$\beta_{IP} = 0.5$ meter

Limit:

- magnet strength
- aperture

$$x = \sqrt{A \cdot \beta} \cdot \sin(\phi)$$

$$x' = \sqrt{\frac{A}{\beta}} \cdot \sin(\phi)$$

Synchrotron Radiation

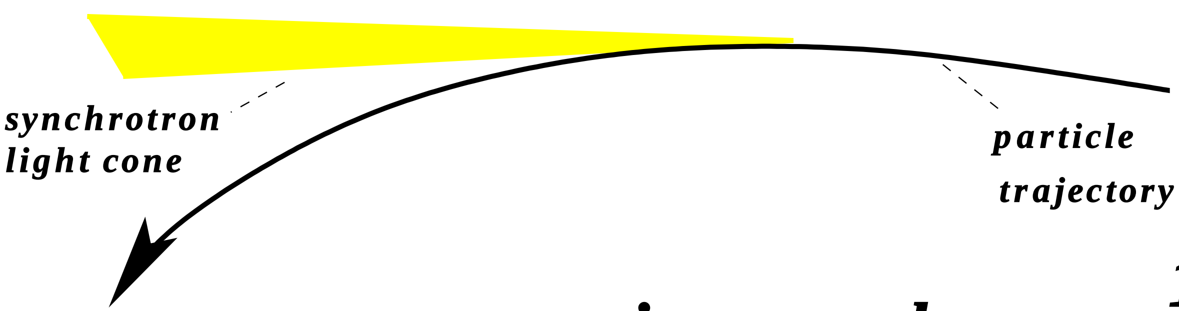
Electro−Magnetic Waves :

- accelerated charge emits electro−magnetic waves

  → radio signal

  → X−rays

- radiation fan in bending plane
  bending plane



opening angle $\propto \frac{1}{\gamma}$

(LEP: $\gamma = 200000$
LHC: $\gamma = 7000$)

- $P \propto \frac{\gamma^4}{\rho^2}$

- $<E_\gamma> \propto \frac{\gamma^3}{\rho}$

Examples

	E [GeV]	ρ [km]	N [10^{12}]	U [MeV]	P [MW]	u_c [keV]
LEP 1	45	3.1	4.7	260	1.2	90
LEP 2	100	3.1	4.7	2900	30	715
LEP2+	110	3.1	312	3900	44	952
LHC	7000	3.1	312	0.007	0.005	0.04

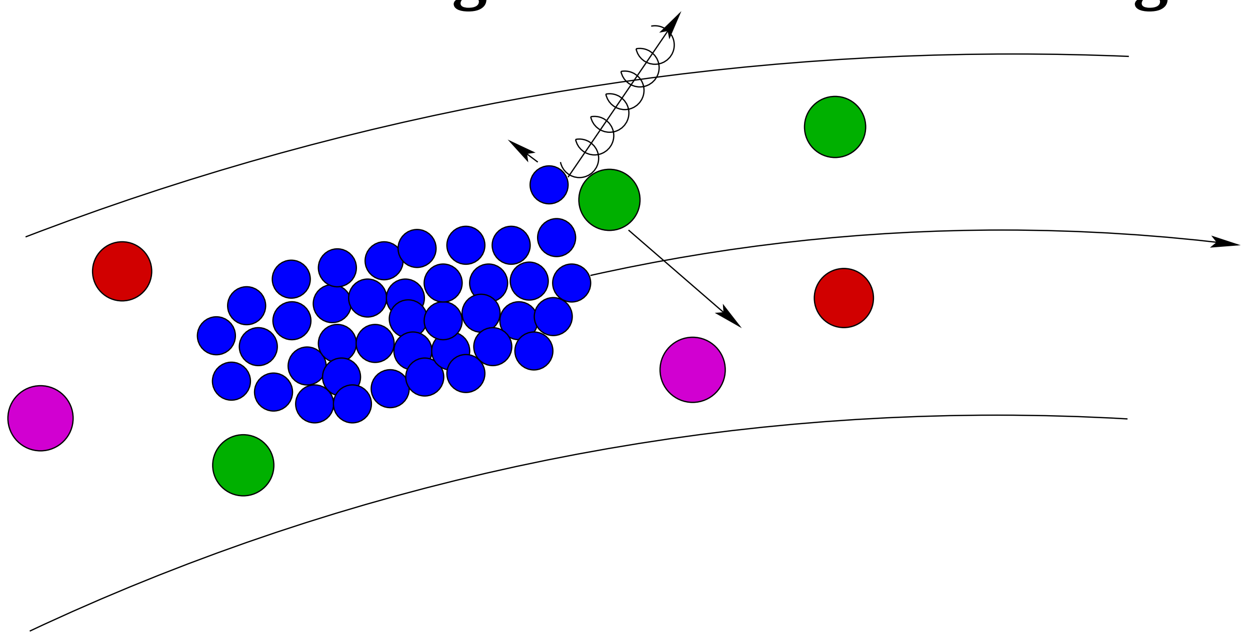
LEP 1 → X−rays

LEP 2 → γ −rays

LHC → UV light

Vacuum

Bremsstrahlung + Coulomb Scattering



beam blow−up

particle loss

background in experiments

loss in luminosity!

equipment damage!

LHC – Beam Parameter

$$L = \frac{N_p^2 \cdot n_b}{\varepsilon \cdot \beta} \cdot \frac{f_{rev}}{2 \cdot \pi}$$

$L = 10^{34} cm^{-2} s^{-1}$

- **Beam–Beam Interaction:**

$$\Delta Q \propto \frac{N_b}{\varepsilon} < 5 \cdot 10^{-3}$$

→ $N_p = 10^{11}$

- **Beam Size:**

magnet quality + aperture → ε

- **β:** quadrupole strength + aperture

→ $\beta = 0.5$ meter

→ $n_b = 2835$

→ $I_{beam} = 0.5\ A$

Beam Power

$E = 300\ MJ$

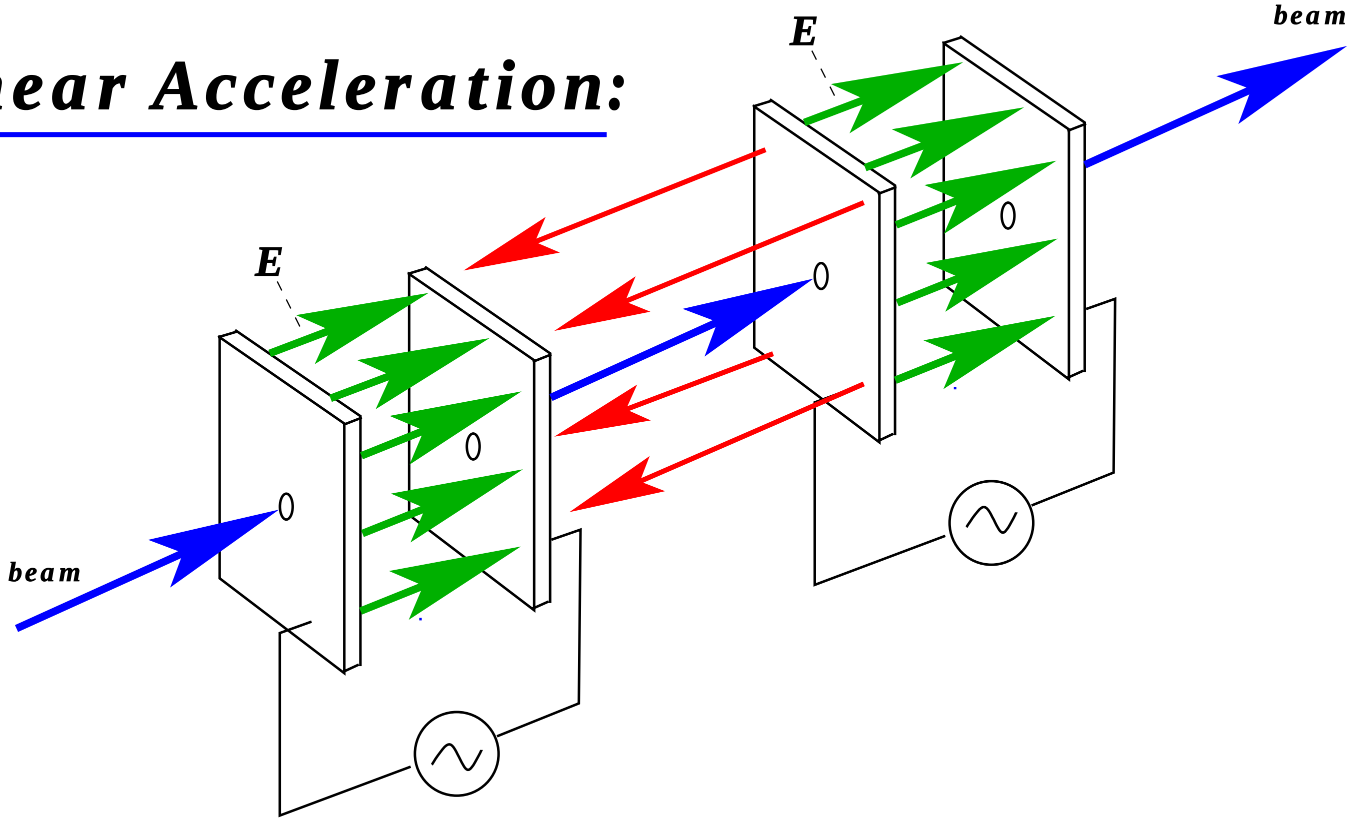
$\hat{=}$ 120 kg TnT

Synchrotron Radiation

$P = 0.5\ W/m$

Time Varying Fields

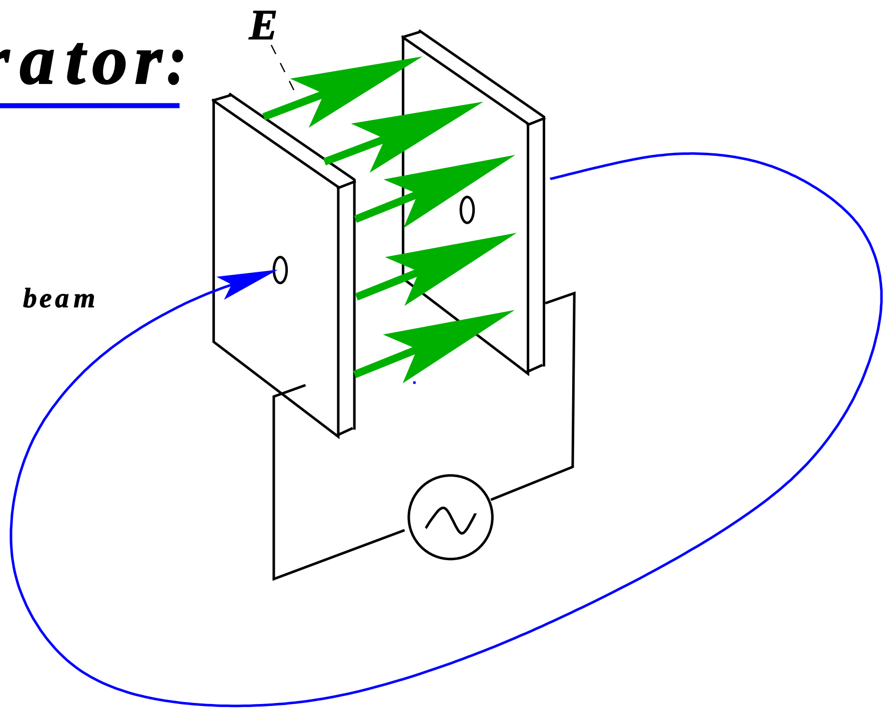
Linear Acceleration:



→ **bunched beam**

→ **long accelerator!**

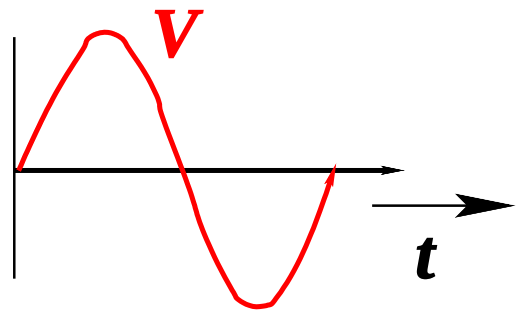
Circular Accelerator:



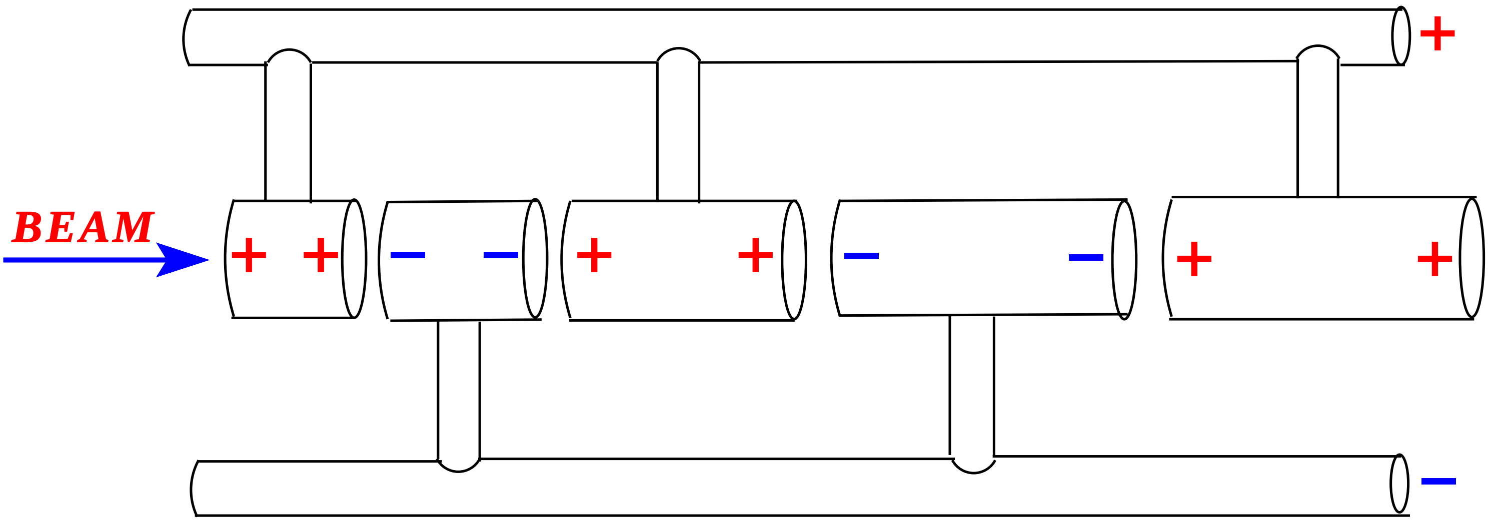
Drift Tubes

- **1924:** Ising

  AC Voltage:



Symmetric line:



→ $l = v_{part} \cdot T/2$

- **1928:** demonstrated by Wideroe

  1MHz, 25kV oscillator

  → 50kV potassium ions

  Lawrance:

  1.3MV mercury ions with 48kV

- **But:** $f < 7$MHz ($l = 21$ meter)!

Time Varying Fields

Maxwell Equations without Sources

a) $\vec{\nabla} * \vec{E} = 0$

b) $\vec{\nabla} \times \vec{E} + \frac{1}{c}\frac{\partial \vec{B}}{\partial t} = 0$

c) $\vec{\nabla} * \vec{B} = 0$

d) $\vec{\nabla} \times \vec{B} - \frac{\mu\varepsilon}{c}\frac{\partial \vec{E}}{\partial t} = 0$

Rotation on b) and d)

plus: $\vec{\nabla} \times (\vec{\nabla} \times \vec{V}) = \vec{\nabla} \cdot (\vec{\nabla} \cdot \vec{V}) - \vec{\nabla} \cdot \vec{V}$

→ ***Wave equation:***

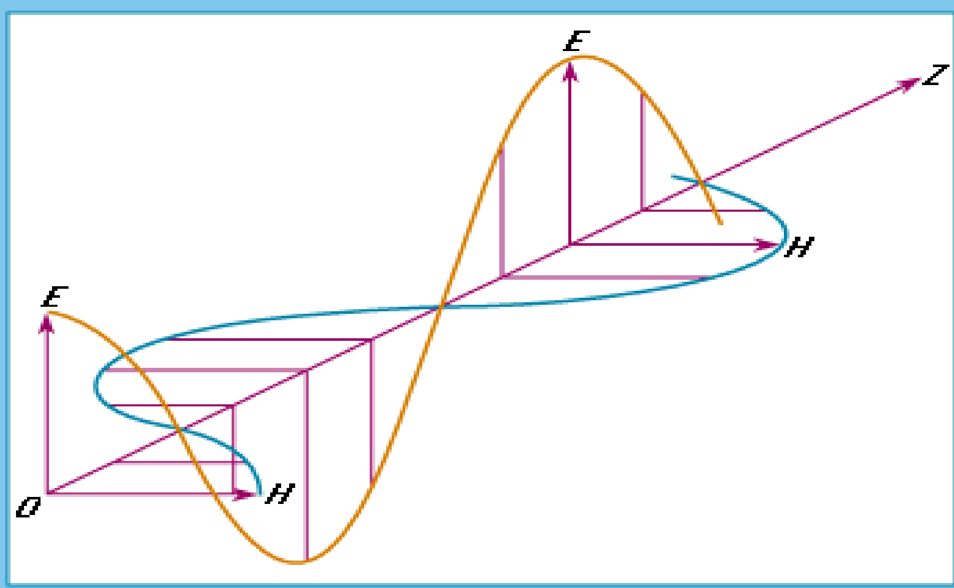
$$\frac{\partial^2 \vec{E}}{\partial t^2} = \frac{c^2}{\mu\varepsilon} \nabla^2 \vec{E} \qquad \frac{\partial^2 \vec{B}}{\partial t^2} = \frac{c^2}{\mu\varepsilon} \nabla^2 \vec{B}$$

Time Varying Fields

Plane Electro Magnetic Wave:

$$\vec{E} = \vec{E}_0 \cdot e^{ik\vec{n}\cdot\vec{x} - \omega t} \qquad \vec{B} = \vec{B}_0 \cdot e^{ik\vec{n}\cdot\vec{x} - \omega t}$$

$$\vec{B}_0 = \sqrt{\mu\varepsilon} \cdot \vec{n} \times \vec{E}_0 \qquad k = \frac{2\pi}{\lambda}$$



→ **No acceleration in the direction of propagation!**

Boundary Conditions I

Transverse Electric Waves (TE):

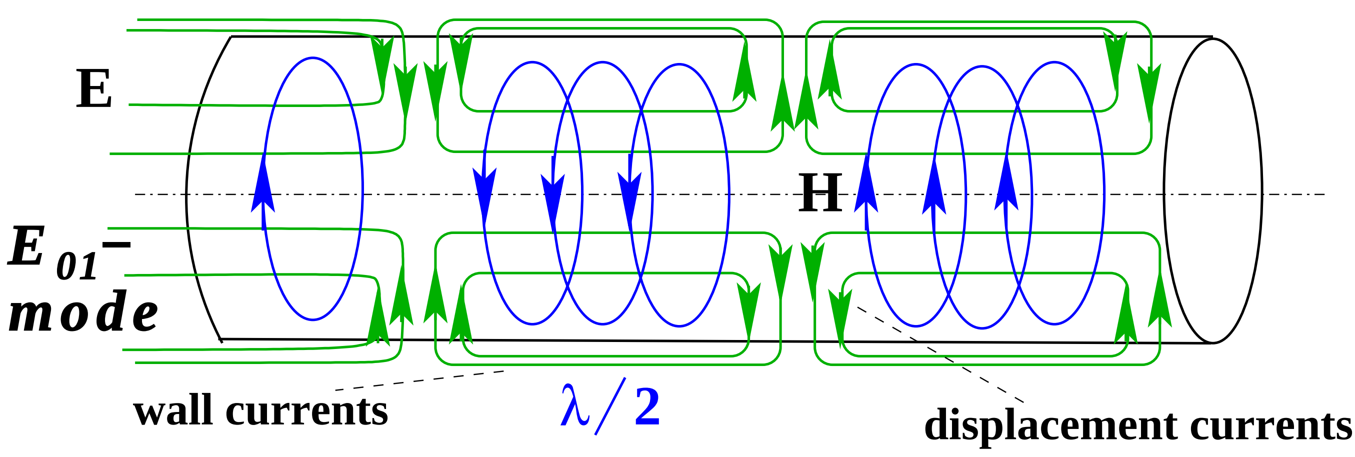
$E_z = 0$ *everywhere;*

Boundary condition: $\left.\frac{\partial B}{\partial n}\right|_s = 0$

Transverse Magnetic Waves (TM):

$B_z = 0$ *everywhere;*

Boundary condition: $\left.E_n\right|_s = 0$

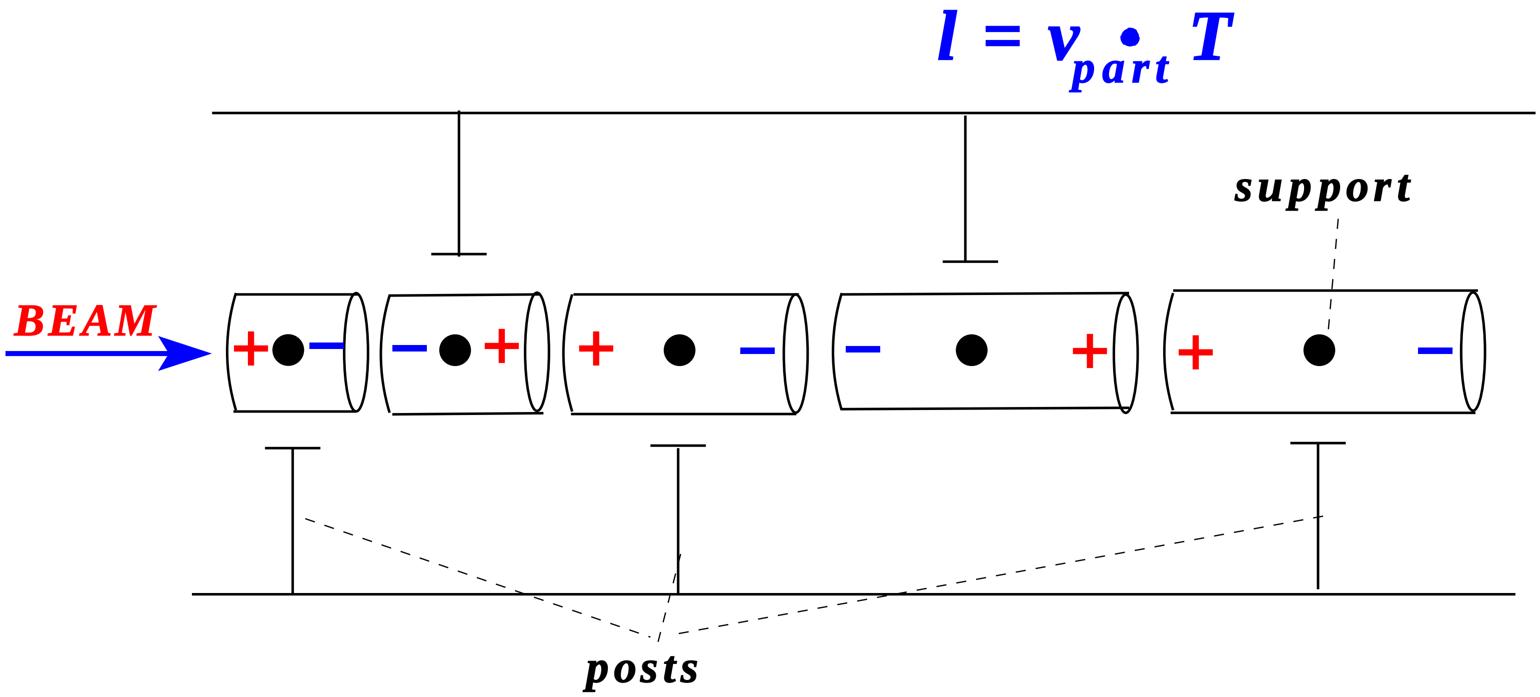


Problem: $v_{ph} > c$

→ **Shielding or change** v_{ph}

Resonance Tank

- **Alvarez:**



- **Tubes are passive**

  → **higher frequencies!**

  (f = 200 MHz gives good tube size)

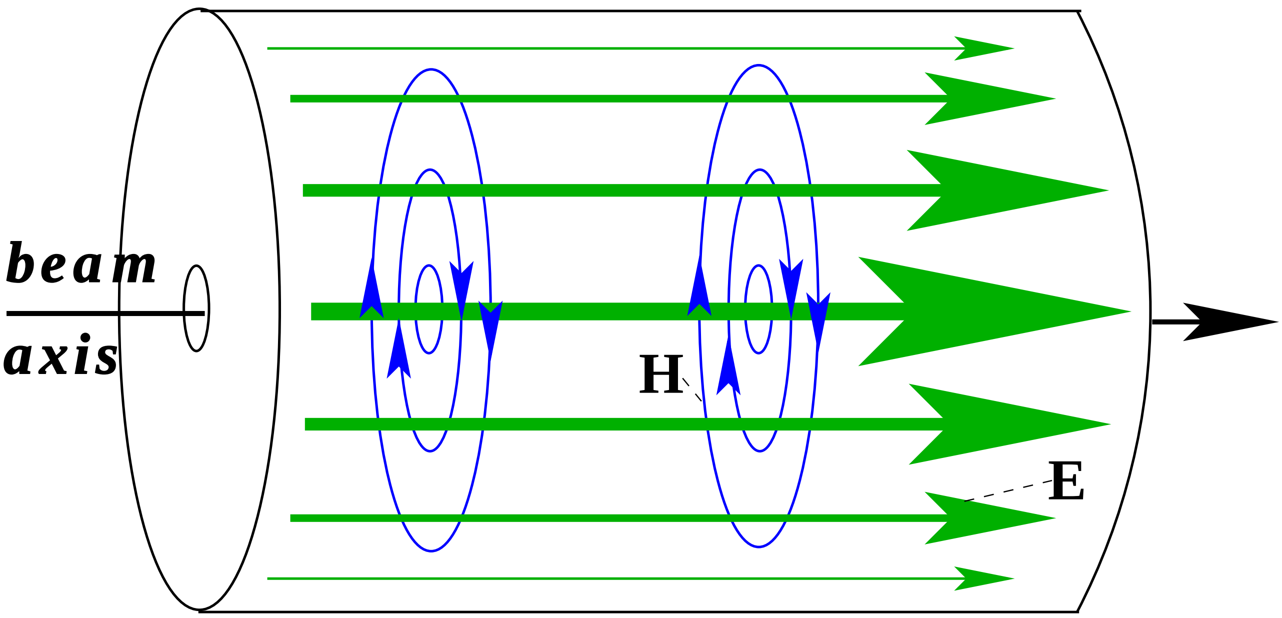
- **Posts** → $v_{gr} \neq 0$

- **Pre−accelerator for most proton acclelerators**

Boundary Conditions II

Cavity Resonator:

TM mode with longitudinal boundary;



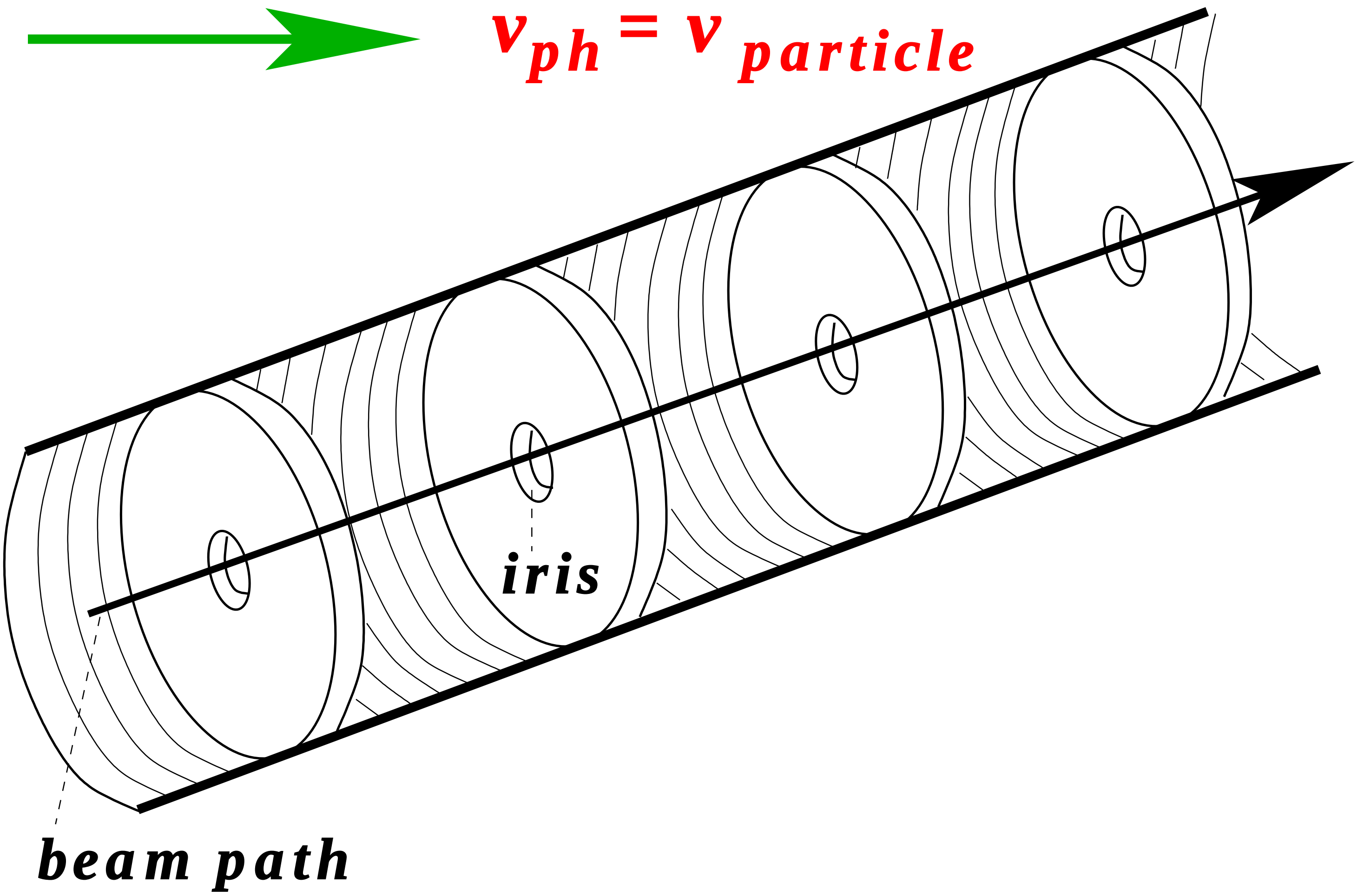
Short Section:

→ multi-cell

→ multi-passage

Boundary Conditions III

Loaded Wave Guide:



But:

Concept of linear acceleration is limited by power of RF generator!

→ Not feasible before World War II