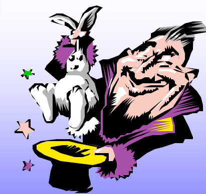
Statistics for HEP

Roger Barlow
Manchester University

Lecture 1: Probability

Definition 1: Mathematical



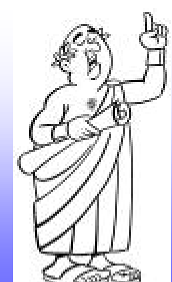
P(A) is a number obeying the Kolmogorov axioms

$$P(A) \geq 0$$
$$P(A_1 \vee A_2) = P(A_1) + P(A_2)$$
$$\sum P(A_i) = 1$$

Problem with Mathematical definition

No information is conveyed by P(A)

Definition 2: Classical



The probability P(A) is a property of an object that determines how often event A happens.

It is given by symmetry for equally-likely outcomes

Outcomes not equally-likely are reduced to equally-likely ones

Examples:

Tossing a coin: P(H)=1/2

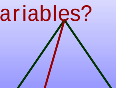
Throwing two dice P(8)=5/36

Problems with the classical definition…

1. When are cases 'equally likely'?
- If you toss two coins, are there 3 possible outcomes or 4?

Can be handled

2. How do you handle continuous variables?
- Split the triangle at random:



Cannot be handled

Bertrand's Paradox

A jug contains 1 glassful of water and between 1 and 2 glasses of wine

Q: What is the most probable wine:water ratio?

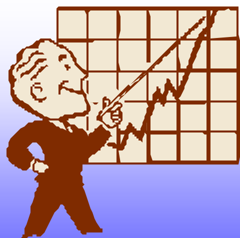
A: Between 1 and 2 →3/2

Q: What is the most probable water:wine ratio?

A: Between 1/1 and 1/2 →3/4

$(3/2)^1 \neq (3/4)^{-1}$

Definition 3: Frequentist



- The probability P(A) is the limit (taken over some ensemble)

$$P(A) \underset{N\to\infty}{=} N(A)/N$$

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Problem (limitation) for the Frequentist definition

P(A) depends on A and the ensemble

Eg: count 10 of a group of 30 with beards.

P(beard)=1/3

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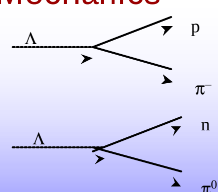
Aside: Consequences for Quantum Mechanics

- QM calculates probabilities
- Probabilities are not 'real' – they depend on the process and the ensemble



PDG:

$P(p\pi^-)=0.639$,

$P(n\pi^0)=0.358$

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Big problem for the Frequentist definition

Cannot be applied to unique events

~~'It will probably rain tomorrow'~~

Is unscientific

`The statement

"It will rain tomorrow"

is probably true.'

Is quite OK

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But that doesn't always work

- Rain prediction in unfamiliar territory
- Euler's theorem
- Higgs discovery
- Dark matter
- LHC completion

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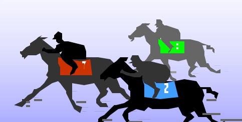
Definition 4: Subjective (Bayesian)



P(A) is your degree of belief in A;

You will accept a bet on A if the odds are better than

1-P to P

A can be Anything :

Beards, Rain, particle decays, conjectures, theories

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Bayes Theorem

Often used for subjective probability

Conditional
Probability P(A|B)

P(A & B)= P(B) P(A|B)

P(A & B)= P(A) P(B|A)

$$P(A|B) = \frac{P(B|A)}{P(B)} P(A)$$

Example:
W=white jacket B=bald
P(W&B)=(2/4)x(1/2)
or (1/4)x(1/1)
P(W|B)
= 1 / (2/4) x (1/4) =1/2

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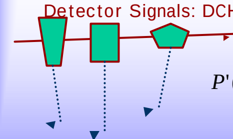
Start digression

Frequentist Use of Bayes Theorem

Example: Particle Identification

Particle types e,π,μ,K,p

Detector Signals: DCH,RICH,TOF,TRD



$$P'(e) = P(e|DCH) = \frac{P(DCH|e)}{P(DCH)} P(e)$$

$$P(DCH) = P(DCH|e)P(e) + P(DCH|m)P(m) + P(DCH|p)P(p)\ldots$$

Then repeat for P(e|RICH) using P'(e) etc

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Warning Notice

To determine P'(e) need P(e), P(μ) etc
('*a priori* probabilities')

If/when you cut on Probability, the Purity depends on these *a priori* probabilities

Example: muon detectors.

P(track| μ)≈0.9 P(track|π) ≈0.015
But P(μ)≈0.01 P(π) ≈1

Quantities like $\frac{P(data|m)}{P(data|e) + P(data|m) + P(data|p) + P(data|K)}$

Have no direct meaning – **use with care!**

End digression

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Bayes' Theorem and subjective probability

$$P(Theory|Result) = \frac{P(Result|Theory)}{P(Result)} P(Theory)$$

Your (posterior) belief in a Theory is modified by experimental result

If P(Result|Theory)=0 belief is killed

Large P(Result|Theory) increases belief, modified by general P(Result)

Applies to successive results

Dependence on prior P(Theory) eventually goes away

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Problem with subjective probability

It is subjective

My P(A) and your P(A) may be different

Scientists are supposed to be objective

Reasons to use subjective probability:

- Desperation ✓
- Ignorance ✗
- Idleness ?

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Can Honest Ignorance justify P(A)=flat?

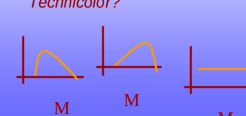
Argument:

- you know nothing
- every value is as believable as any other
- all possibilities equal

How do you count discrete possibilities?

SM true or false?

SM or SUSY or light Higgs or Technicolor?



For continuous parameter (e g. M_{higgs})

Take. $P(M_{higgs})$ as flat

Actually has to be zero as ∫P(M)dM=1 but never mind... 'improper prior'

Working with √M or lnM will give different results

Real Statisticians accept this and test for robustness under different priors.

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'Objective Prior' (Jeffreys)

Transform to a variable q(M) for which the Fisher information is constant

$$I(q) = -\left\langle \frac{\partial^2 \ln P(x;q)}{\partial q^2} \right\rangle = const$$

For a location parameter with P(x;M)=f(x+M) use M

For scale parameter with P(x;M)=Mf(x) use ln M

For a Poisson λ use prior $1/\sqrt{\lambda}$

For a Binomial with probability p use prior $1/\sqrt{p(1-p)}$

This has never really caught on

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Conclusion

What is Probability?

4 ways to define it

- Mathematical
- Classical
- Frequentist
- Subjective

Each has strong points and weak points

None is universally applicable

Be prepared to understand and use them all -

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