Statistics for HEP

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Lecture 5: Errors

Simple Statistical Errors

$$f(x, y)$$

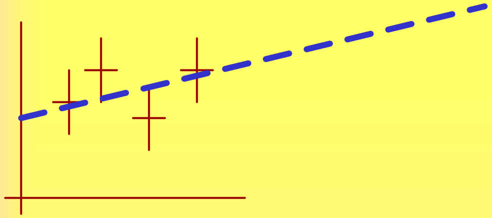
$$V(f) = \left(\frac{\partial f}{\partial x}\right)^2 V(x) + \left(\frac{\partial f}{\partial y}\right)^2 V(y) + 2\left(\frac{\partial f}{\partial x}\right)\left(\frac{\partial f}{\partial y}\right) Cov(x, y)$$

$$V(x) = s_x^2 \qquad V(y) = s_y^2 \qquad Cov(x, y) = r s_x s_y$$

$$\mathbf{f} = \mathbf{G}\mathbf{x}$$

$$\mathbf{V_f} = \mathbf{G}\mathbf{V_x}\tilde{\mathbf{G}}$$

Correlation: examples



$$V(m) = \frac{s^2}{N\left(\overline{x^2} - \bar{x}^2\right)}$$

$$V(c) = \frac{s^2\,\overline{x^2}}{N\left(\overline{x^2} - \bar{x}^2\right)}$$

$$Cov(m,c) = -\frac{s^2\,\bar{x}}{N\left(\overline{x^2} - \bar{x}^2\right)}$$

Extrapolate

$Y=mX+c$

$$V(Y) = \frac{s^2(X^2 + \overline{x^2} - 2X\bar{x})}{N\left(\overline{x^2} - \bar{x}^2\right)}$$

Avoid by using

$y=m(x-\bar{x})+c'$

Efficiency (etc)

$r=N/N_T$

$$V(r) = \left(\frac{1}{N_T}\right)^2 N + \left(\frac{-N}{N_T{}^2}\right)^2 N_T + 2\left(\frac{1}{N_T}\right)\left(\frac{-N}{N_T{}^2}\right)N$$

$$= \frac{N\left(N_T - N\right)}{N_T{}^3}$$

Avoid by using

$r=N/(N+N_R)$

Using the Covariance Matrix

Simple χ^2 : $\sum \left(\frac{x_i - f_i}{s_i} \right)^2$

For uncorrelated data

Generalises to

$$(\tilde{\mathbf{x}} - \tilde{\mathbf{f}})\mathbf{V}^{-1}(\mathbf{x} - \mathbf{f})$$

Multidimensional Gaussian

$$P(\mathbf{x}; \boldsymbol{\mu}, \mathbf{V}) = \frac{1}{(2p)^{N/2}\sqrt{|\mathbf{V}|}} e^{-\frac{1}{2}(\tilde{\mathbf{x}} - \tilde{\boldsymbol{\mu}})\mathbf{V}^{-1}(\mathbf{x} - \boldsymbol{\mu})}$$

Building the Covariance Matrix

Variables x,y,z…

x=A+B

y=C+A+D

z=E+B+D+F

$$\begin{pmatrix} \sigma_A^2+\sigma_B^2 & \sigma_A^2 & \sigma_B^2 \\ \sigma_A^2 & \sigma_A^2+\sigma_C^2+\sigma_D^2 & \sigma_D^2 \\ \sigma_B^2 & \sigma_D^2 & \sigma_E^2+\sigma_B^2+\sigma_D^2+\sigma_F^2 \end{pmatrix}$$

…..

A,B,C,D…

independent

If you can split into separate bits like this then just put the σ^2 into the elements

Otherwise use $V=GVG^T$

Systematic Errors

Systematic Error: reproducible inaccuracy introduced by faulty equipment, calibration, or technique

Bevington

Systematic effects is a general category which includes effects such as background, scanning efficiency, energy resolution, angle resolution, variation of couner efficiency with beam position and energy, dead time, etc. The uncertainty in the estimation of such as systematic effect is called a *systematic error*

Orear

Error=mistake?

Error=uncertainty?

Experimental Examples

- Energy in a calorimeter $E=aD+b$

  a & b determined by calibration expt
- Branching ratio $B=N/(\eta N_T)$

  η found from Monte Carlo studies
- Steel rule calibrated at 15C but used in warm lab

  If not spotted, this is a mistake

  If temp. measured, not a problem

  If temp. not measured guess $\rightarrow$uncertainty

Repeating measurements doesn't help

Theoretical uncertainties

An uncertainty which does not change when repeated does not match a Frequency definition of probability.

Statement of the obvious

Theoretical parameters:

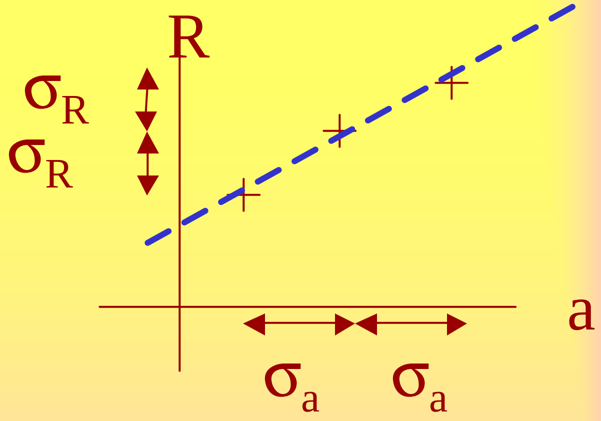
- B mass in CKM determinations
- Strong coupling constant in M_W
- All the Pythia/Jetset parameters in just about everything
- High order corrections in electroweak precision measurements
- etcetera etcetera etcetera.....

No alternative to subjective probabilities

But worry about robustness with changes of prior!

Numerical Estimation

Theory(?) parameter a affects your result R



a is known only with some precision σ_a

Propagation of errors impractical as no algebraic form for R(a)

Use data to find dR/da and σ_a dR/da

Generally combined into one step

The ‘errors on errors’ puzzle

Suppose slope uncertain

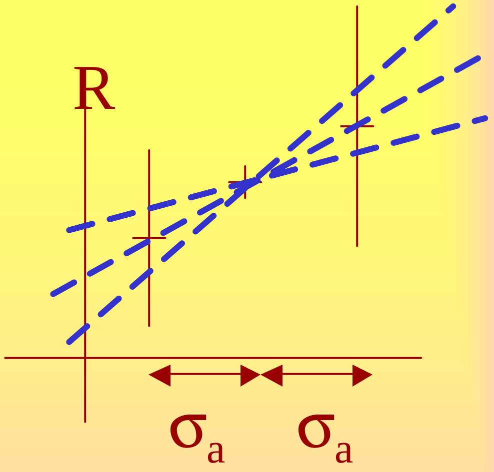
Uncertainty in σ_R.

Do you:

A. Add the uncertainty (in quadrature) to σ_R?

B. Subtract it from σ_R?

C. Ignore it?



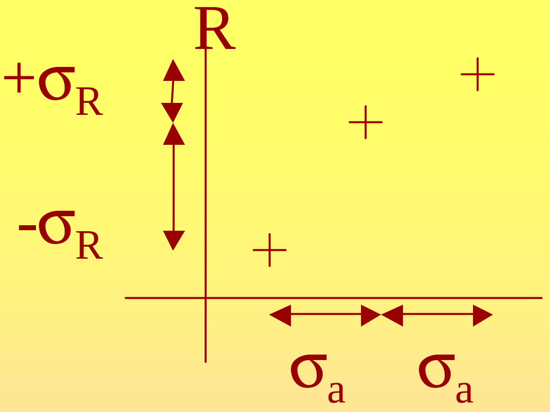
Timid and Wrong

Technically correct but hard to argue

Especially if $\sigma_{\sigma_R} > \sigma_R$

Strongly advised

Asymmetric Errors



Can arise here, or
from non-parabolic likelihoods

Not easy to handle

General technique for $x = y^{+s_y^+}_{-s_y^-} + z^{+s_z^+}_{-s_z^-}$

is to add separately

$$x^{+\sqrt{\left(s_y^+\right)^2+\left(s_z^+\right)^2}}_{-\sqrt{\left(s_y^-\right)^2+\left(s_z^-\right)^2}}$$

Not obviously correct

Introduce only if really justified

Errors from two values

Two models give results: R_1 and R_2

You can quote

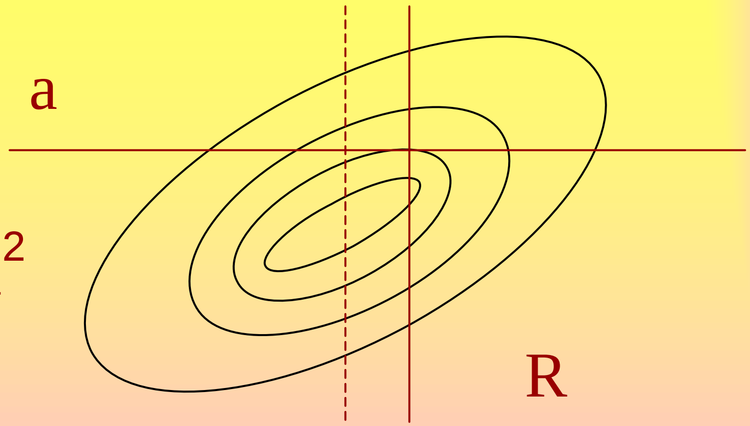
$R_1 \pm | R_1 - R_2 |$ if you prefer model 1

$\frac{1}{2}(R_1+R_2) \pm | R_1 - R_2 | / \sqrt{2}$ if they are equally rated

$\frac{1}{2}(R_1+R_2) \pm | R_1 - R_2 | / \sqrt{12}$ if they are extreme

Alternative: Incorporation in the Likelihood

- Analysis is some enormous likelihood maximisation
- Regard a as 'just another parameter': include $(a-a_0)^2/2\sigma_a^2$ as a chi squared contribution



- Can choose to allow a to vary. This will change the result and give a smaller error. Need strong nerves.
- If nerves not strong just use for errors
- Not clear which errors are 'systematic' and which are 'statistical' but not important

The Traditional Physics Analysis

1. Devise cuts, get result
2. Do analysis for statistical errors
3. Make big table
4. Alter cuts by arbitrary amounts, put in table
5. Repeat step 4 until time/money exhausted
6. Add table in quadrature
7. Call this the systematic error
8. If challenged, describe it as ‘conservative’

Systematic Checks

- Why are you altering a cut?
- To evaluate an uncertainty? Then you know how much to adjust it.
- To check the analysis is robust? Wise move. But look at the result and ask 'Is it OK?

Eg. Finding a Branching Ratio...

- Calculate Value (and error)
- Loosen cut
- Efficiency goes up but so does background. Re-evaluate them
- Re-calculate Branching Ratio (and error).
- Check compatibility

When are differences 'small'?

- It is OK if the difference is 'small' – compared to what?
- Cannot just use statistical error, as samples share data
- 'small' can be defined with reference to the difference in quadrature of the two errors

12±5 and 8 ±4 are OK.

18±5 and 8 ±4 are not

When things go right

DO NOTHING

Tick the box and move on

Do NOT add the difference to your systematic error estimate

- It's illogical
- It's pusillanimous
- It penalises diligence

When things go wrong

1. Check the test
2. Check the analysis
3. Worry and maybe decide there could be an effect
4. Worry and ask colleagues and see what other experiments did

99. Incorporate the discrepancy in the systematic

The VI commandments

Thou shalt never say ‘systematic error’ when thou meanest ‘systematic effect’ or ‘systematic mistake’

Thou shalt not add uncertainties on uncertainties in quadrature. If they are larger than chickenfeed, get more Monte Carlo data

Thou shalt know at all times whether thou art performing a check for a mistake or an evaluation of an uncertainty

Thou shalt not not incorporate successful check results into thy total systematic error and make thereby a shield behind which to hide thy dodgy result

Thou shalt not incorporate failed check results unless thou art truly at thy wits’ end

Thou shalt say what thou doest, and thou shalt be able to justify it out of thine own mouth, not the mouth of thy supervisor, nor thy colleague who did the analysis last time, nor thy mate down the pub.

Do these, and thou shalt prosper, and thine analysis likewise

Further Reading

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http://www.hep.man.ac.uk/~roger

The End