# An Overview of String Theory

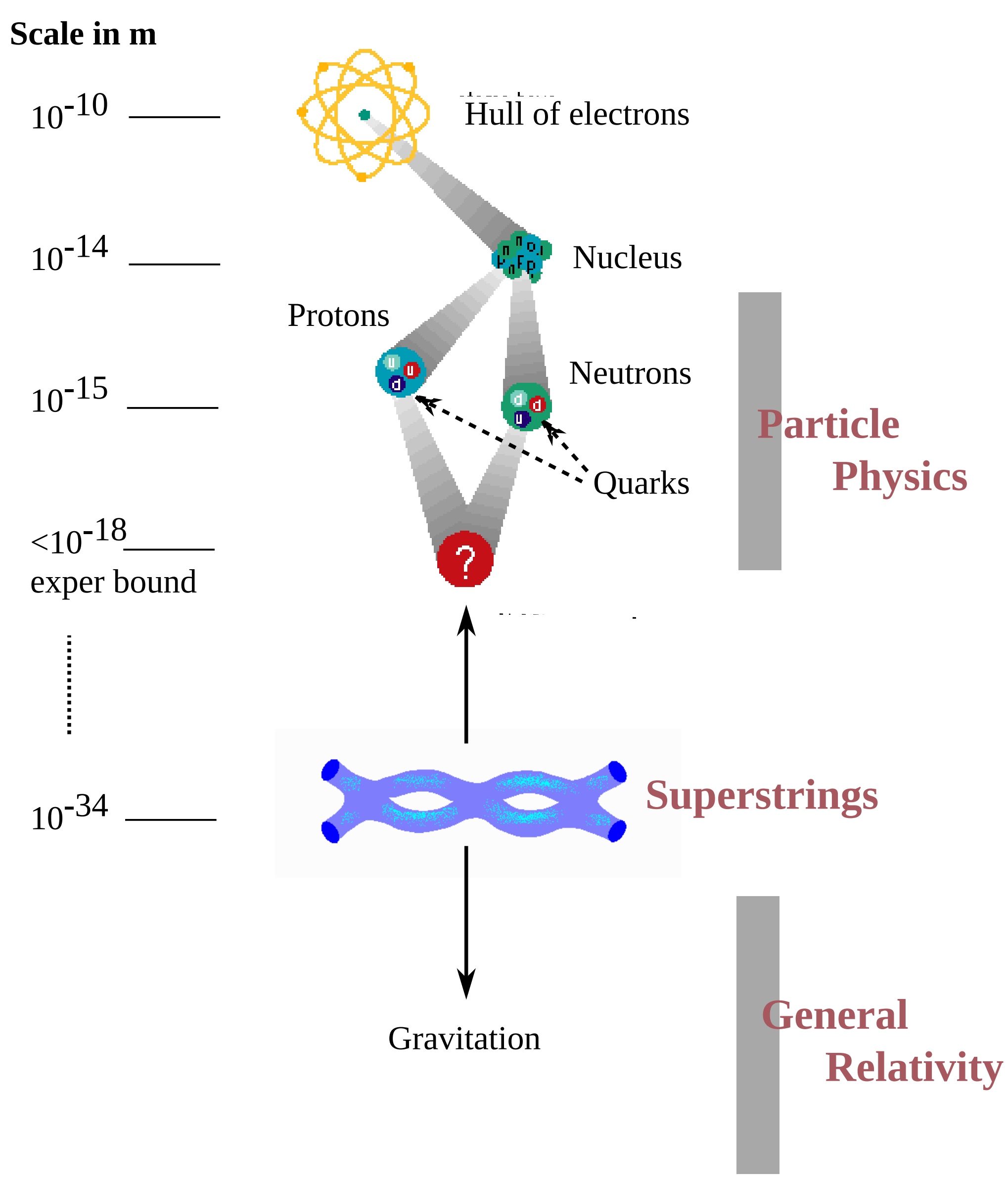
W. Lerche, CERN AcTr, 12/2002
Part 1

- **Perturbative string theories**
  - Motivation: the Standard Model and its Deficiencies
  - String theory as 2d conformal field theory
  - Consistency conditions on string constructions
  - Bosonic and supersymmetric strings in D=26,10
- **Compactification to lower dimensions**
  - T-Duality, minimal length scales
  - Supersymmetry, geometry and zero modes
  - Parameter spaces, geometrization of coupling constants
  - Stringy predictions ?
- **Non-perturbative string dualities**
  - Non-perturbative quantum equivalences
  - S-Duality in SUSY gauge theories
  - D-branes and Stringy Geometry
  - Unification of string vacua
- **Tests and Applications**
  - "Theoretical experiments": tests and consistency checks
  - D-brane approach to QFT
  - Recent developments



for files and text write-up see:
http://nxth21.cern.ch/~lerche/

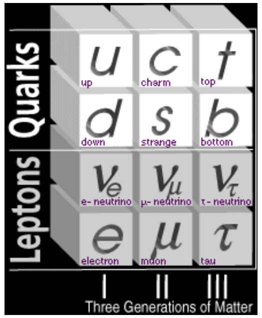
## Fundamental Structure of Matter



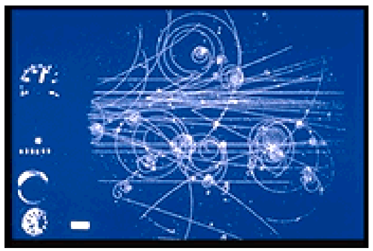
## Physics of Elementary Particles

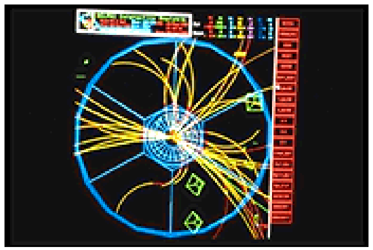
| Interactions (Forces) | Carriers (Gauge Fields) | elementary Matter Fields |
|---|---|---|
| Electromagnetism | Photon | Leptons (eg electrons) |
| weak Interactions (Radioactivity) | W,Z-Bosons | Quarks |
| strong Interactions (binds Nuclei) | Gluons | |

There are three "families" of them:




The "Standard-Model" of perticle physics describes subnuclear phenomena with partly stunning accuracy !







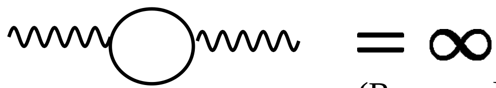
## Deficiencies of the Standard Model

- Its structure is quite ad hoc: are there deeper principles ?
  ("**grand unification**" of all matter and forces)
- ca 25 **free parameters**: determined by what ?

$$\mathcal{L} = \left(\sum \bar{\psi}\gamma(\partial + g_k A)\psi\right) + \left(\sum m_i \bar{\psi}\psi + \varphi_j \bar{\psi}\psi\right) + \dots$$

gauge couplings — massess — Higgs VEVs

- **Gauge hierarchy**:
  why weak scale (100GeV) << Unification scale scale (10^16GeV)
- Instability of parameters through self-interactions

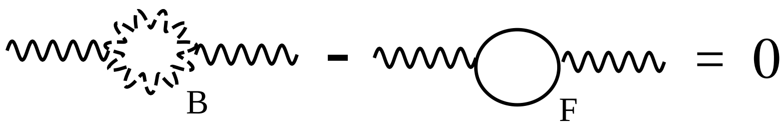


(Renormalization:
large scale hierarchies problematic)

→ **"Supersymmetry"**

Symmetry between bosons and fermions
(roughly: force carriers and matter fields)

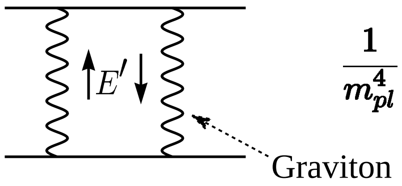
improved divergence structure:



# Problems of Quantum Gravity

- The Standard-Model of Particle Physics is not complete: **gravity** is not included !

The usual quantum field theoretical formulation of gravity does not work, due to incurable divergences (is "not renormalizable"):




$$\frac{1}{m_{pl}^4}\int^{\infty} dE'\,(E')^3 \;\rightarrow\; \infty$$

- There are conceptual difficulties with quantum black holes...

Expressions of a deeply-rooted problem:

**Apparent incompatibility of Quantum Mechanics and general Relativity**

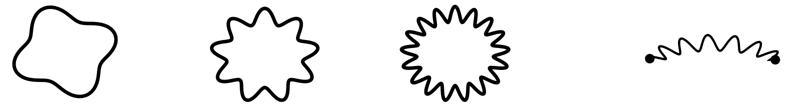
→ New concepts are necessary....like string theory

As a "by-product", it also provides the **grand unification** of all particles and their interactions !



# String-Theory

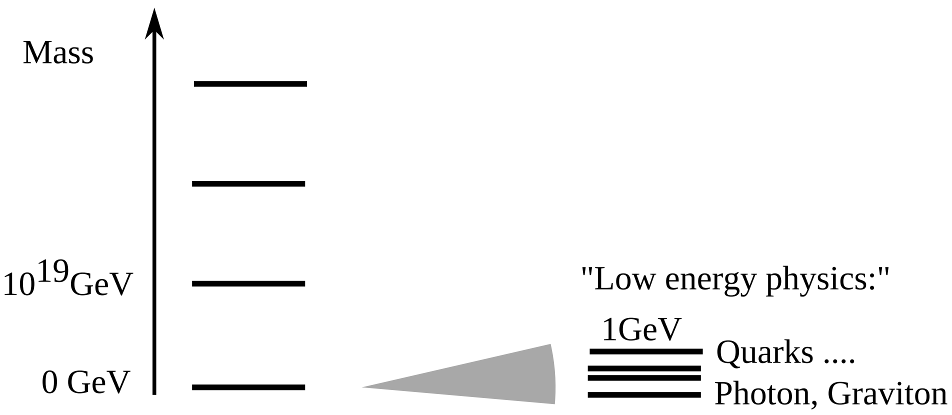
- small ($10^{-34}$m), one-dimensional objects:



scale determined by gravitational coupling:

$$G_N = \frac{1}{m_{\rm planck}^2}\,, \qquad m_{\rm planck} \sim 10^{19}\text{GeV}$$

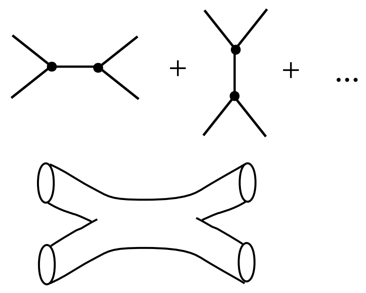
- Excitation spectrum:

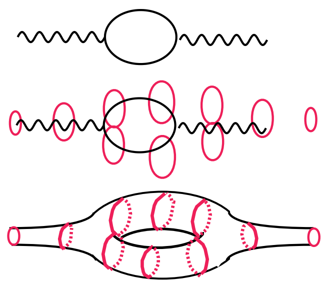



"elementary" Particles of the Standard Model = (almost) massless zero modes

## String Theory as 2d Field Theory

- Strings trace out two-dimensional "world-sheets" $\Sigma_g$:





- Perturbative string theory = 2d field theory on Riemann surfaces

Building blocks: can be eg. free 2d bosons $X^a,\ X_\mu$

$$(X_\mu(z) : \Sigma_g \to \mathbb{R}^D)$$

Variety of field operators in D-dimensional space-time ⟷ Simple combinatorics of 2-d field operators

| | | |
|---|---|---|
| graviton | $g_{\mu\nu}$ | $= \bar\partial X_\mu(\bar z)\,\partial X_\nu(z)$ |
| gauge field | $A^a_\mu$ | $= \bar\partial X_\mu(\bar z)\,\partial X^a(z)$ |
| Higgs boson | $\Phi^{ab}$ | $= \bar\partial X^a(\bar z)\,\partial X^b(z)$ |

⇒ **Intrinsic unification of particles + interactions !**
In particular, gravity is automatically built in.



## The String "Miracle"

- **Perturbative effective action** in D-dimensional space-time:

$$S_{\text{eff}}(g_{\mu\nu}, A_\mu, ..) = \sum_{\Sigma_i} e^{-\phi\chi(\Sigma_i)} \int_{M(\Sigma_i)} \int d\psi dX ..\, e^{\int d^2z \mathcal{L}_{2d}(\psi, X, .., g_{\mu\nu}, A_\mu, ..)}$$

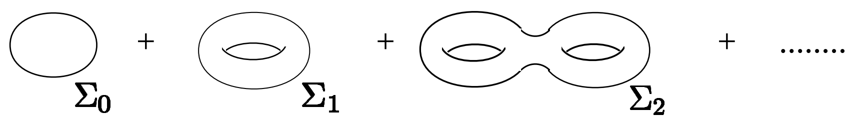
$$= \int d^D x \sqrt{-g}\,[\,R + \mathrm{Tr} F_{\mu\nu}F^{\mu\nu} + \ldots] + \mathcal{O}(m_{\text{planck}}{}^{-1})$$

2d action (eg. free)

general relativity, gauge theory etc

small string corrections $m_{\text{planck}} \sim 10^{19}$GeV
**infinitely many predictions**

''Loop expansion'' = sum over 2d topologies, weighted by $e^{-<\phi>\chi} = \lambda_s{}^{-\chi}$




$\frac{1}{\lambda_s{}^2}$ $\lambda_s{}^0$ $\lambda_s{}^2$

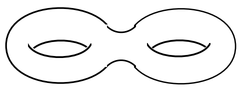
Only one (UV finite) "diagram" at any given order in perturbation theory

- **Discrete reparametriztions** of $\Sigma_i$ have no analog in particle theory; important to make sense eg. of graviton scattering. ''Feynman rules'' are substantially different from particle theory.

String theory, even in perturbation theory, is **more** than just having infinitely many particles plus a cutoff !

## Consistency Conditions

The 2d theory cannot be arbitrary but is highly constrained

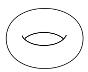


2d properties ⟶ D-dimensional properties

- **Conformal invariance** ⟶ implies gauge and general coordinate invariance in spacetime

  consistency requires D=26

- **Modular invariance** ⟶ strongly constrains spectrum



  absence of UV divergences and anomalies

  consistency allows only few possible spectra

- **2d Supersymmetry** ("Superstrings") ⟶ space-time fermions

  does **not** imply that D-dimensional space-time theory is supersymmetric !

  consistency requires D=10



## Conformal Field Theory

Is a tool set by which one can relatively easily do explicit computations, in terms of simple building blocks

- Stress-Energy Tensor T(z) = generator of conformal transformations $z \to f(z)$

... satisfies an operator product algebra:

$$T(z)\cdot T(0) \sim \frac{c}{2}\frac{1}{z^4} + \frac{2}{z^2}T(0) + \frac{1}{z}\partial T(0)$$

"**central charge c**" is an anomaly, an obstruction to a good transformation behavior of T(z)

For D free 2d scalars: $T_X(z) = \sum :\partial X^\mu \partial X_\mu:(z)$

each contributes 1, so

$$c_X = D$$

- For theories with more symmetries, like supersymmetry or gauge symmetries, one has a corresponding generalization of this current algebra

# The Bosonic String

- Polyakov-action:

$$S_X = \frac{1}{4\pi\alpha'}\int d^2z\,\sqrt{|g|}\,g^{ab}\partial_a X^\mu \partial_b X_\mu \qquad \alpha' \sim (m_{\text{planck}})^{-2}$$

reduces in conformal gauge (2d metric g->1)
to a CFT of D free scalar fields $X_\mu(z,\bar{z}) : \Sigma_g \to \mathbb{R}^D$

- Gauge-fixing of 2d reparametrization symmetries:
Faddeev-Popov ghosts b,c

$$S_{gh} = \frac{1}{2\pi}\int d^2z\, b\bar{\partial}c$$

Ghost stress-tensor $T_{gh}(z) = (\partial b)c - 2\partial(bc)$ has central charge

$$c_{gh} = -26$$

- Consistent quantization requires total central charge to vanish:

$$c_{tot} \equiv c_X + c_{gh} = D - 26 = 0$$

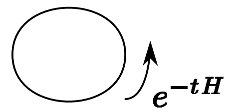
→ Bosonic string exists only in 26 dimensions !

- Quantization also implies a shift of vacuum energy:

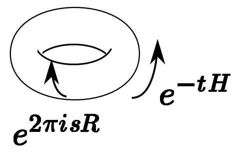
$m^2$
- 2 — massive higher spin fields
- 1 —
- 0 — massless Graviton, Dilaton, Tensorfield
- -1 — Tachyon → No stable vacuum !



# Consistency at 1-loop level

- Particles: 


$$Z(t) = \mathrm{Tr}[e^{-tH}] \quad \text{(QFT partition function)}$$

- Strings: 

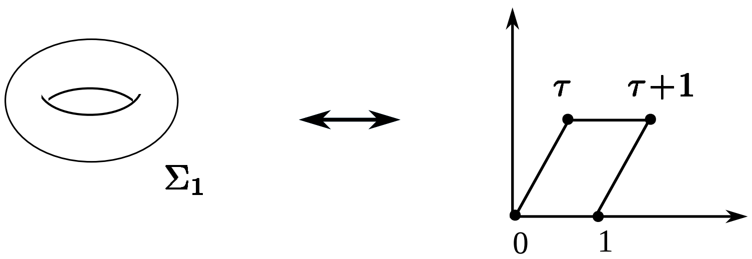

$$Z(t,s) = \mathrm{Tr}[e^{-tH}e^{2\pi isR}]$$
$$= \mathrm{Tr}[q^{L_0}\bar{q}^{\bar{L}_0}]$$

$$q \equiv e^{2\pi i\tau}, \qquad \tau = s + i/2\pi t$$

modular parameter of torus = complexified proper time

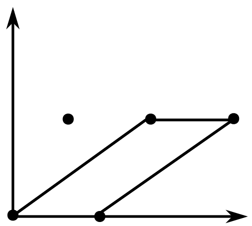
# Modular transformations

- Torus is defined by identifying sides of parallelogram:

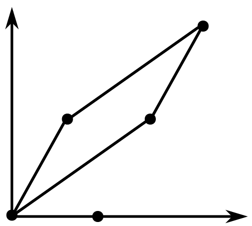



The "modular" parameter $\tau$ determines its shape

- Global reparametrizations:



$\tau \to \tau+1$



$\tau \to \frac{\tau}{\tau+1}$

... yield **equivalent** tori !

... generate the modular group, PSL(2,Z):

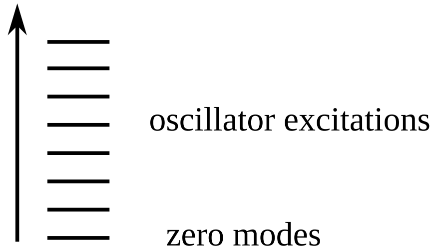
$$\tau \to \frac{a\tau+b}{c\tau+d}\,, \qquad a,b,c,d,\in \mathbf{Z} \qquad ad-bc=1$$

- Physical amplitudes must be **invariant** under such modular transformations !



# Modular Invariance of Partition Function

For bosonic string:




$$Z(q,\bar{q}) \;=\; \left(\sqrt{\mathrm{Im}\tau}\;\eta(q)\eta(\bar{q})\right)^{2-D}$$

where $\eta(q) \;=\; q^{1/24}\prod_{n=1}^{\infty}(1-q^n)$ (Dedekind function)

is the (inverse) oscillator partition function of a scalar field

It has well-defined modular properties, eg: $\eta(\tau+1) \;=\; e^{i\pi/12}\eta(\tau)$

The vacuum amplitude is indeed modular invariant.

- This "global consistency condition" has no analog in particle QFT; and is responsible for many stringy features...

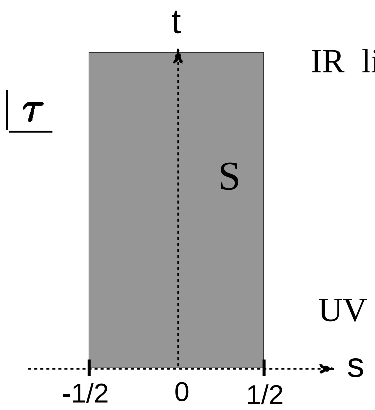
## Vacuum amplitude at 1-loop

- To obtain vacuum amplitude, still need to integrate; try:

$$\mathcal{A} \;=\; \int_{-1/2}^{1/2} ds \int_0^\infty \frac{dt}{t^2}\, Z(s,t) \;=\; \int_{\text{strip } S} \frac{d^2\tau}{\text{Im}\tau^2}\, Z(q,\bar q)$$

projection on R=0

t = proper time
$t \to \infty$




$\frac{dt}{t^2}$ , $t \to 0$

However, this is all-too-naive particle field theory thinking.....

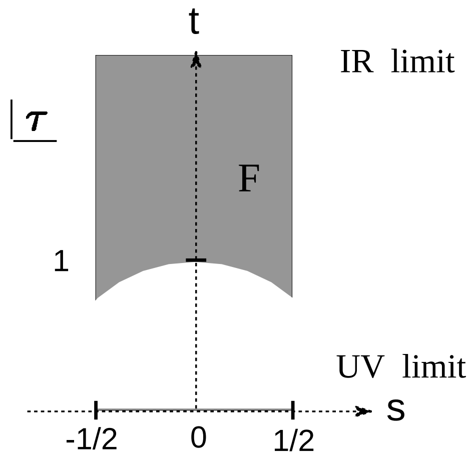
in string theory, $\tau \;=\; s + i/2\pi\, t$

has a definite geometrical meaning,
namely it is the modular parameter of a torus !



## UV and IR Divergences

- In string theory, we need to integrate precisely **once** over **all inequivalent shapes** of the torus.
These are described by the "fundamental region F":




Every point outside F is equivalent (via a modular symmetry transformation) to a point inside F**;** integrating over it would be overcounting

As t=0 is not integrated over, there is no UV divergence possible

(NB: is more than a cutoff...)

- However, there is an IR divergence for large t

$$Z(q,\bar q) \;\sim\; \frac{1}{q\bar q} + 24\left(\frac{1}{q} + \frac{1}{\bar q}\right) + 576 + \ldots\ldots$$

Pole is due to Tachyon state (vacuum instability)

Superstrings !

# Superstrings

- 2d supersymmetry $\{X_\mu, \psi_\mu\}$ --> **space-time** fermions $\Psi_\alpha \sim \sqrt{\psi_\mu}$
  typically, but not necessarily supersymmetric in space-time !

- Typically have no tachyons -- only known consistent theories...

- Two formulations: "Green-Schwarz": covariant, but difficult to quantize

  "Neveu-Schwarz-Ramond": easier to quantize, but not manifestly covariant

- From 2d NSR perspective: CFT -> SuperCFT
  additional ghost fields with c=+11

$$c_{tot} = D + \frac{1}{2}D - 26 + 11 = 0$$

$$X_\mu \quad \psi_\mu \quad b,c \quad \beta,\gamma$$

→ Critical dimension for superstrings is D=10

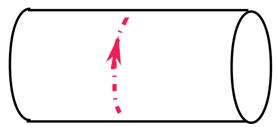
- Main novel technical ingredient:

  boundary conditions of the 2d fermions



# Spin Structures

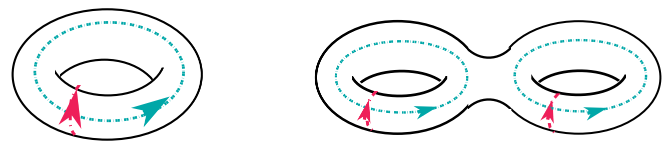
- 2d fermions can have non-trivial boundary conditions:



$$\Psi(e^{2\pi i}z) = \pm\Psi(z) \quad \begin{cases} + & \text{"Neveu-Schwarz"} \\ - & \text{"Ramond"} \end{cases}$$

The **R-sector** leads to space-time **fermions**, the **NS-sector** to **bosons** !

Each cycle can be either periodic (P) or anti-periodic (A)



- The partition function of a fermion depends on the "spin structure":

$$Z_{PP}(q) = \operatorname{Tr}_R[(-1)^F q^{L_0}] = 0$$
$$Z_{PA}(q) = \operatorname{Tr}_R[q^{L_0}] = \prod(1+q^n)$$
$$Z_{AP}(q) = \operatorname{Tr}_{NS}[(-1)^F q^{L_0}] = \prod(1-q^{n-1/2})$$
$$Z_{AA}(q) = \operatorname{Tr}_{NS}[q^{L_0}] = \prod(1+q^{n-1/2})$$

These "theta"-functions have well-defined modular properties .

- Modular invariance of the partition function requires particular, consistent choices for the boundary conditions, and these determine the physical spectrum

→ Modular invariance strongly constrains the possible physical spectra

.... chiral anomalies always cancel

# Modular Invariant Partition Functions

- How to construct modular invariant sums over spin structures ?

$$\sum_{\substack{\text{spin}\\\text{structures}}} \int [dX\, d\Psi]\, e^{-S[X,\Psi]} \;=\; \underset{\substack{\text{spin}\\\text{structures}}}{\mathrm{Tr}} \left[e^{-tH} e^{2\pi i s R}\right]$$

Systematic procedure: map this to certain lattice sums !
**Modular invariance** $\longleftrightarrow$ **self-duality of lattice**,
easy classification of all possibilities

- The result is various possibilities in D=10 :

1) Holomorphic and anti-holomorphic sectors separately invariant

$$Z^{IIA}(q,\bar q) \;=\; Z(\bar q)^{+}\, Z(q)^{-}$$ "Type IIA"

$$Z^{IIB}(q,\bar q) \;=\; Z(\bar q)^{+}\, Z(q)^{+}$$ "Type IIB"

$$Z^{\mathrm{het}}(q,\bar q) \;=\; Z_{\mathrm{bos.}}(\bar q)\, Z(q)^{+}$$ "heterotic"
(two kinds, related to
uniqueness of self-dual lattices:
E8xE8 or SO(32) gauge symm.)

2) Non-trivial correlation of spin structures

This gives various non-supersymmetric theories,
only one of which is tachyon-free

- In addition, there is one more "open" supersymmetric string



# Supersymmetric String Theories in D=10

- By combining superstring (S) and bosonic string (B) building blocks, one can construct five types of string theories in D=10:

| Combination | Name | Gauge group |
|---|---|---|
| $S \otimes \bar S^{\dagger}$ | Type IIA | $U(1)$ |
| $S \otimes \bar S$ | Type IIB | – |
| $S \otimes \bar B$ | Heterotic | $E_8 \times E_8$ |
| $S \otimes \bar B'$ | Heterotic$'$ | $SO(32)$ |
| $(S \otimes \bar S)/Z_2$ | Type I (open) | $SO(32)$ |

- These theories have one dimensionful parameter, the string tension $\alpha' \sim (m_{\mathrm{planck}})^{-2}$ , besides the coupling $\lambda_s = e^{<\Phi>}$.

- They have very different spectra (c.f., gauge groups) !