Tests, Applications & Recent Developments

W. Lerche, CERN AcTr, 12/2002
Part 4

- **Duality** is an extremely useful tool for analyzing in detail many non-trivial string and field theories.
  Supersymmetry facilitates this by virtue of its non-ren. properties, but is perhaps by itself not a fundamentally important feature.

- How do we know that these ideas are correct and make any sense at all ?

  Even though string theory makes **infinitely many predictions**, it is hard to verify with present day experiments ....

Theoretical Experiments: Consistency Checks

- Besides growing circumstantial evidence with varying degree of rigor, there have been numerous non-trivial quantitative tests and consistency checks.

> Not a single test on the dualities has ever failed !

- Apart from aiming for grand unifications, there has also been a lot of highly non-trivial results for gauge and other QFT's !

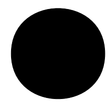
Field theoretical applications

Counting Black Hole Microstates with D-branes

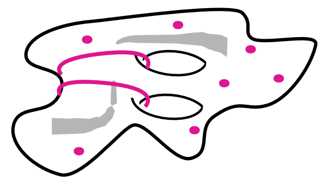
- Example:
  Compute Bekenstein-Hawking entropy (= area of BH horizon) of extremal N=4 supersymmetric black hole in D=5.

$$S_{BE} \equiv \log(d) = 2\pi\sqrt{\frac{q_h q_f{}^2}{2}}$$ q_f, q_h = electric and axionic charges

- Idea:



Duality $\longleftrightarrow$



Large, semi-classical black hole — Type IIB string on K3xS^1

Strongly coupled string theory $\longleftrightarrow$ Weakly coupled string theory

For large circle, this maps to a 2d sigma model on the moduli space of a gas of D0 branes on K3

Counting states in this sigma model, indeed reproduces the above Bekenstein-Hawking entropy for large charges !

> This does not only add credibility to the duality claims, but also tells that there are no "missing" degrees of freedom !

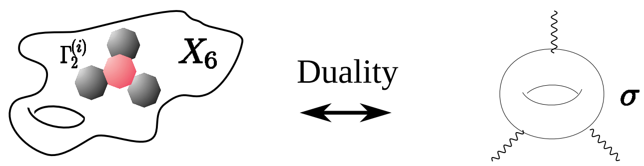
String theory seems to have exactly the right degrees of freedom to make sense of quantum black holes.....

Comparison of Quantum Corrections

- Complicated **perturbative corrections** to effective actions can computed in various different string models, and always give the same answer.

- Example: threshold corrections to gauge coupling in N=2 SUSY in D=4

Type IIA on some Calabi-Yau X_6 versus heterotic string on $K_3 x T_2$



Counting spheres in the Calabi-Yau via mirror symmetry (tree level)

perturbative one-loop diagram ("Borcherds integral")

$$\tau(T,U) = \sum c_{n,m} \log[1 - e^{-nT} e^{-mU}] \qquad \tau(T,U) = \int \frac{d^2\sigma}{\mathrm{Im}\sigma^2} \mathcal{B}(T,U,\sigma)$$

These very non-trivial functions, of completely different origin, match completely !

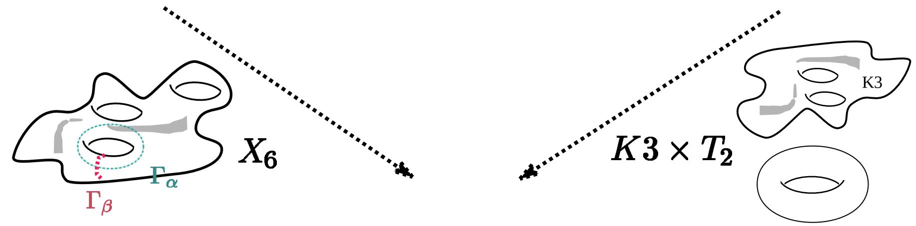
$$\partial_T \tau_{TT}(T,U) = \frac{i}{2\pi} \frac{E_4(T)E_4(U)E_6(U)(E_4(T)^3 - E_6(T)^2)}{E_4(U)^3 E_6(T)^2 - E_4(T)^3 E_6(U)^2}$$

$$\partial_U \tau_{TT}(T,U) = -\frac{i}{2\pi} \frac{E_4(T)^2 E_6(T)(E_4(U)^3 - E_6(U)^2)}{E_4(U)^3 E_6(T)^2 - E_4(T)^3 E_6(U)^2} + \frac{i}{2\pi} \partial_T \ln f_y(q_1, q_3)$$

Recovering Seiberg-Witten Theory from String Duality

Non-perturbative equivalence of type IIB string, compactified on Calabi-Yau manifold X_6, with heterotic string compactified on $K_3 x T_2$

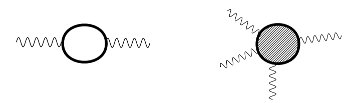
Type IIA/B String | d=10 | Heterotic String



Identical N=2 SUSY theory in d=4

3-branes wrapping 3-cycles Γ_α	$\longleftrightarrow$	fundamental gauge bosons
3-branes wrapping dual 3-cycles Γ_β	$\longleftrightarrow$	SW monopoles
Tree-level physics in gauge sector	$\longleftrightarrow$	Non-perturbative quantum theory
world-sheet instanton effects	$\longleftrightarrow$	space-time gauge instanton effects
classical computation	$\Rightarrow$	exact non-perturbative SW theory, incl gravity corrections !

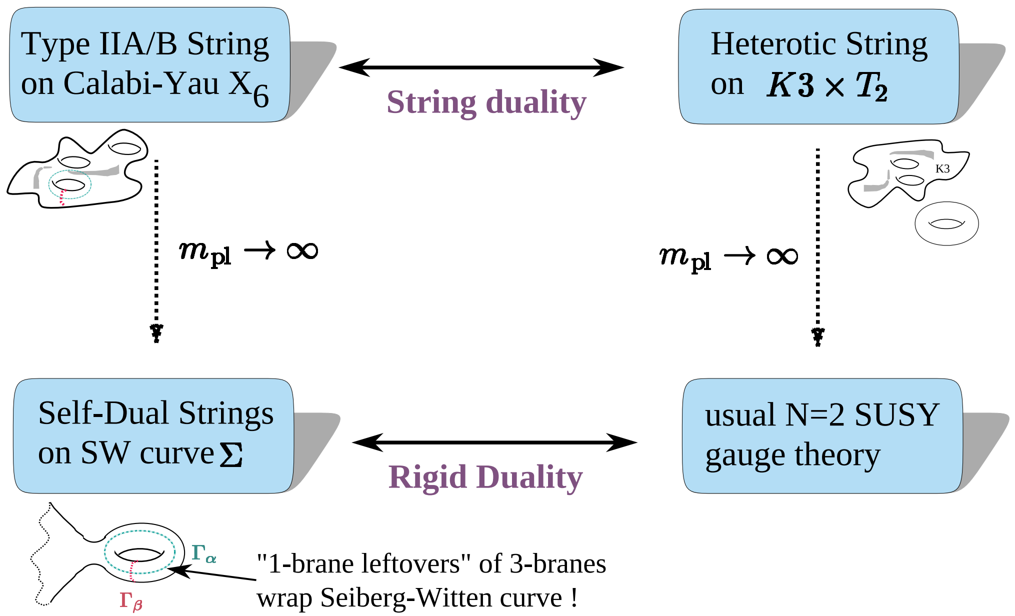
$$\tau_{\mathrm{eff}}(t) = \partial_T \int_{\Gamma_3} \Omega^{(\mathrm{CY}')}(t) \qquad \tau_{\mathrm{eff}}(\phi) = \frac{1}{\pi} \log\left[\frac{\phi^2}{\Lambda^2}\right] - \frac{1}{\pi} \sum_{\ell=1}^{\infty} c_\ell \left(\frac{\Lambda}{\phi}\right)^{4\ell}$$



Stringy Interpretation of Seiberg-Witten Curve Σ

We thus have a natural dual reformulation of N=2 supersymmetric Yang-Mills theory !

It is nothing but a "rigid remnant" of the type II/heterotic string duality, that remains after **decoupling gravity**:



Gauge bosons and monopoles are on equal footing !
They simply correspond to **self-dual strings** wrapping Γ_α and Γ_β cycles on the curve Σ

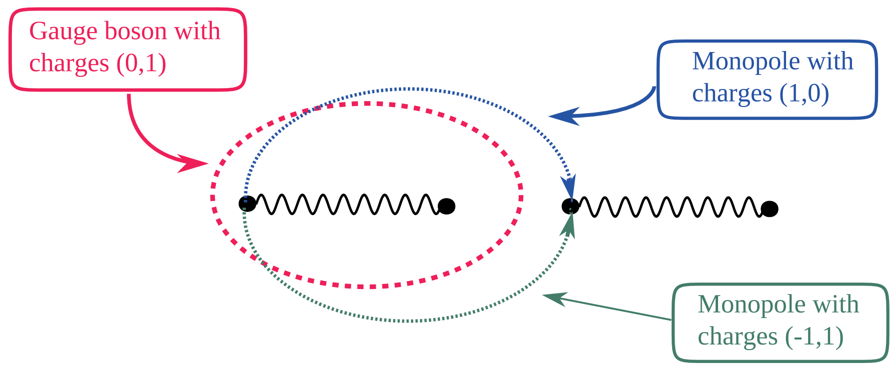
Elementary gauge bosons, but solitonic monopoles (or vice versa)

Can study non-perturbative properties of the N=2 gauge theory, that are extremely hard to get at in ordinary local QFT !

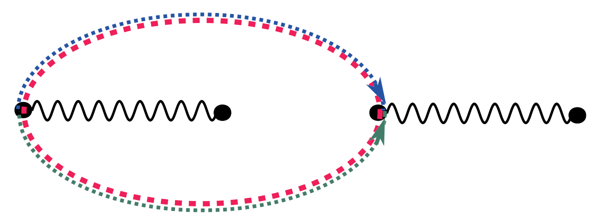
Decay of Gauge Boson into Monopoles

The self-dual string formulation allows to represent the physical states in the gauge theory in terms of homology cycles on the SW Riemann surface Σ

Represent Σ best in terms of branched complex plane; for generic, large vacuum values, the basic states of the theory with (mag,ele) charges are then represented as follows:



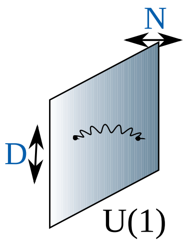
However, for special values of VEV, the string representation degenerates:



The quantum state of the gauge boson becomes indistinguishable of the 2-particle state composed out of two magnetic monopoles!

Not possible to see in ordinary QFT !

Further Applications: D-Brane Technology

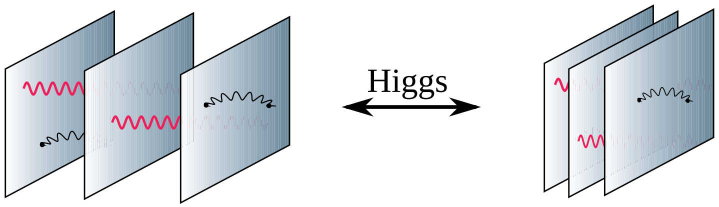


- Recall single D-brane: open strings can end

  Localized degrees of freedom:

  D: U(1) gauge degrees of freedom A_μ

  N: Higgs field, VEV $\langle \Phi \rangle$ = brane position

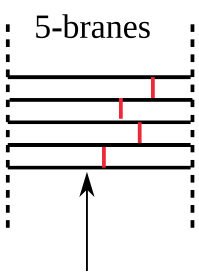


U(1) x U(1) x U(1) + massive U(3) unbroken gauge symmetry

- Decouple gravity and irrelevant string modes:
  get results from string duality also for **ordinary QFT**

"D-brane technology" can be used to model local string geometries which realize, for example, gauge theories with matter.

- Example: In Type IIA string theory, consider



n D4-branes, open strings between them give gauge fields

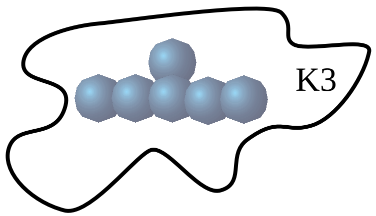
This induces N=2 SUSY SU(n) SYM theory on the world-volume of the D4-branes, reproducing Seiberg-Witten theory

Geometric Engineering

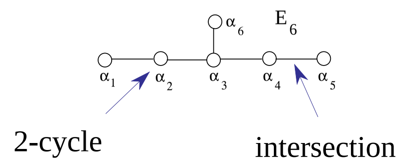
- Instead of **flat** D-branes, we can also use **curved** D-branes wrapped around p-cycles in some Calabi-Yau manifold.

The **intersection topology** determines then gauge group and matter content; it is a very systematic construction which allows to design a huge class of gauge theories, as well as novel QFT.

- Example: In Type IIA string theory on K3, consider D2-branes wrapped around 2-cycles that intersect in a certain pattern:

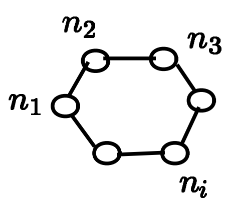


Group theoretical Dynkin diagram:



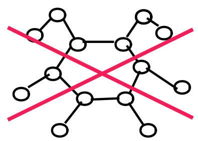
Physically, this yields an E6 gauge theory in D=6 !

- Example: "**Quiver**" N=1 SUSY gauge theory in D=4 each node corresponds to some wrapped D branes:



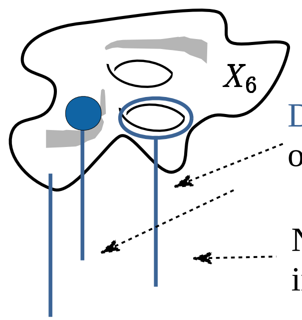
Physically, this yields a gauge group $G = \prod_i U(n_i)$ with matter fields in the reps. $(n_i, \bar{n}_{i+1})$

- Not everything is allowed:



Stringy "Brane Worlds"

- A very general class of N=1 supersymmetric backgrounds can be obtained by placing extra D-branes on a compactification space, eg on a Calabi-Yau space in type II string theory:

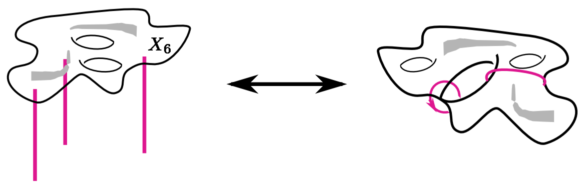


This represents a "**brane world**" where we live on the 3+1 dimensional "left-over" of the D-branes

Generically, this picture is dual to strongly-coupled heterotic strings on large compact dimensions (as naturally suggested by SUSY breaking)

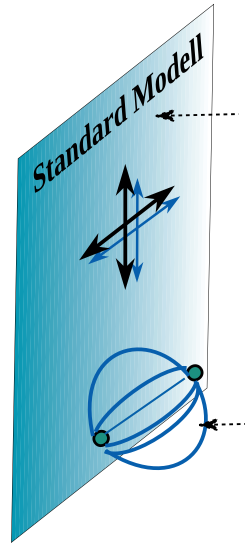
- Note however, that due to stringy geometry, a geometrical interpretation is in general highly ambiguous ...

One and the same effective action may have many different dual interpretations:



Low-Scale Strings ?

- This setup may be phenomenologically very interesting:



- The gravity field lines spread out to more than 4 dimensions, and are "diluted" : gravity appears in the brane world **weaker** than in the bulk !

$$G_N \sim \frac{1}{M^2}$$

This means the scale of gravity (hitherto 10^{19}GeV) can be much smaller, in fact as low as the scale of the weak interactions, or even smaller..

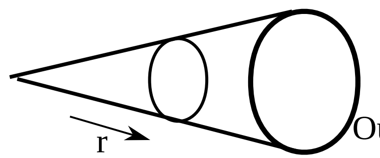
If true, this could be tested at the LHC !

Model Building with "Warped" Geometries

- A related approach rests on the fact that branes can induce a "warped" space time, which is not a direct product of R^4 and an internal space; rather a fiber product with metric:

$$ds^2 = e^{-f(r)}\eta_{\mu\nu}dx^\mu dx^\nu + dr^2$$

Visualize as a cone:

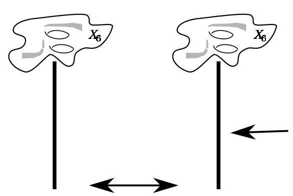


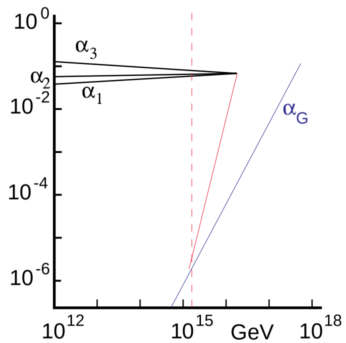
moving along the 5th dimension changes energy in 3+1dim

A warp factor corresponding to a large 5th dimension can make gravity appear much weaker in our 3+1 dimensional world than it is in the "bulk" 5th dimension.

- Horava-Witten scenario

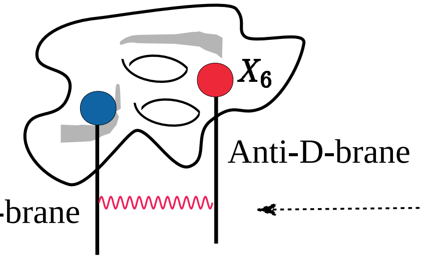
Strong coupling limit of 10dim heterotic string gives 11dim M-theory "compactified" on a line interval, bounded by two "end-of-the-world" branes.



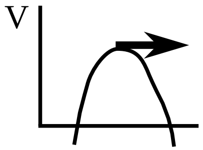


Supersymmetry Breaking

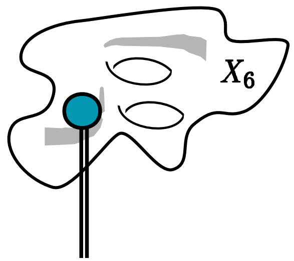
- SUSY breaking can be achieved in various ways, e.g., by putting non-supersymmetric brane configurations on X6:

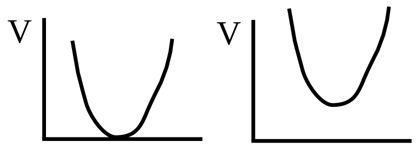


Open string is a **tachyon**, which means an unstable vacuum



System will collapse to new vacuum state, which may or may not be supersymmetric:



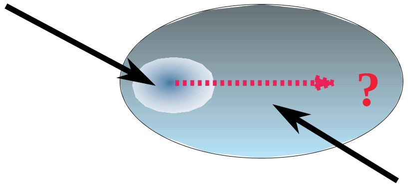


- One would need a reliable computational framework to determine the effective superpotential, and finally to determine the ground state of the system.

In general, there will be complicated non-perturbative corrections to the superpotential....

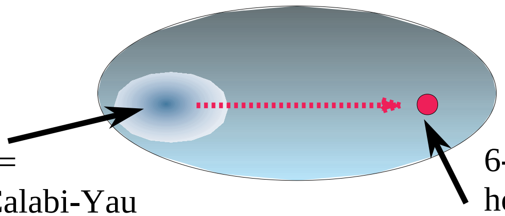
Quantum Geometry of D-Branes

- Classical geometry ("branes wrapping p-cycles") makes sense only in limited regimes of the paramater space, namely at weak coupling or large radius:

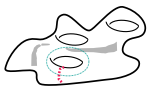


Quantum geometry:
(instanton) corrections wipe out concepts of classical geometry

- Example: "quantum volume"

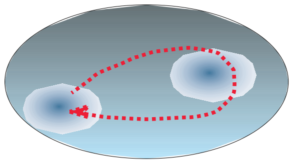


6-cycle = whole Calabi-Yau is large

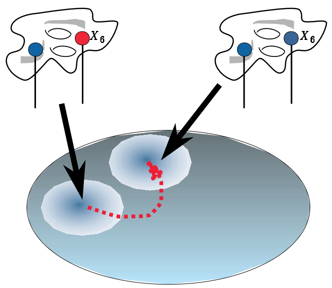


6-cycle -> zero size however: "embedded" 2,4 cycles have non-zero size !

- Example: "monodromy"



Looping around singularieties returns "totally different" brane configurations

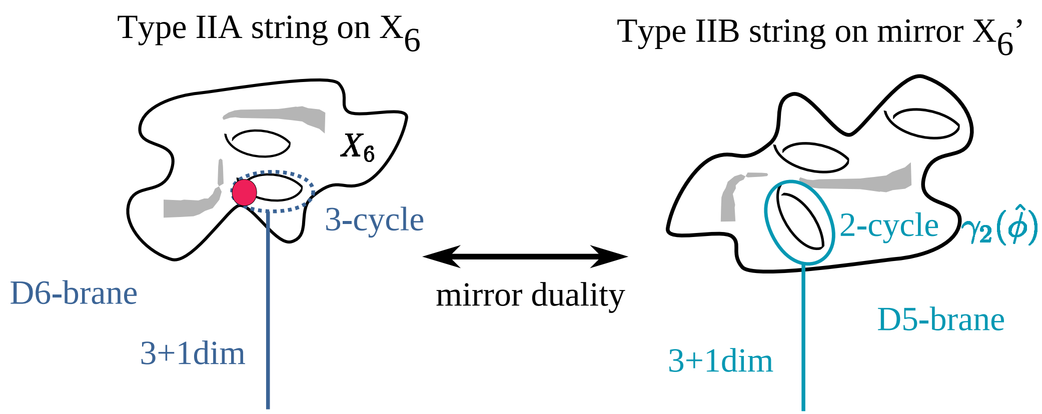


A brane-**anti** brane pair with apparent SUSY breaking, turns into a SUSY preserving brane-brane pair

- To meaningfully address such questions, we need to have **full analytical control** of functions like the superpotential, or gauge coupling

Mirror Symmetry for Open Strings

- Recently, new techniques have been developed to obtain exact non-perturbative superpotentials in N=1 theories



$$W(\phi,\hat{\phi}) = \sum c_{n,m} \mathrm{Li}_2(e^{-n\phi} e^{-m\hat{\phi}})$$

complicated corrections due to open and closed string instantons

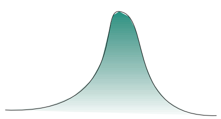
$$W(\phi,\hat{\phi}) = \int_{\gamma_3(\hat{\phi}):\partial\gamma_3=\gamma_2} \Omega(\phi)$$

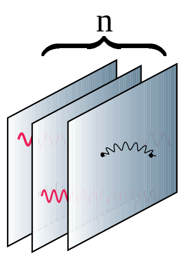
exact result at tree level !

- Such expressions can be extrapolated (analytically continued) and so allow, for example, to find exact vacuum states (minima of the potential) in the strongly coupled, non-perturbative regime.

Duality between Open and Closed Strings

- Recall:





Closed string (Type II)
p-brane solitons

Open string (Type I)
elementary D-p-branes

non-perturbative;
involves gravity

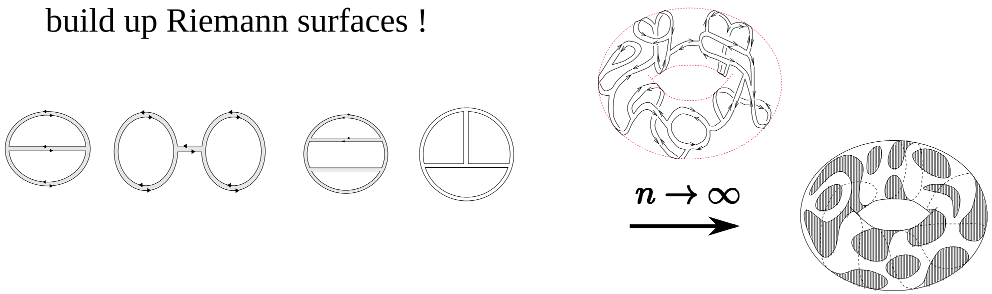
perturbative;
U(n) gauge theory

Is there a deeper relationship between the closed string (gravity) sector, and the open string (gauge theory) sector ?

- An old idea by t'Hooft:

Consider a U(n) gauge theory, and take the limit $n \to \infty$

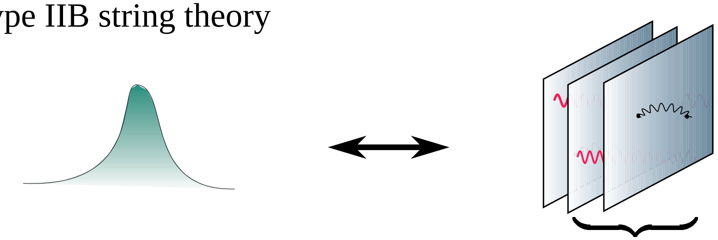
Then only "planar" Feynman digrams dominate, and in fact build up Riemann surfaces !



- The conjecture was that for large n, the gauge theory turns into some sort of string theory - but exactly which one wasn't clear at all, for a long time.

The AdS Correspondence: An exact duality between Gauge and String Theory

- Type IIB string theory



n D3-branes
yield U(n) N=4 SUSY on the 3+1 dimensional brane volumes

Geometry effectively becomes

$S^5 \times AdS_5$

$\sum_{i=1}^{6} x_i^{\,2} \;=\; R^2 \qquad x_0^{\,2} + x_1^{\,2} - \sum_{i=1}^{5} x_i^{\,2} \;=\; R^2$

"anti de Sitter space"

- Claim (Maldacena):

The large-n limit of the U(n) N=4 SUSY gauge theory is exactly dual to the type IIB string on $S^5 \times AdS_5$

Correlators of gauge invariant operators in the N=4 gauge theory are 1:1 to the Green's functions of the type IIB string... many tests !

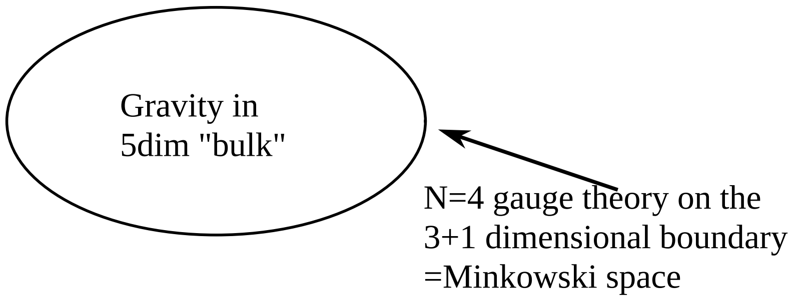
This corresponds to a weak-strong coupling S-duality:

String theory weakly coupled for:		Gauge theory weakly coupled for:	
$g^2 n$	large	$g^2 n$	small
g	small	g	small

- Using this correspondence, many interesting results about Yang-Mills theory were obtained, including theories with less or no SUSY.... like quark-antiquark potential $V(r) \sim g_{YM}\sqrt{n}/r$

The Holographic Principle

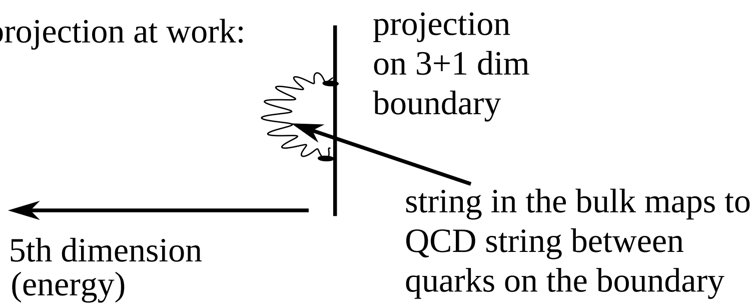
- Geometry of the AdS-space:



How can a theory be equivalent to "another" theory on the boundary ?
It seems to have too many degrees of freedom !

> The "Holographic Principle:"
>
> A theory of quantum gravity is non-local and carries the same number of degrees of freedom as a theory on the boundary !
> (inspired by black holes)

- Holographic projection at work:



(similar features seen in warped string compactification models)

A Holographic Universe ?

- The gauge/gravity correspondence for N=4 gauge theory has been the first example in which the holographic principle is realized, and this has been checked thoroughly.

Thus, another example where an investigation of string or brane physics had a unexpected, stunning outcome....

- Could it be that these ideas apply also to our universe as a whole ?

There are some indications for this, but no clearcut conclusions so far - some, very speculative, ideas are very interesting; for example, that the evolution of our universe between big bang and big crunch may be something like a renormalization group flow between two conformal fixed points.......