# Searching for Supersymmetry at the LHC

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**Lay-out:**

1. **Why supersymmetry**
2. **General features of a supersymmetric theory**
3. **The minimal supersymmetric Standard Model**
4. **Brief introduction to the LHC**
   - **the enviroment**
   - **the detectors**
   - **the main challenges**
5. **The LHC potential for supersymmetry**
   - **Inclusive searches**
   - **Precision measurements**
   - **Reconstruction of the underlying theory**
6. **Conclusions and outlook**

**We will try (as much as possible) to interleave the theoretical and experimental parts.**

**Special emphasis on:**

- **The basic language**
- **Comparison of strategy and potential of various machines**
- **How to perform a SUSY analysis (step by step) at the LHC**
- **What we can (and cannot) learn about SUSY from the LHC**

**In many respects**

**we are happy with the Standard Model**

- **a unitary, renormalizable relativistic quantum field theory**
- **amazingly consistent with data.**

**The problem of spontaneous gauge symmetry breaking (short range of weak interactions) is solved in a simple way by the Higgs mechanism.**

**This also allows breaking of the flavour symmetry in a phenomenologically consistent way (flavour breaking confined in the charged current interaction sector).**

**Achieving the same results without the Higgs mechanism is extremely difficult.**

**At the same time,**

**we are unhappy with the Standard Model.**

**Many unsatisfactory aspects:**

- **What is the origin of flavour symmetry breaking?**
- **Is there a grand unification?**
- **Where is gravitation?**

**The last two points raise further problems:**

- **Hierarchy**
- **Naturalness**

**Two facts:**

- **The mass of the minimal Standard Model Higgs boson is not far from the weak scale, $\sim 200$ GeV.**
- **Much larger energy scales become relevant at some point**

**A first question arises: why is the weak scale so much smaller than the Planck scale, $M_P \sim 10^{19}$ GeV, or the unification scale, $M_{GUT} \sim 10^{16}$ GeV?**

**This is usually referred to as the hierarchy problem.**

**Even worse: The Higgs mass (masses of scalar particles, in general) is strongly sensitive to any large energy scale unless a *fine tuning* of parameters is performed.**

**This is the so-called naturalness problem.**

**There is a very simple reason for this: scalar masses are not naturally small, in the sense that no symmetry is recovered when they are let go to zero.**

**Fermion and vector boson masses *are* naturally small: radiative corrections are proportional to the masses themselves.**

# Naturalness and fine tuning in a simple example

**Consider a theory of two real scalars fields:**

$$\mathcal{L} = \frac{1}{2}\partial^\mu \phi \, \partial_\mu \phi + \frac{1}{2}\partial^\mu \Phi \, \partial_\mu \Phi - V(\phi, \Phi)$$

**with**

$$V(\phi, \Phi) = \frac{m^2}{2}\phi^2 + \frac{M^2}{2}\Phi^2 + \frac{\lambda}{4!}\phi^4 + \frac{\sigma}{4!}\Phi^4 + \frac{\delta}{4}\phi^2\Phi^2$$

**Assume $\lambda$, $\sigma$, $\delta$ are all positive, small and comparable in magnitude, and assume $M^2 \gg m^2 > 0$.**

**Is the mass hierarchy $m^2 \ll M^2$ conserved at the quantum level?**

**Compute one-loop radiative corrections to $m^2$ by taking the second derivatives of the effective potential at the minimum $\phi = \Phi = 0$:**

$$m^2_{\text{one loop}} = m^2(\mu^2) + \frac{\lambda m^2}{32\pi^2}\left(\log\frac{m^2}{\mu^2} - 1\right) + \frac{\delta M^2}{32\pi^2}\left(\log\frac{M^2}{\mu^2} - 1\right)$$

$$\mu^2\frac{\partial m^2}{\partial \mu^2} = \frac{1}{32\pi^2}\left(\lambda m^2 + \delta M^2\right)$$

**Corrections proportional to $M^2$ appear at one loop. One can choose $\mu^2 \sim M^2$ in order to get rid of them, but they reappear through the running of $m^2(\mu^2)$.**

**The mass hierarchy is preserved only if the parameters are such that**

$$\lambda m^2 \sim \delta M^2 \rightarrow \frac{\delta}{\lambda} \sim \frac{m^2}{M^2}$$

**This is what we usually call a *fine tuning* of the parameters.**

**The same thing happens if $m^2 < 0$, $M^2 \gg \left|m^2\right| > 0$. In this case the tree-level potential has a minimum at**

$$\Phi = 0, \quad \phi^2 = -6m^2/\lambda \equiv v^2$$

**and the symmetry $\phi \to -\phi$ is spontaneously broken. The degrees of freedom in this case are $\Phi$ and $\phi' \equiv \phi - v$, with**

$$m_\Phi^2 = M^2 \qquad m_{\phi'}^2 = -2m^2 = \lambda v^2/3$$

**At one loop, the minimization condition $m^2 + \lambda v^2/6 = 0$ is replaced by**

$$m^2 + \frac{\lambda v^2}{6} = -\frac{\lambda}{32\pi^2}\left(m^2 + \frac{\lambda v^2}{2}\right)\left(\log\frac{m^2 + \frac{\lambda v^2}{2}}{\mu^2} - 1\right) - \frac{\delta}{32\pi^2}\left(M^2 + \frac{\delta v^2}{2}\right)\left(\log\frac{M^2 + \frac{\delta v^2}{2}}{\mu^2} - 1\right)$$

**Following the same procedure as in the unbroken case one finds**

$$m_{\phi'}^2 = \frac{\lambda v^2}{3} + \frac{v^2}{32\pi^2}\left[\lambda^2 \log\frac{m^2 + \frac{\lambda}{2}v^2}{\mu^2} + \delta^2 \log\frac{M^2 + \frac{\delta}{2}v^2}{\mu^2}\right]$$

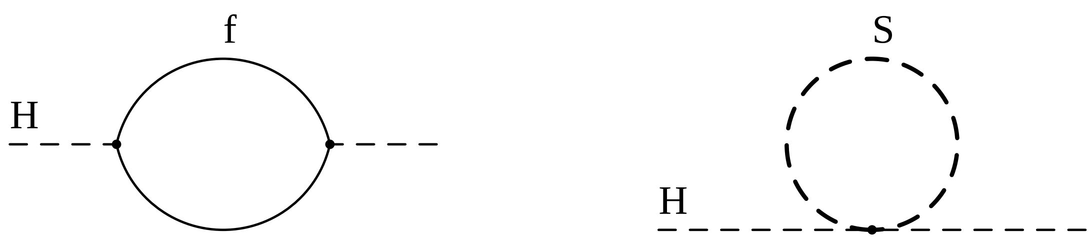
**with $v \sim M$ without a suitable tuning of the parameters.**

# Naturalness: a closer look

**The scalar potential in the Standard Model:**

$$V(\phi) = m^2 |\phi|^2 + \lambda |\phi|^4$$

**One-loop corrections to $m^2$ due to fermionic (a) or bosonic (b) degrees of freedom:**




(a) (b)

$$\left(\Delta m^2\right)_a = \frac{|\lambda_f|^2}{16\pi^2}\left[-2\Lambda^2 + 6m_f^2\,\log\frac{\Lambda}{m_f} + \ldots\right]$$

$$\left(\Delta m^2\right)_b = \frac{\lambda_S}{16\pi^2}\left[\Lambda^2 - 2m_S^2\,\log\frac{\Lambda}{m_S} + \ldots\right]$$

**Here $\Lambda$ is an ultraviolet cut-off, to be identified with the energy scale at which the SM is no longer reliable, and the dots stand for terms that do not grow with $\Lambda$.**

**In dimensional regularization the $\Lambda^2$ term would be absent, but contributions proportional to $m_f^2, m_S^2$ would still be there.**

**Even if the heavy degrees of freedom are not directly coupled to the SM Higgs, it can be shown that similar contributions arise at higher orders.**

**In the absence of very special cancellations, the Higgs boson becomes as heavy as the heaviest degrees of freedom.**

**A symmetry that relates fermions to bosons would do the job, at least at one loop. Suppose there are two scalars for each fermion:**

$$\left(\Delta m^2\right)_{a+b} = \frac{\lambda_S - |\lambda_f|^2}{8\pi^2}\,\Lambda^2 + \ldots$$

**For suitable values of the couplings the quadratic divergence disappears.**

**No surprise: with bosons and fermions in the same multiplet, scalar masses are protected by the same (chiral) symmetry that protects fermion masses from large radiative corrections.**

**Clearly, more restrictions will be needed in order to guarantee that the cancellation takes place at all orders.**

**Such a symmetry is called a supersymmetry:**

$$Q|\text{boson}\rangle = |\text{fermion}\rangle \qquad Q|\text{fermion}\rangle = |\text{boson}\rangle$$

**The symmetry generator $Q$ (and its hermitian conjugate $Q^\dagger$) carry spin $1/2$: it is a space-time symmetry.**

**The form of possible supersymmetry algebras is strongly constrained on the basis of very general theorems in field theory. For example, it is impossible with ordinary symmetry generators (elements of a commutator algebra).**

**There is essentially one possibility:**

$$\{Q, Q^\dagger\} = P^\mu$$
$$\{Q, Q\} = \{Q^\dagger, Q^\dagger\} = 0$$
$$[P^\mu, Q] = [P^\mu, Q^\dagger] = 0$$

**(more on this later). Further specifications:**

- $Q, Q^\dagger$ **transform as spinors under the Lorentz group**
- $Q, Q^\dagger$ **commute with gauge symmetry generators.**

**In principle, we may have more than one** $Q$**:** $Q^i$**,** $i = 1, \ldots, N$ **(extended supersymmetry).**

**A few basic properties of a supersymmetric theory can already be recognized:**

- **particles in the same supersymmetric multiplet (which we will call a supermultiplet) have equal masses and equal gauge transformation properties (electric charge, weak isospin and color)**
- **within the same supermultiplet, there is an equal number of bosonic and fermionic degrees of freedom (a proof on the next slide)**

**A proof (taken from S. Martin):**

$$\left.\begin{array}{l}(-1)^{2s}|\text{boson}\rangle = +|\text{boson}\rangle \\ (-1)^{2s}|\text{fermion}\rangle = -|\text{fermion}\rangle\end{array}\right\} \Rightarrow \left\{(1-)^{2s}, Q\right\} = \left\{(1-)^{2s}, Q^\dagger\right\} = 0$$

**Consider the subspace of states $|i\rangle$ within a supermultiplet with the same eigenvalue $p^\mu$ of the four-momentum operator $P^\mu$. $\sum_i |i\rangle\langle i| = 1$ within this subspace of states.**

$$\begin{aligned}
\sum_i \langle i|(-1)^{2s} P^\mu|i\rangle &= \sum_i \langle i|(-1)^{2s} QQ^\dagger|i\rangle + \sum_i \langle i|(-1)^{2s} Q^\dagger Q|i\rangle \\
&= \sum_i \langle i|(-1)^{2s} QQ^\dagger|i\rangle + \sum_i \sum_j \langle i|(-1)^{2s} Q^\dagger|j\rangle\langle j|Q|i\rangle \\
&= \sum_i \langle i|(-1)^{2s} QQ^\dagger|i\rangle + \sum_j \langle j|Q(-1)^{2s} Q^\dagger|j\rangle \\
&= \sum_i \langle i|(-1)^{2s} QQ^\dagger|i\rangle - \sum_j \langle j|(-1)^{2s} QQ^\dagger|j\rangle \\
&= 0.
\end{aligned}$$

## Fermionic fields: a reminder

**We use the Weyl representation of the Dirac matrices:**

$$\gamma^\mu = \begin{bmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{bmatrix} \qquad \sigma^\mu = (\sigma^0, \vec{\sigma}) \qquad \bar{\sigma}^\mu = (\sigma^0, -\vec{\sigma}) \qquad \gamma_5 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

**where $\vec{\sigma}$ are the three Pauli matrices**

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

**and**

$$\sigma^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

**The familiar lagrangian for a free, massive Dirac spinor is**

$$\mathcal{L} = \bar{\psi}\,(i\partial\!\!\!/ - m)\,\psi \qquad (\partial\!\!\!/ = \gamma^\mu \partial_\mu)$$

$$\psi = \begin{pmatrix} \xi_L \\ \xi_R \end{pmatrix} \quad \psi_L = \begin{pmatrix} \xi_L \\ 0 \end{pmatrix} = \frac{1}{2}(1-\gamma_5)\psi \quad \psi_R = \begin{pmatrix} 0 \\ \xi_R \end{pmatrix} = \frac{1}{2}(1+\gamma_5)\psi$$

**Four-component Dirac spinors realize a reducible representation of the Lorentz group: the two-component spinors $\xi_L$, $\xi_R$ transform independently under Lorentz transformations.**

**In terms of $\xi_L$, $\xi_R$ we have**

$$\mathcal{L} = i\xi_L^\dagger\,\bar{\sigma}^\mu \partial_\mu\,\xi_L + i\xi_R^\dagger\,\sigma^\mu\,\partial_\mu \xi_R - m(\xi_L^\dagger \xi_R + \xi_R^\dagger \xi_L)$$

**It can be shown that the transformation law of the two-component spinor $\xi_R$ under Lorentz transformations are the same as those of $\epsilon\xi_L^*$, where $\epsilon$ is the antisymmetric matrix**

$$\epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} .$$

**It follows that any Dirac spinor may always be written in terms of two left-handed Weyl spinors $\xi, \chi$ as**

$$\psi = \begin{pmatrix} \xi \\ -\epsilon\chi^* \end{pmatrix}$$

**(the minus sign on the second Weyl spinor is conventional).**

**The free lagrangian for a massive Dirac spinor is**

$$\begin{aligned}\mathcal{L} = \bar{\psi}\,(i\partial\!\!\!/ - m)\,\psi &= i\chi^T\,\epsilon\sigma^\mu\epsilon\,\partial_\mu\chi^\dagger + i\xi^\dagger\,\bar{\sigma}^\mu\partial_\mu\xi - m(\chi^T\epsilon\xi - \xi^\dagger\epsilon\chi^*) \\ &= i\chi^\dagger\,\bar{\sigma}^\mu\,\partial_\mu\chi + i\xi^\dagger\,\bar{\sigma}^\mu\partial_\mu\xi - m(\chi^T\epsilon\xi - \xi^\dagger\epsilon\chi^*)\end{aligned}$$

**Note that**

$$\chi^T\epsilon\xi = \xi^T\epsilon\chi$$

**because of the anticommutation properties of fermion fields, and that**

$$(\chi^T\epsilon\xi)^\dagger = \xi^\dagger\epsilon^T\chi^* = -\xi^\dagger\epsilon\chi^*$$

**The shorthand notation**

$$\chi\xi = \xi\chi = i\chi^T\epsilon\xi \qquad \xi^\dagger\chi^\dagger = \chi^\dagger\xi^\dagger = -\xi^\dagger\epsilon\chi^*$$

**is often used.**

**Any theory involving spin-1/2 particles may be written as a collection of left-handed Weyl spinors $\psi_i$.**

**Left-handed fermions in the Standard Model:**

$$u_i, d_i, e_i, \nu_i$$
$$\bar{u}_i, \bar{d}_i, \bar{e}_i$$

**where $i = 1, 2, 3$ is a generation index. For example, the Dirac spinor for the up quark is written in terms of two left-handed Weyl spinors $u$ and $\bar{u}$ as**

$$\psi_u \equiv \begin{pmatrix} u_L \\ u_R \end{pmatrix} = \begin{pmatrix} u \\ -\epsilon \bar{u}^* \end{pmatrix}$$

**(NB: the bar on $\bar{u}$ here has no special meaning, it is just part of the name)**

**The Weyl notation is convenient, since left- and right-handed components of Dirac fermions, which behave differently in weak interactions, are treated separately. Furthermore, the simplest (irreducible) representations of supersymmetry contain Weyl fermions.**

**The simplest supermultiplet candidate:**

**a Weyl fermion $\psi$ and two real (or one complex) scalar $\phi_1, \phi_2$.**

**This is called a chiral or matter supermultiplet.**

**Next-to-simplest possibility: a supermultiplet that contains a massless gauge vector boson $A_\mu^a$. Two bosonic degrees of freedom, so its partner must be again a Weyl fermion, called a *gaugino*, $\lambda^a$ (it cannot be a spin-3/2 field: we want a renormalizable theory).**

**Gauge bosons belong to the adjoint representation of the gauge group, so the same is true for their supersymmetric partners. Thus,**

**gauginos are not chiral fermions**

**because the adjoint representation is equivalent to its conjugate.**

**The pair $A_\mu^a, \lambda^a$ is called a gauge supermultiplet.**

**It turns out that any other representation of the supersymmetry algebra can be reduced to a combination of chiral and gauge supermultiplets, at least if there is only one supersymmetry generator $Q$ ($N = 1$ supersymmetry).**

**We will not consider extended ($N > 1$) supersymmetries, because they cannot accommodate chiral fermions.**

## The particle content of a supersymmetric SM

- No supermultiplet can be formed out of standard particles (e.g. $\gamma, \nu$).
- Matter fermions must belong to chiral supermultiplets, because left and right fermions transform differently under the weak gauge group.
- Gauge bosons must go into gauge supermultiplets.
- At least two Higgs doublets, with their fermionic partners: cancellation of the axial anomaly.

| **spin 0** | **spin 1/2** | **spin 1** | $SU(3)_C$ | $SU(2)_L$ | $U(1)_Y$ |
|---|---|---|---|---|---|
| $\tilde{u}_L, \tilde{d}_L$ | $u_L, d_L$ | | $\mathbf{3}$ | $\mathbf{2}$ | $+\frac{1}{3}$ |
| $\tilde{u}_R$ | $u_R$ | | $\mathbf{3}$ | $\mathbf{1}$ | $+\frac{4}{3}$ |
| $\tilde{d}_R$ | $d_R$ | | $\mathbf{3}$ | $\mathbf{1}$ | $-\frac{2}{3}$ |
| $\tilde{\nu}, \tilde{e}_L$ | $\nu, e_L$ | | $\mathbf{1}$ | $\mathbf{2}$ | $-1$ |
| $\tilde{e}_R$ | $e_R$ | | $\mathbf{1}$ | $\mathbf{1}$ | $-2$ |
| $H_u^+, H_u^0$ | $\tilde{h}_u^+, \tilde{h}_u^0$ | | $\mathbf{1}$ | $\mathbf{2}$ | $+1$ |
| $H_d^0, H_d^-$ | $\tilde{h}_d^0, \tilde{h}_d^-$ | | $\mathbf{1}$ | $\mathbf{2}$ | $-1$ |
| | $\tilde{g}$ | $g$ | $\mathbf{8}$ | $\mathbf{1}$ | $0$ |
| | $\tilde{w}^\pm, \tilde{w}^0$ | $W^\pm, W^0$ | $\mathbf{1}$ | $\mathbf{3}$ | $0$ |
| | $\tilde{b}^0$ | $B^0$ | $\mathbf{1}$ | $\mathbf{1}$ | $0$ |

**Supersymmetric partners of standard particles (e.g. a scalar electron) with the same masses would have been detected in experiments. Since none of them has been observed so far, we must conclude that**

**supersymmetry must be broken**

**in a realistic theory. Recall our formula for scalar mass corrections:**

$$\left(\Delta m^2\right)_{a+b} = \frac{\lambda_S - |\lambda_f|^2}{8\pi^2}\,\Lambda^2 + \ldots$$

**Supersymmetry forces $\lambda_S = |\lambda_f|^2$ to all orders in perturbation theory, so that quadratic divergences are systematically cancelled. This feature must be preserved in the broken theory:**

$$\mathcal{L} = \mathcal{L}_{\text{supersymmetric}} + \mathcal{L}_{\text{soft}}$$

**where $\mathcal{L}_{\text{soft}}$ only contains mass terms and couplings with positive mass dimension.**

**Supersymmetric partners of ordinary particles are so heavy that they have escaped detection so far.**

**Is there a reason for that?**

**All ordinary particles, including the $W, Z$ bosons and the top quark, would be massless in the absence of spontaneous breaking of the electroweak gauge symmetry.**

**The contrary is true for their partners: scalar masses are always allowed by gauge symmetries, and gaugino can be massive because they belong to a real representation of the gauge group.**

**The only exception is the Higgs boson.**

## A rough estimate of superparticle masses

**Call $m_{\text{soft}}$ the largest mass scale present in $\mathcal{L}_{\text{soft}}$. The corrections to $m^2$ arising from $\mathcal{L}_{\text{soft}}$ must vanish as $m_{\text{soft}} \to 0$, so they cannot grow as $\Lambda^2$. Corrections proportional to $m_{\text{soft}}\Lambda$ are also forbidden (UV divergences are either quadratic or logarithmic). Therefore**

$$\Delta m^2 \sim \lambda\, m_{\text{soft}}^2 \log \frac{\Lambda}{m_{\text{soft}}}$$

**($\lambda$ a generic coupling). Furthermore,**

$$m_F^2 - m_B^2 \sim m_{\text{soft}}^2$$

**within a supermultiplet. It follows ($\lambda \sim 1, \Lambda \sim M_P$) that**

$$m_{\text{soft}} \lesssim 1 \text{ TeV}$$

**in order to get the correct value of the Higgs v.e.v.**

# Building a supersymmetric theory

**A simple example: one left-handed Weyl fermion**

$$\psi \equiv P_L\,\psi \qquad P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$$

**and one complex scalar $\phi$, no masses, no interactions:**

$$\begin{aligned} \mathcal{L}_{\text{chiral}} &= \mathcal{L}_f + \mathcal{L}_s \\ & \mathcal{L}_f = \bar{\psi}\, i\partial\!\!\!/\, \psi \\ & \mathcal{L}_s = \partial^\mu \phi^* \partial_\mu \phi \end{aligned}$$

**Is it possible to define a set of transformation rules on the fields, such that scalars are transformed into fermions, under which $\mathcal{L}_{\text{chiral}}$ remains unchanged?**

**Let us consider the scalar field first, and tentatively define**

$$\delta\phi = \bar{\eta}\,\psi \qquad \delta\phi^* = \bar{\psi}\,\eta$$

**where $\eta$ is a constant, infinitesimal, anticommuting spinor. Then**

$$\delta\mathcal{L}_s = (\partial^\mu\bar{\psi})\,(\partial_\mu\phi)\,\eta + \bar{\eta}\,(\partial^\mu\phi^*)\,(\partial_\mu\psi)$$

**How does $\psi$ tranform? $\delta\psi$ must be left-handed, linear in $\eta$, and must contain one derivative. Only one possibility (up to a proportionality factor):**

$$\delta\psi = -i(\partial_\mu\phi)\,\gamma^\mu\eta \qquad \delta\bar{\psi} = i\bar{\eta}\,\gamma^\mu\partial_\mu\phi^*$$

**which tells us that $\eta$ is right-handed, $P_R\eta = \eta$, and gives**

$$\delta\mathcal{L}_f = -\bar{\eta}\,(\partial_\mu\phi^*)\,\gamma^\mu\gamma^\nu\partial_\nu\,\psi + \bar{\psi}\,\gamma^\nu\gamma^\mu\partial_\nu\,(\partial_\mu\phi)\,\eta$$

**Using $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$ and $\partial_\mu \partial_\nu = \partial_\nu \partial_\mu$ we get**

$$\begin{aligned}\delta \mathcal{L}_f &= -\bar{\eta}\,(\partial_\mu \phi^*)\,(\partial^\mu \psi) - (\partial_\mu \bar{\psi})\,(\partial^\mu \phi)\,\eta \\ &\quad -\bar{\eta}\,\partial_\nu[(\partial_\mu \phi^*)\,\gamma^\mu \gamma^\nu\,\psi] + \bar{\eta}\,\partial_\mu[(\partial^\mu \phi^*)\,\psi] + \partial_\mu(\bar{\psi}\,\partial^\mu \phi)\,\eta\end{aligned}$$

**The first row cancels against $\delta \mathcal{L}_s$, and the rest is a total derivative. So,**

$$\delta \int d^4x\, \mathcal{L}_{\text{chiral}} = 0$$

**Not yet enough to declare that $\mathcal{L}_{\text{chiral}}$ is supersymmetric: we must check that the commutator of two transformations is a symmetry of the theory. For the scalar fields we find**

$$(\delta_{\eta_1}\delta_{\eta_2} - \delta_{\eta_2}\delta_{\eta_1})\phi = -(\bar{\eta}_2\,\gamma^\mu\,\eta_1 - \bar{\eta}_1\,\gamma^\mu\,\eta_2)\, i\partial_\mu \phi$$

**Good news: $i\partial_\mu$ is nothing but the four-momentum operator $P_\mu$, the generator of space-time translations, a symmetry of space-time.**

**Now consider the fermion fields. After some tedious algebra,* we get**

$$
\begin{aligned}
(\delta_{\eta_1}\delta_{\eta_2} - \delta_{\eta_2}\delta_{\eta_1})\psi &= -(\bar{\eta}_2\,\gamma^\mu\,\eta_1 - \bar{\eta}_1\,\gamma^\mu\,\eta_2)\,i\partial_\mu\psi \\
&\quad + \frac{1}{2}(\bar{\eta}_2\,\gamma^\mu\,\eta_1 - \bar{\eta}_1\,\gamma^\mu\,\eta_2)\,i\gamma_\mu\not{\partial}\,\psi
\end{aligned}
$$

**The first term is similar to what we got for $\phi$, while the second one vanishes on the mass shell,**

$$\not{\partial}\,\psi = 0.$$

**OK, but it might be useful to close the algebra even off shell.**

* **For those who wish to try: you need the Fierz identities (and some patience).**

**This can be done. Let us introduce an auxiliary scalar field $F$, with the lagrangian**

$$\mathcal{L}_a = F^* F$$

**(note the unusual dimension: $m^2$ instead of $m$). It is harmless: the equation of motion is $F = 0$. Now assume**

$$\delta F = -i\,\bar{\eta}\,\gamma^\mu\,\partial_\mu \psi \qquad \delta F^* = i\,\partial_\mu \bar{\psi}\,\gamma^\mu\,\eta$$

**and modify the fermion transformation rules as follows:**

$$\delta \psi = P_L\,[-i\partial_\mu \phi\,\gamma^\mu \eta + F\eta] \qquad \delta \bar{\psi} = [i\bar{\eta}\,\gamma^\mu \partial_\mu \phi^* + \bar{\eta} F^*]\,P_R$$

**(which amounts to nothing, because the auxiliary fields vanish on shell).**

**Things start getting complicated:**

1. **We can no longer assume that $P_R\eta = \eta$, we must promote it to a full four-component spinor. In order not to increase the number of independent parameters, we assume it is a Majorana spinor:**

$$\eta = \begin{pmatrix} \eta \\ -\epsilon\eta^* \end{pmatrix}$$

2. **We now have to write explicitly the projector $P_L$ in front of $\delta\psi$, in order to keep $\psi$ left-handed.**

**Nevertheless, let us press on ...**

**... It is immediate to check that**

$$\mathcal{L}_{\rm chiral} = \mathcal{L}_s + \mathcal{L}_f + \mathcal{L}_a$$

**is our first supersymmetric lagrangian (not very exciting: it's a non-interacting theory), invariant under the transformation**

$$\delta\phi = \bar\eta\,\psi \qquad \delta\phi^* = \bar\psi\,\eta$$

$$\delta\psi = P_L\left[-i\partial_\mu\phi\,\gamma^\mu\eta + F\eta\right] \qquad \delta\bar\psi = \left[i\bar\eta\,\gamma^\mu\partial_\mu\phi^* + \bar\eta F^*\right]P_R$$

$$\delta F = -i\,\bar\eta\,\gamma^\mu\,\partial_\mu\psi \qquad \delta F^* = i\,\partial_\mu\bar\psi\,\gamma^\mu\,\eta$$

**with**

$$(\delta_{\eta_1}\delta_{\eta_2} - \delta_{\eta_2}\delta_{\eta_1})X = -(\bar\eta_2\,\gamma^\mu P_R\,\eta_1 - \bar\eta_1\,\gamma^\mu P_R\,\eta_2)\,i\partial_\mu X$$

**for $X = \phi, \psi, F$, both on and off the mass shell. This structure can be replicated for an arbitrary number of chiral supermultiplets.**

**The same results they take a much simpler form when recast in Weyl language. Define Weyl spinors through**

$$\psi \to \begin{pmatrix} \psi \\ 0 \end{pmatrix} \qquad \eta \to \begin{pmatrix} \eta \\ -\epsilon\eta^* \end{pmatrix}$$

**Recalling the definitions of $\gamma$ matrices, we get**

$$\delta\phi = \eta\,\psi$$
$$\delta\psi = -i(\sigma^\mu \epsilon\eta^*)\,\partial_\mu\phi + F\eta$$
$$\delta F = -i\,\eta^\dagger\,\bar\sigma^\mu\,\partial_\mu\psi$$

**and**

$$(\delta_{\eta_1}\delta_{\eta_2} - \delta_{\eta_2}\delta_{\eta_1})X = -(\eta_1^\dagger\,\bar\sigma^\mu\,\eta_2 - \eta_2^\dagger\,\bar\sigma^\mu\,\eta_1)\,i\partial_\mu X$$

**Why do we need auxiliary fields? Count fermionic and bosonic degrees of freedom:**

- **on shell:** $\psi$ **has two helicity states (it is a Weyl fermion):** $n_f = 2$
  $\phi$ **is a complex scalar:** $n_b = 2$
- **off shell:** $\psi$ **is a complex two-component spinor:** $n_f = 4$
  $\phi$ **is a complex scalar:** $n_b = 2$
  $F$ **is a complex scalar:** $n_b = 2$

**Without the auxiliary fields, the counting rule** $n_f = n_b$ **would be violated off the mass shell.**

# Ordinary symmetries: a reminder

**Consider a lagrangian density**

$$\mathcal{L}(\phi, \partial\phi)$$

**functions of a set of fields $\phi_i, i = 1, \ldots, n$. The transformation**

$$\delta\phi_i = i\eta^A T^A_{ij}\phi_j; \quad \left(T^A\right)^\dagger = T^A; \quad \left[T^A, T^B\right] = i f^{ABC} T^C$$

**is a symmetry transformation if**

$$\delta\mathcal{L} = \eta^A \partial^\mu K^A_\mu \;\Rightarrow\; \delta S[\phi] = \delta \int d^4x\, \mathcal{L} = 0$$

**As a consequence, a set of *conserved currents* exists:**

$$\partial^\mu J^A_\mu = 0; \quad J^A_\mu = T^A_{ij}\phi_j \frac{\partial\mathcal{L}}{\partial\partial^\mu\phi_i} - K^A_\mu$$

$$Q^A(t) = \int d^3x\, J^A_0(t, \mathbf{x}); \quad \frac{d}{dt}Q^A(t) = 0; \quad \left[Q^A, Q^B\right] = i f^{ABC} Q^C$$

**The same procedure can be applied to our supersymmetry: a straightforward task.**

**The supersymmetry charge $Q$ (a Weyl spinor!) is the space integral of the 0 component of a conserved current $J_\mu$.**

**One finds that its components obey the anticommutator algebra**

$$\{Q, Q^\dagger\} = 2\sigma^\mu P_\mu; \qquad \{Q, Q\} = \{Q^\dagger, Q^\dagger\} = 0$$

**as anticipated.**