2. Neutralinos and charginos

The supersymmetric partner of neutral gauge bosons, $\tilde{w}^0$ and $\tilde{b}$, and of Higgs scalars, $\tilde{h}_u^0, \tilde{h}_d^0$, are not mass eigenstates. We find

$$\mathcal{L}_n = -\frac{1}{2}\,\tilde{\psi}_0^T\, M_n\, \tilde{\psi}_0 + \text{h.c.}$$

where the matrix M_n in the basis $\tilde{b}, \tilde{w}^0, \tilde{h}_d^0, \tilde{h}_u^0$ is given by

$$\begin{bmatrix} M_1 & 0 & -m_Z \cos\beta \sin\theta_W & m_Z \sin\beta \sin\theta_W \\ 0 & M_2 & m_Z \cos\beta \cos\theta_W & -m_Z \sin\beta \cos\theta_W \\ -m_Z \cos\beta \sin\theta_W & m_Z \cos\beta \cos\theta_W & 0 & -\mu \\ m_Z \sin\beta \sin\theta_W & -m_Z \sin\beta \cos\theta_W & -\mu & 0 \end{bmatrix}$$

The mass matrix M_n is symmetric, and can be diagonalized by a unitary transformation N. The four mass eigenstates

$$\tilde{\chi}_i^0 = N_{ij}\,\tilde{\psi}_j^0; \quad M_n = N^T\,\hat{M}_n\,N$$

are called neutralinos. Their coupling are entirely determined by the matrix N, and thus by the four parameters

$$M_1, M_2, \mu, \tan\beta$$

The lightest neutralino is in most models the LSP; neutralinos play a crucial role in phenomenology.

Charged gauginos and higgsinos

$$\tilde{w}^+, \tilde{h}_u^+, \tilde{w}^-, \tilde{h}_d^-$$

mix in a similar way:

$$\mathcal{L}_c = -\frac{1}{2}\,\tilde{\psi}_c^T\, M_c\, \tilde{\psi}_c + \text{h.c.}$$

$$M_c = \begin{bmatrix} 0 & X^T \\ X & 0 \end{bmatrix} \qquad X = \begin{bmatrix} M_2 & \sqrt{2} m_W \sin\beta \\ \sqrt{2} m_W \cos\beta & \mu \end{bmatrix}$$

The mass eigenstates (called charginos) are found by diagonalizing X with a bi-unitary transformation:

$$\begin{pmatrix} \tilde{\chi}_1^+ \\ \tilde{\chi}_2^+ \end{pmatrix} = V \begin{pmatrix} \tilde{w}^+ \\ \tilde{h}_u^+ \end{pmatrix} \qquad \begin{pmatrix} \tilde{\chi}_1^- \\ \tilde{\chi}_2^- \end{pmatrix} = U \begin{pmatrix} \tilde{w}^- \\ \tilde{h}_d^- \end{pmatrix}$$

$$X = U^T \begin{pmatrix} m_{\tilde{\chi}_1^\pm} & 0 \\ 0 & m_{\tilde{\chi}_2^\pm} \end{pmatrix} V$$

$$\begin{aligned} m^2_{\tilde{\chi}_1^\pm, \tilde{\chi}_2^\pm} &= \frac{1}{2}\Big[\left(|M_2|^2 + |\mu|^2 + 2m_W^2 \right) \\ &\mp \sqrt{\left(|M_2|^2 + |\mu|^2 + 2m_W^2 \right)^2 - 4\left|\mu M_2 - m_W^2 \sin 2\beta\right|^2} \Big] \end{aligned}$$

As in the case of neutralinos, the matrices U, V contain all the relevant information about chargino couplings.

It is quite natural to assume that gaugino masses have a common value $m_{1/2}$ at the reference scale Q_0, where the (properly normalized) gauge couplings take approximately the same value $\alpha_U \sim 0.04$.

It turns out that gaugino running masses obey the same renormalization group equations. Therefore, with the above assumption,

$$M_i(Q) = \frac{\alpha_i(Q)}{\alpha_U} m_{1/2}$$

This reduces the number of arbitrary parameters in the gaugino sector.

3. Squarks and sleptons

The scalar partners of matter fermions (squarks and sleptons) have mass matrices with contributions from various terms:

- **quartic Higgs-Higgs-squark-squark vertices**
- **trilinear Higgs-squark-squark couplings**
- **soft-breaking masses**
- D**-terms**

In general, there is mixing among different families, and mixing betweel scalar parners of left- and right-handed fermions. This can be sizeable in the third generation.

Mass matrices for squarks and sleptons in the third generation, in the simplified situation outlined above:

$$\mathbf{s-top:}\quad (\tilde t_L^* \ \ \tilde t_R^*)\begin{pmatrix} m_Q^2+m_t^2+\Delta_Q & m_t(A_t-\mu\cot\beta) \\ m_t(A_t-\mu\cot\beta) & m_U^2+m_t^2+\Delta_U \end{pmatrix}\begin{pmatrix}\tilde t_L \\ \tilde t_R\end{pmatrix}$$

$$\mathbf{s-bottom:}\quad (\tilde b_L^* \ \ \tilde b_R^*)\begin{pmatrix} m_Q^2+m_b^2+\Delta_Q & m_b(A_b-\mu\tan\beta) \\ m_b(A_b-\mu\tan\beta) & m_D^2+m_b^2+\Delta_D \end{pmatrix}\begin{pmatrix}\tilde b_L \\ \tilde b_R\end{pmatrix}$$

$$\mathbf{s-tau:}\quad (\tilde \tau_L^* \ \ \tilde \tau_R^*)\begin{pmatrix} m_L^2+m_\tau^2+\Delta_L & m_\tau(A_\tau-\mu\tan\beta) \\ m_\tau(A_\tau-\mu\tan\beta) & m_E^2+m_\tau^2+\Delta_E \end{pmatrix}\begin{pmatrix}\tilde \tau_L \\ \tilde \tau_R\end{pmatrix}$$

where

$$\Delta = m_Z^2(T_3 - Q\sin^2\theta_W)\cos 2\beta$$

Spontaneous supersymmetry breaking

How are soft supersymmetry-breaking terms generated? We need a mechanism of spontaneous supersymmetry breaking (explicit breaking not acceptable).

A symmetry is said to be spontaneously broken when the vacuum state is not invariant under that symmetry:

$$Q|0\rangle \neq 0$$

Supersymmetry is no exception. This can happen only if some of the auxiliary fields F_i or D^a acquires a non-zero vacuum expectation value.

Proof: the supersymmetry algebra implies

$$H = P^0 = \frac{1}{4}\left(Q_1 Q_1^\dagger + Q_1^\dagger Q_1 + Q_2 Q_2^\dagger + Q_2^\dagger Q_2\right)$$

which implies

$$Q_i|0\rangle \neq 0 \;\Leftrightarrow\; \langle 0|H|0\rangle > 0 \;\Leftrightarrow\; \langle 0|V|0\rangle > 0$$

for a translation-invariant vacuum state. But

$$V = F_i F_i^* + \frac{1}{2} D^a D^a$$

so supersymmetry is spontaneously broken if and only if the system

$$\langle 0|F_i|0\rangle = 0 \quad \langle 0|D^a|0\rangle = 0$$

has no solution.

Spontaneous breaking based on $\langle D^a \rangle \neq 0$ for some D^a is called à la Fayet-Iliopoulos. If there is a $U(1)$ factor in the gauge group, a term

$$\mathcal{L}_{\text{D}-\text{breaking}} = kD$$

can be added to the lagrangian. Then

$$V = \frac{1}{2}D^2 - kD + gD\sum_i q_i \phi_i \phi_i^*$$

$$D = k - g\sum_i q_i \phi_i \phi_i^*$$

D is different from zero if for some reason all scalar fields are zero at the minimum. This is *not* the case in the MSSM.

Not very promising, although not completely ruled out.

Models in which supersymmetry is spontaneously broken by $\langle F_i \rangle \neq 0$ for some F_i are called O'Raifeartaigh models. An explicit example: three chiral superfields, and

$$W = -k\phi_1 + m\phi_2\phi_3 + \frac{y}{2}\phi_1\phi_3^2$$

where ϕ_1 must be a gauge singlet. Then

$$V = |F_1|^2 + |F_2|^2 + |F_3|^2$$

$$F_1 = k - \frac{y}{2}\phi_3^{*2} \quad F_2 = -m\phi_3^* \quad F_3 = -m\phi_2^* - y\phi_1^*\phi_3^*$$

$F_1 = F_2 = F_3 = 0$ has no solution. For $m^2 > yk$, the minimum of V is for $\phi_2 = \phi_3 = 0$, ϕ_1 undetermined (flat direction).

The solution is $F_1 = k$, $V = k^2$ at the minimum. The flat direction is no longer flat after radiative corrections, and the minimum is at $\phi_1 = 0$. The spectrum: six scalars with masses

$$0 \quad 0 \quad m^2 \quad m^2 \quad m^2 - yk \quad m^2 + yk$$

and three fermions with masses

$$0 \quad m \quad m$$

The zero-mass fermion is the analog of a Goldstone boson in broken ordinary symmetry: we call it a goldstino (sic). It is the partner of the auxiliary field F_1, responsible for supersymmetry breaking.

A general conclusion: spontaneous breaking of supersymmetry cannot take place within the MSSM. There is no gauge-singlet supermultiplet whose auxiliary field can take a non-vanishing vacuum expectation value.

Hard to do by simply extending the MSSM with new fields and new renormalizable interactions:

- **gaugino masses cannot arise from renormalizable couplings**
- **the spectrum must include light scalars:**

$$\mathrm{STr}\mathcal{M}^2 = \sum_i (-1)^{(2s)}(2s+1)m_i^2 = 0$$

even after supersymmetry breaking.

Mass terms are generated after spontaneous symmetry breaking as, for example, quark masses in the standard model:

$$y\,\phi\bar{\psi}\psi \;\to\; y\,\langle\phi\rangle\bar{\psi}\psi = m\bar{\psi}\psi$$

In supersymmetry, the same mechanism is at work. A gaugino mass could be generated either by a coupling

$$\phi\lambda\lambda \to \langle\phi\rangle\lambda\lambda$$

but such a term is absent in a supersymmetric theory, or by

$$F\lambda\lambda \to \langle F\rangle\lambda\lambda$$

which is not renormalizable.

Way out: assume that supersymmetry is broken in a hidden sector of non-observable degrees of freedom, that communicate somehow with the observable sector. Historically, two proposals: supersymmetry breaking communicated to the observable sector by

1. **gravitational interactions**
2. **gauge interactions**

The differences between the two cases are due to a different size of the supersymmetry breaking scale

$$\langle 0|F|0\rangle$$

In gravity mediated supersymmetry breaking, soft breaking terms arise from (in general nonrenormalizable) interaction terms (of gravitational strength) of observable fields with combinations of fields in the hidden sector that acquire a non-zero vev. We expect therefore

$$m_{\rm soft} \sim \frac{\langle F \rangle}{M_P}$$

on dimensional grounds: $m_{\rm soft}$ must vanish as $\langle F \rangle$ goes to zero (no spontaneous breaking) and as $M_P \to \infty$ (no interaction between hidden and observable sectors.

For $m_{\rm soft} \sim 1$ TeV we get

$$\sqrt{\langle F \rangle} \sim 10^{11}\ {\rm GeV}$$

In the gauge mediated scenario, superymmetry breaking is communicated to the observable sector by the ordinary gauge interactions via loop effects due to particles called messengers.

A rough estimate of the relevant supersymmetry breaking scale can be obtained. We have in this case

$$m_{\text{soft}} \sim \frac{\alpha}{4\pi} \frac{\langle F \rangle}{M}$$

where M is the typical messenger mass. This corresponds to a much lower value for the supersymmetry order parameter:

$$\sqrt{\langle F \rangle} \sim 10^4 - 10^5 \text{ GeV}$$

if $\langle F \rangle$ and M are assumed to be roughly of the same size.