Monte Carlo event generators for LHC physics

Mike Seymour

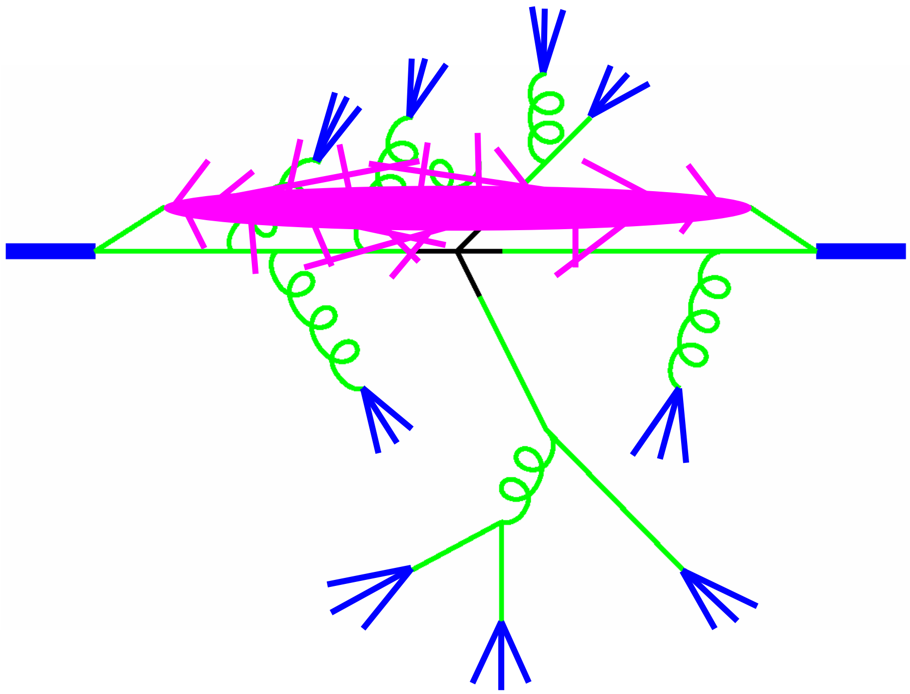
University of Manchester

CERN Academic Training Lectures

July 7th – 11th 2003

Structure of LHC Events

1. Hard process
2. Parton shower
3. Hadronization
4. Underlying event



Monte Carlo for the LHC

1. Basic principles
2. Parton showers
3. Hadronization
4. Monte Carlo programs in practice
5. Questions and answers

Parton Showers: Introduction

QED: accelerated charges radiate.

QCD identical: accelerated colours radiate.

gluons also charged.

à cascade of partons.

= parton shower.

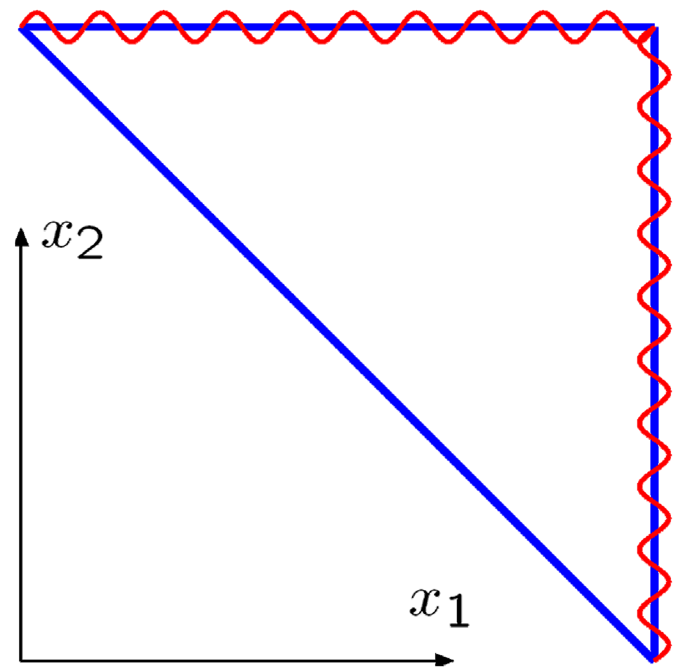
1. e^+e^-annihilation to jets.
2. Universality of collinear emission.
3. Sudakov form factors.
4. Universality of soft emission.
5. Angular ordering.
6. Initial-state radiation.
7. Hard scattering.
8. Heavy quarks.
9. The Colour Dipole Model.

e^+e^- annihilation to jets

Three-jet cross section:

$$\frac{d\sigma}{dx_1 dx_2} = \sigma_0 \, C_F \frac{\alpha_s}{2\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

singular as $x_{1,2} \to 1$



Rewrite in terms of quark-gluon opening angle θ and gluon energy fraction x_3:

$$\frac{d\sigma}{d\cos\theta \, dx_3} = \sigma_0 \, C_F \frac{\alpha_s}{2\pi} \left\{ \frac{2}{\sin^2\theta} \frac{1+(1-x_3)^2}{x_3} - x_3 \right\}$$

Singular as $\sin\theta \to 0$ and $x_3 \to 0$.

can separate into two independent jets:

$$\frac{2\, d\cos\theta}{\sin^2\theta} = \frac{d\cos\theta}{1-\cos\theta} + \frac{d\cos\theta}{1+\cos\theta}$$

$$= \frac{d\cos\theta}{1-\cos\theta} + \frac{d\cos\bar{\theta}}{1-\cos\bar{\theta}}$$

$$\approx \frac{d\theta^2}{\theta^2} + \frac{d\bar{\theta}^2}{\bar{\theta}^2}$$

jets evolve independently

$$d\sigma = \sigma_0 \sum_{\text{jets}} C_F \frac{\alpha_s}{2\pi} \frac{d\theta^2}{\theta^2}\, dz \frac{1+(1-z)^2}{z}$$

Exactly same form for anything $\propto \theta^2$

eg transverse momentum: $k_\perp^2 = z^2(1-z)^2\,\theta^2\,E^2$

invariant mass: $q^2 = z(1-z)\,\theta^2\,E^2$

$$\frac{d\theta^2}{\theta^2} = \frac{dk_\perp^2}{k_\perp^2} = \frac{dq^2}{q^2}$$

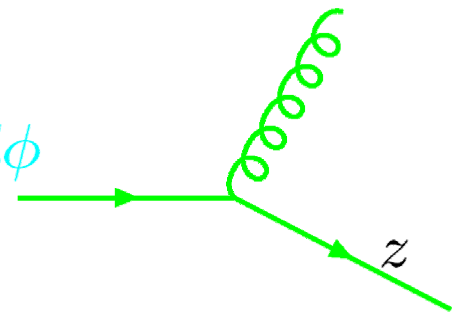
Collinear Limit

Universal:

$$d\sigma = \sigma_0 \, \frac{\alpha_s}{2\pi} \, \frac{d\theta^2}{\theta^2} \, dz \, P(z, \phi) d\phi$$

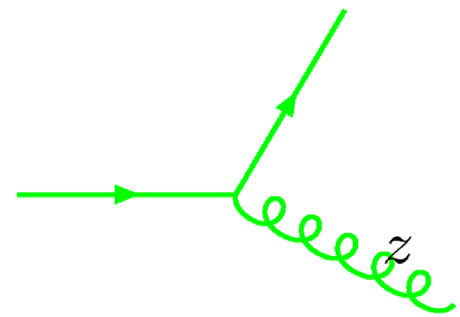
$$P(z, \phi) =$$

Dokshitzer-Gribov-Lipatov-Altarelli-Parisi splitting kernel: dependent on flavour and spin



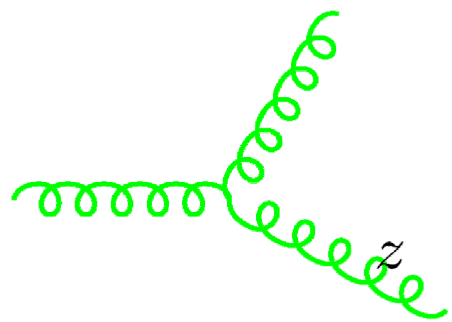
$q \to qg$

$C_F \frac{1+z^2}{1-z}$



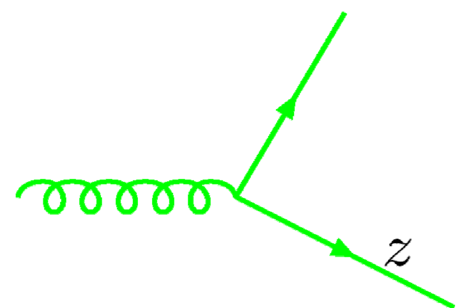
$q \to gq$

$C_F \frac{1+(1-z)^2}{z}$



$g \to gg$

$C_A \frac{z^4+1+(1-z)^4}{z(1-z)}$



$g \to q\bar{q}$

$T_R \left(z^2 + (1-z)^2 \right)$

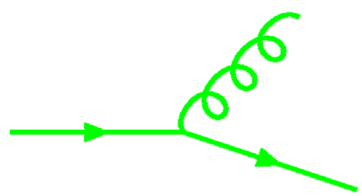
Resolvable partons

What is a parton?

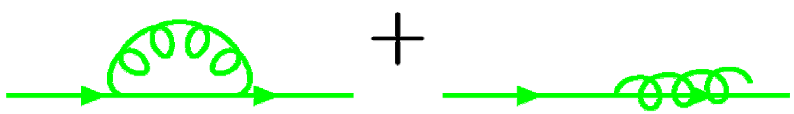
Collinear parton pair $\longleftrightarrow$ single parton

Introduce resolution criterion, eg $k_\perp > Q_0$.

Virtual corrections must be combined with unresolvable real emission



Resolvable emission
Finite



Virtual + Unresolvable emission
Finite

Unitarity: P(resolved) + P(unresolved) = 1

Sudakov form factor

Probability(emission between q^2 and $q^2 + dq^2$)

$$d\mathcal{P} = \frac{\alpha_s}{2\pi}\frac{dq^2}{q^2}\int_{Q_0^2/q^2}^{1-Q_0^2/q^2} dz\; P(z) \equiv \frac{dq^2}{q^2}\bar{P}(q^2).$$

Define probability(no emission between Q^2 and q^2) to be $\Delta(Q^2, q^2)$. Gives evolution equation

$$\frac{d\Delta(Q^2,q^2)}{dq^2} = \Delta(Q^2,q^2)\frac{d\mathcal{P}}{dq^2}$$

$$\Rightarrow \Delta(Q^2,q^2) = \exp - \int_{q^2}^{Q^2}\frac{dk^2}{k^2}\bar{P}(k^2).$$

c.f. radioactive decay

atom has probability λ per unit time to decay.

Probability(no decay after time T) = $\exp - \int^T dt\,\lambda$

Sudakov form factor

Probability(emission between q^2 and q^2+dq^2)

$$d\mathcal{P} = \frac{\alpha_s}{2\pi}\frac{dq^2}{q^2}\int_{Q_0^2/q^2}^{1-Q_0^2/q^2} dz\ P(z) \equiv \frac{dq^2}{q^2}\bar{P}(q^2).$$

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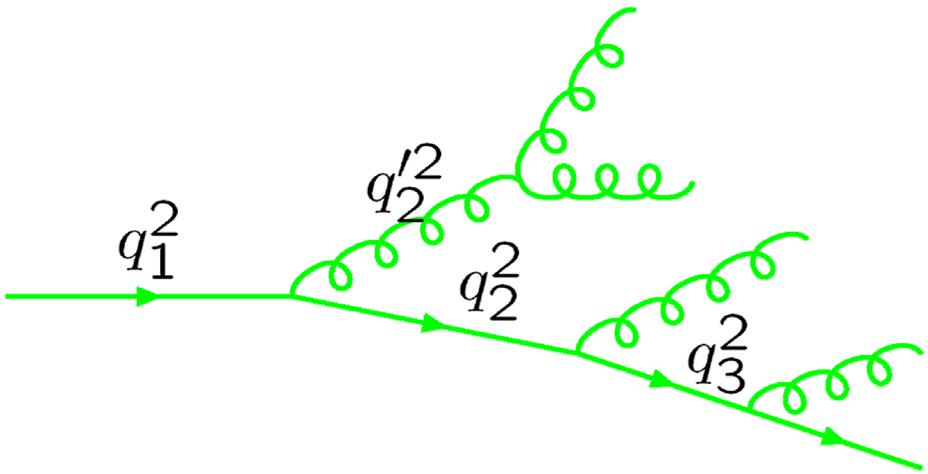
$$\Rightarrow \Delta(Q^2,q^2) = \exp - \int_{q^2}^{Q^2}\frac{dk^2}{k^2}\bar{P}(k^2).$$

$\Delta(Q^2,Q_0^2) \equiv \Delta(Q^2)$ Sudakov form factor

=Probability(emitting no resolvable radiation)

$$\Delta_q(Q^2) \sim \exp - C_F\frac{\alpha_s}{2\pi}\log^2\frac{Q^2}{Q_0^2}$$

Multiple emission



$$q_1^2 > q_2^2 > q_3^2 > \dots$$
$$q_1^2 > q_2'^2 \dots$$

But initial condition? $q_1^2 < ???$

Process dependent

Monte Carlo implementation

Can generate branching according to

$$d\mathcal{P} = \frac{dq^2}{q^2}\bar{P}(q^2)\,\Delta(Q^2, q^2)$$

By choosing $0 < \rho < 1$ uniformly:

If $\rho < \Delta(Q^2)$ no resolvable radiation, evolution stops.

Otherwise, solve $\rho = \Delta(Q^2, q^2)$

for q^2 =emission scale

Considerable freedom:

Evolution scale: $q^2/k_\perp^2/\theta^2$?

z: Energy? Light-cone momentum?

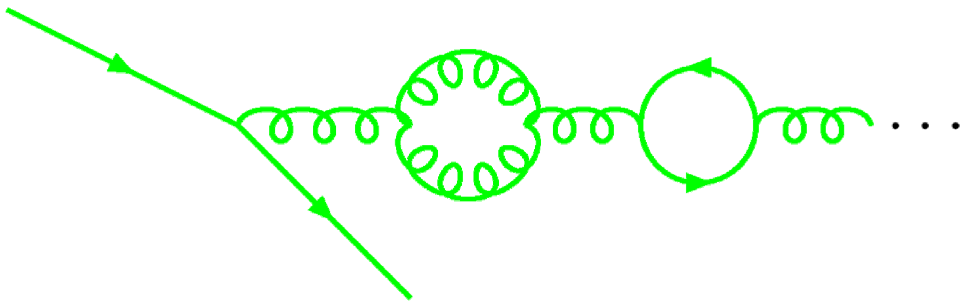
Massless partons become massive. How?

Upper limit for q^2?

All formally free choices, but can be very important numerically

Running coupling

Effect of summing up higher orders:



absorbed by replacing α_s by $\alpha_s(k_\perp^2)$.

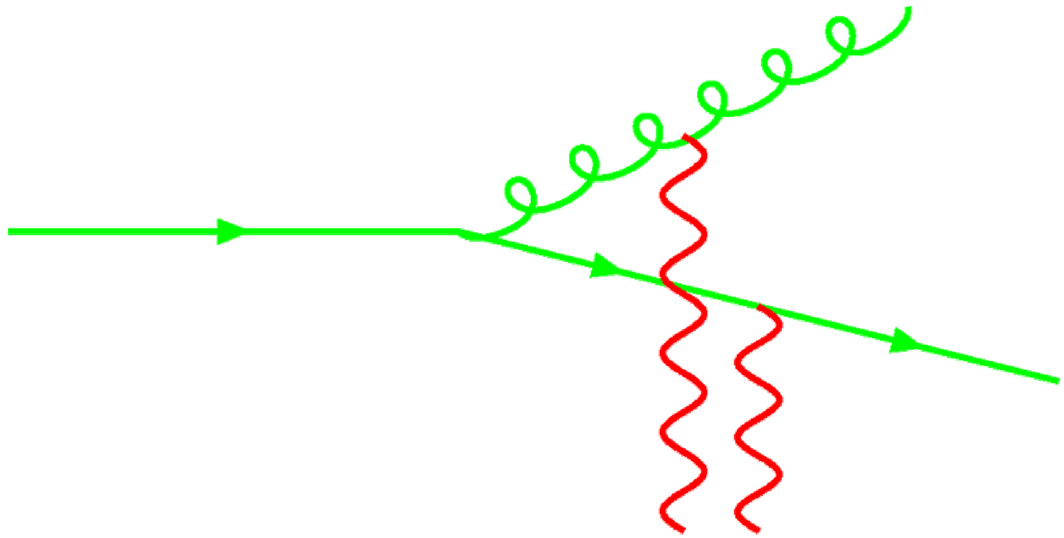
Much faster parton multiplication – phase space fills with soft gluons.

Must then avoid Landau pole: $k_\perp^2 \gg \Lambda^2$.

Q_0 now becomes physical parameter!

Soft limit

Also universal. But at amplitude level…



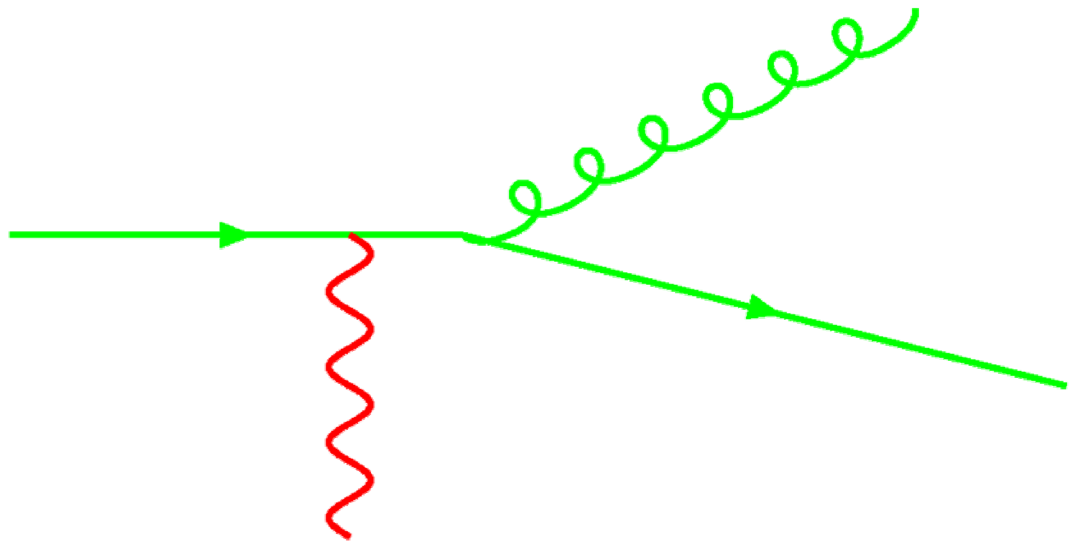
soft gluon comes from everywhere in event.

à Quantum interference.

Spoils independent evolution picture?

Angular ordering

NO:



outside angular ordered cones, soft gluons sum coherently: only see colour charge of whole jet.

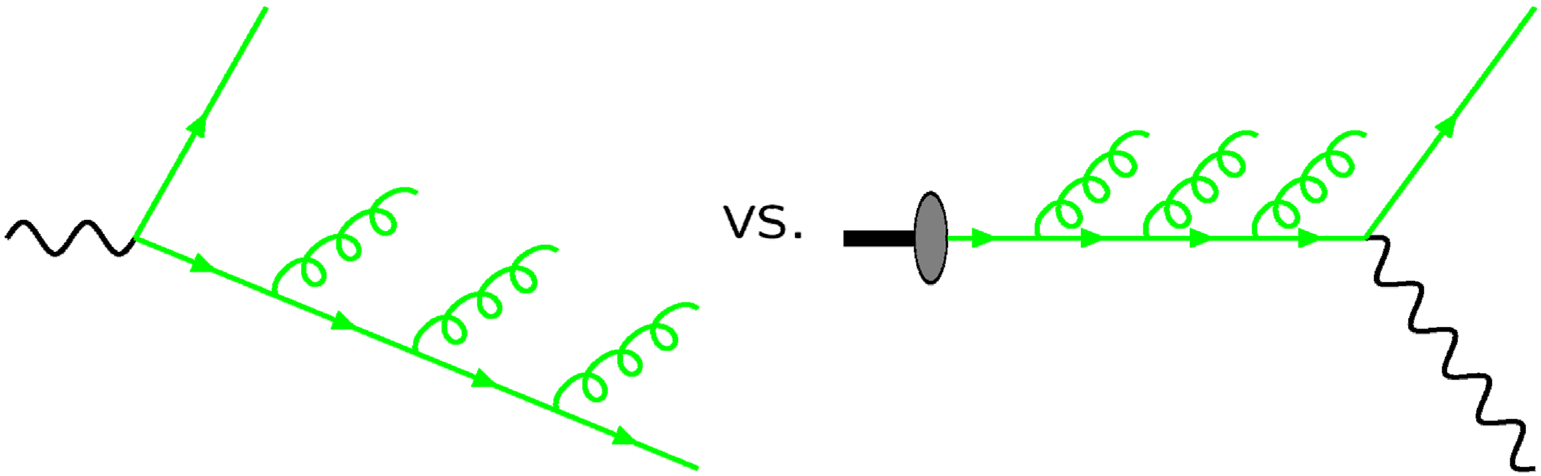
Soft gluon effects fully incorporated by using θ^2 as evolution variable: angular ordering

First gluon not necessarily hardest!

Initial state radiation

In principle identical to final state (for not too small x)

In practice different because both ends of evolution fixed:



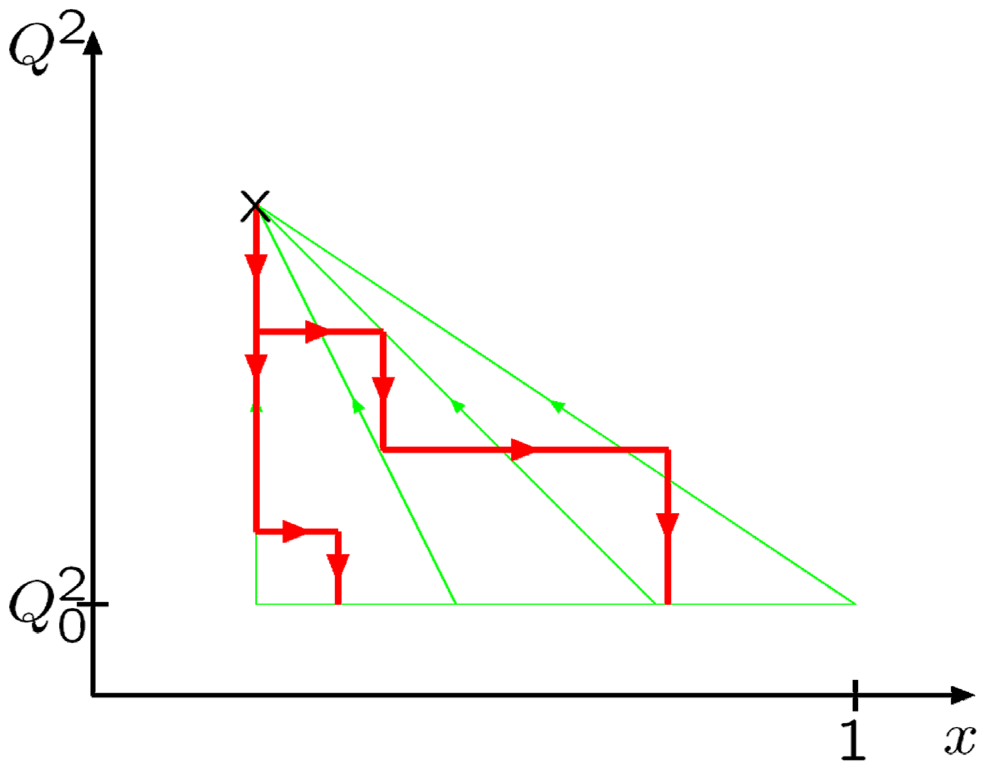
Use approach based on evolution equations…

Backward evolution

DGLAP evolution: pdfs at (x, Q^2) as function of pdfs at $(> x, Q_0^2)$:

Evolution paths sum over all possible events.

Formulate as backward evolution: start from hard scattering and work down in q^2, up in x towards incoming hadron.

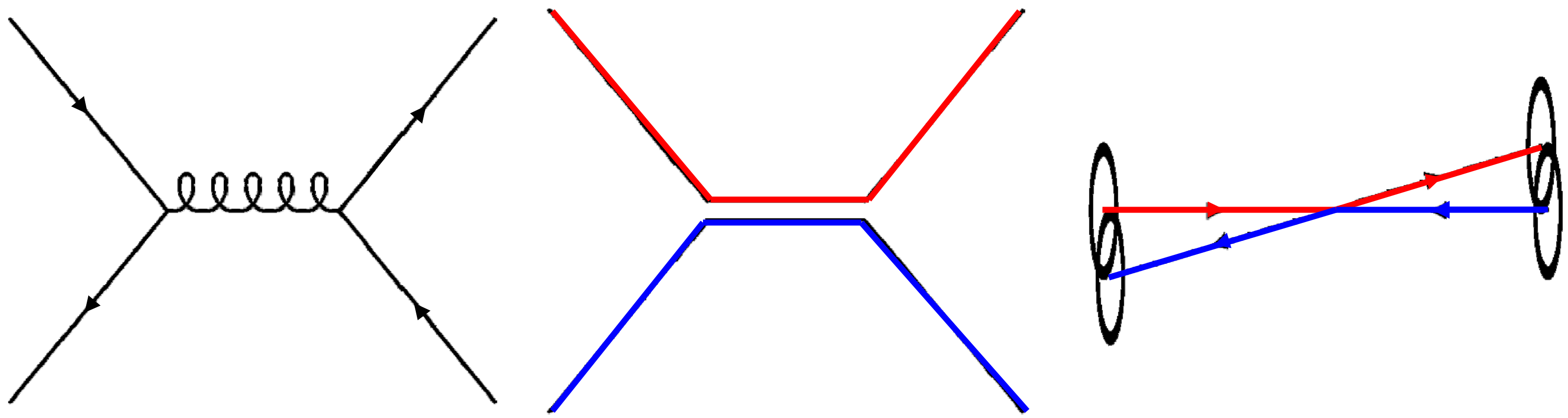


Algorithm identical to final state with $\Delta_i(Q^2, q^2)$ replaced by $\Delta_i(Q^2, q^2)/f_i(x, q^2)$.

Hard Scattering

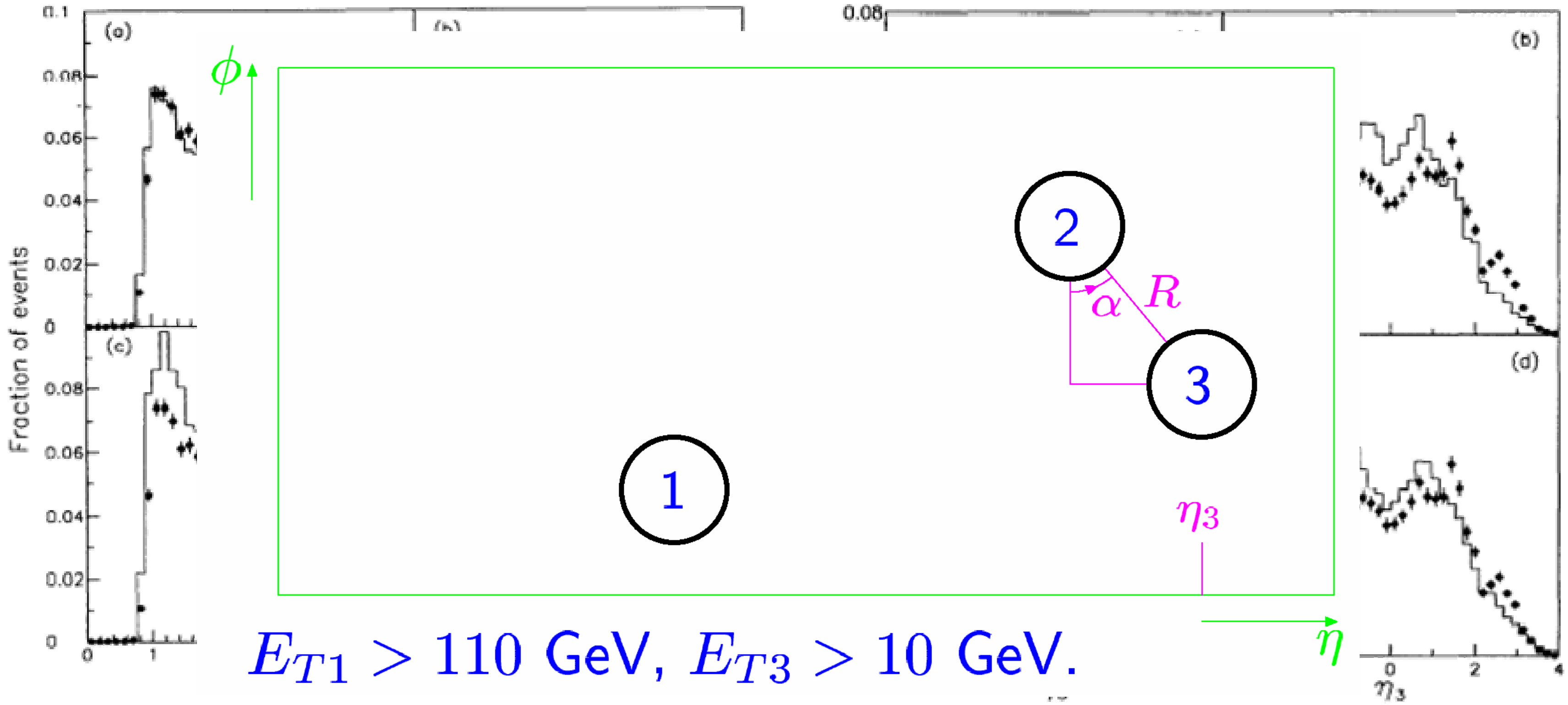
Sets up initial conditions for parton showers.

Colour coherence important here too.



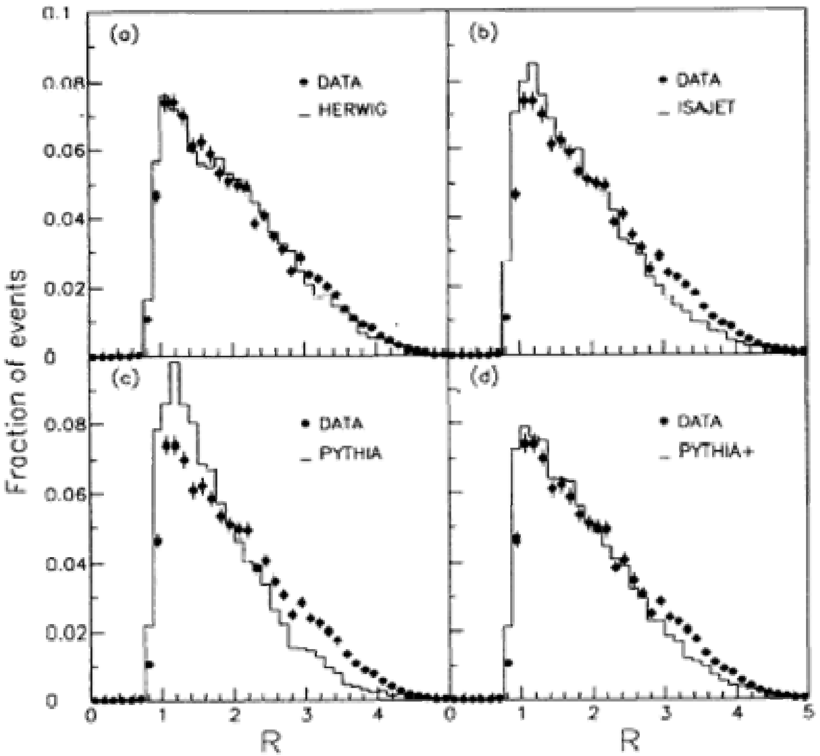
Emission from each parton confined to cone stretching to its colour partner

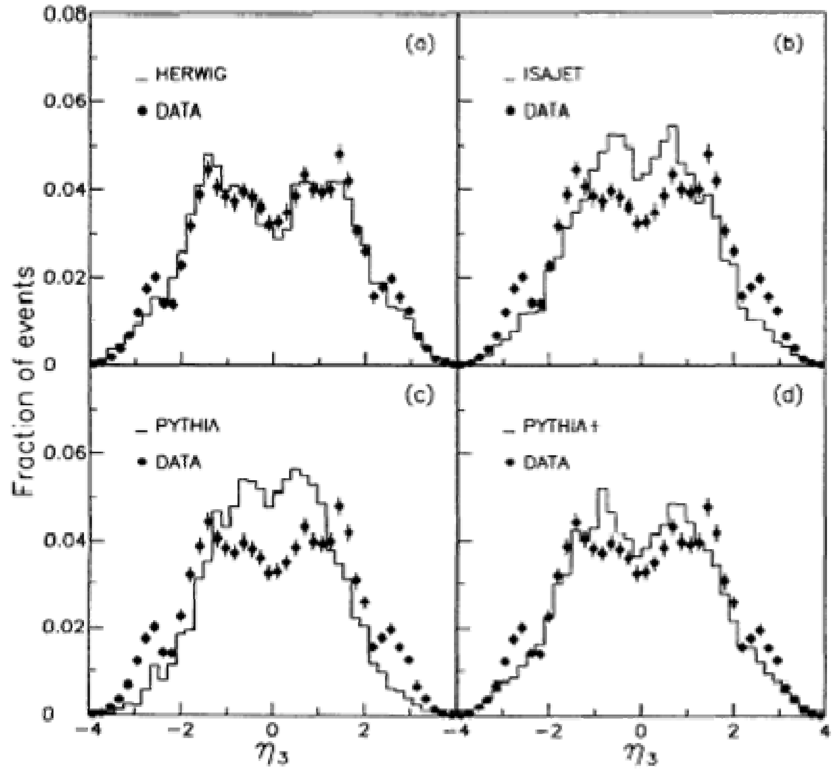
Essential to fit Tevatron data…



Distributions of third-hardest jet in multi-jet events

HERWIG has complete treatment of colour coherence, PYTHIA+ has partial



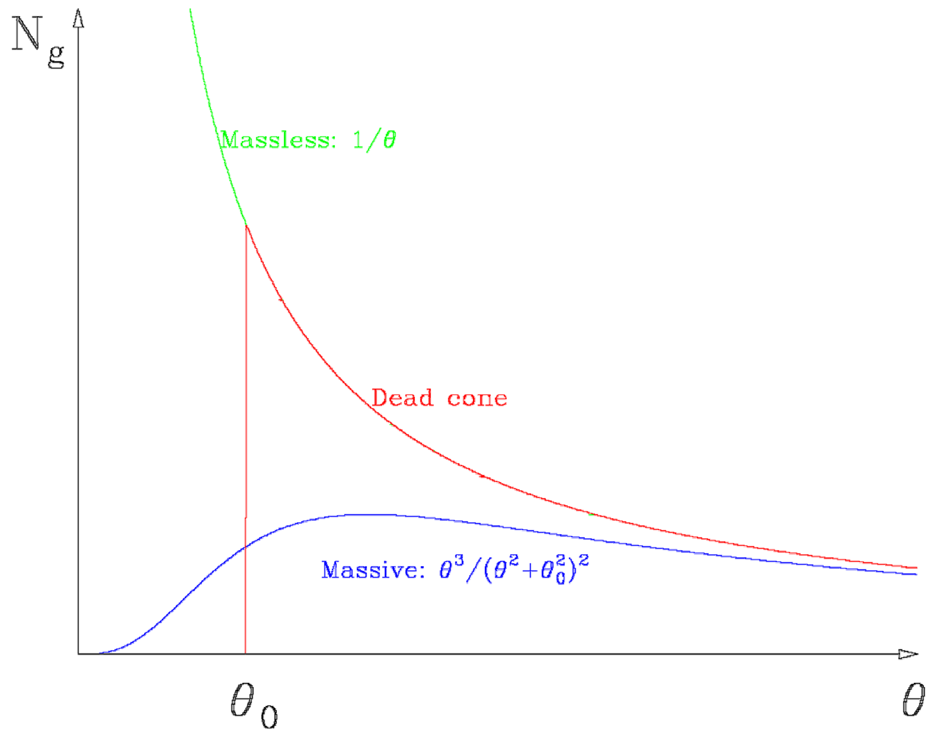


Distributions of third-hardest jet in multi-jet events

HERWIG has complete treatment of colour coherence, PYTHIA+ has partial

Heavy Quarks/Spartons

look like light quarks at large angles, sterile at small angles:



implemented as energy-dependent cutoff: $\theta > \theta_0 = \frac{m_q}{E_q}$.

The ‘dead cone’. Too extreme?

The Colour Dipole Model

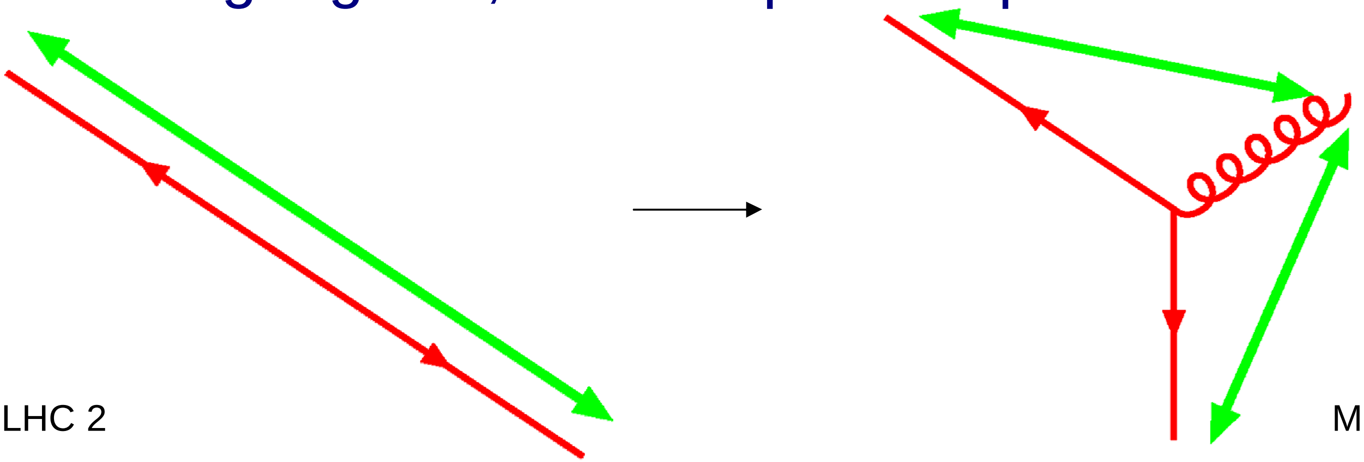
Conventional parton showers: start from collinear limit, modify to incorporate soft gluon coherence

Colour Dipole Model: start from soft limit

Emission of soft gluons from colour-anticolour dipole universal (and classical):

$$d\sigma \approx \sigma_0 \, \tfrac{1}{2} C_A \frac{\alpha_s(k_\perp)}{2\pi} \frac{dk_\perp^2}{k_\perp^2} \, dy, \qquad y = \text{rapidity} = \log\tan\theta/2$$

After emitting a gluon, colour dipole is split:

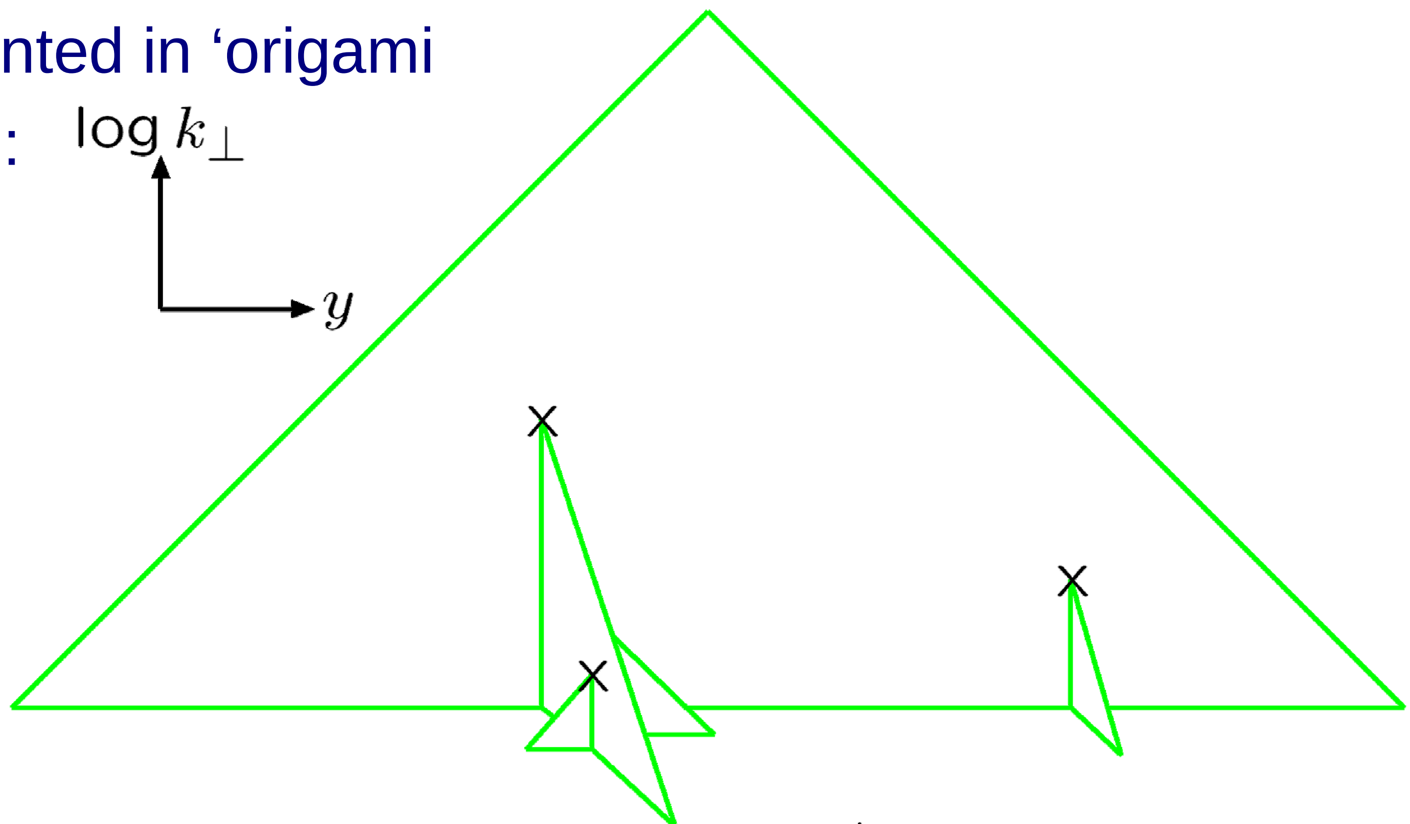


Subsequent dipoles continue to cascade

c.f. parton shower: one parton à two

CDM: one dipole à two = two partons à three

Represented in ‘origami diagram’:



Similar to angular-ordered parton shower for e^+e^- annihilation

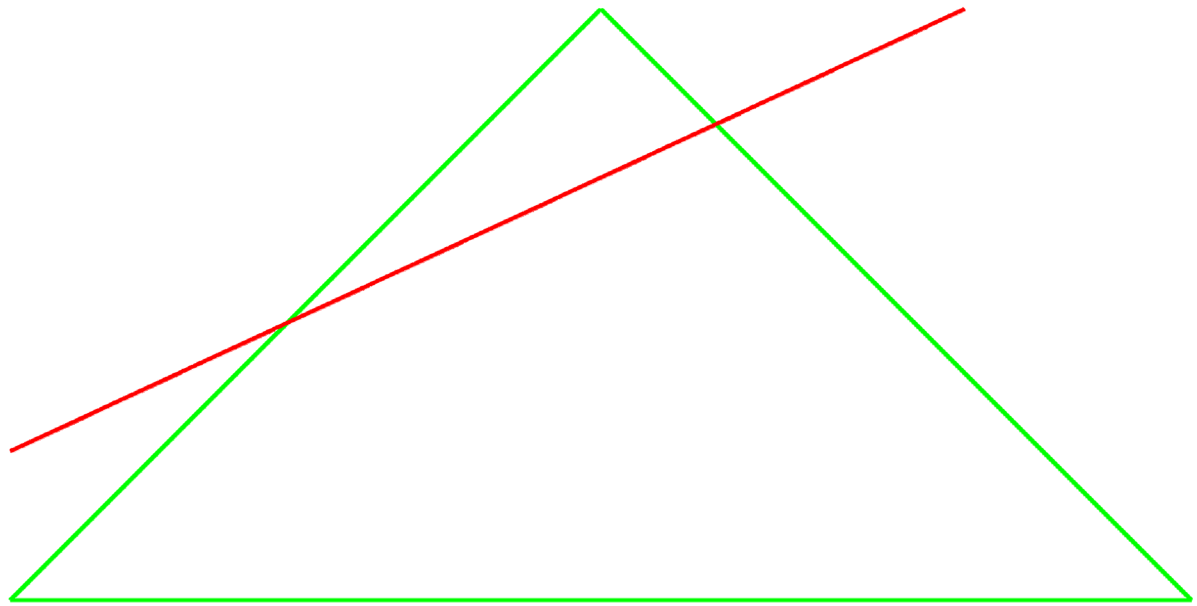
Initial-state radiation in the CDM

There is none!

Hadron remnant forms colour dipole with scattered quark.

Treated like any other dipole.

Except remnant is an extended object: suppression



Biggest difference relative to angular-ordered à more radiation at small x

The Programs

- ISAJET: q^2 ordering; no coherence; huge range of hard processes.
- PYTHIA: q^2 ordering; veto of non-ordered final state emission; partial implementation of angular ordering in initial state; big range of hard processes.
- HERWIG: complete implementation of colour coherence; NLO evolution for large x; smaller range of hard processes.
- ARIADNE: complete implementation of colour dipole model; best fit to HERA data; interfaced to PYTHIA for hard processes.

Summary

- Accelerated colour charges radiate gluons.

  Gluons are also charged à cascade.

- Probabilistic language derived from factorization theorems of full gauge theory.

  Colour coherence is a fact of life: do not trust those who ignore it!

- Modern parton shower models are very sophisticated implementations of perturbative QCD, but would be useless without hadronization models…

Reminder – FAQs

Lecture 5, Friday 11th July:

Question and Answer session

Email questions to: M.H.Seymour@rl.ac.uk

Cutoff: Thursday 10th July, 2pm