The Standard Model of Fundamental Interactions

Achievements and Prospects

Gilles Cohen-Tannoudji

Quod Erat Demonstrandum

The QED perturbative expansion

- The **whole information** necessary for the QED (electromagnetic interactions of electrons) **perturbation expansion** is encoded in the **QED Lagrangian**

$$Z = \int \mathcal{D}\psi \mathcal{D}A \exp\left\{ \frac{i}{\hbar} \int d^4x \mathcal{L}_{\text{QED}} \left(\psi(x), A(x) \right) \right\}$$

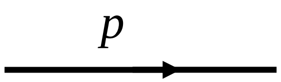
$$\mathcal{L}_{QED} = \bar{\psi}\left(i\gamma^\mu \partial_\mu - m \right)\psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - e\bar{\psi}\gamma^\mu A_\mu \psi$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

- Generating functional

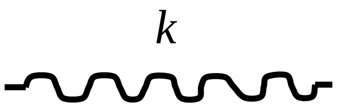
$$\mathcal{K}(j_\psi, j_{\bar\psi}, j_A) = \int \mathcal{D}\psi \mathcal{D}\bar\psi \mathcal{D}A \exp\left\{ \frac{i}{\hbar} \int d^4x (\mathcal{L}_{\text{QED}} + j_\psi \psi + j_{\bar\psi} \bar\psi + j_A A) \right\}$$

- Feynman's diagrams and rules
  - Electron propagator



$$\frac{\not{p} + m}{p^2 - m^2}$$

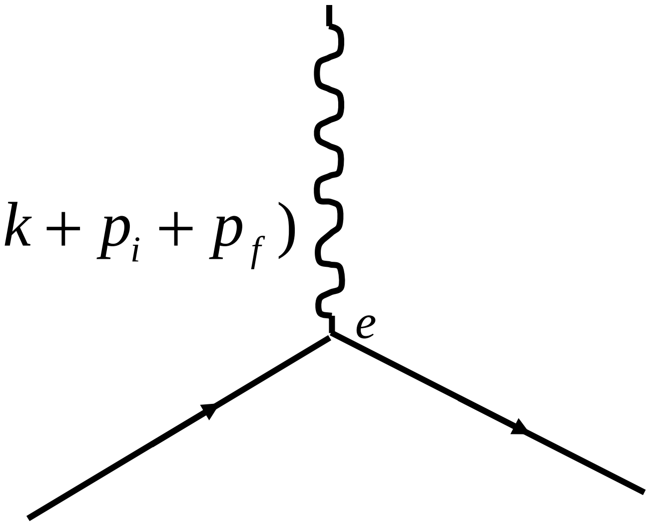
  - Photon propagator



$$\frac{1}{k^2}$$

  - Interaction vertex

$$\gamma^\mu \delta(k + p_i + p_f)$$

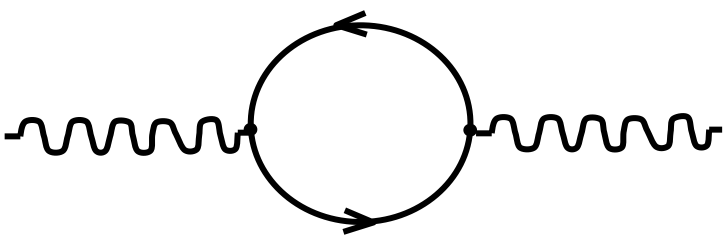


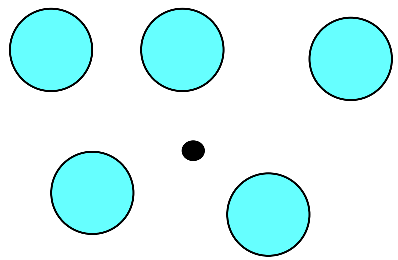
The divergence problem

- All Feynman amplitudes defined in terms of **ordinary integrals**
- In general, integrals implied by diagrams involving loops **diverge**
- These divergences reflect the **conflict** between **locality** (i.e. relativity + causality) and **quantum mechanics**: quantum fields are operator valued **distributions whose products at the same point is ill-defined**
- In QED, all divergences are **logarithmic**

Quantum vacuum as a dielectric: a heuristic picture of renormalisation

- Quantum fluctuations and vacuum polarization.
  - In the Fock space of the electromagnetic field, vacuum is the state with 0 photon. According to Heisenberg's inequalities, when the number of quanta is determined (0 in the vacuum) the value of the field is completely undetermined; it fluctuates
  - One such fluctuation corresponds to the **appearance followed by the disappearance of an electron-positron pair**
  - These fluctuations produce a **screening effect**, leading to the notion of an **effective charge depending on the resolution**

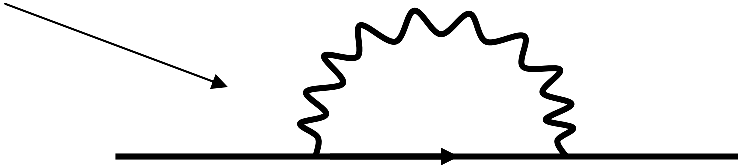




Effect of the interactions on the parameters of the theory

- The charge and the mass of the electron (the parameters of the theory) are **affected by the interaction with the electromagnetic field**.
- **self energy diagram**

An electron emits and reabsorbs a photon: the electron quantum field is modified by the electromagnetic fields it generates



- The parameters of the theory are split into "**bare**" parameters and "**dressed**" parameters, i.e. "**renormalized"** by the interaction
- Renormalized parameters are **effective parameters depending on the resolution**, one says that they are **renormalized at scale** μ

The "miracle" of Renormalizability

- **Renormalization**= redefinition of the fundamental parameters of the theory
  - Electric **charge** *e* split into
    - e_0 "**bare charge**", i.e. the charge if there were no interaction
    - e physical, "**renormalized**" charge
  - Electron **mass** *m* split into
    - m_0 "**bare mass**", i.e. the mass if there were no interaction
    - m physical, "**renormalized**" mass
- **Regularization**: With an energy **cutoff** Λ one removes the divergences in the expressions of e, m and any amplitude in terms of e_0 and m_0.

- One can **invert** these expressions

$$e_0 = e + 1/2\beta_2 e^3 \,\mathrm{Log}\left(\frac{\Lambda}{m}\right) + O(e^5)$$

$$m_0 = m + \gamma_1 m e^2 \,\mathrm{Log}\left(\frac{\Lambda}{\mathrm{m}}\right) + O(e^4)$$

and express **all amplitudes in terms of *e* and *m***

- The "**miracle of renormalizability**" is then that in these expressions, for any amplitude, and at all orders of perturbation expansion, **the limit when Λ goes to infinity is finite,** which means that one can express perturbatively all observables **in terms of two and only two physical parameters** which can be measured experimentally.
- The theory/experiment agreement is **amazingly good**

  For example, for the magnetic moment of the electron one finds

  Exp: 2,00231930482+/- 40
  Th: 2,00231930476+/- 52

The renormalization group

- If the electron mass *m* was equal to zero, it would have been necessary to introduce a new energy scale μ, called the **renormalization energy**, to deal with the logarithmic divergence,

$$e_0 = e + 1/2\beta_2 e^3 \,\mathrm{Log}\left(\frac{\Lambda}{\mu}\right) + O(e^5)$$

and when one inverts this relation, *e* becomes a function of μ

- When $m^{1}\ 0$

a renormalization energy is still necessary, but it is implicitly set equal to *m* : $e(\mu = m) = e\,; m(\mu = m) = m$

- The renormalization energy being arbitrary, physics must be independent on its value, which means that the renormalized charge must obey a **differential equation, called the renormalization group equation**, expressing this independence:

$$\mu \frac{de^2(\mu)}{d\mu} = \beta\left(e^2(\mu)\right)$$

where $\beta(e^2) = \beta_2 e^4 + O(e^6)$

is finite when Λ goes to infinity (β_2 is a c-number, depending only on the QED Lagrangian)

- As a consequence, parameters in QED have to be considered as **effective parameters**, depending on the resolution

$$\alpha(\mu = m_e) = 1/137\,;\, \alpha(\mu = m_Z) = 1/128$$

Gauge invariance

- In Maxwell theory, **gauge invariance** $(A_\mu \to A_\mu + \partial_\mu \Lambda(x))$ **is equivalent to current conservation**
- In QED, the following transformation leaves the QED Lagrangian invariant

$$\psi(x) \to e^{-i\alpha(x)}\psi(x)$$

$$\overline{\psi(x)} \to e^{+i\alpha(x)}\overline{\psi(x)} \qquad \alpha(x) = e\Lambda(x)$$

$$A_\mu \to A_\mu + \partial_\mu \Lambda(x)$$

- Conversely, one can impose invariance through a **local** change of the phase of the electron field by introducing a gauge invariant field, the electromagnetic field: **local gauge invariance determines the structure of the interaction**

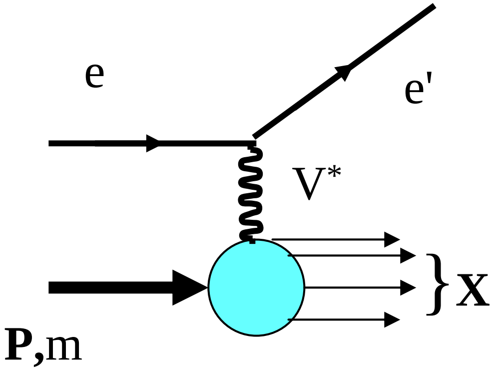
- The **Ward identities**, consequences of **gauge invariance** and local conservation of currents are **crucial for renormalizability** of QED.
- This feature suggests to look for **gauge invariant theories for other interactions**
- **Yang and Mills** generalization of gauge invariance to **non Abelian** symmetry groups.
- Failure of such generalization for strong interaction because of **hadron proliferation** and **absence of massless hadronic vector boson**
- Search for a **subhadronic structure**.

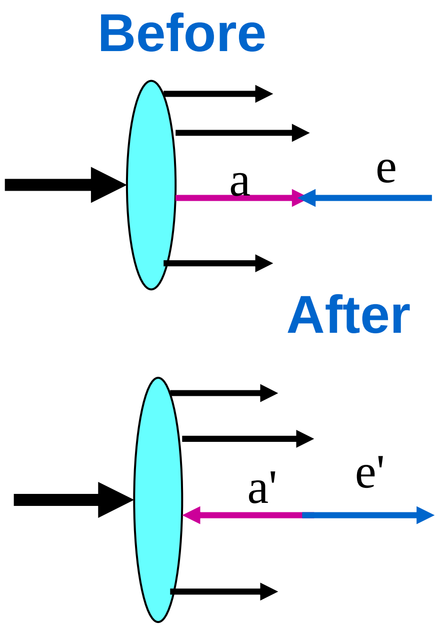
Quantum Chromodynamics

The quark-parton model

- The colored quark model:
  - Baryons are quark-quark-quark color singlet bound states
  - Mesons are quark-antiquark color singlet bound states
  - Color symmetry to insure Fermi statistics for the quarks
- Partons: generic constituents of hadrons which are elementary in weak and EM interactions
- Lepton-hadron deep inelastic scattering (DIS) inelastic lepton-hadron reaction with a large transfer of energy and of momentum allowing to probe a parton structure of the hadron
- Asymptotic freedom assumption: partons are weakly coupled at high energy (i.e. at short distance)

DIS in the Breit frame





•Meaning of the Bjorken variable *x:* one and only one active parton

$$p_a = x_a p = Q/2$$
$$x_a = Q/2p = x$$

•Asymptotic freedom and factorization

$$\sigma(l + p \to l' + X) = F_{p\to a}(x) \times \sigma(a + l \to a' + l')$$

$$F_{p\to a}(x) = \Pr\{x_a = x\}$$

•Scale invariance of the structure function

•Quarks and antiquarks are partons

•There are other partons, the gluons

QCD as a non Abelian Gauge theory

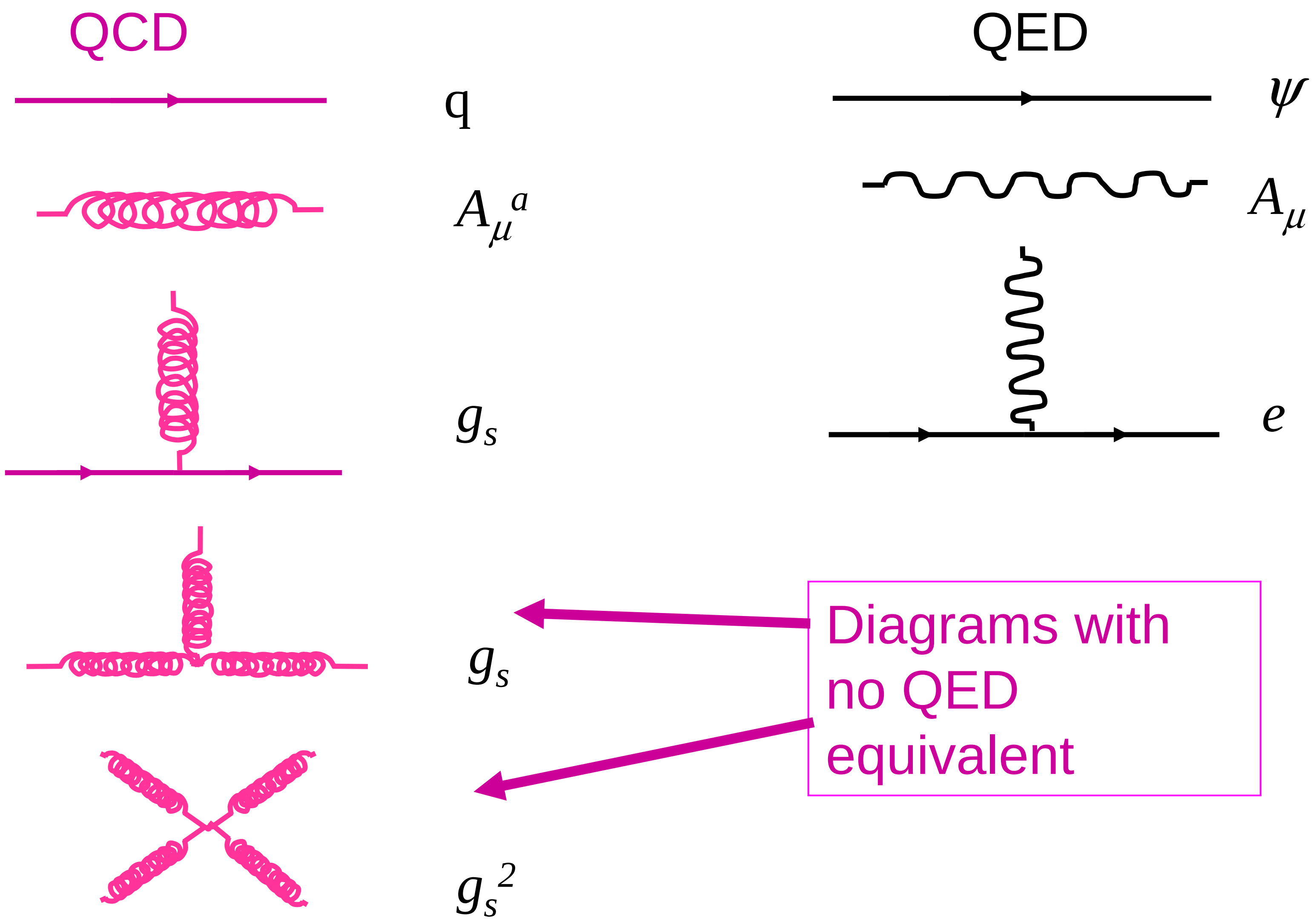
- One imposes the ***SU(3)* color** symmetry as a **local symmetry**
- **Triplets of quarks** (red blue green) belong to the fundamental representation of the group
- **Octets of gluons** (color-anticolor) belong to the adjoint representation of the group
- Lagrangian:

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4}\mathbf{F}_{\mu\nu}.\mathbf{F}^{\mu\nu} + \sum_{\alpha=1}^{N_f} \bar{\mathbf{q}}_\alpha \left(i\mathbf{D}_\mu \gamma^\mu - m_\alpha\right)\mathbf{q}_\alpha$$

$\alpha = 1, \cdots N_f$: flavors

color indices are omitted

Feynman's rules



Renormalization and asymptotic freedom

- QCD is renormalizable
- At one loop order, one has:

$$\alpha_s = g_s^{\ 2} / \hbar c$$

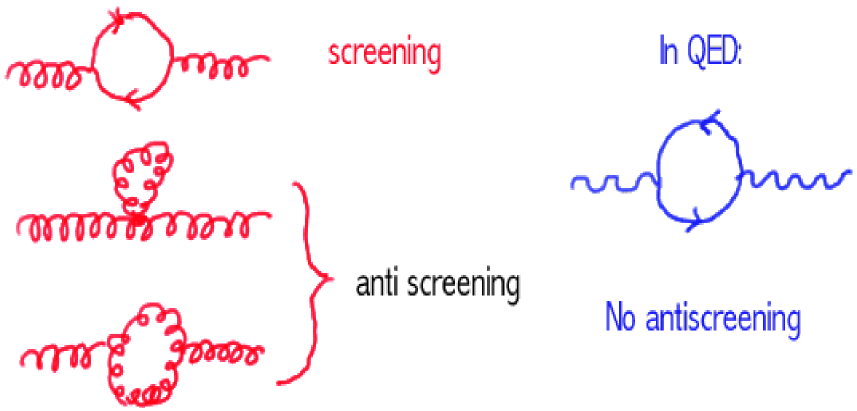
$$\alpha_s(\mu^2) = \frac{\alpha_s(\mu_0^{\ 2})}{1 + \frac{\alpha_s(\mu_0^{\ 2})}{12\pi}(33 - 2N_f)\,\mathrm{Log}\left(\frac{\mu^2}{\mu_0^{\ 2}}\right)}$$

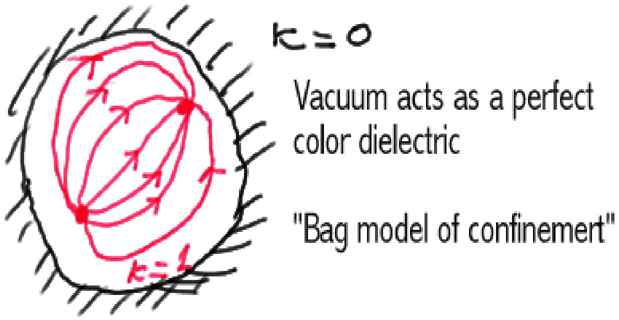
$$\alpha_s(\mu^2) = \frac{1}{b\,\mathrm{Log}\frac{\mu^2}{\Lambda^2}}$$

$$b = \frac{33 - 2N_f}{12\pi}$$

- In DIS, $\mu = \mathrm{Q}$, the coupling constant goes to zero when Q goes to infinity, that is asymptotic freedom (provided that the number of flavors N_f is less than 17)
- Since the coupling constant depends on Q, scale invariance is broken but in a calculable way. Experiment agrees very well with theoretical predictions.
- When $\mu = \Lambda$ (about 150 MeV) the coupling constant goes to infinity, which means that perturbative QCD breaks down at low energy

The problem of confinement





Varying picture of the hadron

- At medium Q: the quark parton model
- At very high Q: a "fractal" parton model
- At small Q: a hadronic "bag" or "string" model

The Electroweak theory

Prehistory of Weak Interaction Theory

- The Fermi model

$$\mathcal{L}_{Fermi} = -\frac{G}{\sqrt{2}} J_\lambda J^{\lambda\dagger}$$

$$G = (1.026 \pm 0.001) 10^{-5} m_p^{-2}$$

$$J_\lambda^{lept} = \overline{\nu_\mu} \gamma_\lambda (1-\gamma_5) \mu + \overline{\nu_e} \gamma_\lambda (1-\gamma_5) e + h.c.$$

$$J_\lambda^{had} = \overline{u} \gamma_\lambda (1-\gamma_5) (\cos\theta d + \sin\theta s) + h.c.$$

- Cabibbo mixing angle
- Maximal violation of parity

Structure of weak and EM currents when fermions are massless

- Organization of fundamental fermions in doublets
- Parity vs. chirality

$$\psi_L = \frac{1-\gamma_5}{2}\psi \, ; \psi_R = \frac{1+\gamma_5}{2}\psi$$

- Any mass term mixes the two chiralities

$$m\overline{\psi}\psi = m(\overline{\psi_R}\psi_L + \overline{\psi_L}\psi_R)$$

- Parity violation in WI: only left fermions are involved
- Parity conservation in EM interactions: left fermions and right fermions have the same electric charge

Weak neutral currents

- Weak neutral currents and neutrino experiments
- Absence of flavor changing neutral currents (FCNC)
- The Glashow-Iliopoulos-Maiani (GIM) mechanism, and charm

$$J_\lambda^{N-had} = \overline{N}\gamma_\lambda(1-\gamma_5)N + \overline{N'}\gamma_\lambda(1-\gamma_5)N'$$

$$N \equiv \begin{pmatrix} u \\ d_\theta = d\cos\theta + s\sin\theta \end{pmatrix}; N' = \begin{pmatrix} c \\ s_\theta = -d\sin\theta + s\cos\theta \end{pmatrix}$$

- Three generations and the Cabibbo-Kobayashi-Maskawa matrix: a possible mechanism for *CP* violation

The Electroweak gauge theory (before symmetry breaking)

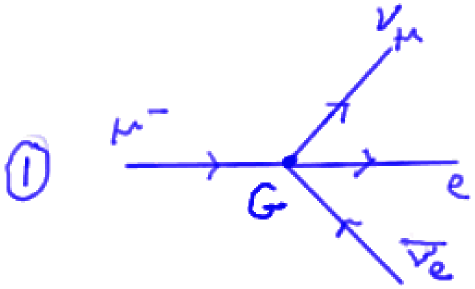
- In order to account for
  - **Doublet** organization of fermions
  - **Gauge** invariance of EM interactions

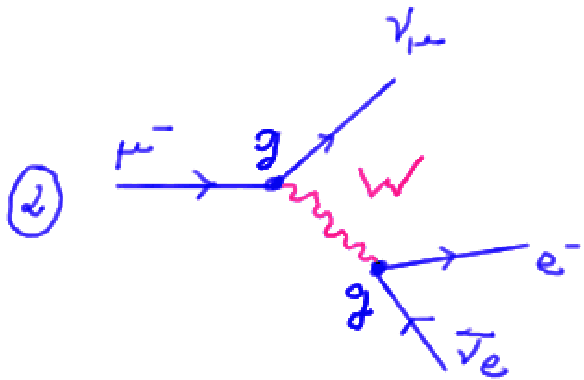
one tries as gauge group, the product of weak isospin *SU(2)* by weak hypercharge *U(1)*

- One attributes different quantum numbers to left and right fermions
  - **Left** fermions are weak isospin **doublets**
  - **Right** fermions are weak isospin **singlets**
  - The **Gell-Mann Nishijima relation** between charge, hypercharge and third component of weak isospin insures the **equality of the charges of left and right fermions**.

Fermion	**Charge:** $Q=I_3+Y/2$	**Isospin** I_3	**Hypercharge** Y
u_G	2/3=1/2+1/6	1/2	1/3
d_G	-1/3=-1/2+1/6	-1/2	1/3
u_D	2/3=0+2/3	0	4/3
d_D	-1/3=0-1/3	0	-2/3
ν_G	0=1/2-1/2	½	-1
e_G	-1=-1/2-1/2	½	-1
ν_D (?)	0=0+0	0	0
e_D	-1=0-1	0	-2

The problem of the mass of the VIB





$$A_1 = G$$

$$A_2 = \frac{g^2}{q^2 - M_W^2}$$

$$q = p_e + p_{\bar{\nu}_e}$$

$$|q^2| \ll M_W^2$$

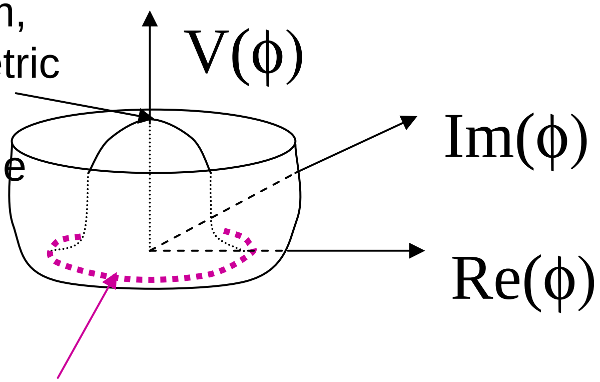
$$G \sim \frac{g^2}{M_W^2}$$

$$A_1 \sim A_2$$

$$M_W \sim 80\ \mathrm{GeV} \Longrightarrow g^2 \sim e^2$$

- The Fermi model is not renormalizable
- The vector intermediate boson (VIB) model makes sense only if intermediate bosons are massive
- If VIB are not gauge bosons, the model is not renormalizable
- If VIB are gauge bosons, they are necessarily massless !

Spontaneous global symmetry breaking



$$\mathbf{L} = \partial_\mu \phi^* \partial^\mu \phi - V(\phi)$$

$$\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2); V = \frac{\lambda}{6}(\phi^*\phi - \eta)^2 \; ; \eta \text{ real}$$

- Situation of spontaneous symmetry breaking: equations are symmetric solutions are not.
- In QFT, Lagrangian is symmetric, vacuum (and all other states) are not symmetric
- The possible symmetric vacuum would be unstable
- There are degenerate, stable, non symmetric vacua
- There is a massless scalar boson (the Goldstone boson) allowing to move from one vacuum to another with no energy expense

Spontaneous local symmetry breaking

- If the symmetry which is spontaneously broken is a local Abelian gauge symmetry, there are two massless particles, the gauge vector boson and the Goldstone scalar boson.
- The two transverse modes of the gauge boson and the Goldstone boson get mixed to form a massive vector boson
- There remains a massive scalar boson, the Higgs boson

The Brout Engler Higgs mechanism

- Spontaneous symmetry breaking of the $SU_L(2)xU_Y(1)$ local gauge symmetry down to $U_{EM}(1)$
- Higgs fields: an isospin doublet and its conjugate (4 degrees of freedom)

$$\left(\phi^+,\phi^0\right)\ \left(\overline{\phi^0},\phi^-\right)$$

- Three Goldstone bosons get mixed with transverse *SU(2)* gauge bosons and give rise to massive VIB
- The fourth Higgs degree of freedom becomes the massive Higgs boson
- The neutral VIB and the hypercharge gauge boson get mixed to lead to a massive *Z* boson and a massless photon
- Z/photon mixing and parity violations in nuclear and atomic physics

Predictions of Electroweak SM (Glashow Salam Weinberg)

- At the Born approximation
  - The Weinberg mixing angle

$$A_\mu = \cos\theta_W B_\mu + \sin\theta_W W_\mu^3$$

$$Z_\mu = -\sin\theta_W B_\mu + \cos\theta_W W_\mu^3$$

  - Predictions

$$e = g\sin\theta_W$$

$$\tan\theta_W = g'/g$$

$$\frac{G}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

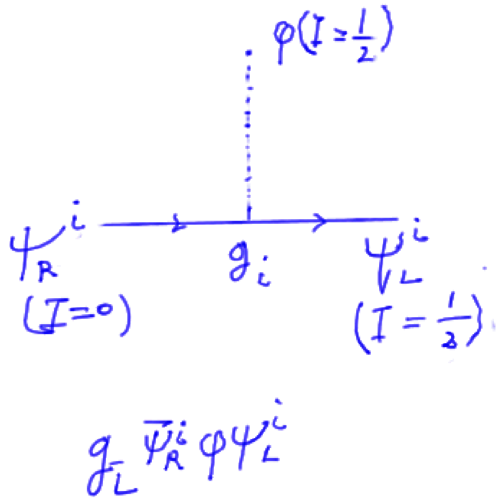
$$M_W^2 = M_Z^2\cos^2\theta_W$$

$$G_N = G$$

- Gauge invariance is not lost, it is "hidden"; currents are conserved
- Spontaneously broken gauge theories have been proven to be renormalizable ('tHooft, Veltman, Lee, Zinn-Justin)
- Radiative (quantum) corrections can be calculated and tested with high precision experiments performed at LEP: excellent agreement
- The mass of the Higgs boson is not predicted by theory. The Higgs boson remains to be discovered

Masses of the fermions

Before symmetry breaking, introduce Yukawa couplings



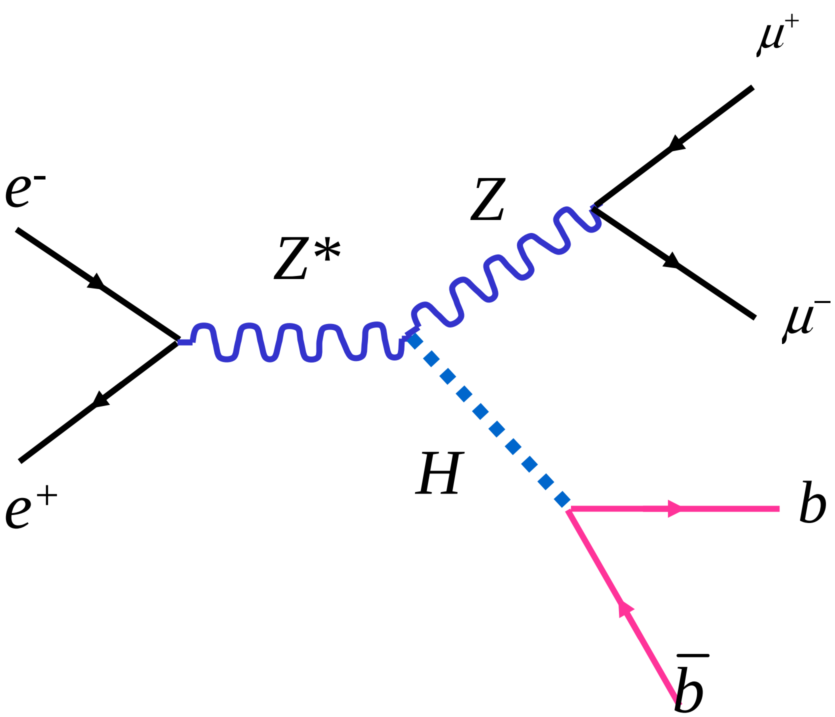
- After symmetry breaking, replace the Higgs field by its vacuum expectation value (VEV), η

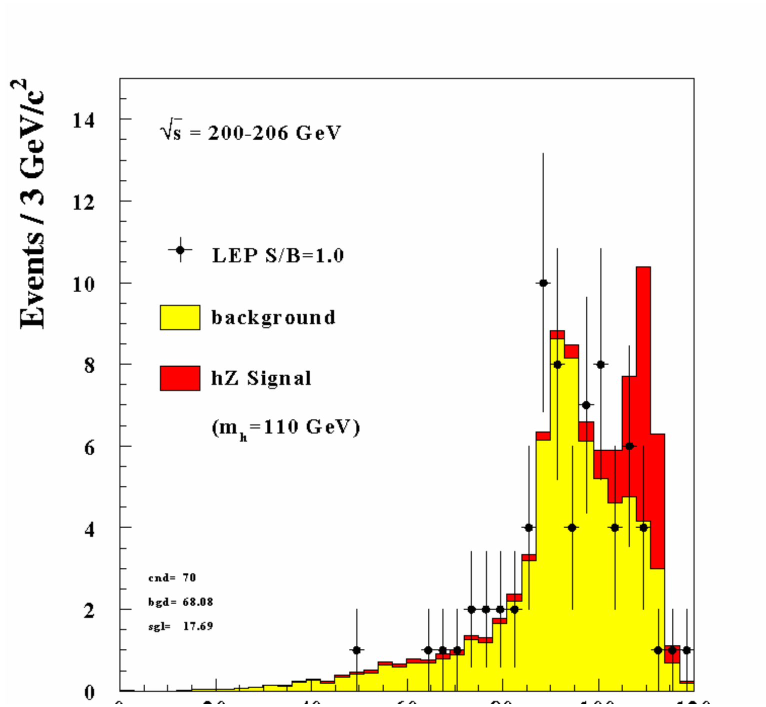
$$g_i \overline{\psi}_R{}^i \phi \psi_L{}^i \rightarrow g_i \eta \overline{\psi}_R{}^i \psi_L{}^i$$

$$g_i = \frac{m_i}{\eta}$$

- In SM right neutrinos, with zero isospin, zero hypercharge and zero charge, would not interact: they can be ignored. Thus, in SM, neutrinos are massless. Any indication of a neutrino mass is an indication of physics beyond the SM
- Generation of the mass of baryons through spontaneous chiral symmetry breaking

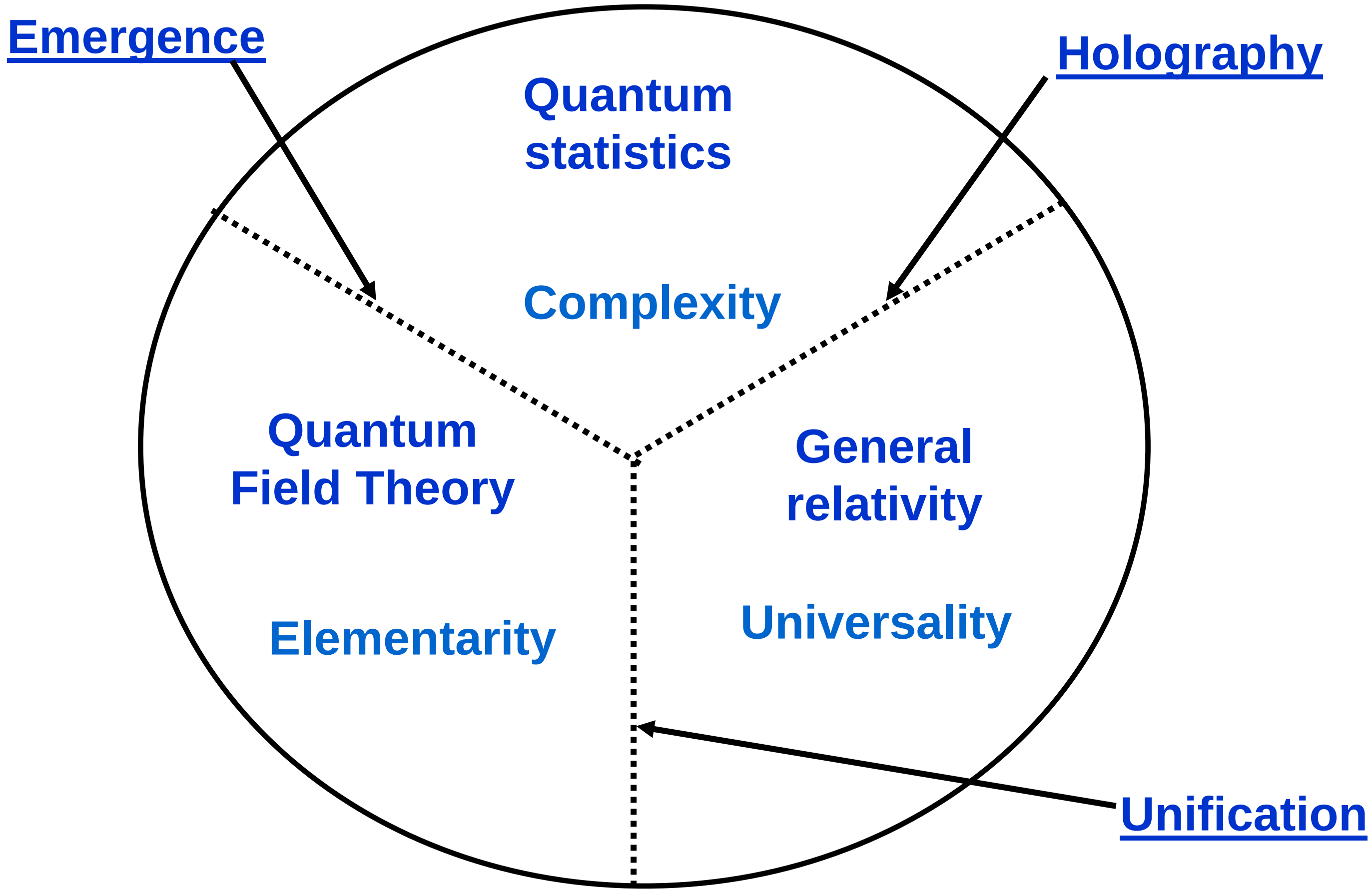
Recherche du boson de Higgs au LEP





Conclusion and outlook

The new horizons



Unification

- The direct way towards quantum gravity
  - Unification of all fondamental interactions, including gravitation
  - The key concept: supersymmetry, leading to supergravity and superstrings.
  - Crucial role of LHC experiments : Higgs boson and emergence of masses, MSSM
  - New dimensions of space
  - The branes
  - Quantum loop gravity
  - Non commutative geometry

Emergence

- Modern interpretation of quantum physics: emergence, through decoherence of classical physics from quantum physics
- Effective theories
- Quantum Field Theory et Finite Temperature, (h,c,k)
- Deconfinement and search of the quark gluon plasma
- Non perturbative physics, lattice gauge theories, computer experiments in QCD

Holography

- Gravity and **thermodynamics of horizons**
  - Gravity is the only interaction capable of producing **horizons** hiding to the observer information lying beyond them
  - Since **hidden information is an entropy**, one can attributes an entropy to a horizon
  - The Hawking Bekenstein **area law for black hole horizon**

$$S = k \frac{1}{4} \frac{A_{\text{horizon}}}{A_P} = \frac{kc^3}{hG} \frac{1}{4} A_{\text{horizon}}$$

- **Holography** and a possible **thermodynamic route towards quantum cosmology** (T. Padmanabhan, A. Patel, arXiv gr-qc/0309053