

The Standard Model of Fundamental Interactions Achievements and Prospects

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Quod Erat Demonstrandum

The QED perturbative expansion

- The **whole information** necessary for the QED (electromagnetic interactions of electrons) **perturbation expansion** is encoded in the **QED Lagrangian**

$$Z = \int \mathcal{D}\psi \mathcal{D}A \exp \left\{ \frac{i}{\hbar} \int d^4x \mathcal{L}_{\text{QED}}(\psi(x), A(x)) \right\}$$

$$\mathcal{L}_{\text{QED}} = \bar{\psi} (i\gamma^\mu \partial_\mu - m) \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - e \bar{\psi} \gamma^\mu A_\mu \psi$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

- Generating functional

$$\mathcal{K}(j_\psi, j_{\bar{\psi}}, j_A) = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A \exp \left\{ \frac{i}{\hbar} \int d^4x (\mathcal{L}_{\text{QED}} + j_\psi \psi + j_{\bar{\psi}} \bar{\psi} + j_A A) \right\}$$

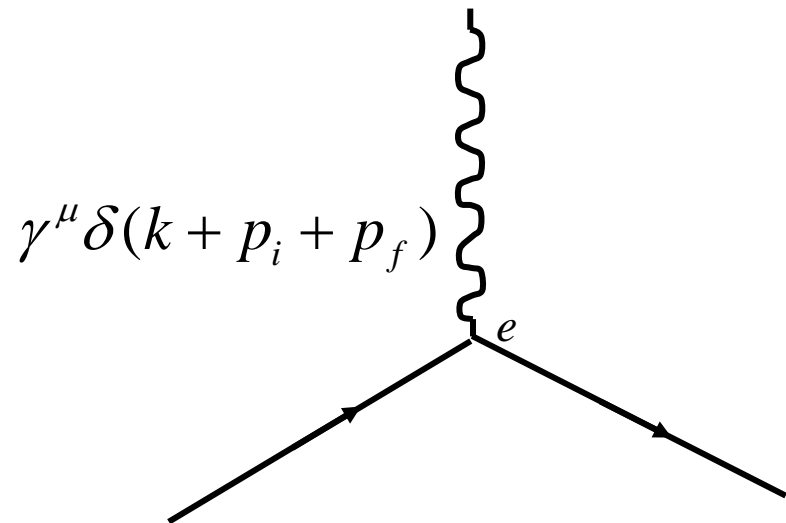
- Feynman's diagrams and rules
 - Electron propagator

$$\begin{array}{c} p \\ \longrightarrow \\ \hline \end{array} \quad \frac{\not{p} + m}{p^2 - m^2}$$

- Photon propagator

$$\begin{array}{c} k \\ \text{~~~~~} \\ \hline \end{array} \quad \frac{1}{k^2}$$

- Interaction vertex

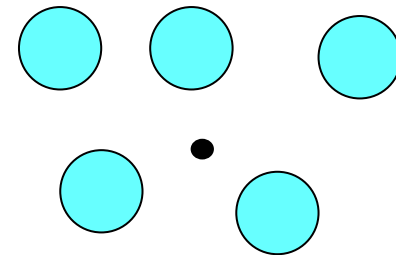
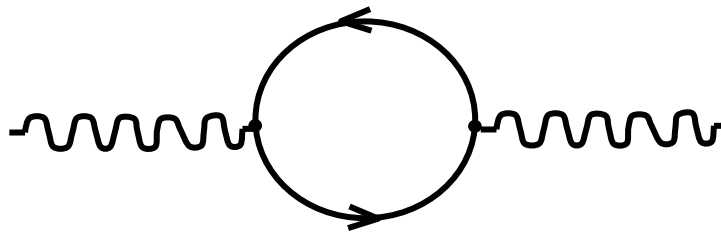


The divergence problem

- All Feynman amplitudes defined in terms of **ordinary integrals**
- In general, integrals implied by diagrams involving loops **diverge**
- These divergences reflect the **conflict** between **locality** (i.e. relativity + causality) and **quantum mechanics**: quantum fields are operator valued **distributions whose products at the same point is ill-defined**
- In QED, all divergences are **logarithmic**

Quantum vacuum as a dielectric: a heuristic picture of renormalisation

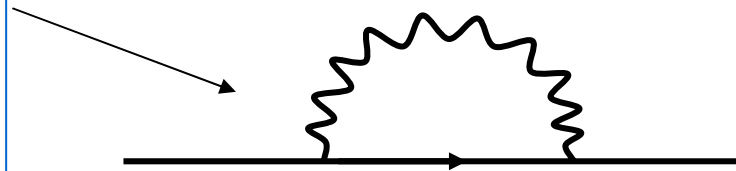
- Quantum fluctuations and vacuum polarization.
 - In the Fock space of the electromagnetic field, vacuum is the state with 0 photon. According to Heisenberg's inequalities, when the number of quanta is determined (0 in the vacuum) the value of the field is completely undetermined; it fluctuates
 - One such fluctuation corresponds to the **appearance followed by the disappearance of an electron-positron pair**
 - These fluctuations produce a **screening effect**, leading to the notion of an **effective charge depending on the resolution**



Effect of the interactions on the parameters of the theory

- The charge and the mass of the electron (the parameters of the theory) are **affected by the interaction with the electromagnetic field**.
- **self energy diagram**

An electron emits and reabsorbs a photon: the electron quantum field is modified by the electromagnetic fields it generates



- The parameters of the theory are split into "**bare**" parameters and "**dressed**" parameters, i.e. "**renormalized**" by the interaction
- Renormalized parameters are **effective parameters depending on the resolution**, one says that they are **renormalized at scale μ**

The "miracle" of Renormalizability

- **Renormalization**= redefinition of the fundamental parameters of the theory
 - Electric **charge** e split into
 - e_0 "**bare charge**", i.e. the charge if there were no interaction
 - e physical, "**renormalized**" charge
 - Electron **mass** m split into
 - m_0 "**bare mass**", i.e. the mass if there were no interaction
 - m physical, "**renormalized**" mass
- **Regularization**: With an energy **cutoff** Λ one removes the divergences in the expressions of e , m and any amplitude in terms of e_0 and m_0 .

- One can **invert** these expressions

$$e_0 = e + 1/2\beta_2 e^3 \text{Log}\left(\frac{\Lambda}{m}\right) + O(e^5)$$

$$m_0 = m + \gamma_1 m e^2 \text{Log}\left(\frac{\Lambda}{m}\right) + O(e^4)$$

and express **all amplitudes in terms of e and m**

- The "**miracle of renormalizability**" is then that in these expressions, for any amplitude, and at all orders of perturbation expansion, **the limit when Λ goes to infinity is finite**, which means that one can express perturbatively all observables **in terms of two and only two physical parameters** which can be measured experimentally.

- The theory/experiment agreement is **amazingly good**

For example, for the magnetic moment of the electron one finds

Exp: 2,00231930482+/- 40

Th: 2,00231930476+/- 52

The renormalization group

- If the electron mass m was equal to zero, it would have been necessary to introduce a new energy scale μ , called the **renormalization energy**, to deal with the logarithmic divergence,

$$e_0 = e + 1/2\beta_2 e^3 \text{Log}\left(\frac{\Lambda}{\mu}\right) + O(e^5)$$

and when one inverts this relation, e becomes a function of μ

- When $m \neq 0$

a renormalization energy is still necessary, but it is implicitly set equal to m : $e(\mu = m) = e$; $m(\mu = m) = m$

- The renormalization energy being arbitrary, physics must be independent on its value, which means that the renormalized charge must obey a **differential equation, called the renormalization group equation**, expressing this independence:

$$\mu \frac{de^2(\mu)}{d\mu} = \beta(e^2(\mu))$$

where $\beta(e^2) = \beta_2 e^4 + O(e^6)$

is finite when Λ goes to infinity (β_2 is a c-number, depending only on the QED Lagrangian)

- As a consequence, parameters in QED have to be considered as **effective parameters**, depending on the resolution

$$\alpha(\mu = m_e) = 1/137 ; \alpha(\mu = m_Z) = 1/128$$

Gauge invariance

- In Maxwell theory, **gauge invariance** ($A_\mu \rightarrow A_\mu + \partial_\mu \Lambda(x)$) **is equivalent to current conservation**

- In QED, the following transformation leaves the QED Lagrangian invariant

$$\psi(x) \rightarrow e^{-i\alpha(x)} \psi(x)$$

$$\overline{\psi(x)} \rightarrow e^{+i\alpha(x)} \overline{\psi(x)} \quad \alpha(x) = e\Lambda(x)$$

$$A_\mu \rightarrow A_\mu + \partial_\mu \Lambda(x)$$

- Conversely, one can impose invariance through a **local** change of the phase of the electron field by introducing a gauge invariant field, the electromagnetic field: **local gauge invariance determines the structure of the interaction**

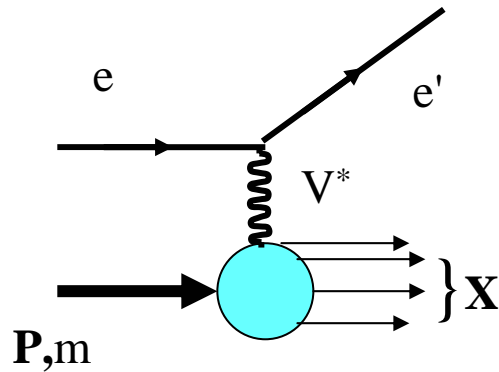
- The **Ward identities**, consequences of **gauge invariance** and local conservation of currents are **crucial for renormalizability** of QED.
- This feature suggests to look for **gauge invariant theories for other interactions**
- **Yang and Mills** generalization of gauge invariance to **non Abelian** symmetry groups.
- Failure of such generalization for strong interaction because of **hadron proliferation** and **absence of massless hadronic vector boson**
- Search for a **subhadronic structure**.

Quantum Chromodynamics

The quark-parton model

- The **colored quark model**:
 - Baryons are quark-quark-quark **color singlet bound states**
 - Mesons are quark-antiquark **color singlet bound states**
 - **Color** symmetry to insure Fermi statistics for the quarks
- **Partons**: generic constituents of hadrons which are elementary in weak and EM interactions
- **Lepton-hadron deep inelastic scattering (DIS)**
inelastic lepton-hadron reaction with a large transfer of energy and of momentum allowing to **probe a parton structure** of the hadron
- **Asymptotic freedom assumption**: partons are weakly coupled at high energy (i.e. at short distance)

DIS in the Breit frame



- Meaning of the Bjorken variable x : one and only one active parton

$$p_a = x_a p = Q/2$$

$$x_a = Q/2p = x$$

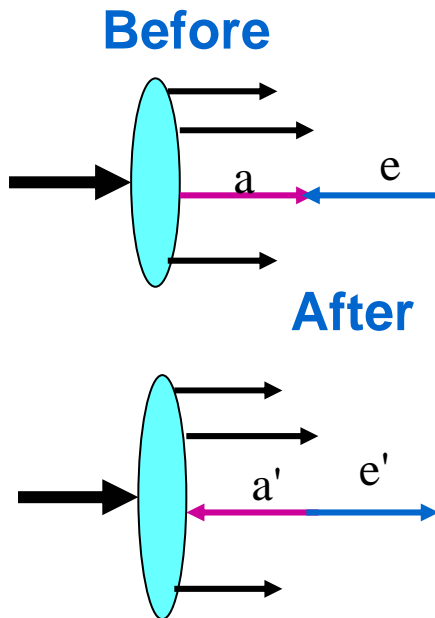
- **Asymptotic freedom** and factorization $\sigma(l + p \rightarrow l' + X) = F_{p \rightarrow a}(x) \times \sigma(a + l \rightarrow a' + l')$

$$F_{p \rightarrow a}(x) = \Pr\{x_a = x\}$$

- **Scale invariance** of the **structure function**

- **Quarks and antiquarks are partons**

- There are other partons, the **gluons**



QCD as a non Abelian Gauge theory

- One imposes the **$SU(3)$ color** symmetry as a **local symmetry**
- **Triplets of quarks** (red blue green) belong to the fundamental representation of the group
- **Octets of gluons** (color-anticolor) belong to the adjoint representation of the group
- Lagrangian:

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4} \mathbf{F}_{\mu\nu} \cdot \mathbf{F}^{\mu\nu} + \sum_{\alpha=1}^{N_f} \bar{\mathbf{q}}_{\alpha} (i\mathbf{D}_{\mu} \gamma^{\mu} - m_{\alpha}) \mathbf{q}_{\alpha}$$

$\alpha = 1, \dots, N_f$: flavors

color indices are omitted

Feynman's rules

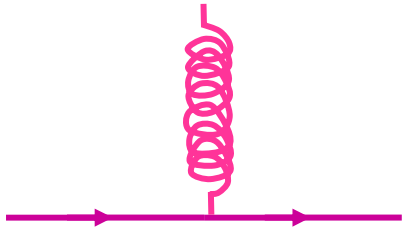
QCD



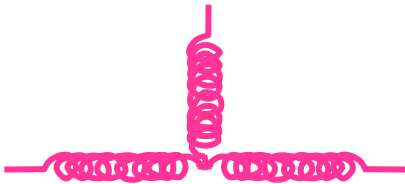
q



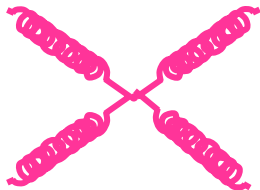
A_μ^a



g_s



g_s



g_s^2

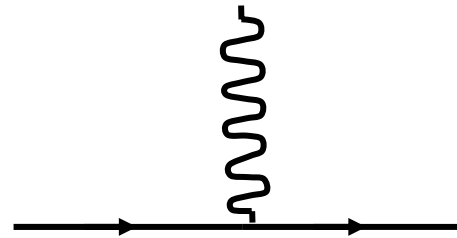
QED



ψ



A_μ



e

Diagrams with no QED equivalent

Renormalization and asymptotic freedom

- QCD is renormalizable
- At one loop order, one has:

$$\alpha_s = g_s^2 / \hbar c$$

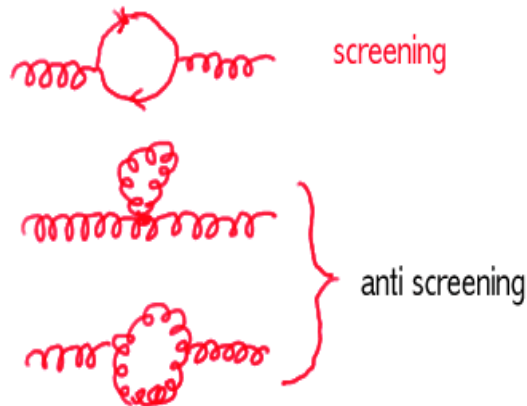
$$\alpha_s(\mu^2) = \frac{\alpha_s(\mu_0^2)}{1 + \frac{\alpha_s(\mu_0^2)}{12\pi} (33 - 2N_f) \text{Log} \left(\frac{\mu^2}{\mu_0^2} \right)}$$

$$\alpha_s(\mu^2) = \frac{1}{b \text{Log} \frac{\mu^2}{\Lambda^2}}$$

$$b = \frac{33 - 2N_f}{12\pi}$$

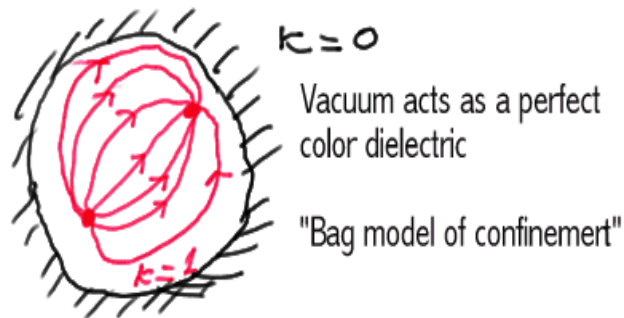
- In DIS, $\mu = Q$, the coupling constant goes to zero when Q goes to infinity, that is **asymptotic freedom** (provided that the number of flavors N_f is less than 17)
- Since the coupling constant depends on Q , **scale invariance is broken but in a calculable way**. Experiment agrees very well with theoretical predictions.
- When $\mu = \Lambda$ (about 150 MeV) the coupling constant goes to infinity, which means that **perturbative QCD breaks down at low energy**

The problem of confinement



Varying picture of the hadron

- At medium Q : the quark parton model
- At very high Q : a "fractal" parton model
- At small Q : a hadronic "bag" or "string" model



The Electroweak theory

Prehistory of Weak Interaction Theory

- The Fermi model

$$\mathcal{L}_{Fermi} = -\frac{G}{\sqrt{2}} J_\lambda J^{\lambda\dagger}$$

$$G = (1.026 \pm 0.001) 10^{-5} m_p^{-2}$$

$$J_\lambda^{lept} = \bar{\nu}_\mu \gamma_\lambda (1 - \gamma_5) \mu + \bar{\nu}_e \gamma_\lambda (1 - \gamma_5) e + h.c.$$

$$J_\lambda^{had} = \bar{u} \gamma_\lambda (1 - \gamma_5) (\cos \theta d + \sin \theta s) + h.c.$$

- Cabibbo mixing angle
- Maximal violation of parity

Structure of weak and EM currents when fermions are massless

- Organization of fundamental fermions in doublets
- Parity vs. **chirality**

$$\psi_L = \frac{1-\gamma_5}{2}\psi; \psi_R = \frac{1+\gamma_5}{2}\psi$$

- Any mass term mixes the two chiralities

$$\overline{m\psi\psi} = m(\overline{\psi_R}\psi_L + \overline{\psi_L}\psi_R)$$

- Parity violation in WI: **only left fermions are involved**
- Parity conservation in EM interactions: **left fermions and right fermions have the same electric charge**

Weak neutral currents

- Weak neutral currents and neutrino experiments
- Absence of flavor changing neutral currents (FCNC)
- The Glashow-Iliopoulos-Maiani (GIM) mechanism, and charm

$$J_{\lambda}^{N-had} = \bar{N}\gamma_{\lambda}(1-\gamma_5)N + \bar{N}'\gamma_{\lambda}(1-\gamma_5)N'$$
$$N \equiv \begin{pmatrix} u \\ d_{\theta} = d \cos \theta + s \sin \theta \end{pmatrix}; N' = \begin{pmatrix} c \\ s_{\theta} = -d \sin \theta + s \cos \theta \end{pmatrix}$$

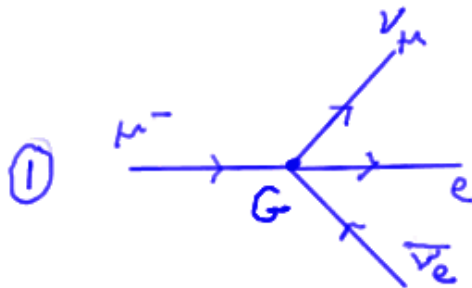
- Three generations and the Cabibbo-Kobayashi-Maskawa matrix: a possible mechanism for CP violation

The Electroweak gauge theory (before symmetry breaking)

- In order to account for
 - **Doublet** organization of fermions
 - **Gauge** invariance of EM interactionsone tries as gauge group, the product of **weak isospin $SU(2)$** by **weak hypercharge $U(1)$**
- One attributes **different quantum numbers to left and right fermions**
 - **Left** fermions are weak isospin **doublets**
 - **Right** fermions are weak isospin **singlets**
 - The **Gell-Mann Nishijima relation** between charge, hypercharge and third component of weak isospin insures the **equality of the charges of left and right fermions.**

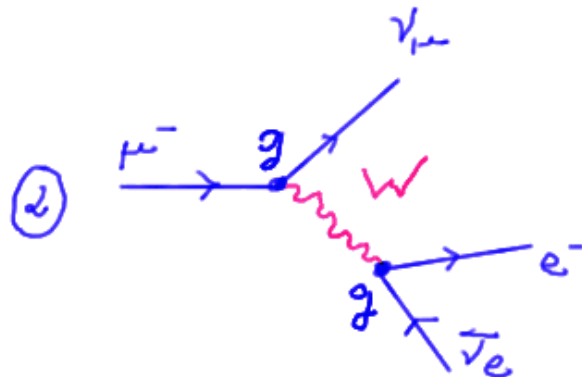
Fermion	Charge: $Q=I_3+Y/2$	Isospin I_3	Hypercharge Y
u_G	$2/3=1/2+1/6$	$1/2$	$1/3$
d_G	$-1/3=-1/2+1/6$	$-1/2$	$1/3$
u_D	$2/3=0+2/3$	0	$4/3$
d_D	$-1/3=0-1/3$	0	$-2/3$
ν_G	$0=1/2-1/2$	$1/2$	-1
e_G	$-1=-1/2-1/2$	$1/2$	-1
$\underline{\nu}_D(?)$	$0=0+0$	0	0
e_D	$-1=0-1$	0	-2

The problem of the mass of the VIB



$$A_1 = G$$

$$A_2 = \frac{g^2}{q^2 - M_W^2}$$



$$q = p_e + p_{\bar{\nu}_e}$$

$$|q^2| \ll M_W^2$$

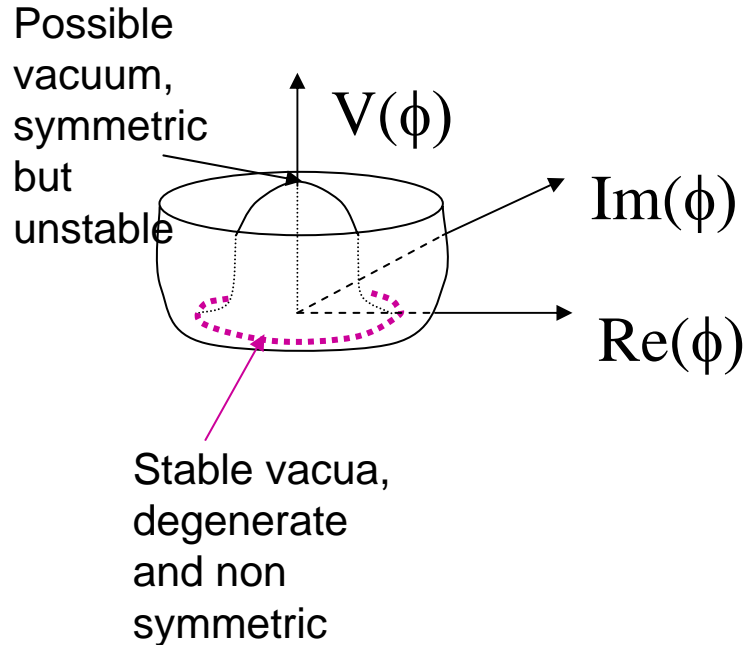
$$G \sim \frac{g^2}{M_W^2}$$

$$A_1 \sim A_2$$

$$M_W \sim 80 \text{ GeV} \Rightarrow g^2 \sim e^2$$

- The Fermi model is **not renormalizable**
- The **vector intermediate boson (VIB)** model makes sense only if intermediate bosons are **massive**
- If **VIB** are not gauge bosons, the **model is not renormalizable**
- If **VIB** are **gauge bosons**, they are necessarily **massless** !

Spontaneous **global** symmetry breaking



- Situation of **spontaneous symmetry breaking**: equations are symmetric solutions are not.
- In QFT, **Lagrangian is symmetric, vacuum** (and all other states) are not **symmetric**
- The possible **symmetric vacuum** would be **unstable**
- There are **degenerate, stable, non symmetric vacua**
- There is a **massless scalar boson (the Goldstone boson)** allowing to move from one vacuum to another with no energy expense

$$\mathbf{L} = \partial_{\mu} \phi^* \partial^{\mu} \phi - V(\phi)$$

$$\phi = \frac{1}{\sqrt{2}} (\phi_1 + i\phi_2); V = \frac{\lambda}{6} (\phi^* \phi - \eta)^2; \eta \text{ real}$$

Spontaneous **local** symmetry breaking

- If the symmetry which is spontaneously broken is a **local Abelian gauge symmetry**, there are two massless particles, the **gauge vector boson** and the **Goldstone scalar boson**.
- The **two transverse modes** of the gauge boson and the Goldstone boson get **mixed** to form a **massive vector boson**
- There remains a massive scalar boson, **the Higgs boson**

The Brout Engler Higgs mechanism

- Spontaneous symmetry **breaking** of the $SU_L(2) \times U_Y(1)$ local gauge symmetry **down** to $U_{EM}(1)$
- **Higgs fields**: an isospin **doublet** and its conjugate (4 degrees of freedom)

$$\left(\phi^+, \phi^0 \right) \left(\overline{\phi^0}, \phi^- \right)$$

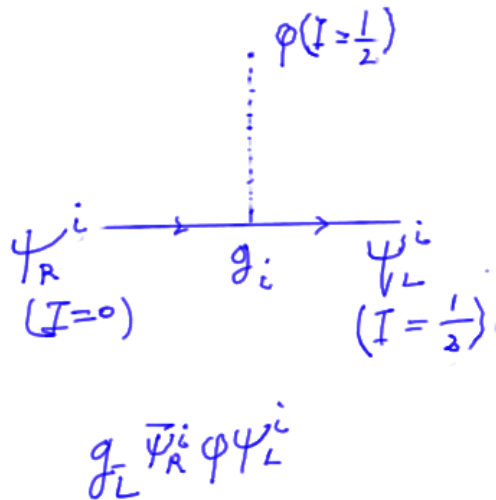
- **Three Goldstone** bosons get mixed with transverse $SU(2)$ gauge bosons and give rise to **massive VIB**
- The fourth Higgs degree of freedom becomes the massive **Higgs boson**
- The neutral VIB and the hypercharge gauge boson get mixed to lead to a **massive Z boson** and a **massless photon**
- Z/photon mixing and **parity violations** in nuclear and atomic physics

Predictions of Electroweak SM (Glashow Salam Weinberg)

- At the Born approximation
 - The Weinberg mixing angle
$$A_\mu = \cos \theta_W B_\mu + \sin \theta_W W_\mu^3$$
$$Z_\mu = -\sin \theta_W B_\mu + \cos \theta_W W_\mu^3$$
 - Predictions
$$e = g \sin \theta_W$$
$$\tan \theta_W = g' / g$$
$$\frac{G}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$
$$M_W^2 = M_Z^2 \cos^2 \theta_W$$
$$G_N = G$$
- Gauge invariance is not lost, it is "hidden"; currents are conserved
- Spontaneously broken gauge theories have been proven to be renormalizable ('tHooft, Veltman, Lee, Zinn-Justin)
- Radiative (quantum) corrections can be calculated and tested with high precision experiments performed at LEP: excellent agreement
- The mass of the Higgs boson is not predicted by theory. The Higgs boson remains to be discovered

Masses of the fermions

Before symmetry breaking,
introduce Yukawa couplings



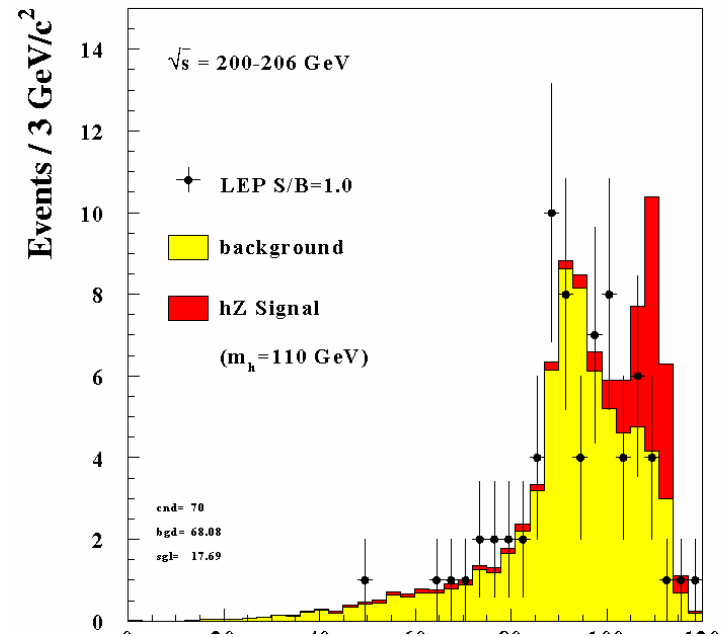
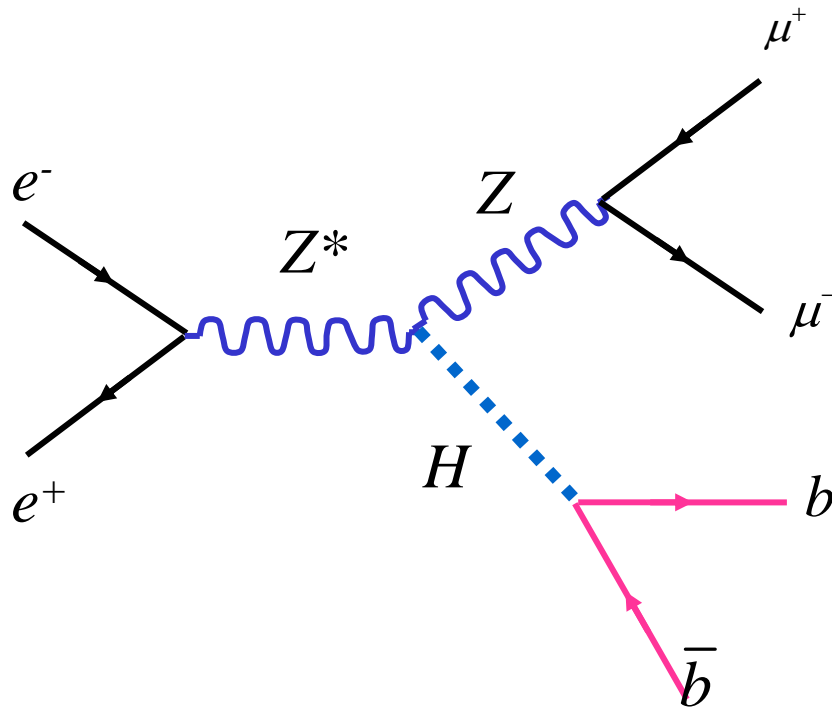
- After symmetry breaking, replace the Higgs field by its vacuum expectation value (VEV), η

$$g_i \bar{\psi}_R^i \phi \psi_L^i \rightarrow g_i \eta \bar{\psi}_R^i \psi_L^i$$

$$g_i = \frac{m_i}{\eta}$$

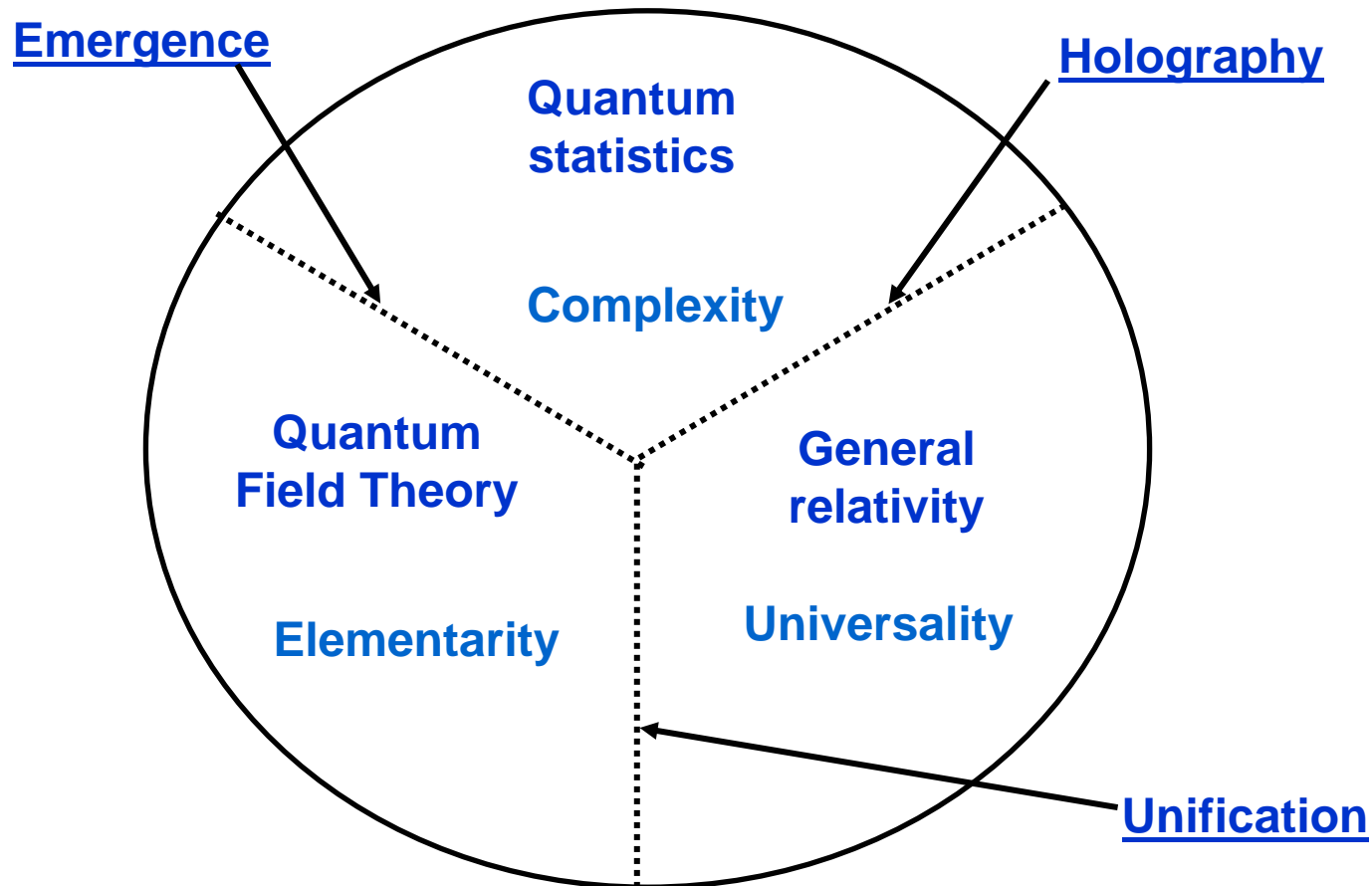
- In SM right neutrinos, with zero isospin, zero hypercharge and zero charge, would not interact: they can be ignored. Thus, in SM, neutrinos are massless. Any indication of a neutrino mass is an indication of physics beyond the SM
- Generation of the mass of baryons through spontaneous chiral symmetry breaking

Recherche du boson de Higgs au LEP



Conclusion and outlook

The new horizons



Unification



- The direct way towards quantum gravity
 - Unification of all fundamental interactions, including gravitation
 - The key concept: supersymmetry, leading to supergravity and superstrings.
 - Crucial role of LHC experiments : Higgs boson and emergence of masses, MSSM
 - New dimensions of space
 - The branes
 - Quantum loop gravity
 - Non commutative geometry

Emergence



- Modern interpretation of quantum physics: emergence, through decoherence of classical physics from quantum physics
- Effective theories
- Quantum Field Theory at Finite Temperature, (\hbar, c, k)
- Deconfinement and search of the quark gluon plasma
- Non perturbative physics, lattice gauge theories, computer experiments in QCD

Holography

- Gravity and **thermodynamics of horizons**
 - Gravity is the only interaction capable of producing **horizons** hiding to the observer information lying beyond them
 - Since **hidden information is an entropy**, one can attributes an entropy to a horizon
 - The Hawking Bekenstein **area law for black hole horizon**

$$S = k \frac{1}{4} \frac{A_{\text{horizon}}}{A_p} = \frac{kc^3}{hG} \frac{1}{4} A_{\text{horizon}}$$

- **Holography** and a possible **thermodynamic route towards quantum cosmology** (T. Padmanabhan, A. Patel, arXiv gr-qc/0309053)