The Standard Model of Fundamental Interactions

Achievements and Prospects

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Introduction

The scope of the Standard Model

- The Standard Model of Particle Physics (SM), the main focus of the present lectures, is the theory of reference of elementary constituents of matter and of the fundamental interactions in which they are involved, the electromagnetic,strong and weak interactions.
- Except at extremely high energy (10^{19} GeV, the Planck energy), the fourth fundamental interaction, gravity is negligible in particle physics.
- Gravity is dominant in the Universe at large scale; it is described by means of general relativity, which is the basis of the Big Bang Model, the cosmological standard model.

- General relativity is incompatible with quantum mechanics. Quantization of gravitation, which has to be dealt with at the Planck energy, is still an outstanding open question.
- Apart from phenomena which occurred within a few 10^{-44} second (i.e. the Planck time), after the big bang, the SM and the big bang model provide a theoretical description of all phenomena with no contradiction with observation and /or experiment.

Coarse grain history of the Standard Model

Dates	Cadre théorique	Gravitation	Électro magnétisme	Interaction faible	Interaction forte
17ème siècle	Galilée, Newton	Newton			
19ème siècle	Euler, Lagrange, Jacobi, Hamilton		Maxwell		
1895-1898			Rayons X, électron, radioactivité		
1900-1930	Mécanique quantique				
1905-1915	Relativité	Einstein			
1930-1950	Théorie quantique des champs		QED	Fermi	Yukawa
1970-2000	Théories de jauge	Big bang	Théorie électrofaible de Glashow, Salam et Weinberg		QCD
2003- …	Supersymétrie Supercordes, Gravitation quantique?	Inflation	MSSM ?		

Fundamental fermions

Generation / Type	1st generation	2nd generation	3rd generation
q=2/3 quarks	**Up** ***u*** (W EM S)	**Charm** ***c*** (W EM S)	**Top** ***t*** (W EM S)
q=-1/3 quarks	**Down** ***d*** (W EM S)	**Strange** ***s*** (W EM S)	**Beauty** ***b*** (W EM S)
Neutral leptons (neutrinos)	**Electron neutrino** ν_e (W)	**Muon neutrino** ν_μ (W)	**Tauon neutrino** ν_τ (W)
Charged leptons	**Electron** ***e*** (W EM)	**Muon** μ (W EM)	**Tauon** τ (W EM)

Fundamental interactions

Interaction	Involved particles	Charge	Boson
Strong	Quarks	Color	Gluons
Electromagnetic	Quarks, charged leptons	Electric charge	Photon
Weak	Quarks, charged leptons and neutrinos	Weak Isospin	Vector intermediate bosons, W^+, W^- Z^0
Gravity	All particles	Energy	Graviton

The theoretical framework of the standard model

- The theoretical "tripod": three theories taking into account pairs of universal constants
  - **General Relativity**: *c* and *G*
  - **Quantum Field Theory**: *h* and *c*
  - **Quantum Statistics**: *h* and *k*
- The basic concepts the SM owes to these three theories
  - From general relativity: the concepts of **relativistic field** and of **events localized in space-time**
  - From quantum statistics: the concept of **Fock space**
  - From quantum field theory: the Feynman's **Path Integral Methodology**

Relativistic fields and events localized in space-time

Relativity and the theoretical status of space

- **Einstein's restricted relativity**
  - Principle of **relativity**: all inertial frames are equivalent
  - Constancy of the velocity of light
  - Elimination of the concept of ether, replaced by the concept of **field**: "*The field thus becomes an irreducible element of physical description, irreducible in the same sense as the concept of matter in the theory of Newton*" (Einstein, 1952)
  - Apart from the field, there exists matter, in the form of material points, possibly carrying electric charge
  - "*The description of physical states postulates space as being initially given and as existing independently*" (Einstein, 1952)
  - **Space-time**: "*It appears therefore more natural to think of physical reality as a four-dimensional existence, instead of, as hitherto, the* evolution *of a three-dimensional existence.*" (Einstein, 1952)

- **General relativity**
  - Extension of the theory of relativity to arbitrary change of reference frame, through a detour to gravitation and with the help of the equivalence principle:
    - Any arbitrary change of reference frame can **locally** be replaced by an adequate gravitational field
    - Any gravitational field can **locally** be replaced by an adequate change of reference frame.
  - General relativity is thus a **geometric theory of gravitation**: matter and the gravitational field it generates are replaced by a **non-Euclidean space-time**, the **metric** of which is a universal field.
  - "*On the basis of the general theory of relativity, space, as opposed to 'what fills space', has no separate existence.(…) There exists no space empty of field*" (Einstein, 1952)
  - The "**dualism problem**": in a consistent theory the concepts of fields and of material points cannot coexist.

From the rigid body of reference to the concept of encounter event

- In classical mechanics, as well as in special relativity, space (time) co-ordination can be performed by means of **rigid bodies**
- In general relativity, because of the non-Euclidean character of space-time, this is no more possible: the body of reference would be rather (as Einstein says) a **mollusk of reference**
- In the non-Euclidean space-time the **Gaussian co-ordinates** are not interpreted in terms of measuring rods, but in terms of **encounter events** *"Every physical description resolves itself into a number of statements, each of which refers to the space-time coincidence of two events A and B. In terms of Gaussian by the agreement of their four co-ordinates* x_1, x_2, x_3, x_4*."*

- The "hole" no-go argument against general covariance
  - In 1913, Einstein was on the point to give up the elaboration of a generally covariant theory because of the "**hole no-go argument**": two distributions of metric and matter related by a "hole transformation", namely a transformation leaving them invariant except in the hole, would be theoretically and experimentally impossible to distinguish outside the "hole"; which would mean that the theory in question would be incomplete.
  - The way out found by Einstein: **the physical content of a theory is exhausted by the catalog of all encounter events it authorizes**; events that are localized in space-time, such as the intersection of two world lines or **the point-like coupling of two or several fields** are the only entities that can be defined in an **invariant way**.

Aim of the present lectures

- To show that the answer to the Einstein's objections to quantum mechanics given by quantum field theory is analogous to the one he gave to the "hole argument"
- The validation by experiment of the SM thus appears as a reliable validation of the whole of quantum physics.
- To take up the challenge of quantization of gravity experimentally testing the SM and discovering physics lying beyond it are compulsory stages

Free quantum fields, the Fock space

What is Quantum Field Theory?

- Quantum Field Theory (QFT) is the result of the **merging** of **quantum mechanics** and **restricted relativity**.
- In ordinary quantum mechanics (QM), the space coordinates of a particles are **operators** whereas time is a **continuous parameter**. **QM is local in time but non local in space**
- In QFT, the four space-time coordinates are **continuous parameters**, just as in classical relativistic field theory. QFT can be made **local in space-time**
- The **relativistic generalizations of the Schrödinger equation**,
  - the **Dirac** equation $\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi(x) = 0$
  - the **Klein Gordon** equation $\left(\Box - m^{2}\right)\psi(x) = 0$

admit **negative energy solutions** difficult to interpret if $\psi(x)$ is the wave function of an isolated particle

- The problem of negative energy solutions is solved if Dirac or Klein Gordon equations apply to **quantum fields** rather than to wave functions of isolated particles.
- A quantum field is a field of **operators**, creating or annihilating, at any point of space-time, a particle or an antiparticle.
- Particles and antiparticles are **not material points**.
- The creation or the annihilation of a particle or an antiparticle is a **quantum event** that causality and relativity force us to treat as **strictly localized** in space-time.
- **Heisenberg indeterminacy inequalities** imply that such strictly localized events **cannot be individually predicted in a deterministic way**.
- Quantum statistics deals with **ensembles of events.**

Fundamental implications of QFT

- **Indistinguishability** of particles and/or antiparticles
- In a fundamental interaction **the number of particles is not conserved**
- **Spin/statistics connection theorem**:
  - **Fermions** have a **half integer spin**
  - **Bosons** have an **integer spin**
- **The *PCT* theorem**: all interactions are invariant under the ***PCT* symmetry**, which is the product of **space parity** *P* by **time reversal** *T* and by **charge conjugation** *C.*
- A particle and its antiparticle have the same mass, the same spin, but **opposite charges**.
- A particle with negative energy going backward in time is replaced by its antiparticle, with positive energy going forward in time

The Fock space

- In absence of interactions (i.e. for free fields) the equations of motion (Dirac, Klein Gordon or Maxwell) can be solved entirely.
- In momentum space, the free field Hamiltonian is an **infinite sum of independent harmonic oscillators**.
- **Quantum statistics** (Bose-Einstein or Fermi-Dirac) are most easily taken into account by means of the **Fock space**, a Hilbert space in which the quantum states of the field are defined in terms of the **occupation numbers**, i.e. the number of field quanta with given four momenta.

In non relativistic quantum statistical physics

$$\text{Vacuum : } |0\rangle = |0_{k_1} 0_{k_2} \ldots 0_{k_i} \ldots\rangle$$

annihilation operators a_{k_i}

creation operators $a_{k_i}^\dagger$

commutation rules: $\left[a_k, a_{k'}^\dagger\right] = \delta_{k,k'}$

$$a_{k_i}|0\rangle = 0 \quad \forall k_i$$

$$H|n_{k_1} \ldots n_{k_i} \ldots\rangle = \left[\sum_i \hbar\omega_{k_i}\left(n_{k_i} + \frac{1}{2}\right)\right]|n_{k_1} \ldots n_{k_i} \ldots\rangle$$

$$|n_{k_1} \ldots n_{k_i} \ldots\rangle = \frac{1}{\sqrt{n_{k_1}! \ldots n_{k_i}! \ldots}} \left(a_{k_1}^\dagger\right)^{n_{k_1}} \ldots \left(a_{k_i}^\dagger\right)^{n_{k_i}} \ldots |0\rangle$$

Neutral scalar field

- Klein Gordon Lagrangean and equation

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m^2}{2}\phi^2$$

$$\left(\Box + m^2\right)\phi = 0$$

$$\Box = \partial_\mu\partial^\mu = \frac{\partial^2}{\partial t^2} - \nabla^2$$

- Fock space

$$k \equiv \left(k_0, \mathbf{k}\right); k_0 = \omega_k = \sqrt{\mathbf{k}^2 + m^2} > 0$$

$$d\tilde{k} = \frac{d^3k}{\left(2\pi\right)^3\omega_k} = \frac{d^4k}{\left(2\pi\right)^4}2\pi\delta\left(k^2 - m^2\right)\theta\left(k_0\right)$$

$$\left[a_\alpha(k), a_\alpha^\dagger(k')\right] = \left(2\pi\right)^3 2\omega_k\delta^3(\mathbf{k} - \mathbf{k}')\delta_{\alpha\beta}$$

$$\left[a_\alpha(k), a_\beta(k')\right] = \left[a_\alpha^\dagger(k), a_\beta^\dagger(k')\right] = 0$$

$\alpha, \beta,$internal quantum numbers

Charged scalar fields (the antiparticle mechanism)

$$\phi = \frac{\phi_1 + i\phi_2}{\sqrt{2}}$$

$$\phi(x) = \int d\tilde{k}\left[a(k)\exp(-ikx) + b^\dagger(k)\exp(ikx)\right]$$

$$\phi^\dagger(x) = \int d\tilde{k}\left[b(k)\exp(-ikx) + a^\dagger(k)\exp(ikx)\right]$$

$$a(k) = \frac{a_1(k) + ia_2(k)}{\sqrt{2}};\, a^\dagger(k) = \frac{a_1(k) - ia_2^\dagger(k)}{\sqrt{2}}$$

$$b(k) = \frac{a_1(k) - ia_2(k)}{\sqrt{2}};\, b^\dagger(k) = \frac{a_1(k) + ia_2^\dagger(k)}{\sqrt{2}}$$

$$\left[a(k), a^\dagger(k)\right] = \left[b(k), b^\dagger(k)\right] = (2\pi)^3 2\omega_k \delta^3(\mathbf{k} - \mathbf{k}')$$

$$\phi\begin{cases}\text{destroys } a \\ \text{creates } b\end{cases} \quad \phi^\dagger\begin{cases}\text{destroys } b \\ \text{creates } a\end{cases}$$

The photon

- Maxwell Lagrangean and equation

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}; F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\Box A^\mu = 0; (m = 0)$$

- Fock space

$$A_\mu(x) = \int d\tilde{k} \sum_{\lambda=0}^{3} \left[a^{(\lambda)}(k) \varepsilon_\mu^{(\lambda)}(k) \exp(-ikx) + a^{(\lambda)\dagger} \varepsilon_\mu^{(\lambda)*}(k) \exp(ikx) \right]$$

$$d\tilde{k} = \frac{d^3 k}{2k_0 (2\pi)^3}; k_0 = |\mathbf{k}|$$

$$\left[a^{(\lambda)}(k), a^{(\lambda')\dagger}(k') \right] = -g^{\lambda\lambda'} 2k_0 (2\pi)^3 \delta^3(\mathbf{k} - \mathbf{k}')$$

$\varepsilon_\mu^{(\lambda)}(k)$ polarisation vector

$\left\{ n, \varepsilon_\mu^{(\lambda)}(k) \right\}$ independent tetrad

n time axis

$$\varepsilon^{(0)} = n; \varepsilon^{(1)}, \varepsilon^{(2)} \perp k, n; \varepsilon^{(\lambda)}(k).\varepsilon^{(\lambda')}(k') = -\delta_{\lambda\lambda'}; \lambda, \lambda' = 1, 2$$

$$\varepsilon^{(3)} \in (k, n); \perp n; \varepsilon^{(3)}(k)^2 = -1$$

Free spinorial field

- **Dirac Lagrangean and equation**

$$\mathcal{L} = \frac{i}{2}\left[\bar{\psi}\gamma^{\mu}\left(\partial_{\mu}\psi\right)-\left(\partial_{\mu}\bar{\psi}\right)\gamma^{\mu}\psi\right]-m\bar{\psi}\psi$$

$$\left(i\gamma.\vec{\partial}-m\right)\psi \equiv \left(i\not{\partial}-m\right)\psi = 0;\bar{\psi}\left(i\gamma.\overleftarrow{\partial}+m\right)=0$$

"square root" of K.G. equation

Dirac matrices

$$\left\{\gamma^{\mu},\gamma^{\nu}\right\}=2g^{\mu\nu}$$

e.g. $\gamma^{0}=\begin{pmatrix}\mathbf{1} & 0\\ 0 & -\mathbf{1}\end{pmatrix};\gamma^{i}=\begin{pmatrix}0 & \sigma^{i}\\ -\sigma^{i} & 0\end{pmatrix}$

- Fock space

$$\psi(x)=\int \frac{d^3k}{(2\pi)^3}\frac{m}{k_0}\sum_{\alpha=1,2}\left[b_\alpha(k)u^{(\alpha)}(k)\exp(-ikx)+d_\alpha^\dagger(k)v^{(\alpha)}(k)\exp(ikx)\right]$$

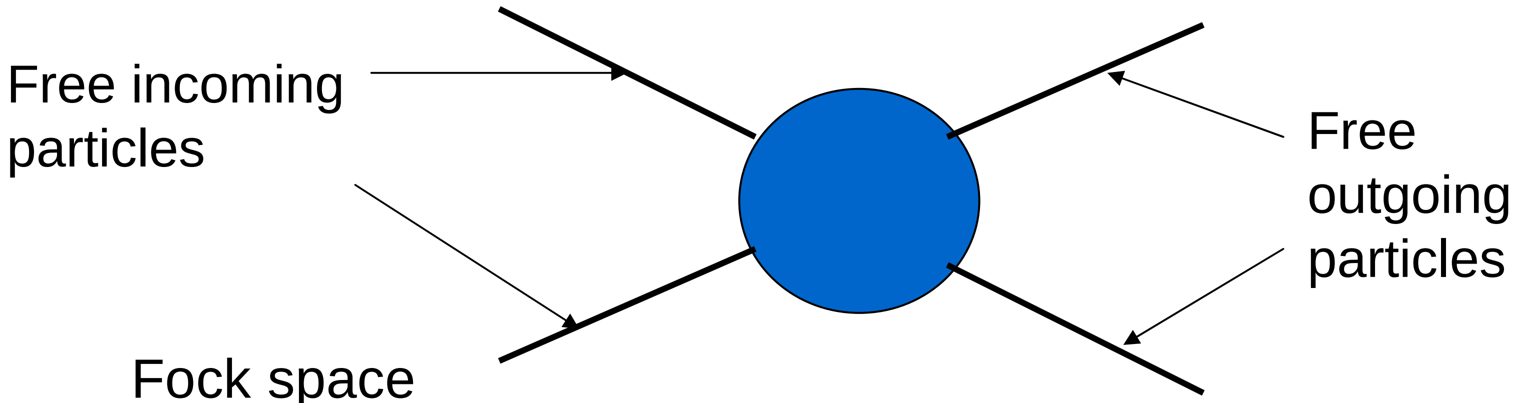
$$u^{(\alpha)}(k)=\frac{\not{k}+m}{\sqrt{2m(E+m)}}u^{(\alpha)}(m,0);v^{(\alpha)}(k)=\frac{\not{k}+m}{\sqrt{2m(E+m)}}v^{(\alpha)}(m,0)$$

$$u^{(1)}(m,0)=\begin{pmatrix}1\\0\\0\\0\end{pmatrix}u^{(2)}(m,0)=\begin{pmatrix}0\\1\\0\\0\end{pmatrix}\ v^{(1)}(m,0)=\begin{pmatrix}0\\0\\1\\0\end{pmatrix}v^{(2)}(m,0)=\begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

$$\overline{\psi}(x)=\int \frac{d^3k}{(2\pi)^3}\frac{m}{k_0}\sum_{\alpha=1,2}\left[b_\alpha^\dagger(k)\overline{u}^{(\alpha)}(k)\exp(ikx)+d_\alpha(k)\overline{v}^{(\alpha)}(k)\exp(-ikx)\right]$$

$$\left\{b_\alpha(k),b_\beta^\dagger(q)\right\}=(2\pi)^3\frac{k_0}{m}\delta^3(\mathbf{k}-\mathbf{q})\delta_{\alpha\beta};\left\{d_\alpha(k),d_\beta^\dagger(q)\right\}=(2\pi)^3\frac{k_0}{m}\delta^3(\mathbf{k}-\mathbf{q})\delta_{\alpha\beta}$$

The S matrix and phenomenology of fundamental interactions



Fock space

$$\left| \begin{matrix} in \\ out \end{matrix} \right\rangle \text{Free fields} \begin{matrix} t=-\infty \\ t=+\infty \end{matrix}$$

$$|out\rangle = S^{-1}|in\rangle = S^{\dagger}|in\rangle$$

$$|in\rangle = S|out\rangle$$

Vacuum

$$\langle 0out|0in\rangle = \langle 0in|S|0in\rangle = \langle 0out|S|0out\rangle$$

Unitarity: conservation of probabilities

$$SS^{\dagger} = S^{\dagger}S = \mathbf{1}$$

The path integral methodology

A re-formulation and a re-interpretation of quantum physics

- In *Space-Time Approach to Non-Relativistic Quantum Mechanics (*Rev. of Modern Physics, 1948), Feynman re-formulates and re-interprets quantum mechanics as a **field theory**:
  - Principle of **superposition of probability amplitudes**
  - Interpretation of the wave function as an amplitude evaluated by means of the **Huygens principle applied** to a field
  - **Probabilistic interpretation of the field intensity**
- This reformulation can be extended to the quantization of relativistic field theories.

QFT and Path Integral

- In QFT, the Path Integral (PI) is a **functional integral**, i.e. an integral over the fields, of the exponential of ***i* times the "classical" action of the theory in units of $\hbar$**
- The "classical" action integral is the invariant integral over space-time of the Lagrangian, expressed in terms of c-number fields, which would lead through the **least action principle** to the "classical" equations of motion.
- In absence of interactions, one recovers with PI the results of the standard quantization method for Klein-Gordon, Dirac and Maxwell fields.
- When the Lagrangian involves an interaction term consisting of a product of fields multiplied by a **small coupling constant**, PI reduces to an expansion in powers of this small coupling constant the coefficients of which involve only ordinary integrals, the **perturbative expansion**.

The perturbative expansion in QFT

Academic case of a single self coupled neutral scalar field:
generating functional of the perturbative expansion

$$K = \int D\phi \exp\left\{\frac{i}{\hbar}\int d^4x \mathcal{L}\left(\phi(x), \partial_\mu\phi(x)\right)\right\}$$

$$\mathcal{L} = \mathcal{L}_0 + j\phi$$

$$j = \text{source of field } \phi$$

$$\mathcal{L}_0 = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 + g\phi^r$$

Generating functional: expansion in powers of *j*;

Perturbative expansion: expansion in powers of *g*

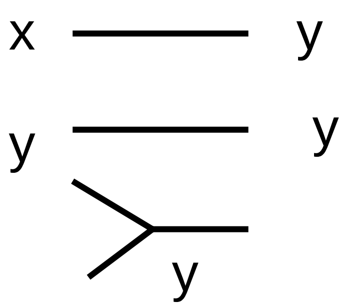
- Feynman's rules and diagrams
  - In configuration space

The **Green's functions** are the coefficients of the expansion in powers of j

For N sources (***N*** **point function**), the **perturbative expansion** is an expansion in powers of g : the coefficient of g^n is a sum **Feynman amplitudes** associated each, by means of **Feynman rules** to a **Feynman diagram** with n **vertices**.

Vertices are the space time points where the r interacting fields are coupled.

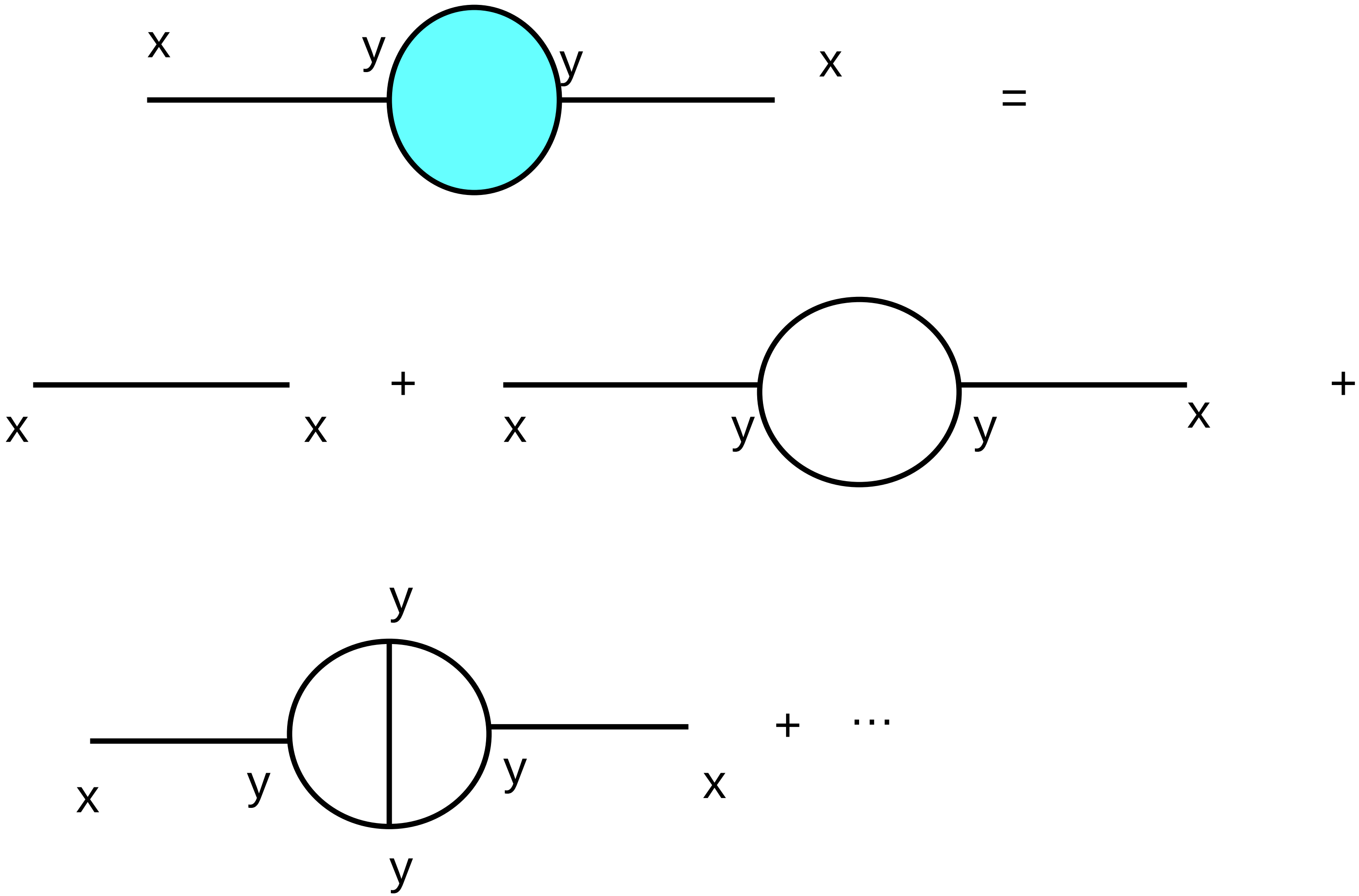
Internal lines relating two vertices, and **external** relating a vertex to a source are **propagators** corresponding to **free particle Green's functions**



N external lines

l internal lines

N vertices with r incident lines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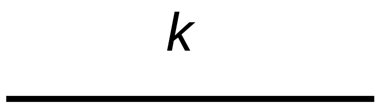
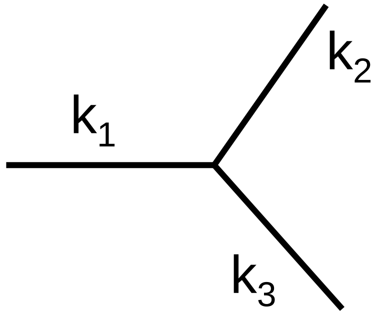
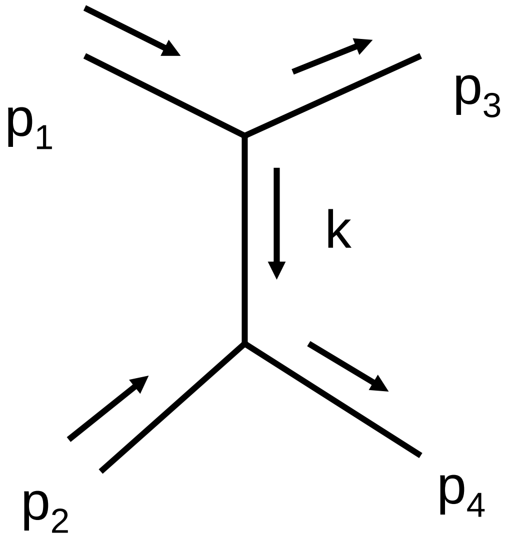
- Feynman rules (in momentum space):integration over vertices Y



$$\frac{1}{k^2 - m^2}$$



$$g\delta(k_1 + k_2 + k_3)$$

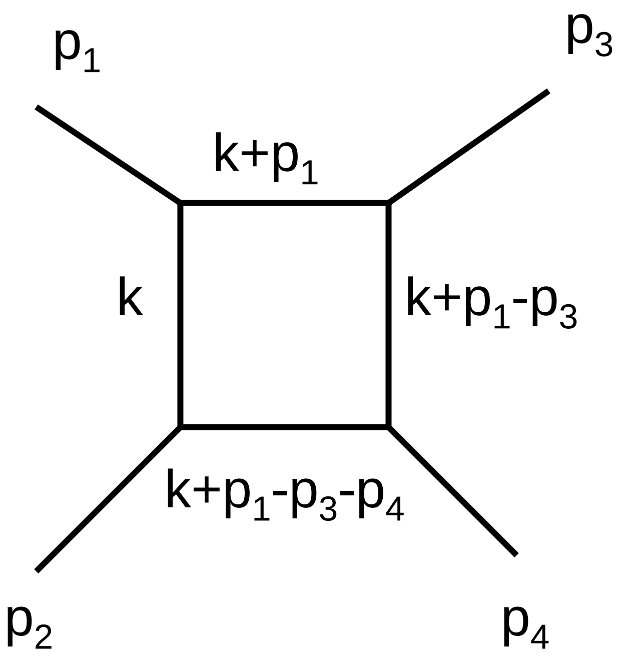


Born approximation

$$A^{(B)} = \frac{g^2}{k^2 - m^2} = \frac{g^2}{\left(p_1 - p_3\right)^2 - m^2} = \frac{g^2}{t - m^2}$$

Fourier transform of $V(r) = g^2 \frac{\exp(-mr)}{r}$

Loop integral

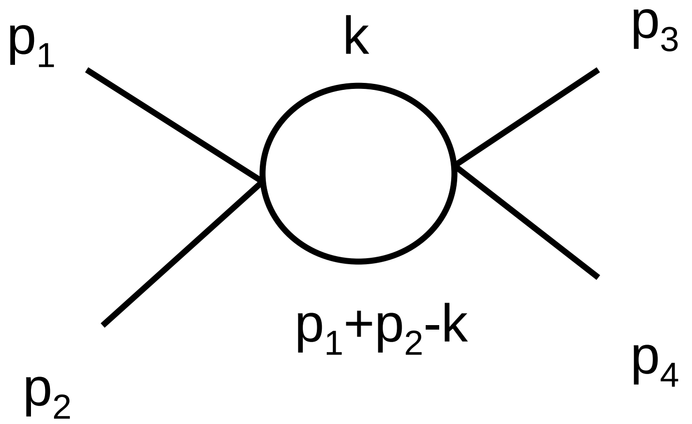


In the considered case,the integral converges; but what happens when r = 4?

Quantum fluctuation

(expansion in powers of $\hbar$)

$$A^{\text{loop}} = g^4 \int d^4k \frac{1}{\left(k^2 - m^2\right)\left((k+p_1)^2 - m^2\right)\left((k+p_1-p_3)^2 - m^2\right)\left((k+p_1-p_3-p_4)^2 - m^2\right)}$$



The loop integral diverges. One needs to renormalise!

$$r = 4$$

$$A^{\text{loop}} = g^2 \int d^4k \frac{1}{\left(k^2 - m^2\right)\left((p_2 + p_2 - k)^2 - m^2\right)}$$

The QED perturbative expansion

- The **whole information** necessary for the QED (electromagnetic interactions of electrons) **perturbation expansion** is encoded in the **QED Lagrangian**

$$Z = \int \mathcal{D}\psi \mathcal{D}A \exp\left\{ \frac{i}{\hbar} \int d^4x \mathcal{L}_{\mathrm{QED}}\left(\psi(x), A(x)\right) \right\}$$

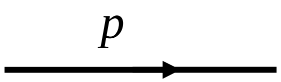
$$\mathcal{L}_{QED} = \overline{\psi}\left(i\gamma^\mu \partial_\mu - m\right)\psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - e\overline{\psi}\gamma^\mu A_\mu \psi$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

- Generating functional

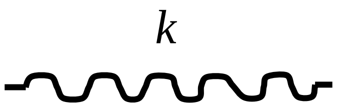
$$\mathcal{K}(j_\psi, j_{\bar\psi}, j_A) = \int \mathcal{D}\psi \mathcal{D}\bar\psi \mathcal{D}A \exp\left\{\frac{i}{\hbar}\int d^4x(\mathcal{L}_{\text{QED}} + j_\psi\psi + j_{\bar\psi}\bar\psi + j_A A)\right\}$$

- Feynman's diagrams and rules
  - Electron propagator



$$\frac{\not p + m}{p^2 - m^2}$$

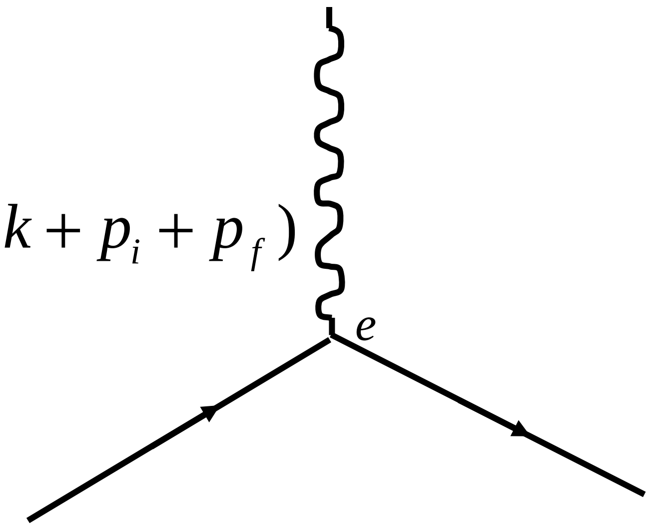
  - Photon propagator



$$\frac{1}{k^2}$$

  - Interaction vertex

$$\gamma^\mu \delta(k + p_i + p_f)$$

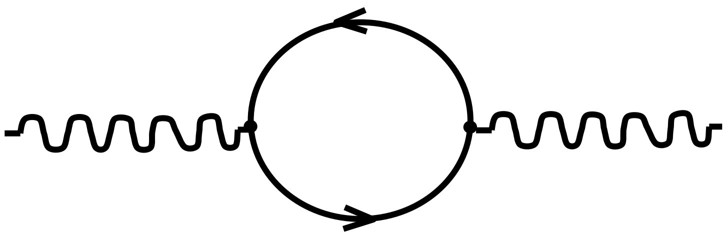


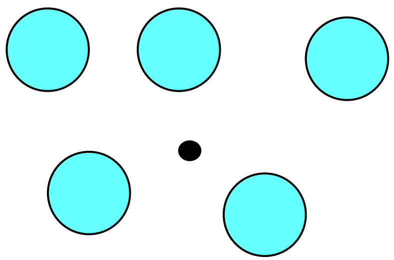
The divergence problem

- All Feynman amplitudes defined in terms of **ordinary integrals**
- In general, integrals implied by diagrams involving loops **diverge**
- These divergences reflect the **conflict** between **locality** (i.e. relativity + causality) and **quantum mechanics**: quantum fields are operator valued **distributions whose products at the same point is ill-defined**
- In QED, all divergences are **logarithmic**

Quantum vacuum as a dielectric: a heuristic picture of renormalisation

- Quantum fluctuations and vacuum polarization.
  - In the Fock space of the electromagnetic field, vacuum is the state with 0 photon. According to Heisenberg's inequalities, when the number of quanta is determined (0 in the vacuum) the value of the field is completely undetermined; it fluctuates
  - One such fluctuation corresponds to the **appearance followed by the disappearance of an electron-positron pair**
  - These fluctuations produce a **screening effect**, leading to the notion of an **effective charge depending on the resolution**

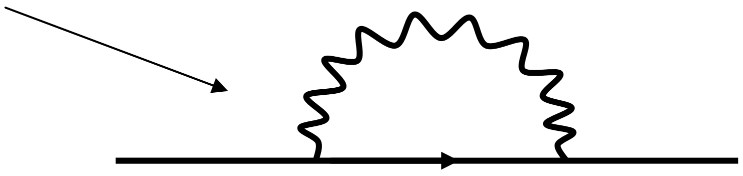




Effect of the interactions on the parameters of the theory

- The charge and the mass of the electron (the parameters of the theory) are **affected by the interaction with the electromagnetic field**.
- **self energy diagram**

An electron emits and reabsorbs a photon: the electron quantum field is modified by the electromagnetic fields it generates



- The parameters of the theory are split into "**bare**" parameters and "**dressed**" parameters, i.e. "**renormalized"** by the interaction
- Renormalized parameters are **effective parameters depending on the resolution**, one says that they are **renormalized at scale μ**

The "miracle" of Renormalizability

- **Renormalization**= redefinition of the fundamental parameters of the theory
  - Electric **charge** *e* split into
    - e_0 "**bare charge**", i.e. the charge if there were no interaction
    - *e* physical, "**renormalized**" charge
  - Electron **mass** *m* split into
    - m_0 "**bare mass**", i.e. the mass if there were no interaction
    - *m* physical, "**renormalized**" mass
- **Regularization**: With an energy **cutoff** Λ one removes the divergences in the expressions of *e, m* and any amplitude in terms of e_0 and m_0.

- One can **invert** these expressions

$$e_0 = e + 1/2\beta_2 e^3 \,\mathrm{Log}\left(\frac{\Lambda}{m}\right) + O(e^5)$$

$$m_0 = m + \gamma_1 m e^2 \,\mathrm{Log}\left(\frac{\Lambda}{\mathrm{m}}\right) + O(e^4)$$

and express **all amplitudes in terms of *e* and *m***

- The "**miracle**" is then that in these expressions, for any amplitude, and at all orders of perturbation expansion, **the limit when Λ goes to infinity is finite**, which means that one can express perturbatively all observables **in terms of two and only two physical parameters** which can be measured experimentally.
- The theory/experiment agreement is **amazingly good**

For example, for the magnetic moment of the electron one finds

Exp: 2,00231930482+/- 40
Th: 2,00231930476+/- 52

The renormalization group

- If the electron mass *m* was equal to zero, it would have been necessary to introduce a new energy scale μ, called the **renormalization energy**, to deal with the logarithmic divergence,

$$e_0 = e + 1/2\,\beta_2 e^3\,\mathrm{Log}\left(\frac{\Lambda}{\mu}\right) + O(e^5)$$

and when one inverts this relation, *e* becomes a function of μ

- When $m \neq 0$

a renormalization energy is still necessary, but it is implicitly set equal to *m* : $e(\mu = m) = e\,; m(\mu = m) = m$

- The renormalization energy being arbitrary, physics must be independent on its value, which means that the renormalized charge must obey a **differential equation, called the renormalization group equation**, expressing this independence:

$$\mu \frac{de^2(\mu)}{d\mu} = \beta\left(e^2(\mu)\right)$$

where $\beta(e^2) = \beta_2 e^4 + O(e^6)$
is finite when Λ goes to infinity (β_2 is a c-number, depending only on the QED Lagrangian)

- As a consequence, parameters in QED have to be considered as **effective parameters**, depending on the resolution

$$\alpha(\mu = m_e) = 1/137\,;\, \alpha(\mu = m_Z) = 1/128$$

Gauge invariance

- In Maxwell theory, **gauge invariance** $(A_\mu \to A_\mu + \partial_\mu \Lambda(x))$ **is equivalent to current conservation**
- In QED, the following transformation leaves the QED Lagrangian invariant

$$\psi(x) \to e^{-i\alpha(x)}\psi(x)$$

$$\overline{\psi(x)} \to e^{+i\alpha(x)}\overline{\psi(x)} \qquad \alpha(x) = e\Lambda(x)$$

$$A_\mu \to A_\mu + \partial_\mu \Lambda(x)$$

- Conversely, one can impose invariance through a **local** change of the phase of the electron field by introducing a gauge invariant field, the electromagnetic field: **local gauge invariance determines the structure of the interaction**

- The **Ward identities**, consequences of **gauge invariance** and local conservation of currents are **crucial for renormalizability** of QED.
- This feature suggests to look for **gauge invariant theories for other interactions**
- **Yang and Mills** generalization of gauge invariance to **non Abelian** symmetry groups.
- Failure of such generalization for strong interaction because of **hadron proliferation** and **absence of massless hadronic vector boson**
- Search for a **subhadronic structure**.