(INTRODUCTION TO)
PRIMORDIAL COSMOLOGY

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OUR PROGRAM

1- Introduction:

What is the size of the universe? What is cosmology? Models and hypothesis

2- The standard hot big-bang model

Friedmann equations. Dynamics. Successes and problems.

3- Gravitational dynamics and large scale structures

Newtonian regime. General properties. Gravity waves.

4- Observing the large scale structures

Galaxy catalogs. CMB. Lensing. Summary of the observational status.

5- Inflation: a scenario for the origin of structures

6- The input of high energy physics

dark energy. dark matter. Extra-dimension and string inspired cosmology

7- Conclusion

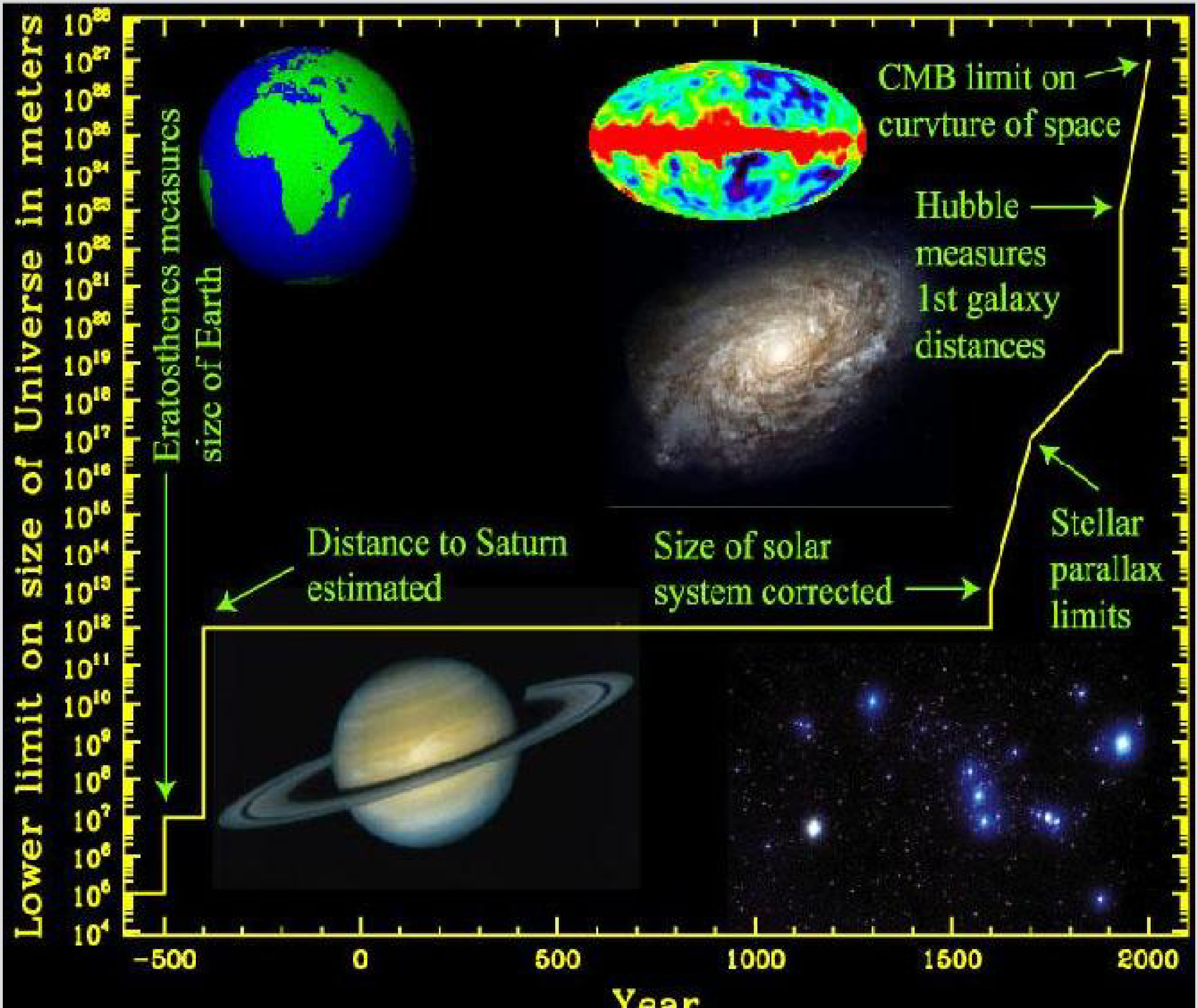
Open issues and questions.- exotica

PART I: COSMOLOGY

Main topics

- *What is the size of the universe?*
- *What is cosmology?*
- *Models and hypothesis*

VHAT IS THE SIZE OF THE UNIVERSE?



WHAT IS COSMOLOGY?

Cosmology is a **description** of what we think is the universe

It requires to define the framework of its study:

What is the universe?

It is developed within a precise **geometrical theory**

(Euclid, Descartes, Riemann,…)

which serves as a test bank for **new physical theories**

(Aristotle, Newton, Einstein, …)

Observations allow to distinguish, draw new questions,…

(planetary orbits, cosmic microwave background...)

ARTICULARITIES

Cosmology is however peculiar:

- we observe only a *small* part of the universe from a *fixed point* of spacetime,
- only *one* universe is observed by us.

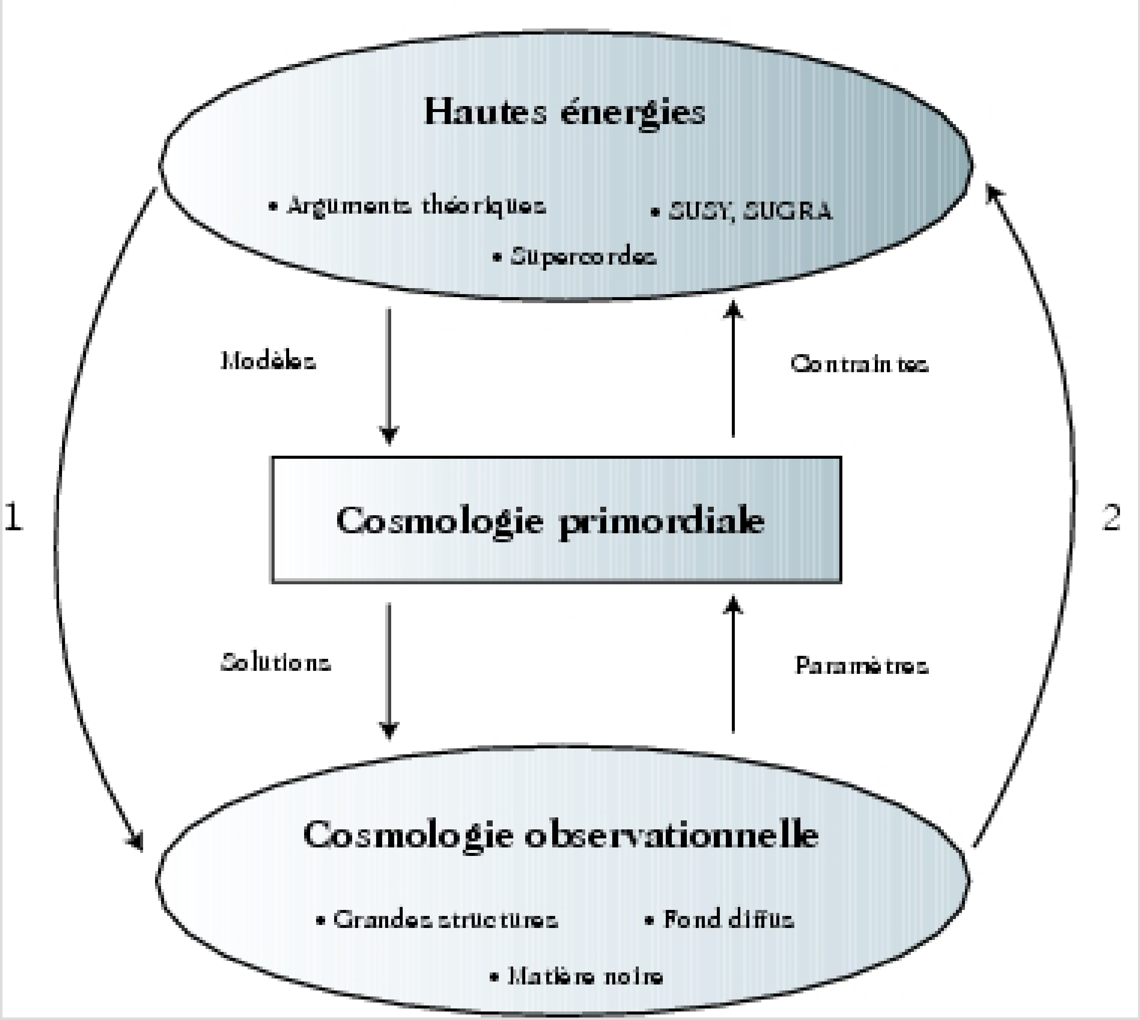
t implies that *some* hypothesis *cannot be tested*.

We start from the standard fundamental theories (as tested in laboratory and accelerator).

This theories are extrapolated to higher energies in the early energies.

- we can constrain various extension of this fundamental setting.
- it is important to diagnostic the failures of these extrapolations in order to infer the more fruitful extensions

RIMORDIAL COSMOLOGY



Hautes énergies

- Arguments théoriques
- SUSY, SUGRA
- Supercordes

Modèles

Contraintes

1

Cosmologie primordiale

2

Solutions

Paramètres

Cosmologie observationnelle

- Grandes structures
- Fond diffus
- Matière noire

High energy theories provide *models* of the primordial universe
Cosmology allows to set constraints on various extensions of the standard fundamental theorie

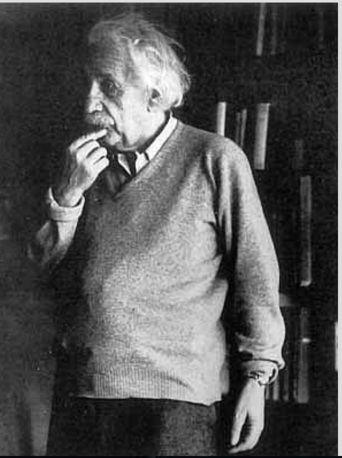
Each models must provide unambiguous observational predictions

PART II: THE STANDARD BIG-BANG MODEL

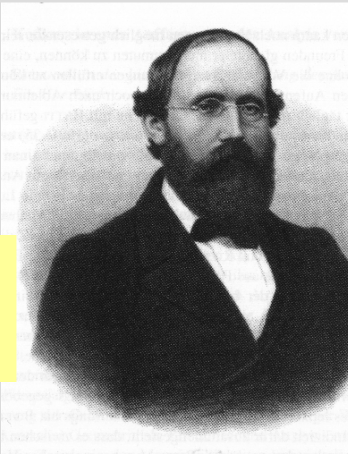
Main topics

- *Friedmann-Lemaître universes*
- *dynamics and properties*
- *Successes*
    - *Hubble diagram*
    - *Age of the universe*
    - *Relics and BBN*
    - *Cosmic microwave background*
- *Problems*

ELATIVISTIC COSMOLOGY



Physics : Einstein
Geometry : Riemann



Formulation of General Relativity (1915)

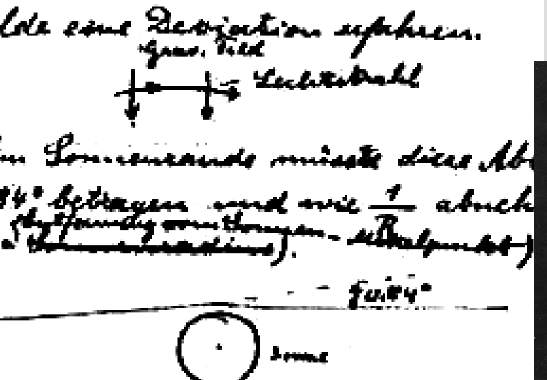
First cosmological models (1917-1927)
static or in *evolution* ?

Development of physics in expanding universe (>1948)

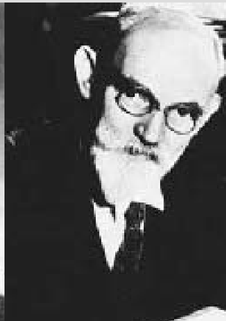
Hot big bang model

Links with high-energy physics (>1981)

Inflation, string cosmology



lde eine Deviation erfahren.
Grav. Feld
Lichtstrahl
m Sonnenrande müsste diese Ab
84" betragen und wie $\frac{1}{R}$ abneh
[illegible]
f=0,84"
Sonne



HYPOTHESIS

- gravitation is described by general relativity,

- physical laws are universal
- we are not seating at the particular place in the universe:
  cosmological principle - Copernician principle

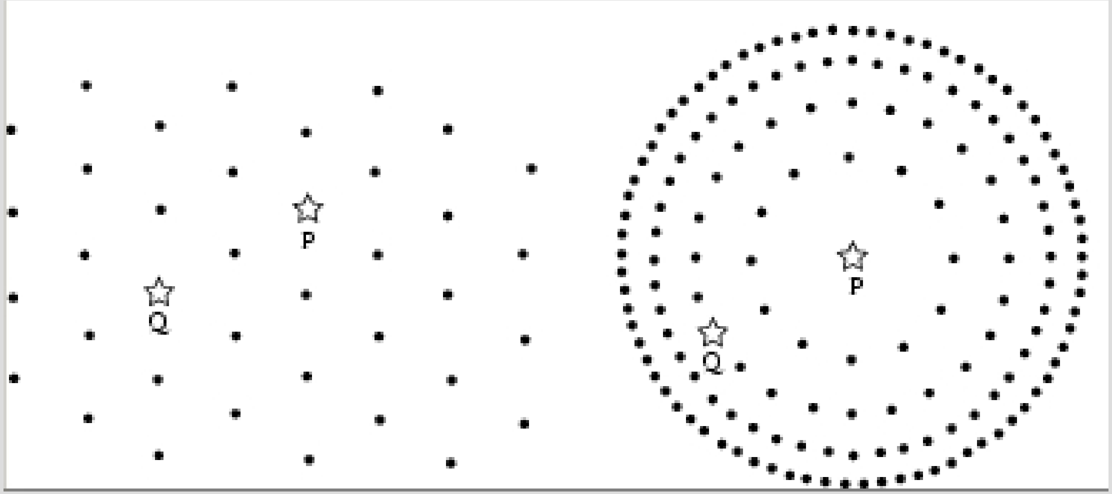
- matter contains
  - radiation
  - dust (fluid of galaxies without pressure)

Conservative hypothesis !

OSMOLOGICAL PRINCIPLE

Cosmological principle: the universe is homogeneous and isotropic

Copernician principle: we do not occupy a particular place in the universe



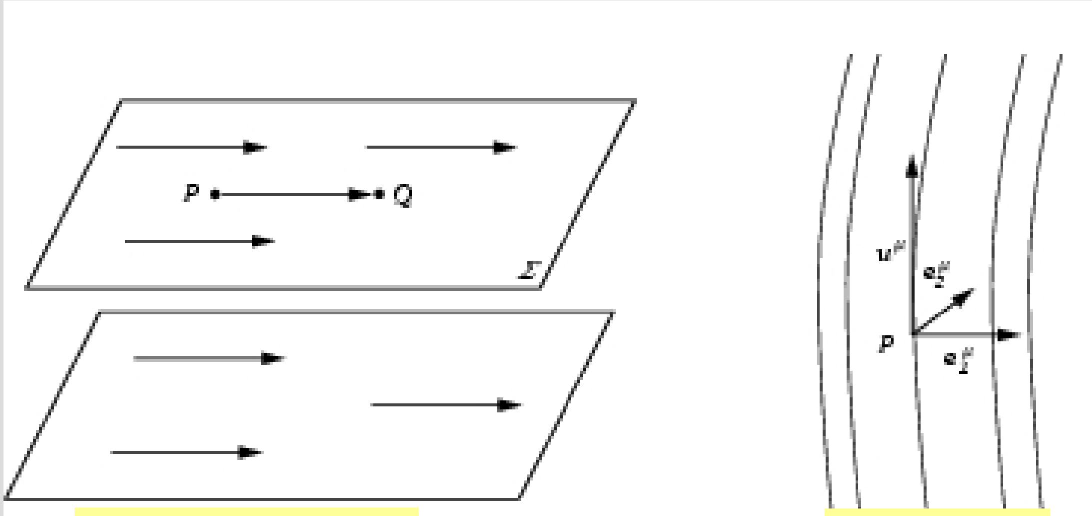
Distribution statistically isotropic
Around each point
P and Q are equivalent

Distribution statistically isotropic
around P alone
P and Q are not equivalent

Copernician + isotropy → Cosmological principle

OMOGENEITY AND ISOTROPY

Homogeneity: at any time, any point of space is equivalent to any other
Isotropy: the universe is seen isotropic from any point



homogeneity

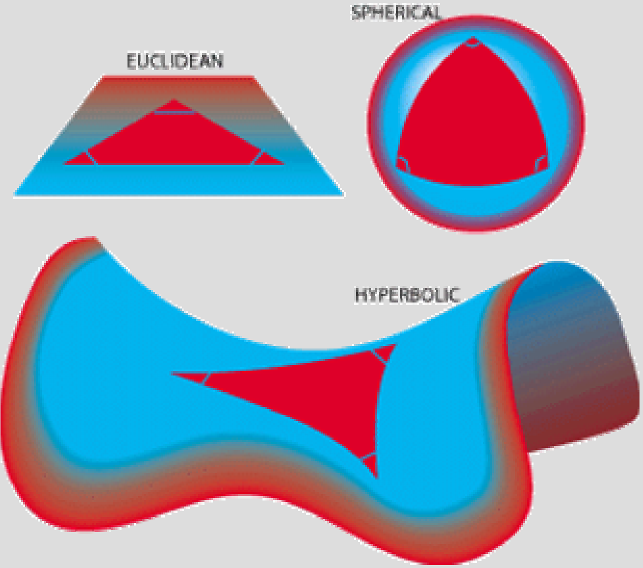
isotropy

pace can be foliated by hypersurfaces Σ uch that on each hypersurface there exists n isometry bringing P to Q

It exists a geodesic flow with tangent Vector u^μ such that at any P, there is an isometry letting P and u^μ invariant which makes a rotation of the directions of observation

RIEDMANN-LEMAITRE UNIVERSE

Homogeneity + isotropy → constant curvature space



There are only three *3D* constant curvature spaces

$$\mathrm{d}s_3^2 = \gamma_{ij}(x^k)\mathrm{d}x^i\mathrm{d}x^j$$

The most general *4D* metric satisfying these hypotheses is

$$\mathrm{d}s^2 = -\,\mathrm{d}t^2 + a^2(t)\gamma_{ij}(x^k)\mathrm{d}x^i\mathrm{d}x^j$$

scale factor

cosmic time

comoving coordinates

Friedmann-Lemaître spacetim

IRST PREDICTION: *HUBBLE LAW*

Space is expanding

The relative distance between 2 comoving observers

$$\mathbf{r}_{12} = a(t)(\mathbf{x}_1 - \mathbf{x}_2)$$

evolves as

$$\boxed{\dot{\mathbf{r}}_{12} = H\mathbf{r}_{12} \quad H \equiv \dot{a}/a}$$

This is the Hubble law (Lemaître, 1924!)

- Purely kinematic consequence of the expansion
- The function H(a) will be central to describe the dynamics

SECOND PREDICTION: *REDSHIFT*

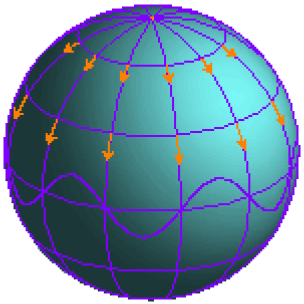
From the geodesic equation for photon, one can deduce that the energy of the photons satisfy

$$\dot{E}/E = -H \;\Rightarrow\; E \propto 1/a$$

<u>Interpretation:</u> the wavelength of the photon is stretched by the expansion

redshift

$$\boxed{1 + z \equiv \frac{E(t_{obs})}{E(t_{em})} = \frac{a(t_{em})}{a(t_{obs})}}$$



<u>Consequence:</u> achromatic stretching of atomic spectra

RIEDMANN EQUATIONS

Start from the Einstein equations

$$G_{\mu\nu} \equiv R_{\mu\nu} - \tfrac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

for the Friedmann-Lemaître metric

The Cosmological Principle implies that $T_{\mu\nu} = \mathrm{diag}(-\rho, P, P, P)$

So that the Friedmann equations take the form

$$\boxed{\begin{aligned} 3\left(H^2 + \frac{K}{a^2}\right) &= 8\pi G\rho \\ \frac{\ddot{a}}{a} &= -\frac{4\pi G}{3}(\rho + 3P) \end{aligned}}$$

The conservation equation gives

$$\boxed{\dot{\rho} + 3H(\rho + P) = 0}$$

RIEDMANN EQUATIONS: MATTER DESCRIPTION

Only 2 equations are independent

To describe the matter, we need to specify an equation of state

$$P(\rho) = w\rho$$

At the cosmological level

- pressureless matter: $P=0$
- radiation $P=\rho/3$

3 variables (a,P,ρ) -1 relation - 2 equations = OK

The conservation equation implies

$$\boxed{\rho = \rho_0 (\frac{a}{a_0})^{-3(1+w)} = (1+z)^{3(1+w)}}$$

RIEDMANN EQUATIONS: ADDING A COSMOLOGICAL CONSTANT

One can add a constant term to the einstein action

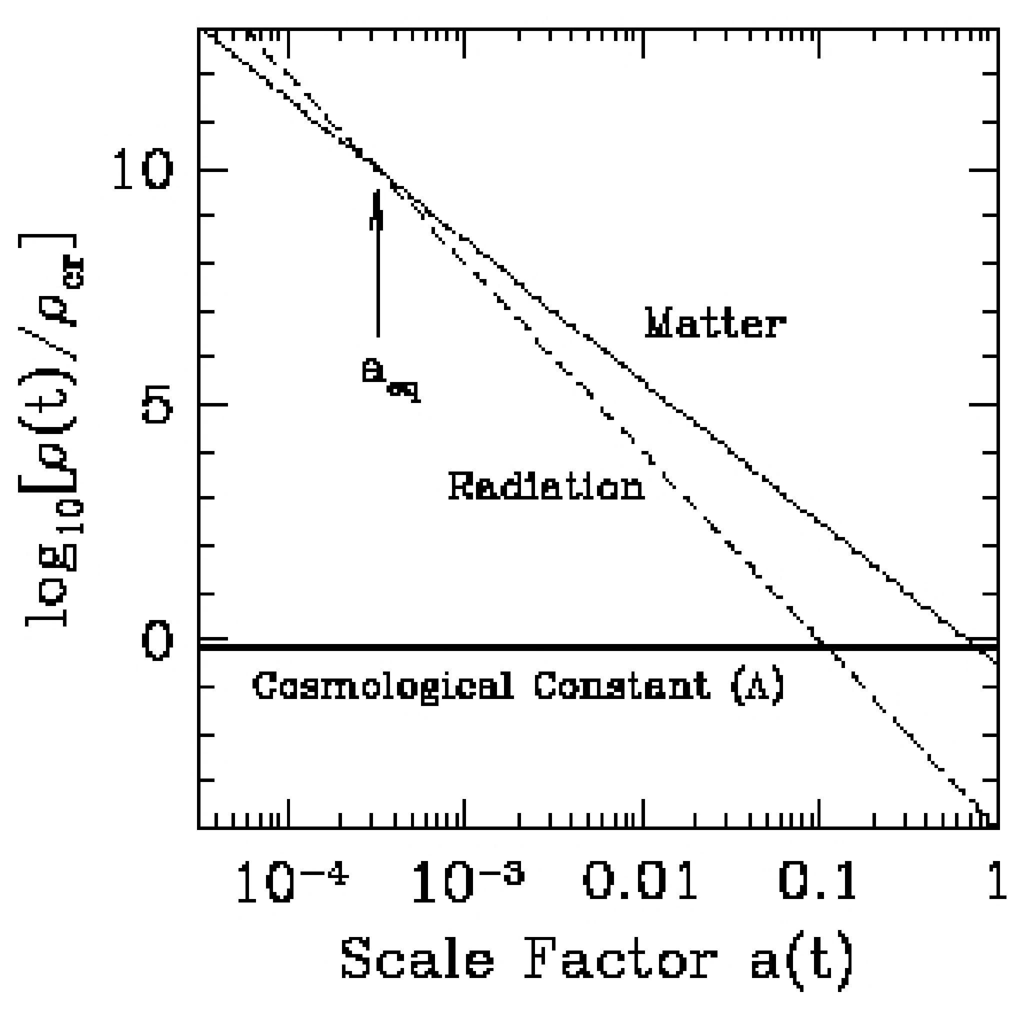
$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

So that the Friedmann equations become

$$\boxed{\begin{aligned} 3\left(H^2 + \frac{K}{a^2}\right) &= 8\pi G\rho + \Lambda \\ \frac{\ddot{a}}{a} &= -\frac{4\pi G}{3}(\rho + 3P) + \frac{\Lambda}{3} \end{aligned}}$$

- Interpretation of this term to be discussed later
- Equivalent to a fluid with *w=-1*

RIEDMANN EQUATIONS: CONSEQUENCE



The universe was dominated by radiation in the past
HOT BIG BANG MODEL

RIEDMANN EQUATIONS: REWRITING

It is common to rewrite the Friedmann equations with adimensional quantities

Density parameters

$$\Omega = \frac{8\pi G\rho}{3H^2} \qquad \Omega_\Lambda = \frac{\Lambda}{3H^2} \qquad \Omega_K = -\frac{K}{a^2H^2}$$

Friedmann equation

$$E^2(z) \equiv \left(\frac{H}{H_0}\right)^2 = \sum \Omega_0(1+z)^{3(1+w)} + \Omega_{\Lambda 0} + \Omega_{K0}(1+z)^2$$

Constraint

$$\sum \Omega_0 + \Omega_{\Lambda 0} + \Omega_{K0} = 1$$

RIEDMANN EQUATIONS: EXERCISE

Find the evolution of the scale factor when K=0 and w≠-1

$$\rho \propto a^{-3(1+w)} \longrightarrow H^2 \propto a^{-3(1+w)} \longrightarrow \dot{a}/a \propto a^{-3(1+w)/2}$$

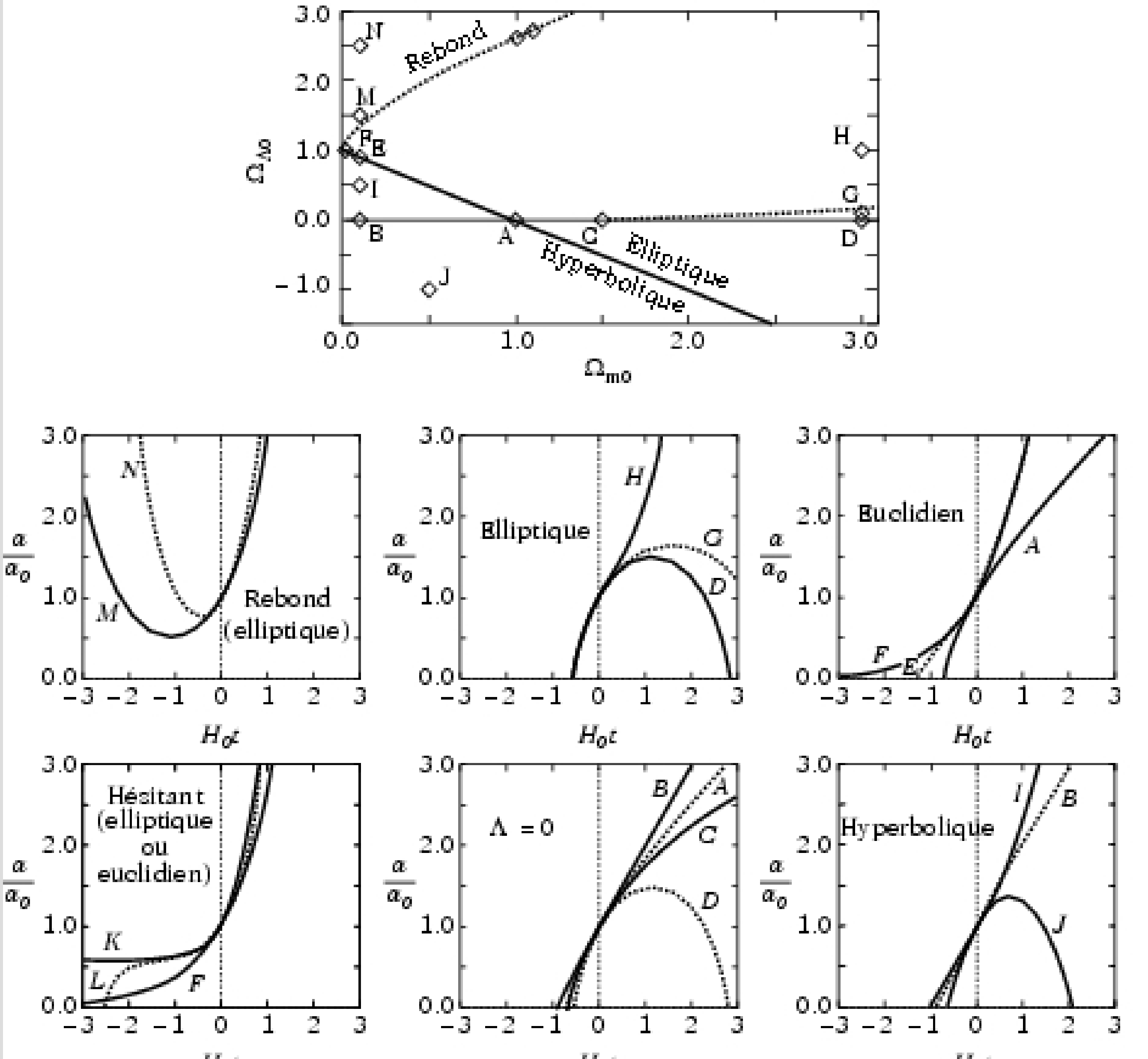
$$\boxed{a(t) \propto t^{2/3(1+w)}}$$

Find the evolution of the scale factor when K=0 and w=-1

$$\rho \propto a^0 \longrightarrow H^2 \propto a^0 \longrightarrow \dot{a} \propto Ha$$

$$\boxed{a(t) \propto \exp Ht}$$

RIEDMANN EQUATIONS: DYNAMICS



IME AND DISTANCE SCALES

H_0 sets a typical time and distance scales for the universe

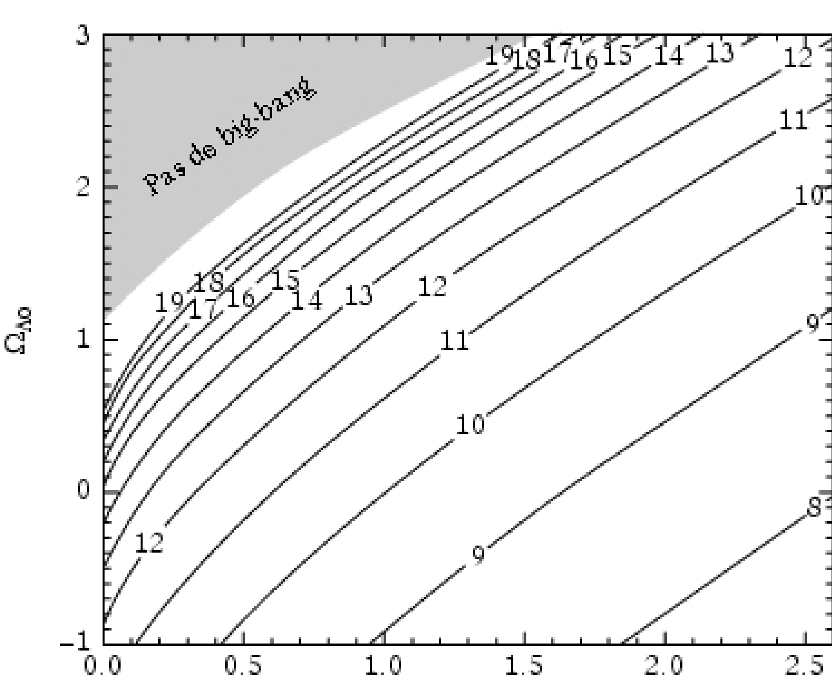
$$H_0 = 100\,h \ \mathrm{km/s/Mpc}$$

$$D_{H_0} = c/H_0 = 3000 h^{-1}\,\mathrm{Mpc} = 2.9 h^{-1} \times 10^{25}\,\mathrm{m}$$

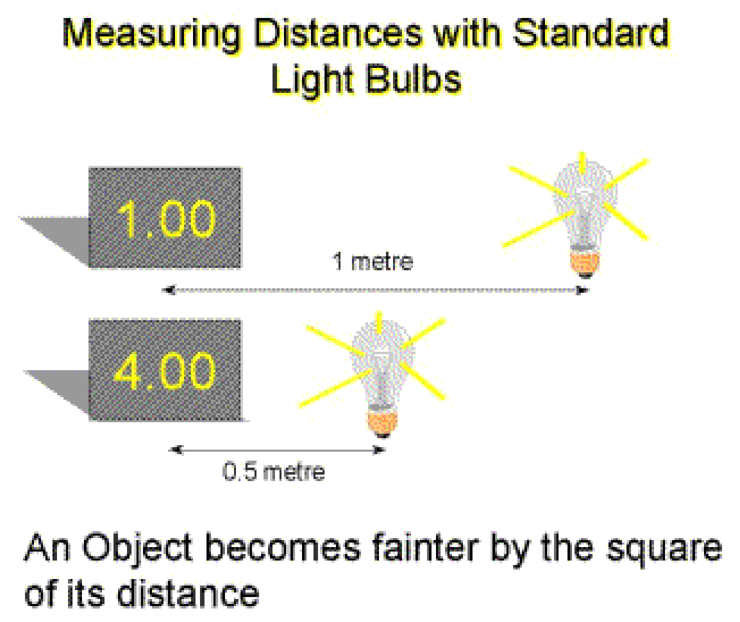
$$t_{H_0} = 1/H_0 = 9.78\,h^{-1} \times 10^{9}\,\mathrm{yr}$$

Dynamical age of the universe

$$\mathrm{d}t = \frac{\mathrm{d}a}{\dot{a}} = t_{H_0} \frac{\mathrm{d}a}{aE(a)}$$



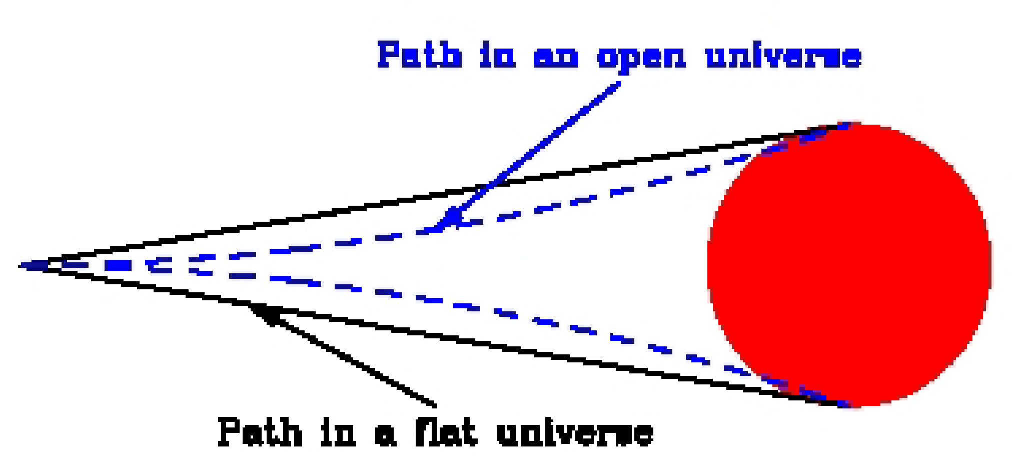
DISTANCES



Measuring Distances with Standard Light Bulbs

An Object becomes fainter by the square of its distance

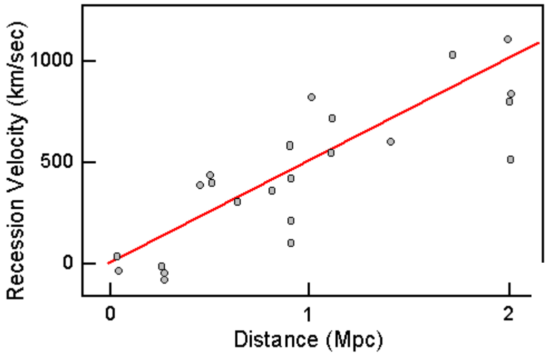
$$\phi_{\mathrm{obs}} = \frac{L_{\mathrm{source}}}{4\pi D_L^2}$$

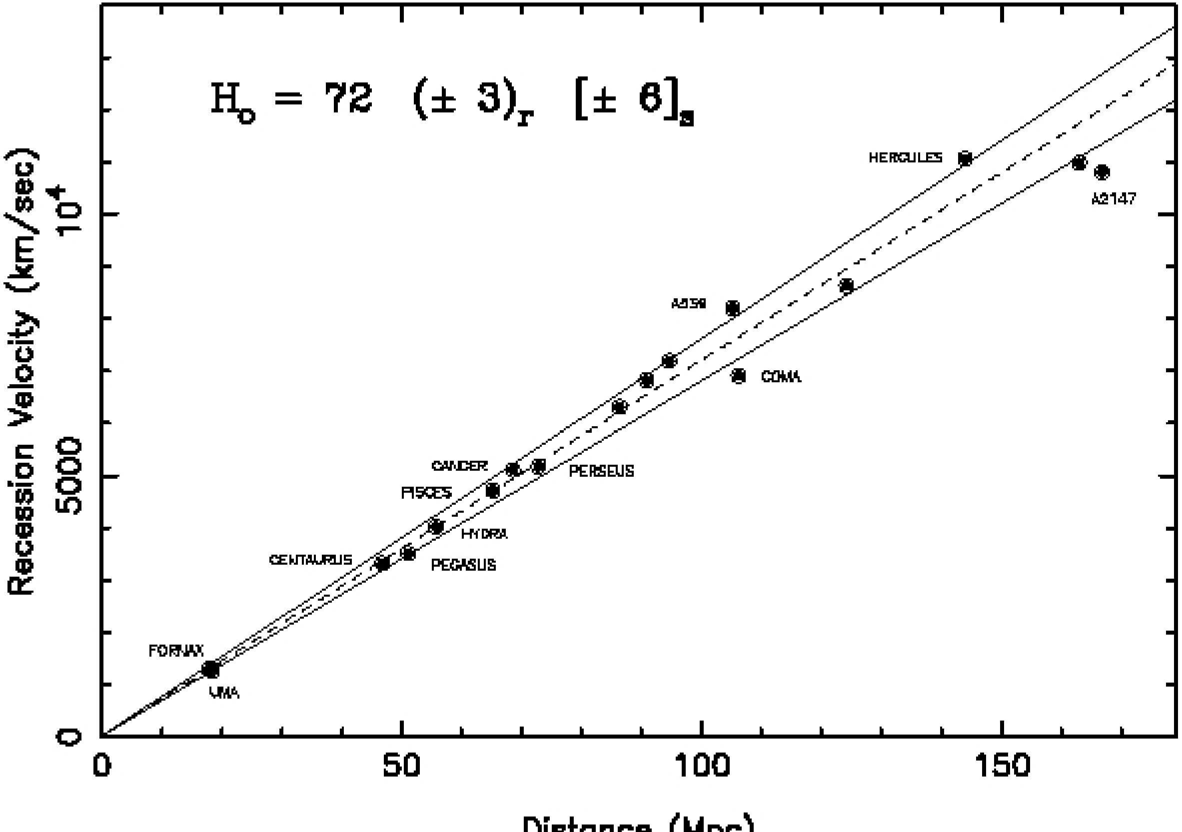


$$D_A^2 = \frac{\mathrm{d}S^{\mathrm{phys}}_{\mathrm{source}}}{\mathrm{d}\Omega_{\mathrm{obs}}}$$

SUCCESSES: EXPANSION

Hubble’s Data (1929)





UCCESSES: THERMAL HISTORY

Consider a particle of mass M.

T>M It is in thermal equilibrium with its anti-particle

T<M Annihilation implies that it disappears $X + \bar{X} \leftrightarrow l + \bar{l}$

But, particles are diluted by expansion

Competition between dilution and annihilation

H $\Gamma \sim n\langle \sigma v \rangle$

$H < \Gamma$ Interaction effect

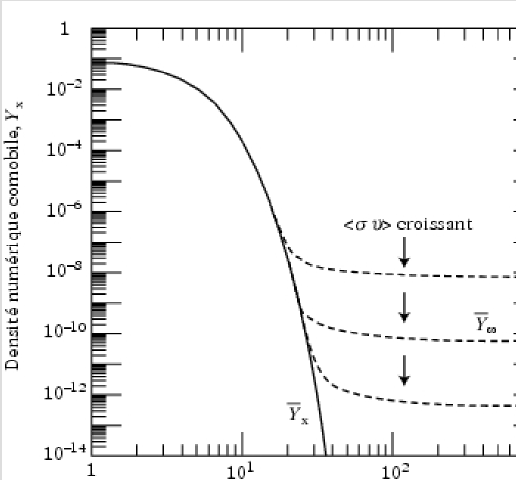
$H > \Gamma$ Interaction froze

In radiation era $H \propto T^2$

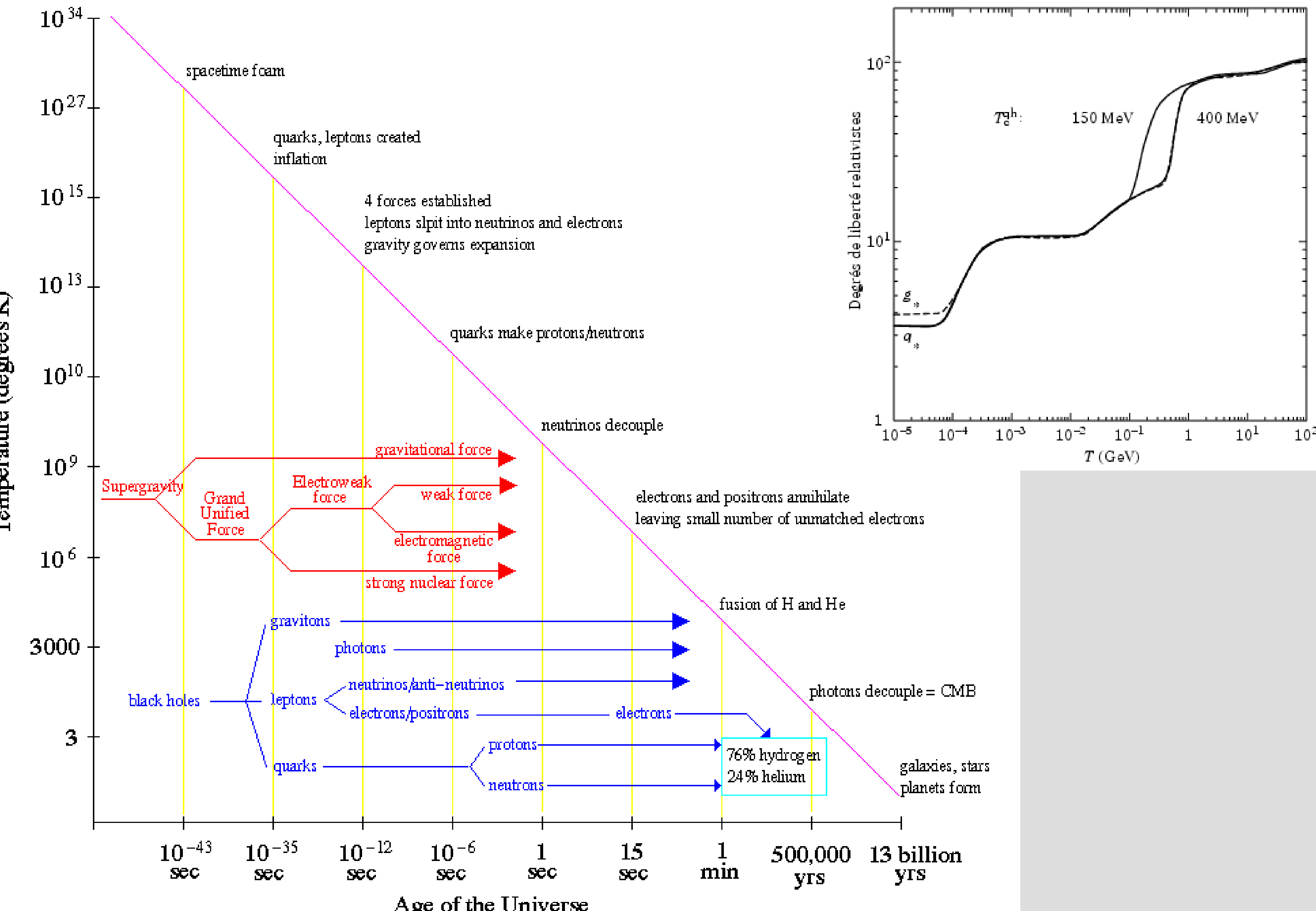
If $\Gamma \propto T^{n+3}$ (n=2 for weak interaction)

There ALWAYS a temperature below which an interaction is frozen

The universe has a THERMAL HISTORY



UCCESSES: THERMAL HISTORY



UCCESSES: BIG-BANG NUCLEOSYNTHESIS

T>> 100 Mev

Electron, positron, neutrinos and photons: UR / proton, neutron: NR

T>> 1 Mev

Neutron and proton in equilibrium by weak interaction

$$(n/p)_{eq} \sim \exp(-Q/T) \sim 1, \quad Q = m_n - m_p \sim 1.3\,\mathrm{MeV}$$

T=1-0.7 MeV

Weak interaction freezes

$$\Gamma_{\text{weak}} \sim H \Leftrightarrow T_f \sim 0.8\,\mathrm{MeV}$$

$$(n/p)_f \sim \exp(-Q/T_f) \sim 1/5$$

Free neutrons decay in 887 sec

T=0.7-0.05 MeV

Light nuclei are formed by a series of nuclear reactions

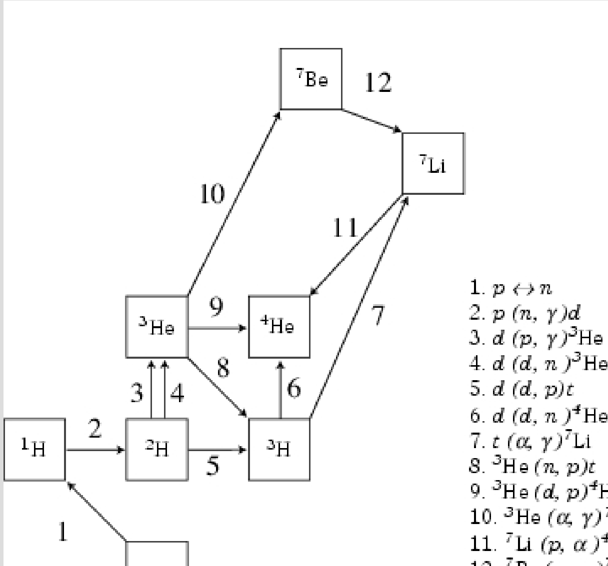
$$n + p \to \mathrm{D} + \gamma$$

D can be produced only when T<0.066MeV is low enough so that photo-dissociation negligible is negligible

$$(n/p)_N \sim 1/7$$

Helium-4

$$Y = \frac{2n}{n+p}\bigg|_N \sim 0.25$$



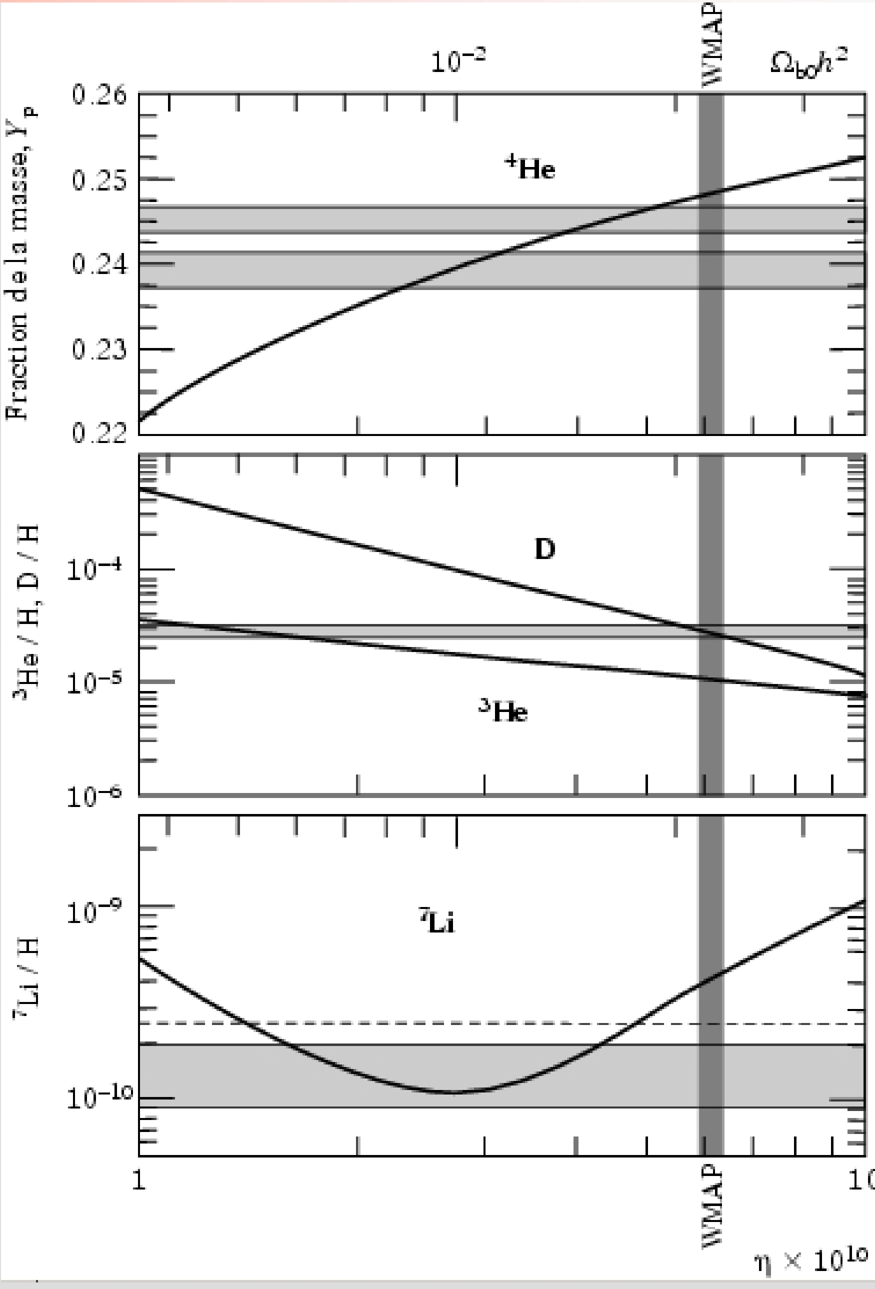
UCCESSES: BIG-BANG NUCLEOSYNTHESIS

Parameters:

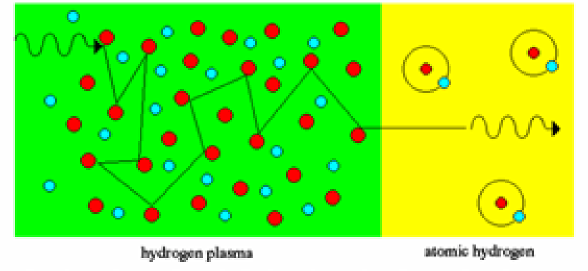
- -Number of relativistic particles
- - lifetime of neutron
- - $\eta = n_{baryon}/n_{\gamma}$
- - $G, G_F, \alpha, \ldots$

Allows to test extensions of the standard model of particle physics

$N_{\nu} = 3$

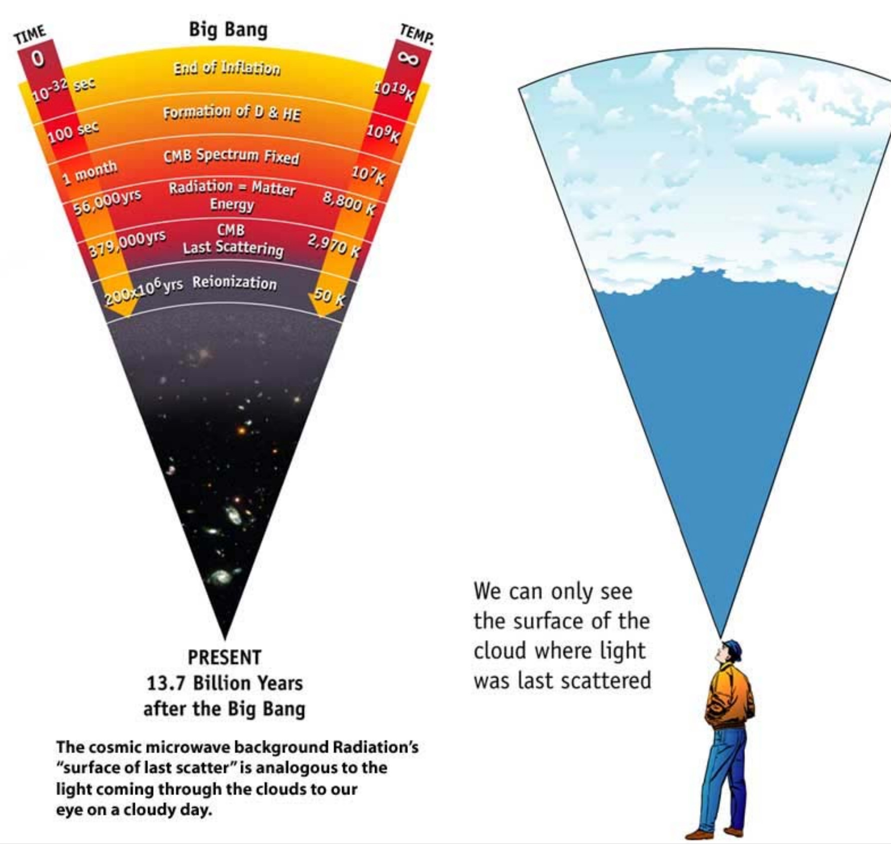


UCCESSES: COSMIC MICROWAVE BACKGROUND



The universe cools during expansion

Around **T~4000 K**, protons and électrons
Combine to form hydrogene.
The universe becomes *transparent*



The cosmic microwave background Radiation's "surface of last scatter" is analogous to the light coming through the clouds to our eye on a cloudy day.

Gamow argument

Knowing the baryonic density today

$$n_{b0} \sim 10^{-7}\,\mathrm{cm}^{-3}$$

and at BBN

$$n_b \sim 10^{18}\,\mathrm{cm}^{-3}$$

he inferred the redshift at BBN

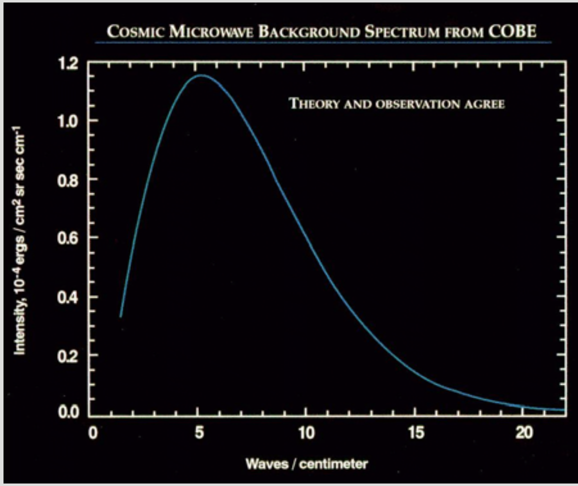
$$1 + z_{BBN} \sim (n_b/n_{b0})^{1/3} \sim 2 \times 10^8$$

and the temperature of the photon bath today

$$T_{\gamma 0} = \frac{T_{BBN}}{1+z_{BBN}} \sim 5K$$

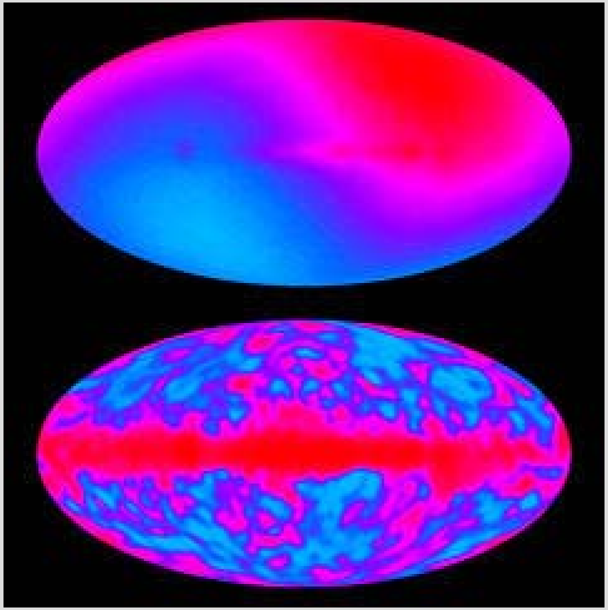
SUCCESSES: COSMIC MICROWAVE BACKGROUND

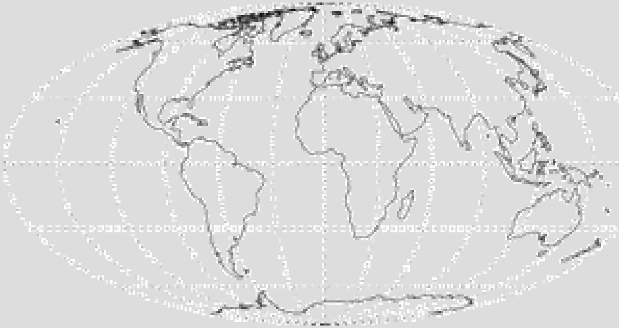
Emission d’un fond de photons avec un spectre de corps noir à une température de 2.725K aujourd’hui.



COBE observation

Dipole after
Monopole substraction

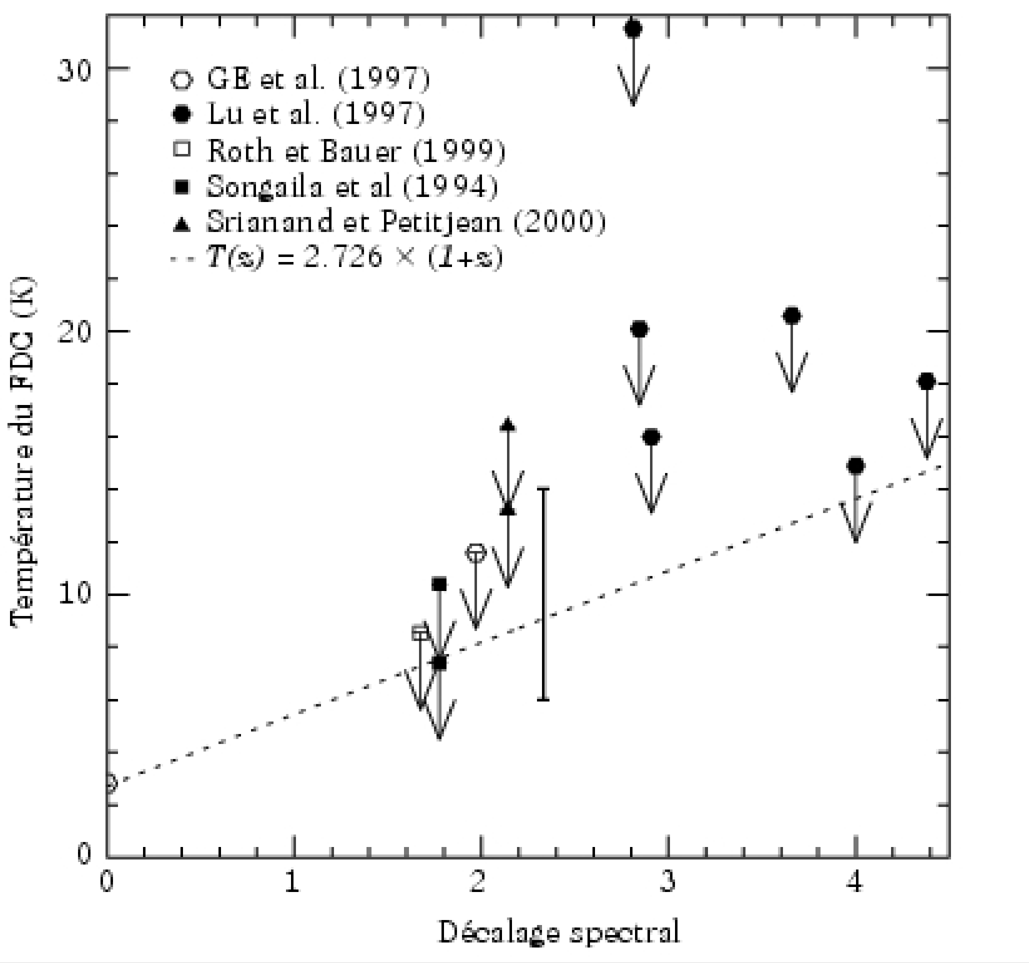




After dipole
Substraction:
Fluctuation of order μK

UCCESSES: COSMIC MICROWAVE BACKGROUND

Temperature of CMB scales as 1/(1+z)



ARAMETERS OF THE MODEL

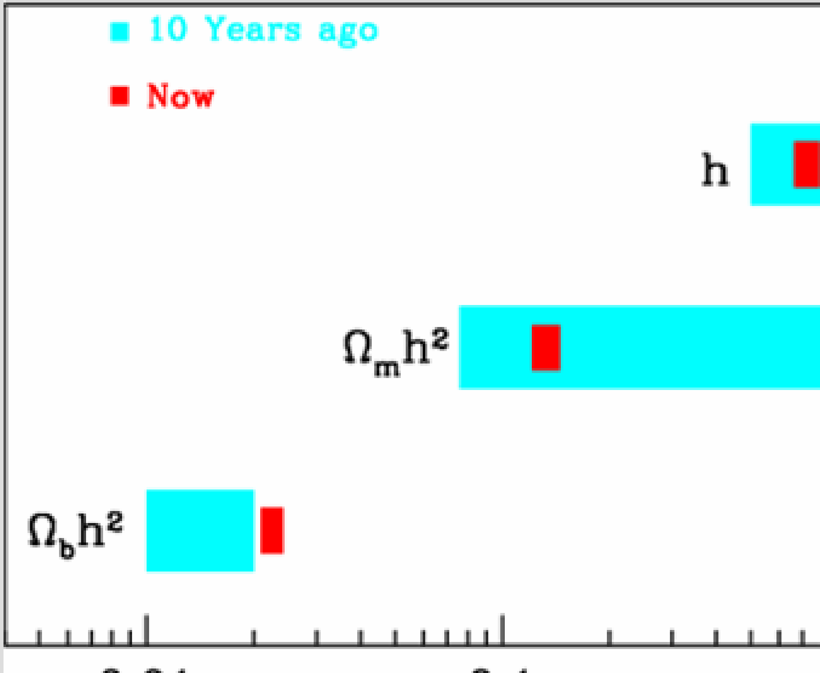
4 numbers to describe the dynamics of the universe

$$\Omega_m, \quad \Omega_r, \quad \Omega_K, \quad \Omega_\Lambda, \quad H_0$$

They start to be accurately measured
- we shall see how later
- ``precision cosmology’’
- This is a very successful model

But…

The universe cools down from a hot thermal equilibrium state



Successes:

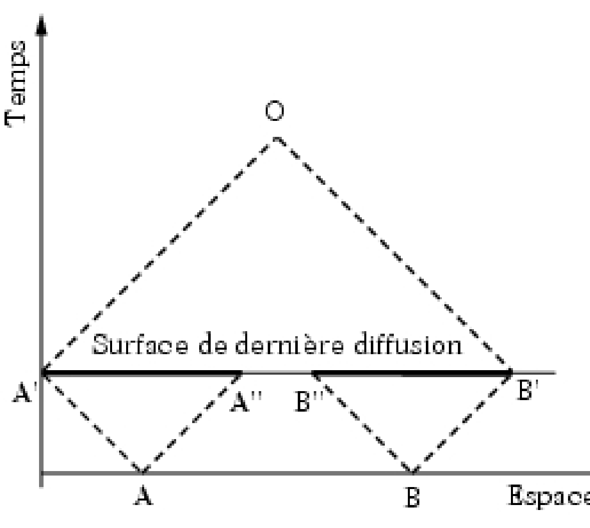
-expansion observed (Hubble law and redshift)
- light nuclei abundances (RG and weak interaction)
- CMB (RG and electromagnetism)

ROBLEMS AND QUESTIONS

Flatness

$$|\Omega_{K0}| < 0.1 \Rightarrow |\Omega_K(z_{pl})| < 10^{-60}$$

Horizon



CMB isotropic but corresponds to 10^{87} causal zones.
How do they reach thermal equilibrium?

Origin of structures

The universe is obviously not smooth. Where do the structure come from?

Dark sector

$$\Omega_r : \Omega_b : \Omega_m : \Omega_\Lambda \sim 10^{-3} : 1 : 5 : 14$$

Good description up to approx. 10^{16} GeV

- Effect of the GUT unification scheme on the particle content
- Topological defects…
- Close to 10^{19} GeV, we expect to have effect of quantum gravity
- We have a window on energies not accessible in accelerator!

PART III: GRAVITATIONAL DYNAMICS AND LARGE SCALE STRUCTURES

Main topics

- *Newtonian regime*
- *General cosmological perturbation*

- *Description and parameters*

GRAVITATIONAL DYNAMICS IN A STATIC SPACETIME

In a static spacetime, hydrodynamics takes its standard form

$$\boxed{\begin{aligned}&\partial_t\rho + \nabla.(\rho\mathbf{v}) = 0\\ &\partial_t\mathbf{v} + (\mathbf{v}.\nabla)\mathbf{v} = -\tfrac{1}{\rho}\nabla P - \nabla\Phi\end{aligned}}$$

If we expand quantities as $P = \bar{P} + \delta P, \ldots$ we obtain the wave equation

$$\partial_t^2\delta\rho - \Delta\delta P = \bar{\rho}\Delta\Phi$$

Use the Poisson equation

$$\Delta\Phi = 4\pi G\delta\rho$$

and the definition of the sound speed

$$c_s^2 = (\delta P/\delta\rho)_s$$

You get the propagation equation

$$\partial_t^2\delta\rho - c_s^2\Delta\delta\rho = 4\pi G\bar{\rho}\delta\rho$$

EANS LENGTH

If we decompose in plane waves exp[i(ωt-k.x)], we get the dispersion relation

$$\omega^2 = \frac{4\pi c_s^2}{\lambda_J^2}\left(\frac{\lambda_J^2}{\lambda^2} - 1\right)$$

$$\lambda_J \equiv c_s\sqrt{\frac{\pi}{G\bar{\rho}}}$$ is the Jean length

$\lambda > \lambda_J$ Gravity dominates, density perturbations grow

$\lambda < \lambda_J$ Gravity negligible, sound waves

$$t_{\mathrm{sound}} \sim \lambda c_s, \quad t_{\mathrm{collapse}} \sim \sqrt{G\bar{\rho}}$$

$t_{\mathrm{sound}} \ll t_{\mathrm{collapse}}$ Pressure can compensate gravitational collapse

EWTONIAN DYNAMICS IN A EXPANDING UNIVERSE

In an expanding universe, one can follow the same route for wavelengths smaller than the Hubble radius, H^{-1},

Physical coordinates are time dependent $\mathbf{r} = a(t)\mathbf{x}$

It follows that $\begin{cases} \nabla_{\mathbf{r}} \rightarrow \frac{1}{a}\nabla_{\mathbf{x}} \\ \partial_t \rho(\mathbf{r}, t) \rightarrow \partial_t \rho(\mathbf{x}, t) - H\mathbf{x}.\nabla_{\mathbf{x}}\rho \end{cases}$

The wave equation becomes

$$\ddot{\delta} + \boxed{2H\dot{\delta}} = \frac{c_s^2}{\boxed{a^2}}\Delta\delta + 4\pi G\boxed{\bar{\rho}}\delta$$

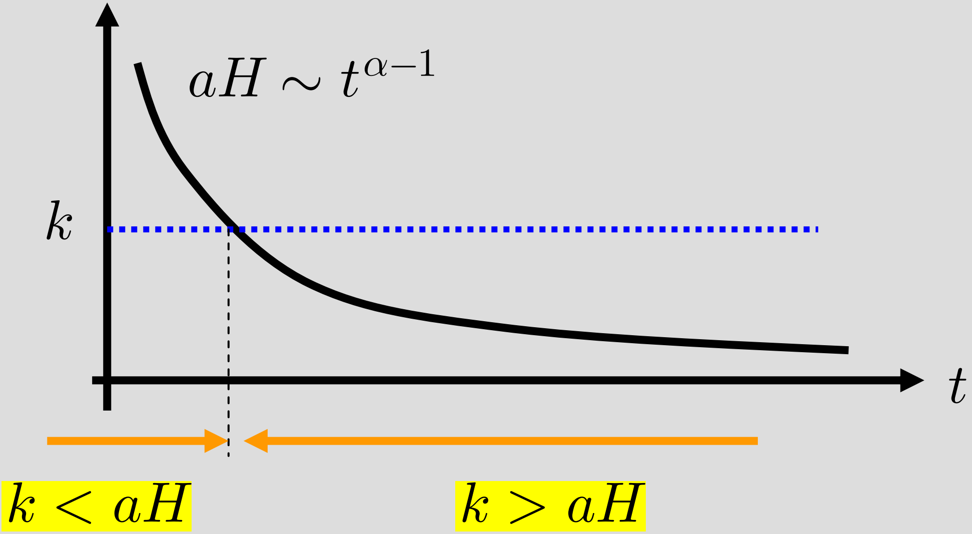
Friction due to expansion

Jeans length becomes time dependent

ONG WAVELENGTHS

All wavelength are redshifted $\lambda_{\text{phys}} \propto a$

Wavenumbers $k_{\text{phys}} = k/a$



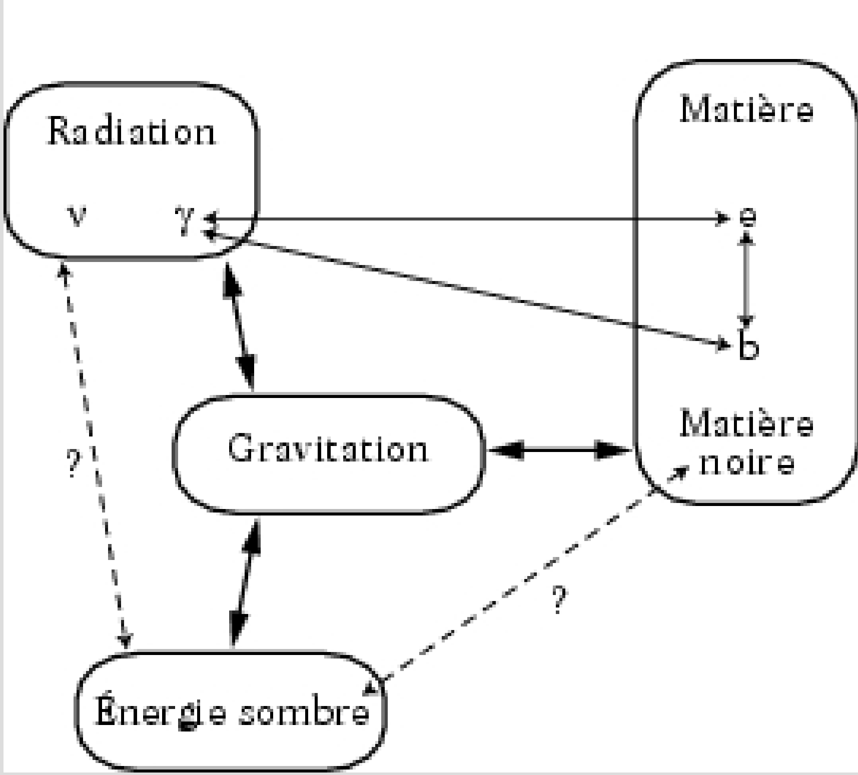
$k < aH$ Super-Hubble mode. Pressure negligible.

$k > aH$ Sub-Hubble mode. Expansion negligible.

All modes was SUPER-HUBBLE in the past

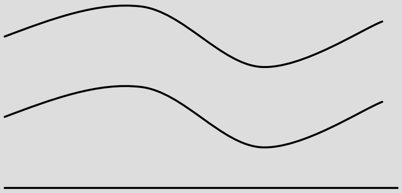
HE SYSTEM TO DESCRIBE

System of coupled differential equations



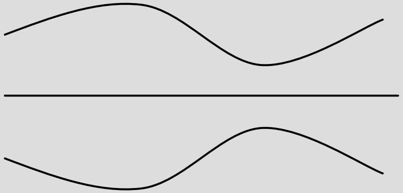
Nature of the initial conditions

adiabatic



$$\delta\rho_i \propto \delta\rho_j$$

isocurvature



$$\delta\rho_i + \delta\rho_j = 0$$

NITIAL SPECTRUM AND OBSERVED SPECTRUM

Harrison-Zel'dovich initial power spectrum:

Initial power spectrum of fluctuations on super-Hubble scales

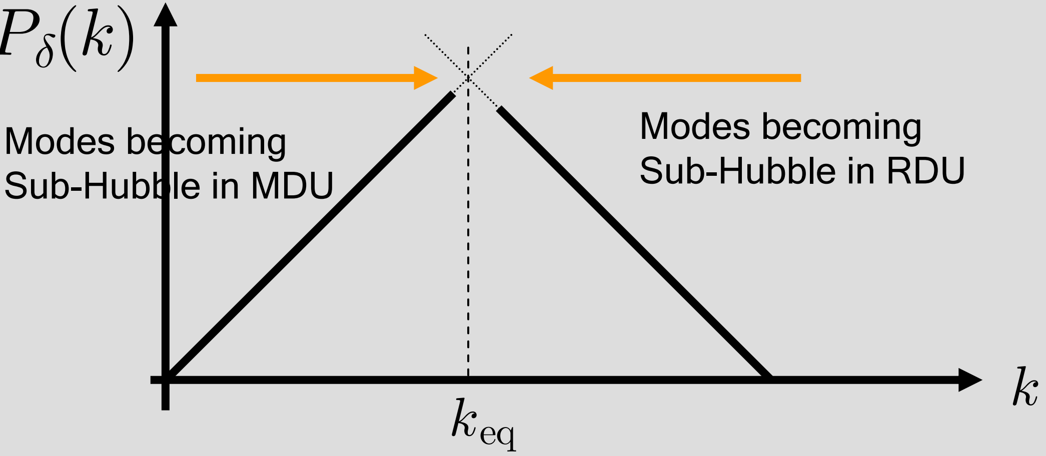
$$P_\Phi(k) \propto k^{-3} \Longleftrightarrow P_\delta(k) \propto k$$

No justification at this level. Origin of the spectrum?

Observed spectrum

Modes that became sub-Hubble during radiation era remained constant until equalit
There amplitude is thus suppressed by a factor $(k_{eq}/k)^2$.

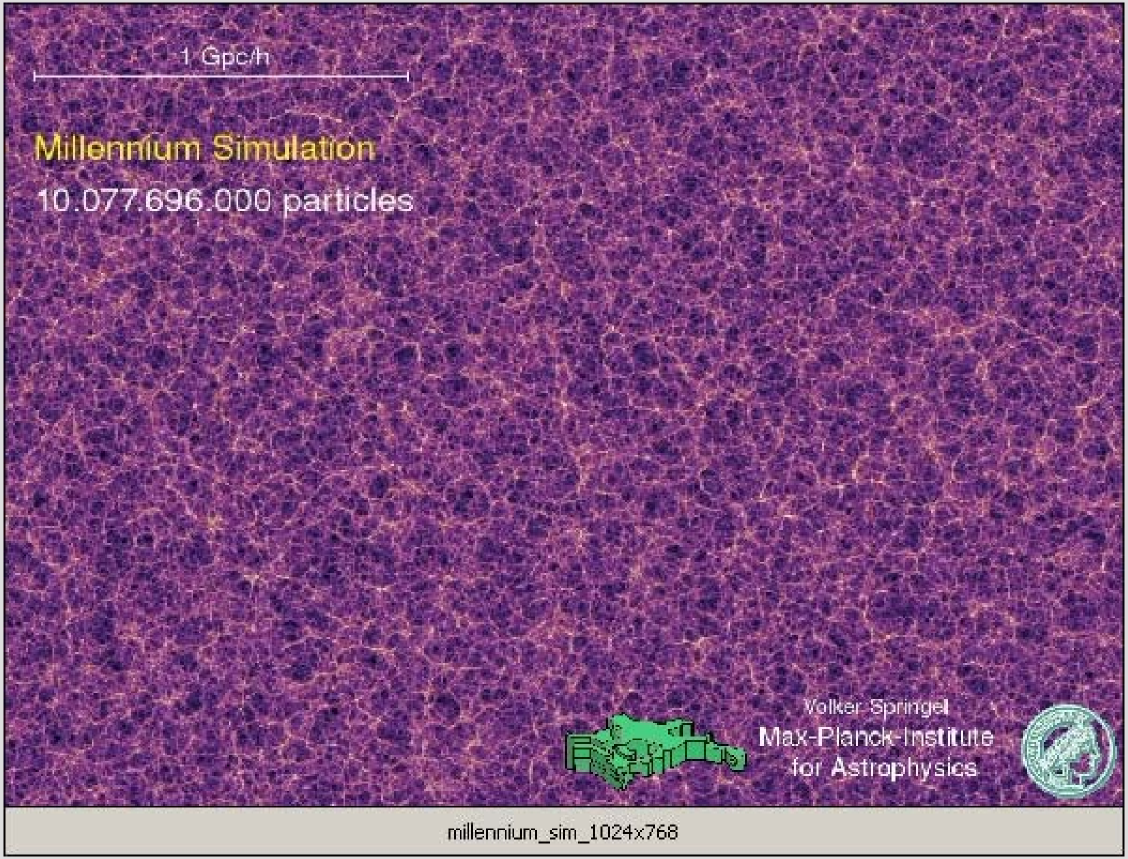
$$P_\delta(k) \propto k^{-1} \qquad (k < k_{\mathrm{e}}$$



Modes becoming
Sub-Hubble in MDU

Modes becoming
Sub-Hubble in RDU

UMERICAL SIMULATION



millennium_sim_1024x768

ARAMETERS OF THE MODEL

5 numbers to describe the dynamics of the background universe

$$\Omega_m, \quad \Omega_b, \quad \Omega_r, \quad \Omega_K, \quad \Omega_\Lambda, \quad H_0$$

1 function to describe the scalar perturbation

$$P(k): \quad A, \;\; n_S, \;\; \alpha_S \;\; (?)$$

1 function to describe the gravity waves

$$P_T(k): T/S, \;\; n_T, \;\; \alpha_T \;\; (?)$$

Reionisation parameter (needed for CMB)

$$\tau$$

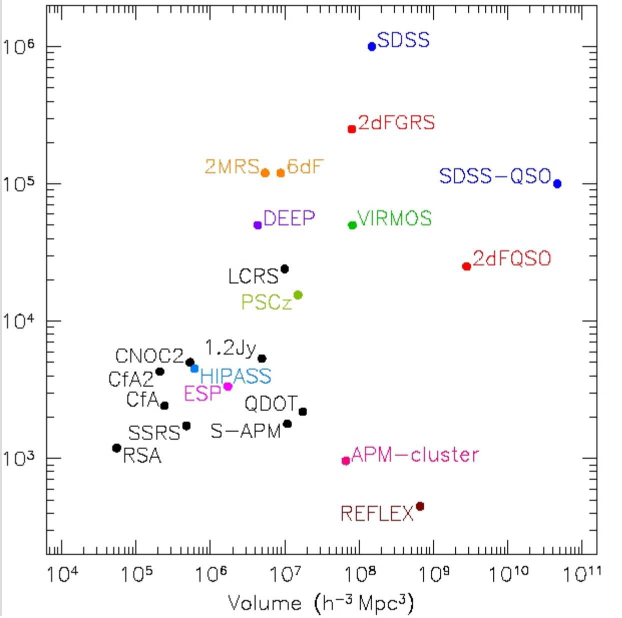
PART IV: OBSERVING THE LARGE SCALE STRUCTURE

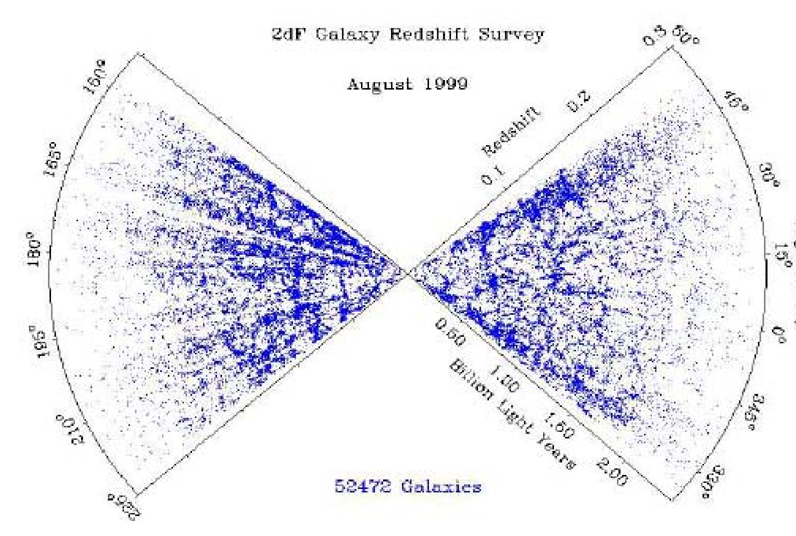
Main topics

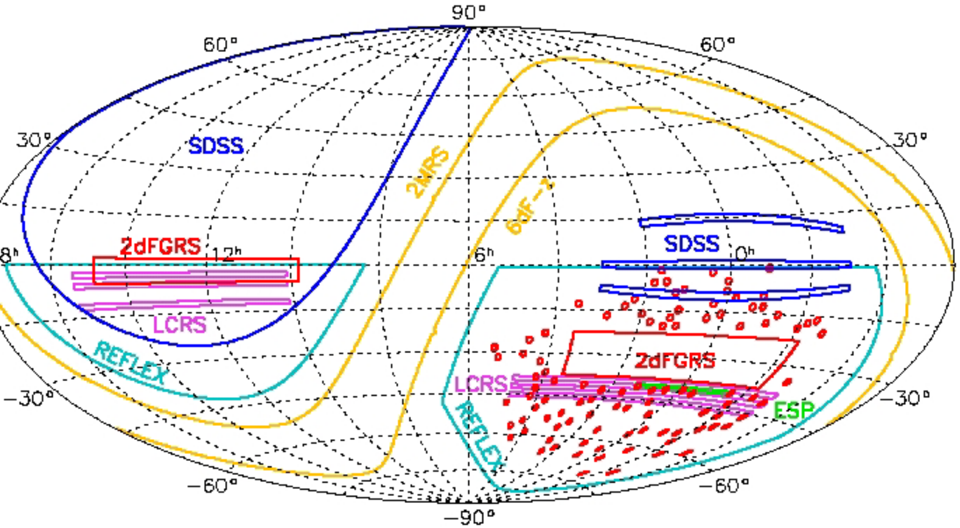
- *Galaxy catalogs*
- *Cosmic microwave background*
- *Weak lensing*

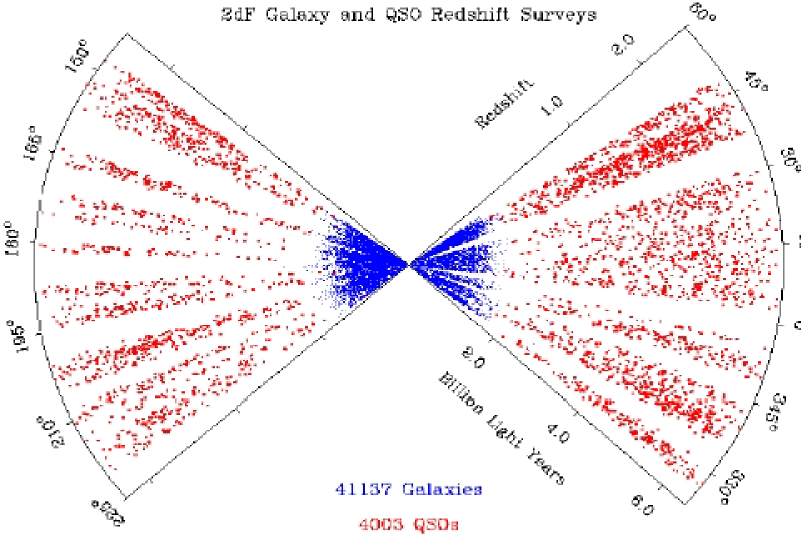
- *Observational status*

ALAXY CATALOGS







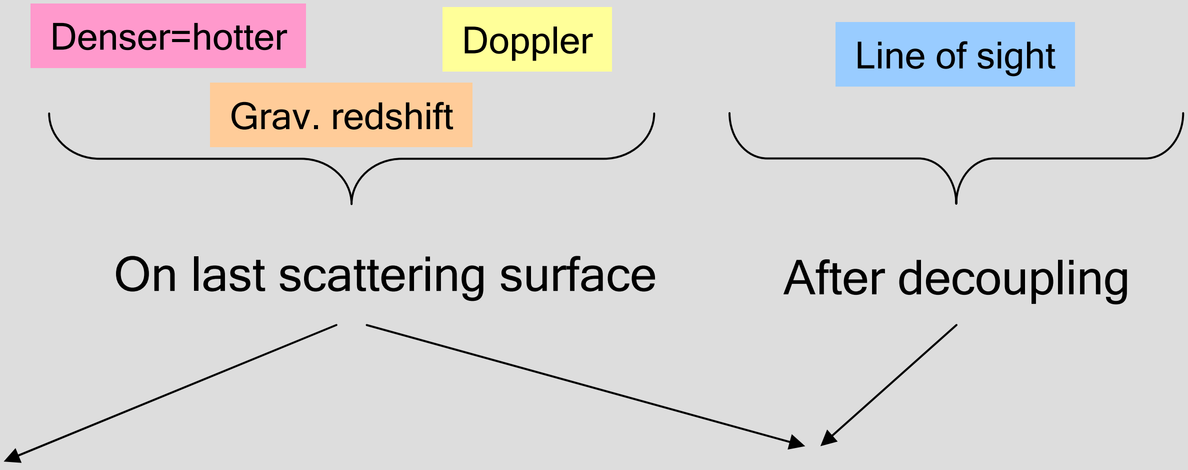


MB ANISOTROPIES: ORIGIN

COBE has detected small temperature fluctuation in the CMB of order 10^{-5}.

Sachs-Wolfe relation

$$\frac{\delta T}{T}(\mathbf{e}) = \frac{1}{4}\delta_\gamma + \Phi + \mathbf{e}.\mathbf{v}_b + \int(\Phi' + \Psi')d\eta$$



Denser=hotter

Doppler

Line of sight

Grav. redshift

On last scattering surface

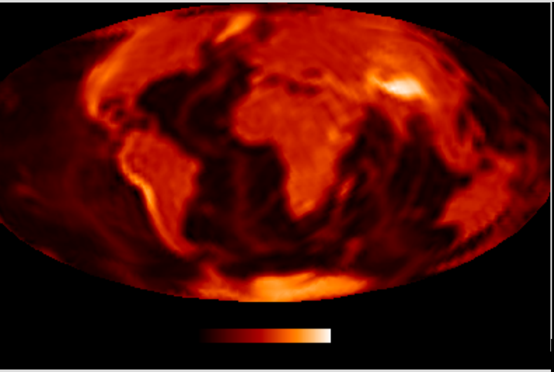
After decoupling

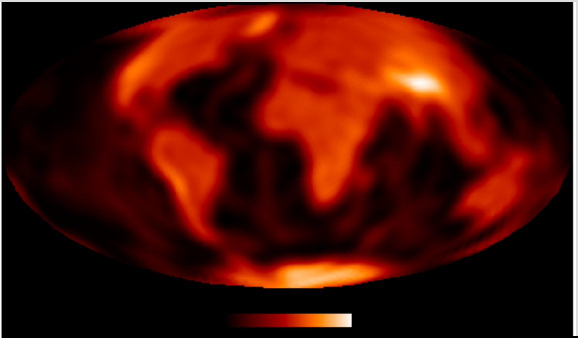
On large scales: observed modes were super-hubble at the time of decoupling. Initial conditions!

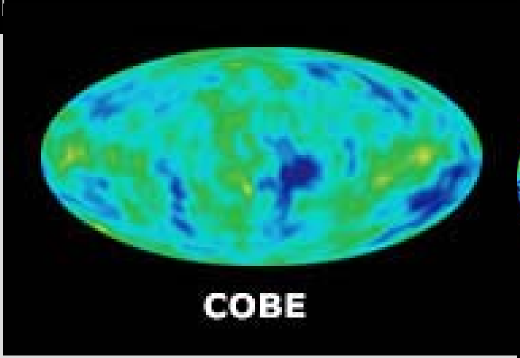
Evolution effects

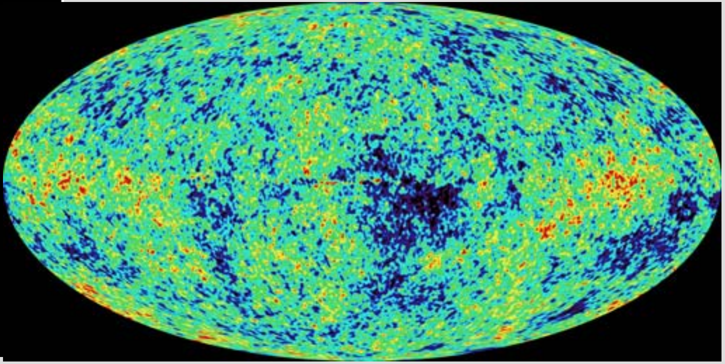
CMB: OBSERVATION

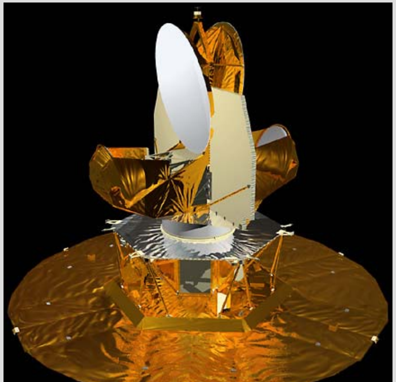
This temperature fluctuations have been observed with increasing resolution











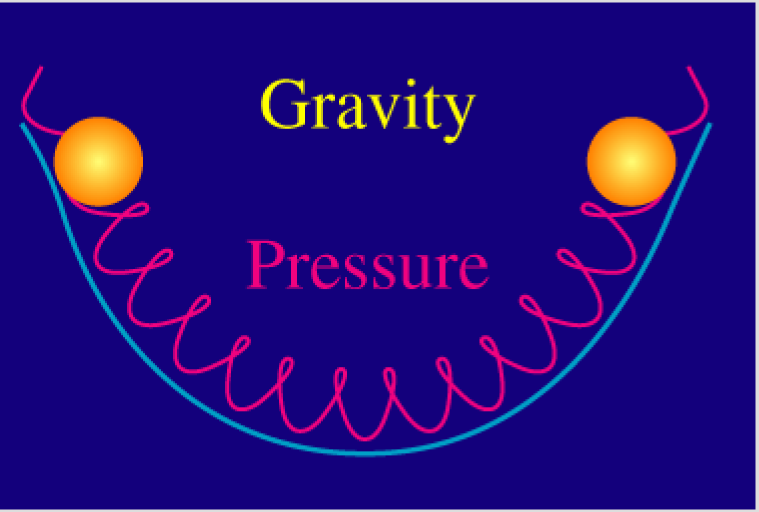
MB: ANGULAR POWER SPECTRUM

Angular power spectrum

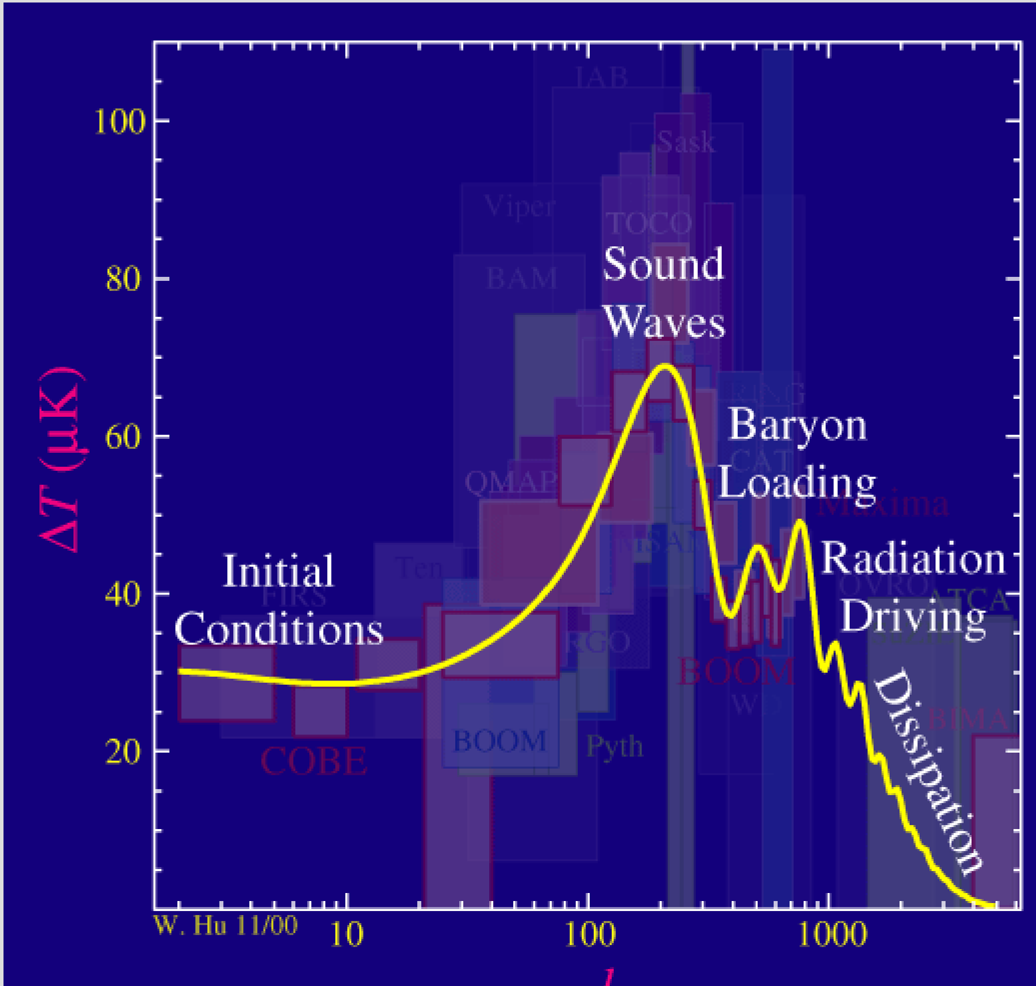
$$\langle \delta T(\mathbf{e}_1) \delta T(\mathbf{e}_2) \rangle = \sum \tfrac{2\ell+1}{4\pi} P_\ell(\cos\theta)$$

Before decoupling

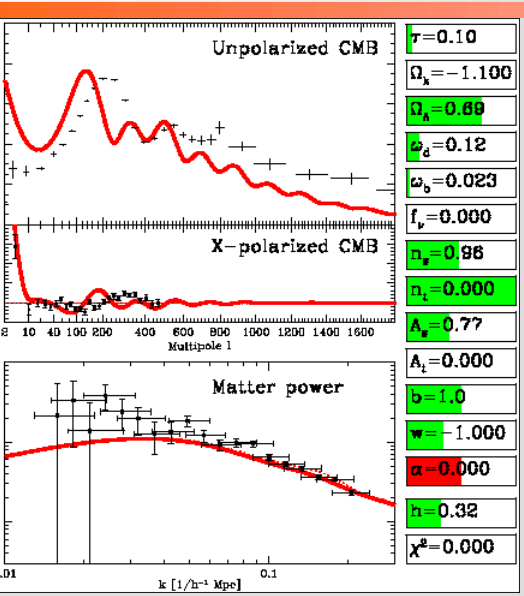
Oscillation in the photon-baryon plasma



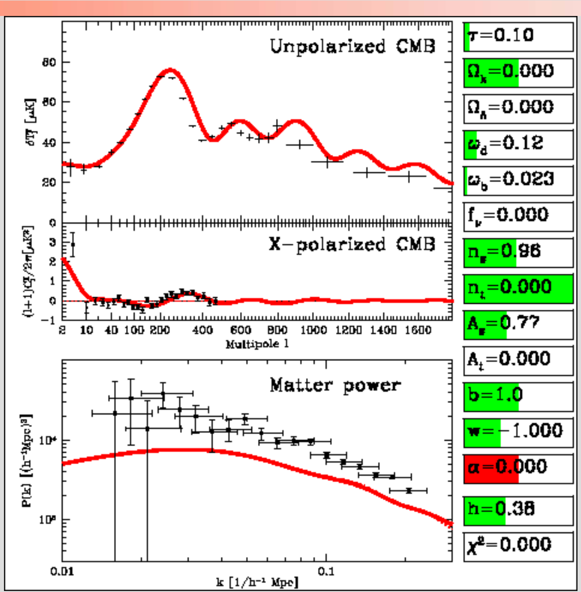
Characteristic scale



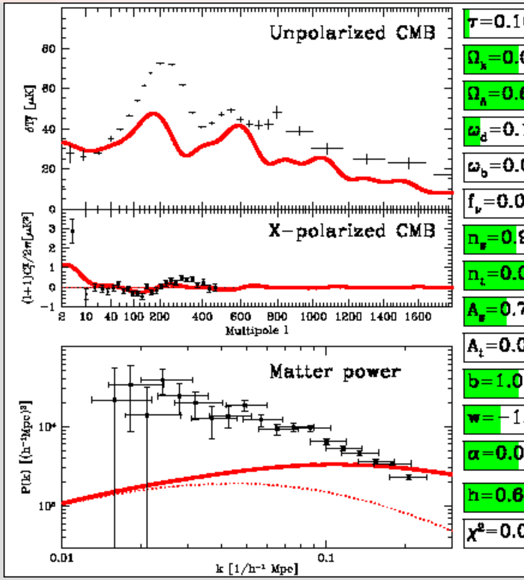
MB: EFFECT OF COSMOLOGICAL PARAMETERS



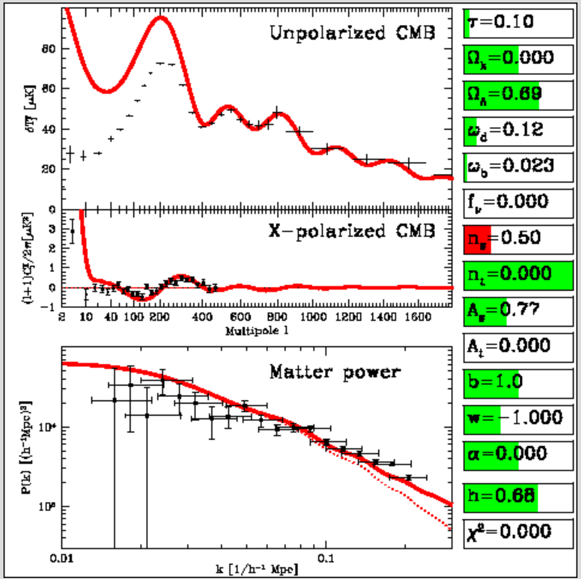
Cosmological constant



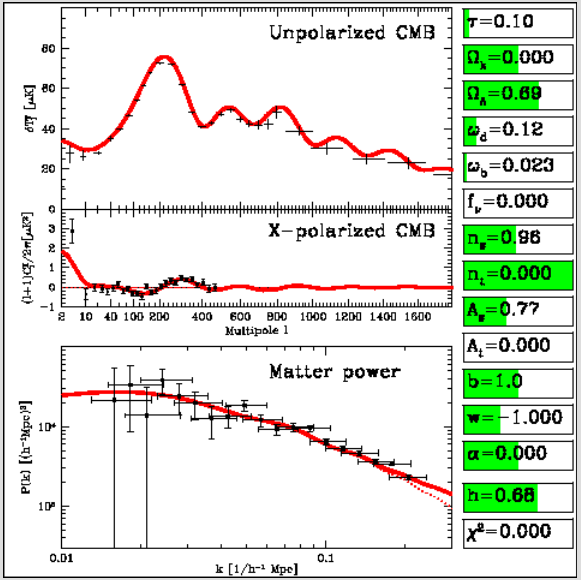
Curvature



baryons



Spectral index



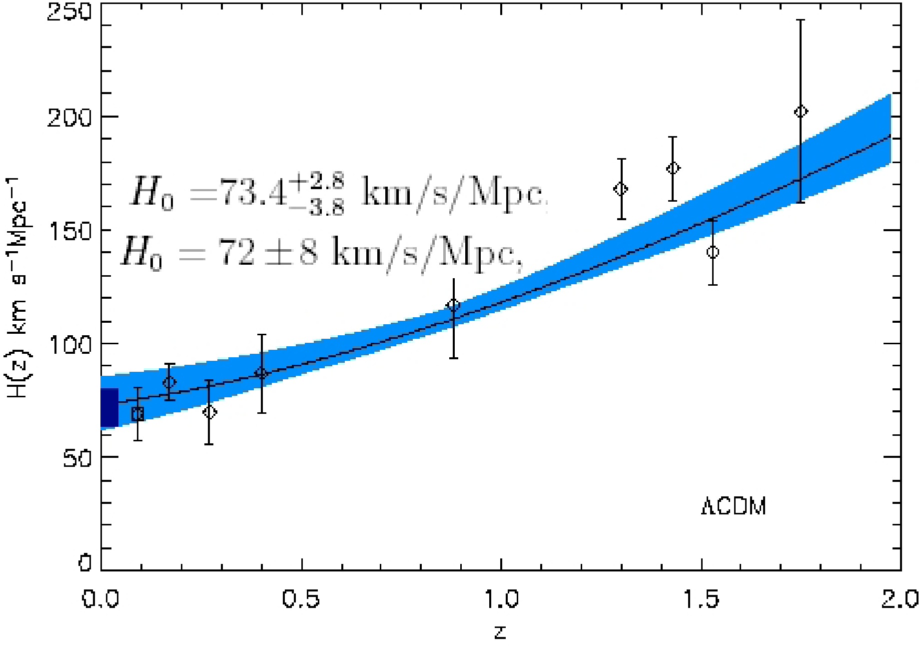
neutrinos

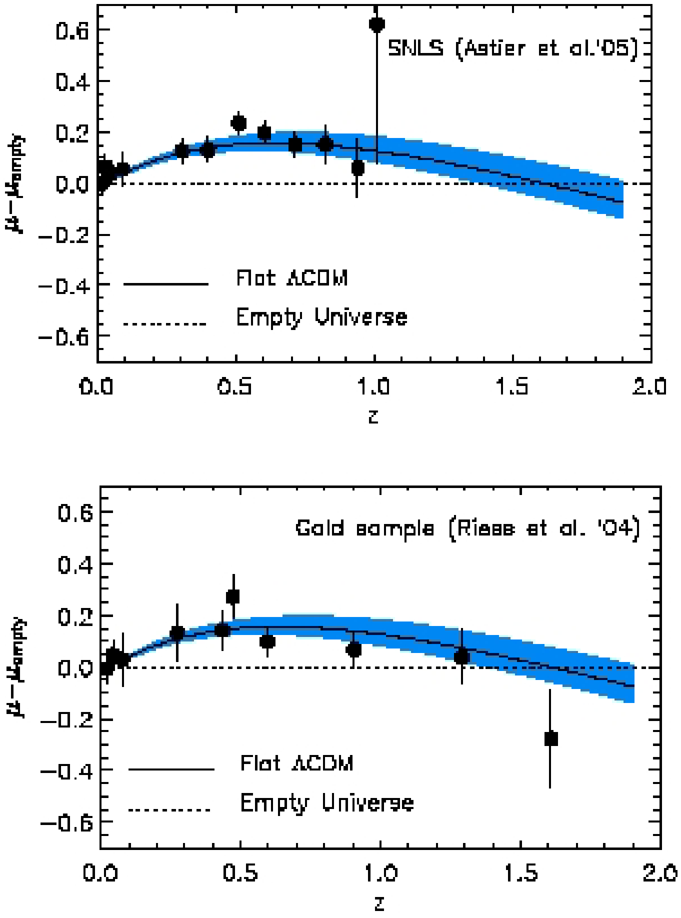
WMAP- DYNAMICS OF THE UNIVERSE

Analysis of a ΛCDM model with 6 parameters
(h, Ω_m, Ω_b, τ, n_s, σ_8)

Spergel et al. , astro-ph/0603449

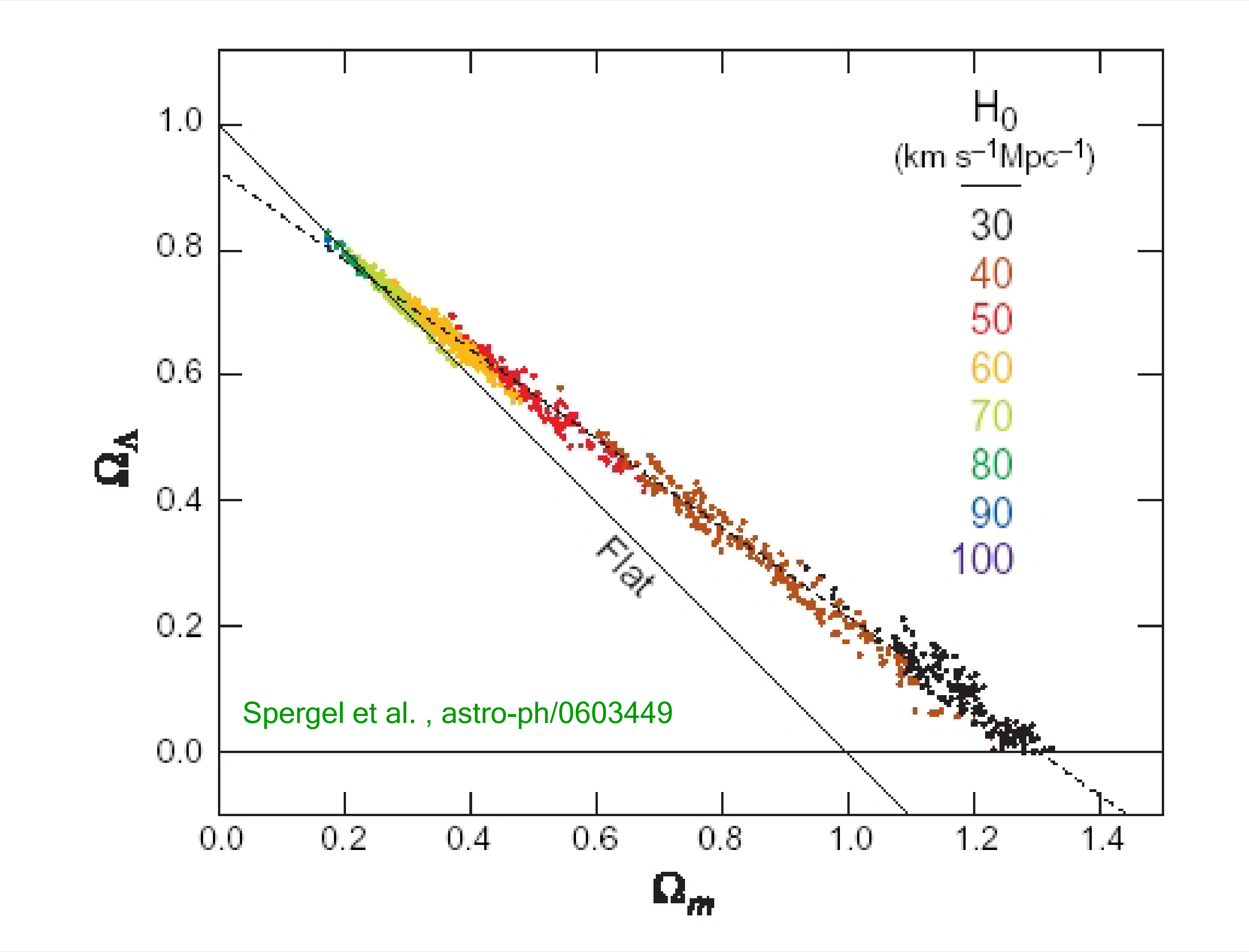
$$t_0 = 13.73^{+0.13}_{-0.17}\ \mathrm{Gyr}$$





WMAP- CURVATURE OF SPACE

Non-flat ΛCDM *models are* compatible with WMAP

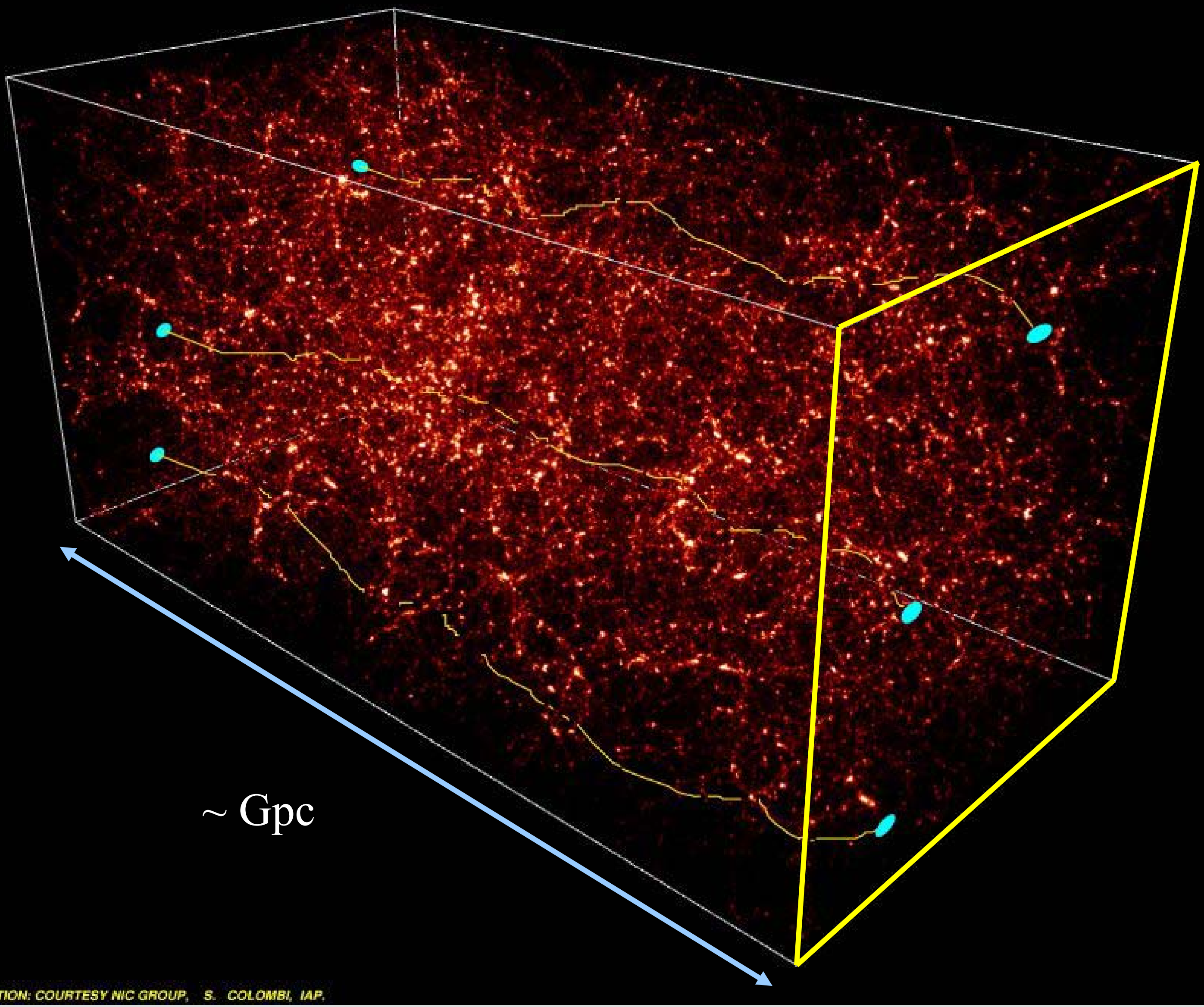


Spergel et al. , astro-ph/0603449

GRAVITATIONAL LENSING: PRINCIPLE

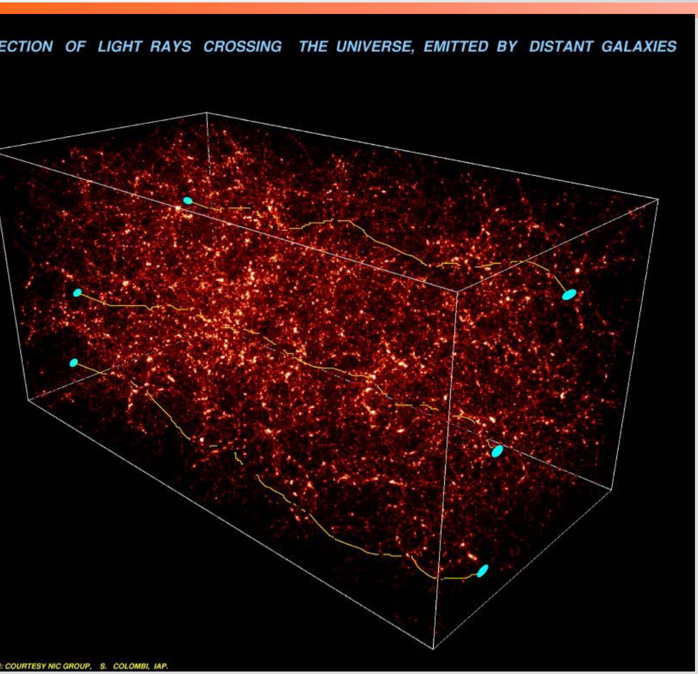
Cluster Abell 1698, HST/ACS

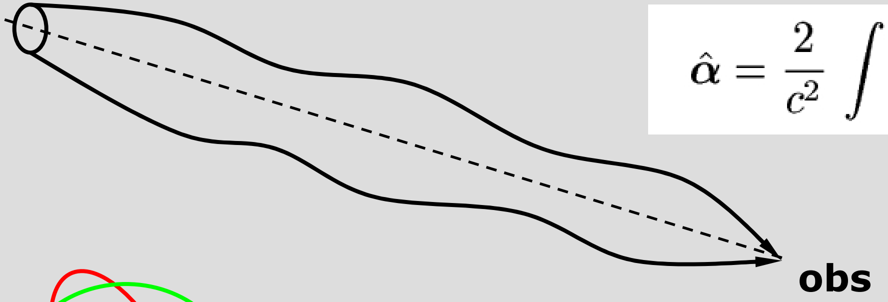
DEFLECTION OF LIGHT RAYS CROSSING THE UNIVERSE, EMITTED BY DISTANT GALAXIES



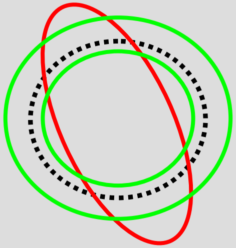
SIMULATION: COURTESY NIC GROUP, S. COLOMBI, IAP.

VEAK LENSING





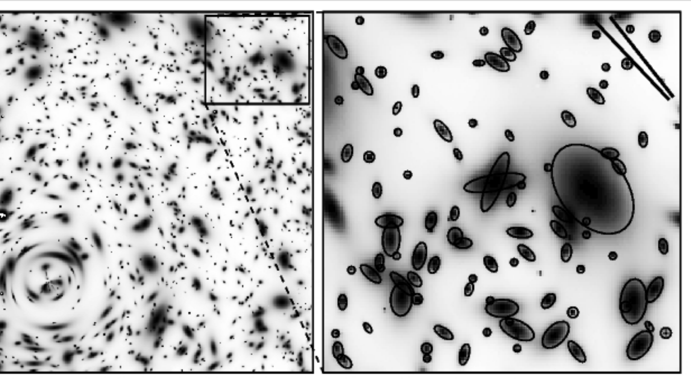
$$\hat{\boldsymbol{\alpha}} = \frac{2}{c^2} \int \nabla_{\perp} \Phi \ \mathrm{d}l$$



convergence κ

shear (γ_1, γ_2)

$$A = \begin{pmatrix} 1-\kappa-\gamma_1 & \gamma_2 \\ \gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix}$$



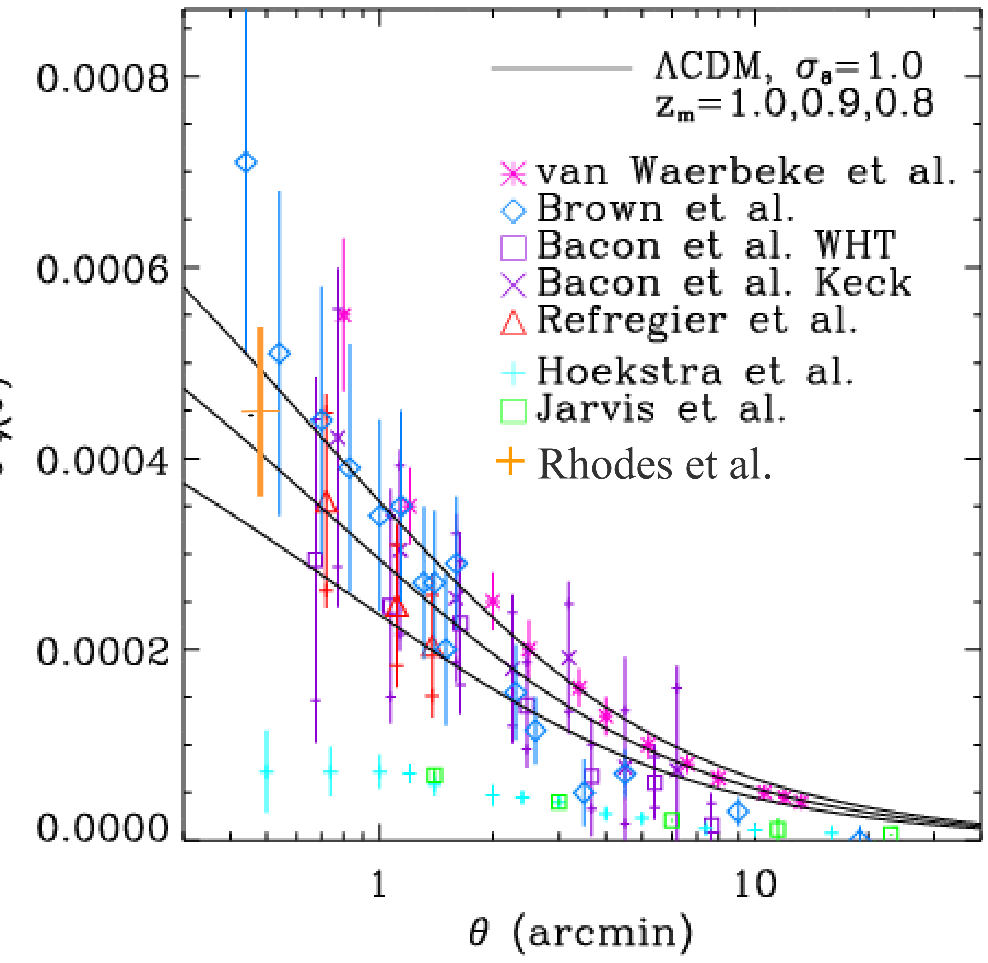
$$\delta = \frac{2\gamma\ (1-\kappa)}{(1-\kappa)^2 + |\gamma|^2} \quad = \frac{a^2-b^2}{a^2+b^2}$$

$$M_{ij} = \frac{\int I(\boldsymbol{\theta})\, \theta_i\ \theta_j\ \mathrm{d}^2\theta}{\int I(\boldsymbol{\theta})\ \mathrm{d}^2\theta}$$

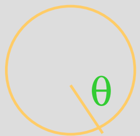
$$\frac{a^2-b^2}{a^2+b^2}$$

$$\boxed{\delta = 2\gamma}$$

EAK LENSING OBSERVATIONS



Shear variance

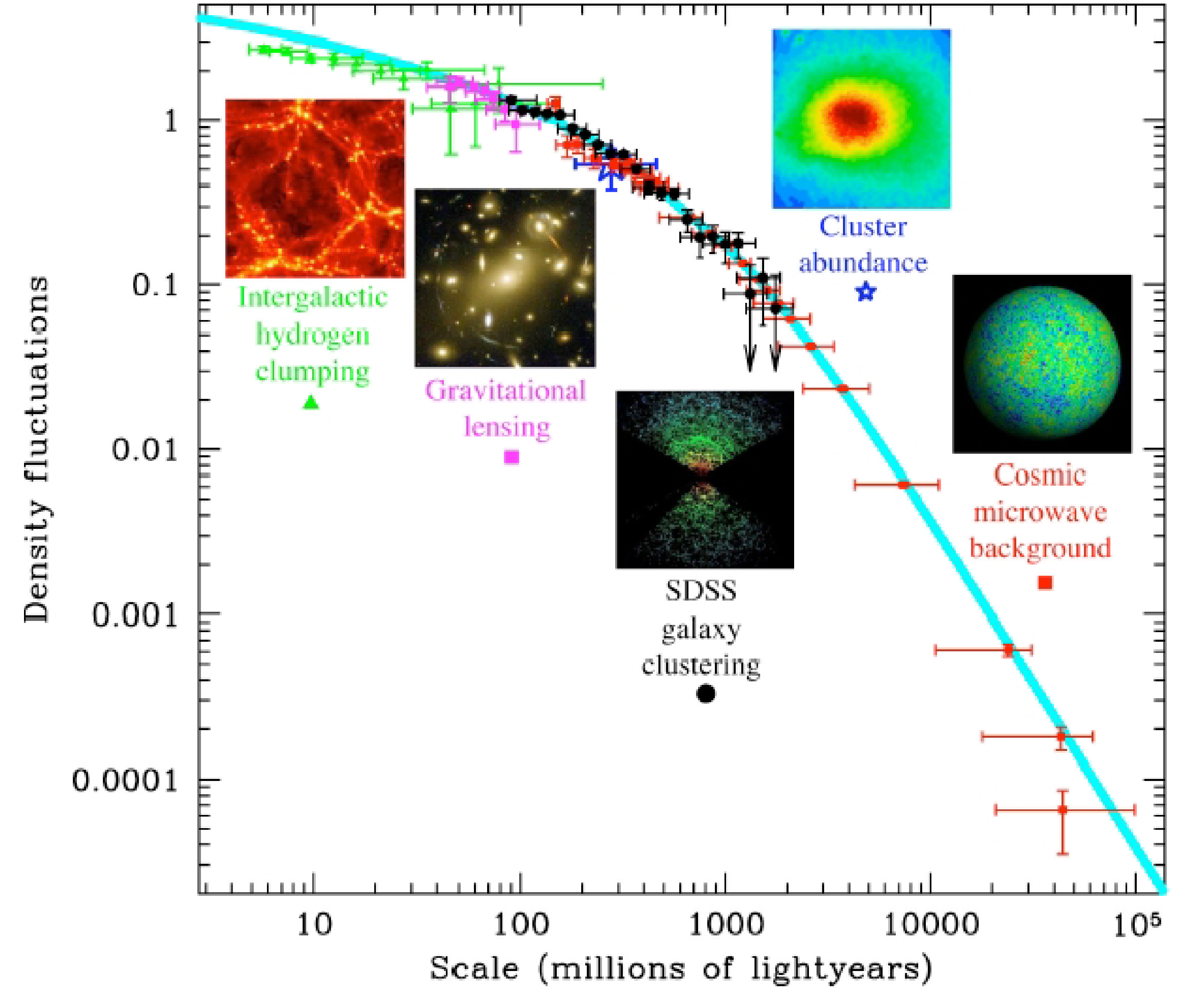


$\sigma^2_\gamma(\theta)=<\gamma^2>$

Bacon, Refregier & Ellis 2000*
Bacon, Massey, Refregier, Ellis 2001
Kaiser et al. 2000*
Maoli et al. 2000*
Rhodes, Refregier & Groth 2001*
Refregier, Rhodes & Groth 2002
van Waerbeke et al. 2000*
van Waerbeke et al. 2001, 2005
Wittman et al. 2000*
Hammerle et al. 2001*
Hoekstra et al. 2002 *
Brown et al. 2003
Hamana et al. 2003 * * not shown
Jarvis et al. 2003
Casertano et al 2003*
Rhodes et al 2004
Massey et al. 2004
Heymans et al 2004*

In agreement with a ΛCDM and the gravitational Instability paradigm

ONCLUSION



1
0.1
0.01
0.001
0.0001
Density fluctuations
Intergalactic hydrogen clumping
▲
Gravitational lensing
■
Cluster abundance
☆
SDSS galaxy clustering
●
Cosmic microwave background
■
10
100
1000
10000
10^5
Scale (millions of lightyears)

PART V: INFLATION

Main topics

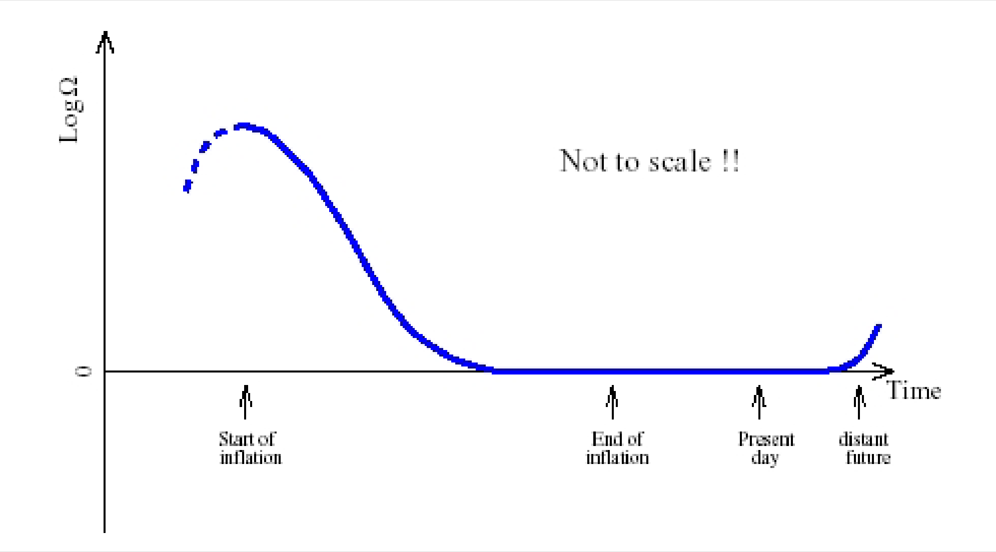
- *Principle*
- *Resolution of the BB problems*
- *Origin of perturbations*
- *Eternal inflation*
- *Observational constraints*

NFLATION: BASICS

The origin of the flatness problem is clear: $\Omega_K = -\frac{K}{a^2H^2}$

During the cosmological evolution *aH* decreases

Assume there is a primordial phase during which aH increases $\Omega_K \to 0$



Inflation = primordial phase of accelerated expansion

$$aH \nearrow \quad \Leftrightarrow \quad \ddot{a} > 0 \quad \Leftrightarrow \quad \rho + 3P < 0$$

LATNESS PROBLEM

If the inflation phase is long enough then WK is brought very close to 0, explaining the flatness of our universe.

E-fold

We define the number of e-fold of inflation by

$$N = \ln\left(\frac{a_{\rm final}}{a_{\rm init}}\right)$$

Solving the problem

If H is almost constant during this phase then

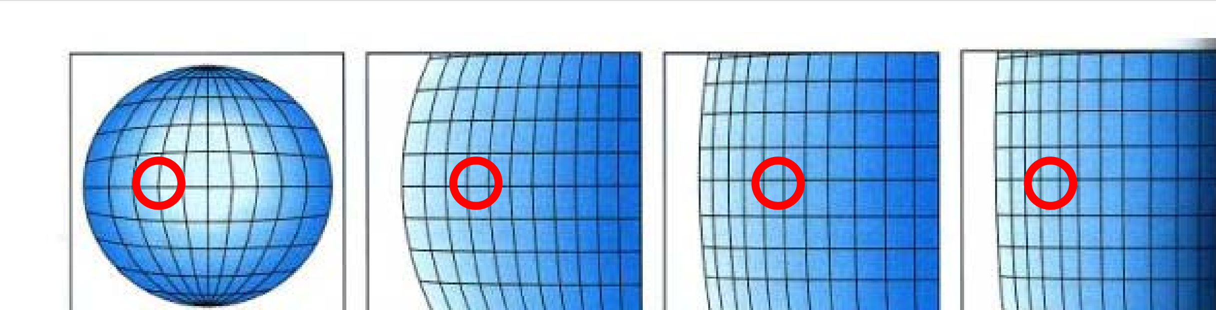
$< 10^{-60}$

Order 1

$$\frac{\Omega_K(t_f)}{\Omega_K(t_i)} = \left(\frac{a_{\rm final}}{a_{\rm init}}\right)^2 = \exp(-2N)$$

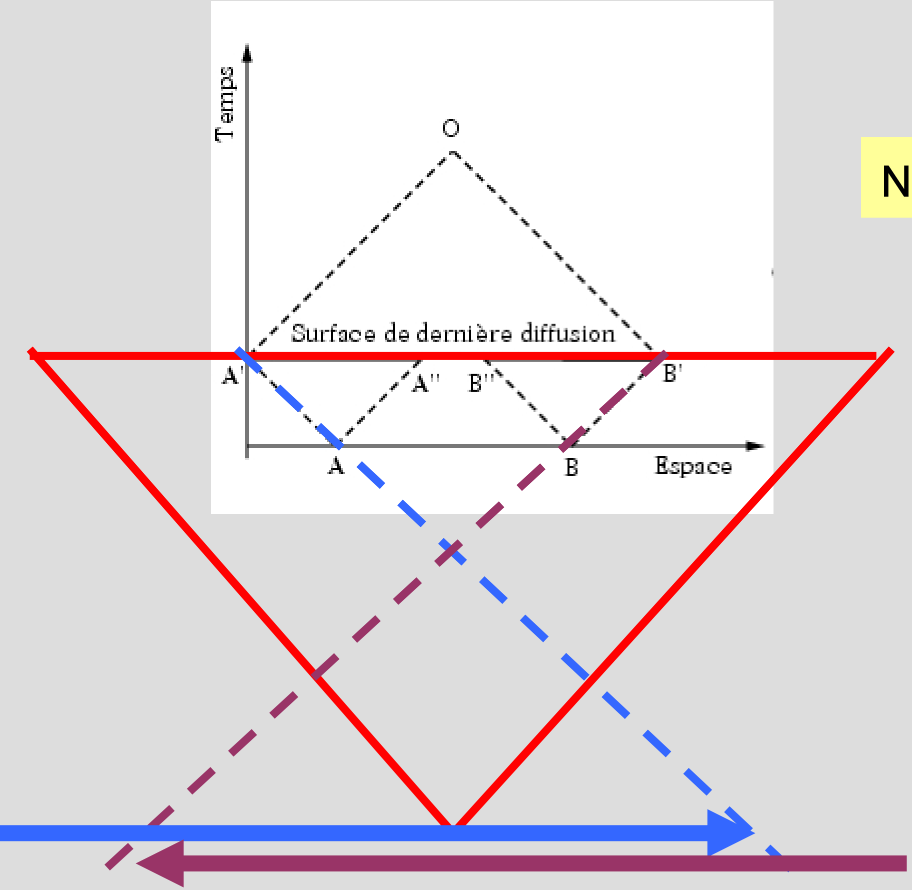
Thus, we need at least

$$N > 60 - 70$$

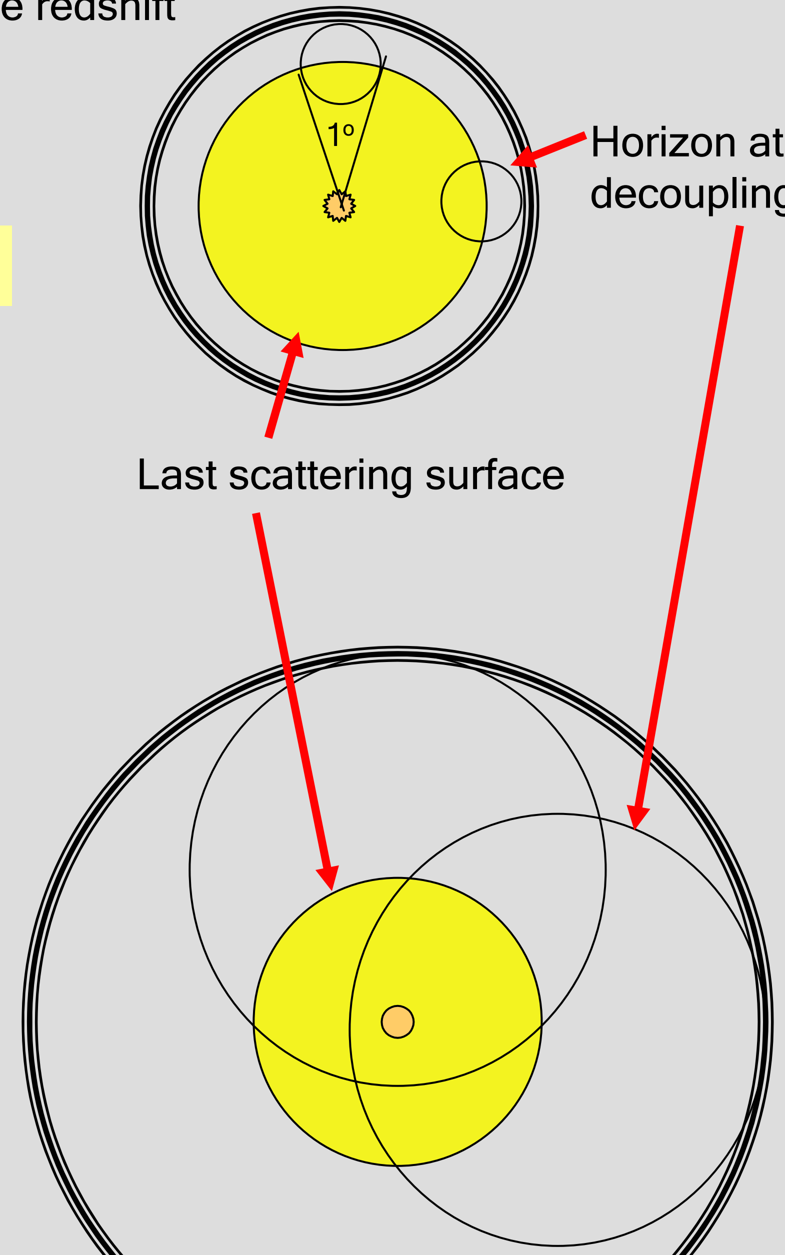


ORIZON PROBLEM

Vithout Inflation



N~10^{87}



Infinite redshift

Horizon at decoupling

Last scattering surface

Vith Inflation

MPLEMENTATION

We need matter satisfying ρ + 3P <0

Cosmological constant: de Sitter phase
exponential expansion

Scalar field:

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad P = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

so that

$$\rho + 3P = -2V(\phi) \times \left(1 - \frac{\dot{\phi}^2}{V}\right)$$

The field must be in slow-roll

SLOW-ROLL CONDITIONS

The 2 slow-roll conditions

$$\dot{\phi}^2 \ll V \qquad \ddot{\phi}^2 \ll 3H\dot{\phi}$$

Evolution equations obtained from the Friedmann and Klain-Gordon equations

$$H^2 \simeq \frac{8\pi G}{3}V \qquad 3H\dot{\phi} \simeq -V'$$

The expansion will be quasi-exponential

Validity conditions

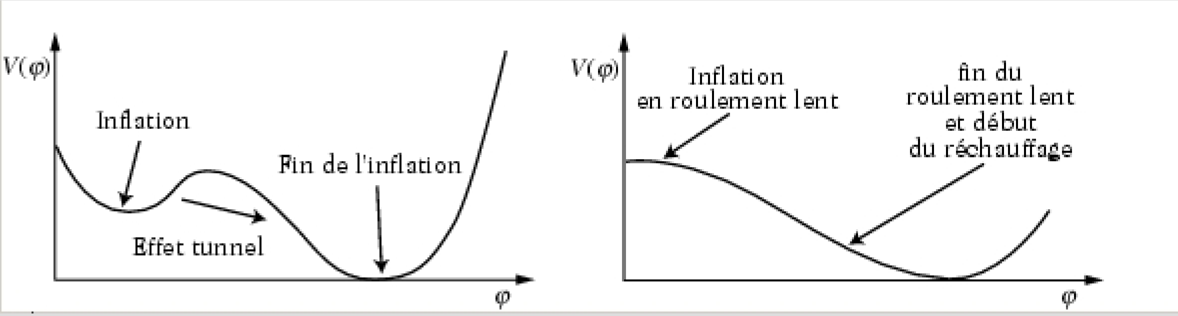
The slow-roll conditions are fulfilled if

$$(V'/V)^2 \ll 24\pi G, \qquad V''/V \ll 24\pi G$$

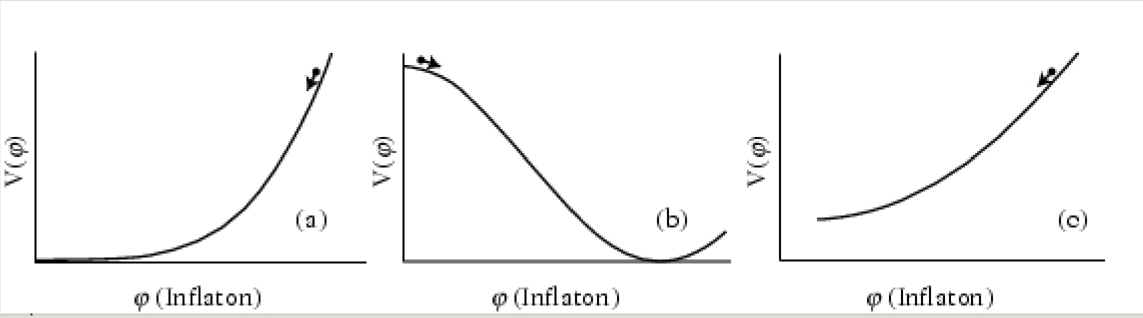
The potential must be flat

HE ZOO OF POTENTIALS

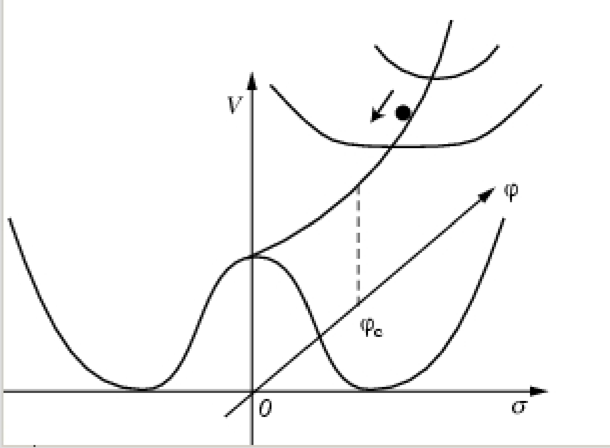
Old and new inflation



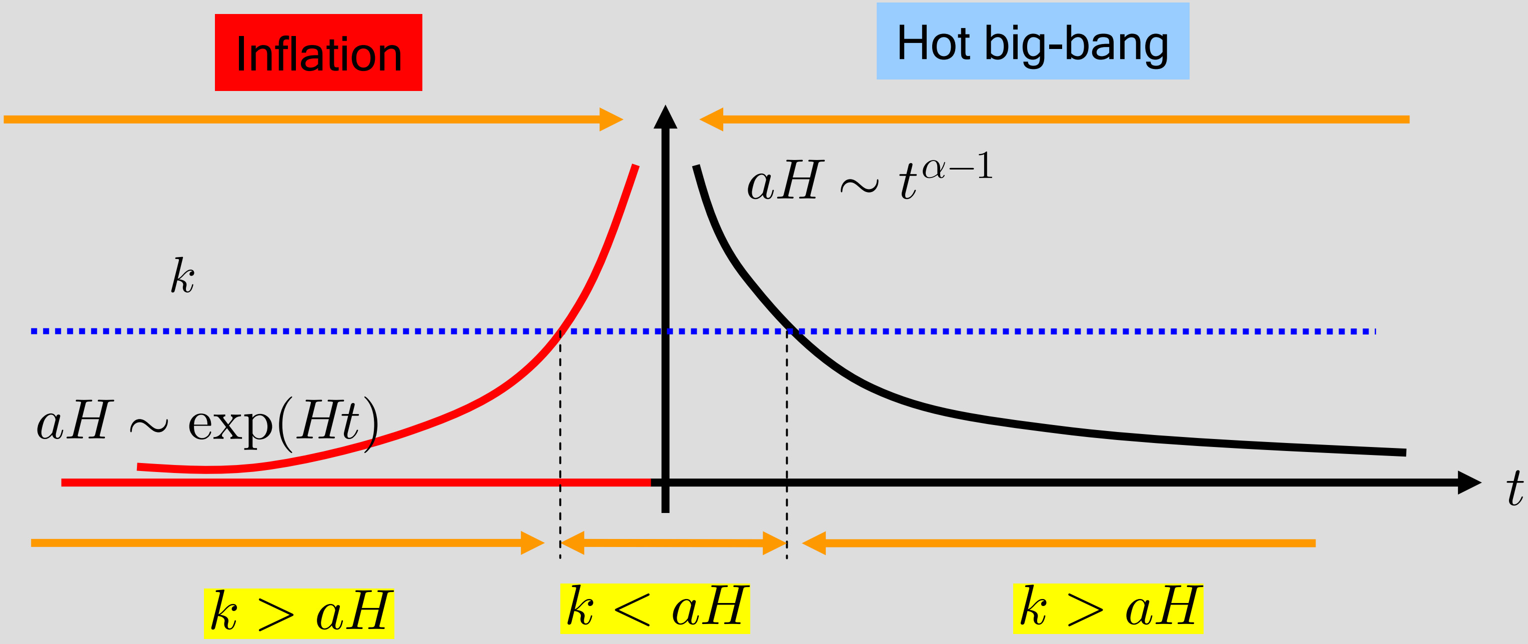
Large/small field



Hybrid inflation



ORIGIN OF FLUCTUATION: MODE EVOLUTION



The modes
- start sub-Hubble
- exit the Hubble radius during inflation
- enters the Hubble radius during the matter or radiation era

EST FIELD IN DE SITTER

To grasp the origin of perturbation, consider a test massless scalar field in de Sitter

Its equation of evolution is the Klein-Gordon equation

$$\ddot{\chi} + 3H\dot{\chi} + \frac{k^2}{a^2}\chi = 0$$

$k > aH$ Friction negligible - harmonic oscillator.

$k < aH$ Gradient negligible - constant mode

Setting v=aχ, and using η=–exp(-Ht)/H the Klein-Gordon equation becomes

$$v'' + \left(k^2 - \frac{2}{\eta^2}\right)\chi = 0$$

Its solution is

$$v = A(k)\left(1 + \frac{1}{ik\eta}\right)\exp(-\,ik\eta) + B(k)c.c.$$

NITIAL CONDITION

$k\eta \gg 1$ The curvature is negligible

We can quantify as in Minkowski space

Initial conditions

$$v \longrightarrow \frac{\exp(-ik\eta)}{\sqrt{2k}} \qquad k\eta \longrightarrow -\infty$$

Solution

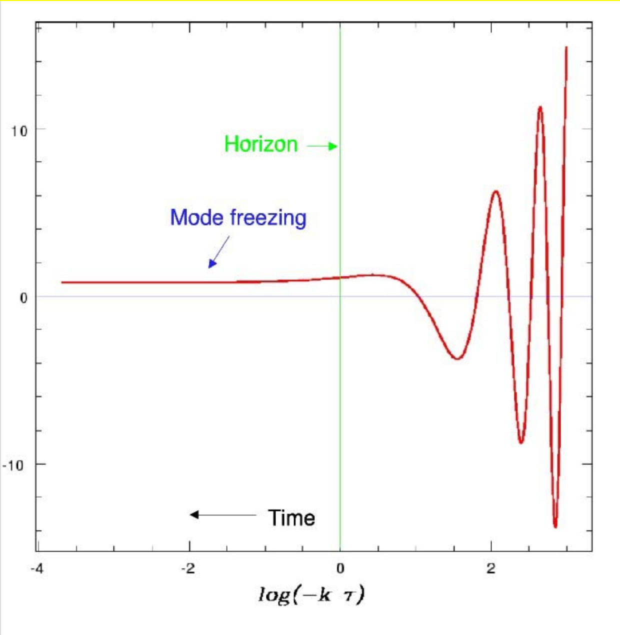
$$\chi = \frac{H\eta}{\sqrt{2k}}\left(1+\frac{1}{ik\eta}\right)\exp(-\,ik\eta)$$

Frozen super-Hubble fluctuations

$$\chi_k \sim \frac{H}{\sqrt{2k^3}}$$

10^{-28} cm

amplitude ~$10^{-5}M_4$



Plane wave on small scales

$$\chi_k \xrightarrow[k\eta \gg 1]{} \frac{e^{-ik\eta}}{\sqrt{2k}}$$

10^{-33} cm

OPHISTICATIONS

The inflaton also fluctuates

But its fluctuations are coupled to gravity

What is the variable to quantify?

Heuristically

Because of the initial fluctuations in the scalar field, inflation last more or less longer from one Hubble patch to another.

The curvature perturbation from one patch to the other is

$$\delta\mathcal{R} \sim H\delta t \sim H\delta\phi/\dot{\phi}$$

The fluctuation in the inflaton are of order

$$\delta\phi/\dot{\phi} \sim H/2\pi$$

$$\frac{\delta\rho}{\rho} \sim \delta\mathcal{R} \sim \frac{H^2}{2\pi\dot{\phi}}$$

Gravity waves

One produces GW with an amplitude of order

$$h \sim \frac{H}{2\pi M_p}$$

CALE OF INFLATION

On super-Hubble scales, the CMB anisotropy is of order

$$\frac{\delta T}{T} \sim \frac{1}{3}\Phi$$

The amplitude of the CMB temperature anisotropies allow to calibrate the spectrum

$$\frac{\delta T}{T} \sim 2 \times 10^{-6} \ \ K \longrightarrow \left(\frac{V}{\varepsilon}\right)^{1/4} \sim 5.7 \times 10^{16} \ \ \mathrm{GeV}$$

From WMAP data, we can infer the bound

$$\varepsilon < 0.032 \Leftrightarrow \frac{H_{inf}}{M_p} < 1.4 \times 10^{-5}$$

$\varepsilon \equiv (V'/V)^2/16\pi G$

ENERIC PREDICTIONS

1- universe is *flat*: $\Omega=1$
2- classical inhomogeneities are erased
Justification of the *cosmological principle*

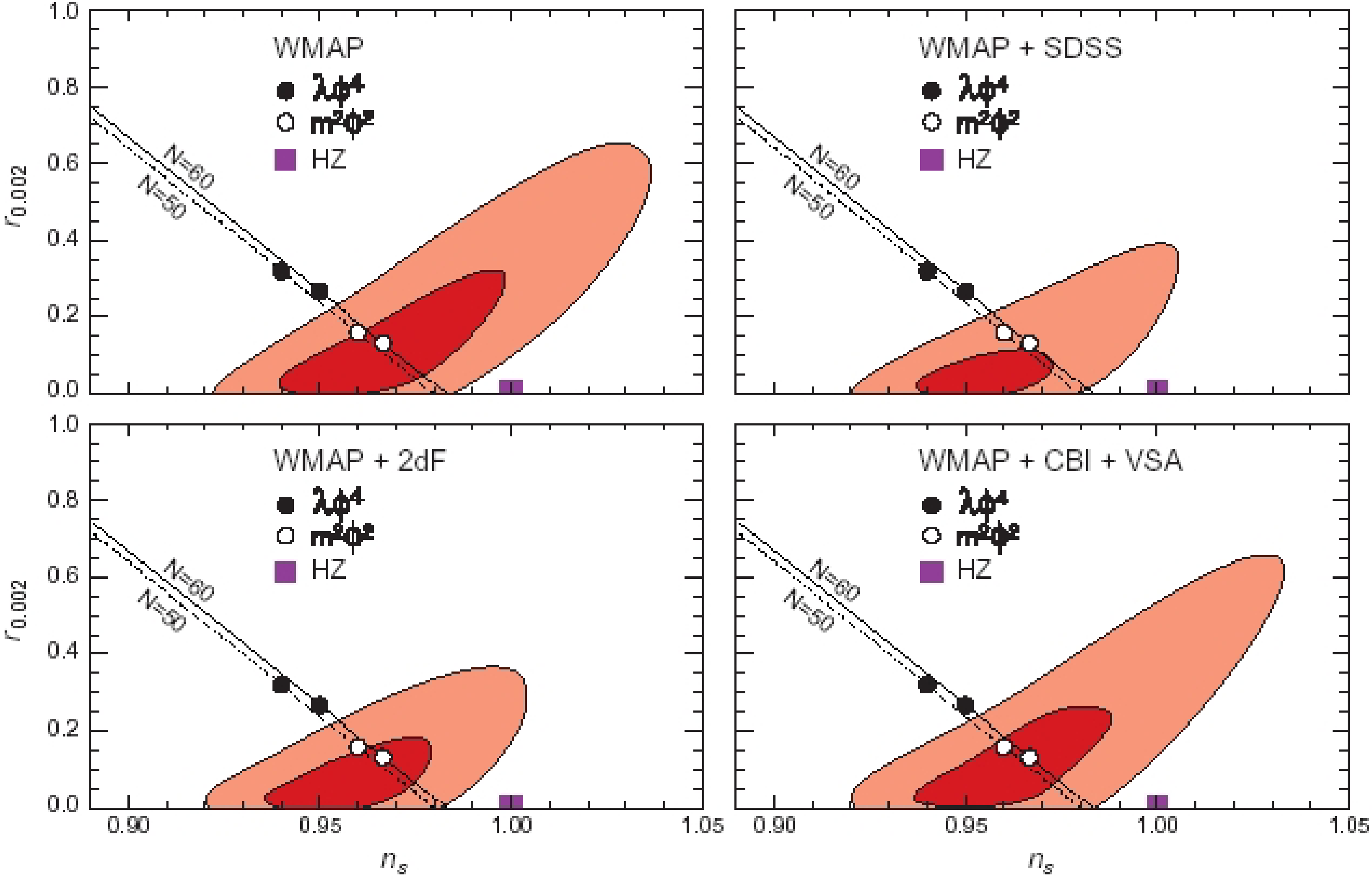
3- metric perturbations are Gaussian, almost scale invariant
origin of the Harrison-Zel'dovich spectrum
4- no vector perturbation
5- tensor perturbations are almost scale invariant

6- there exists a *consistency relation* between T/S, n_t et n_s

extensions

- multi-field
  non-gaussianity, isocurvature modes

WMAP CONSTRAINTS



TERNAL INFLATION

Consider a potential $V = \frac{1}{2}m^2\phi^2$

In slow-roll, during a time step $\delta t \sim H^{-1}$

Classical motion

$$\Delta\phi_{cl} \sim \dot{\phi}\delta t \sim \dot{\phi}/H \sim -M_p^2/4\pi\phi$$

Quantum fluctuation

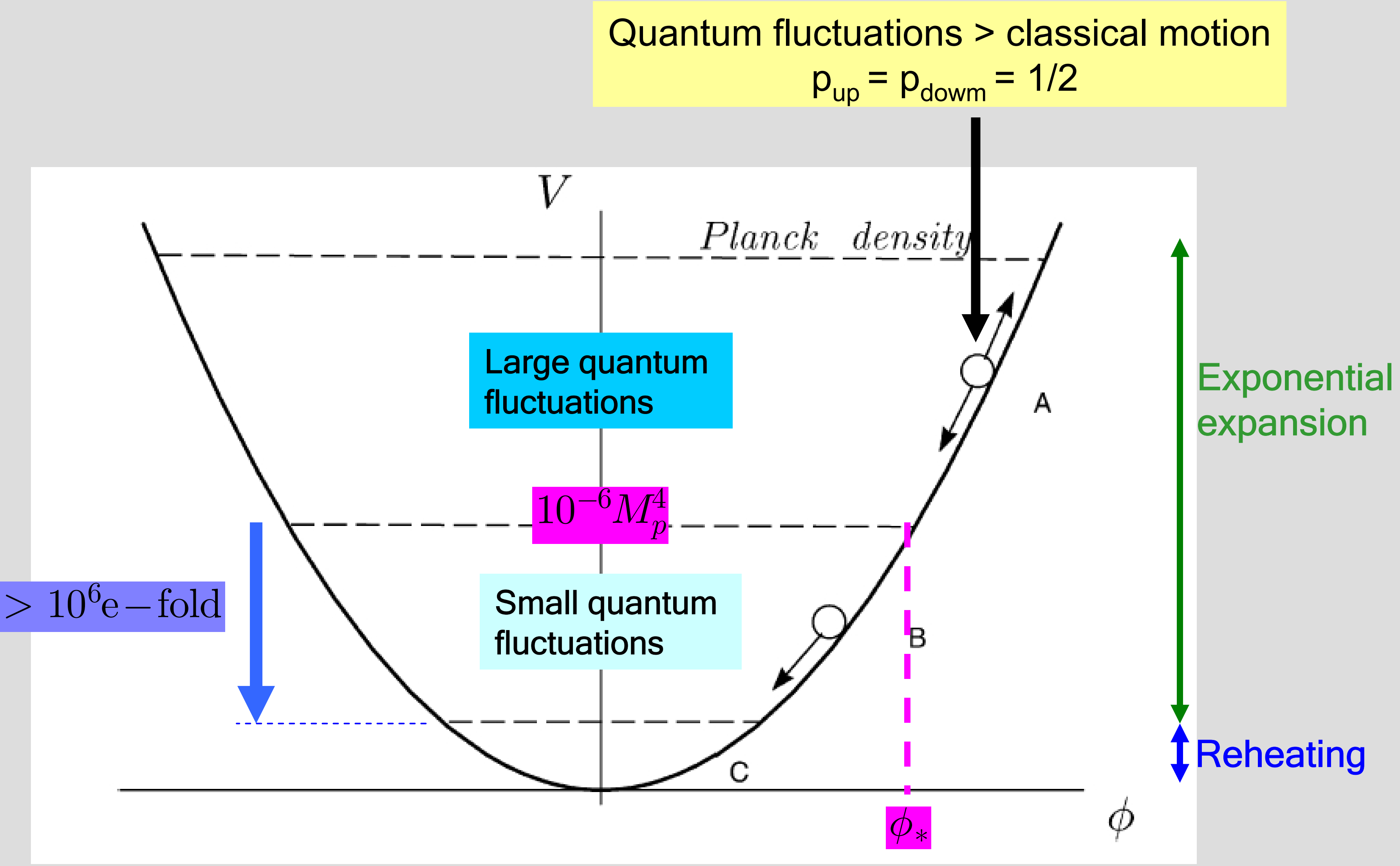
$$\delta\phi_q \sim H/2\pi$$

The fluctuations are of same amplitude when

$$\phi \sim \phi_* = \frac{M_p}{2}\sqrt{\frac{M_p}{m}}$$

Note that $V(\phi_*) \sim 10^{-6}M_p^4$

TERNAL INFLATION



Quantum fluctuations > classical motion

$p_{up} = p_{dowm} = 1/2$

Large quantum fluctuations

$10^{-6}M_p^4$

Small quantum fluctuations

$> 10^6 \mathrm{e-fold}$

ϕ_*

Exponential expansion

Reheating

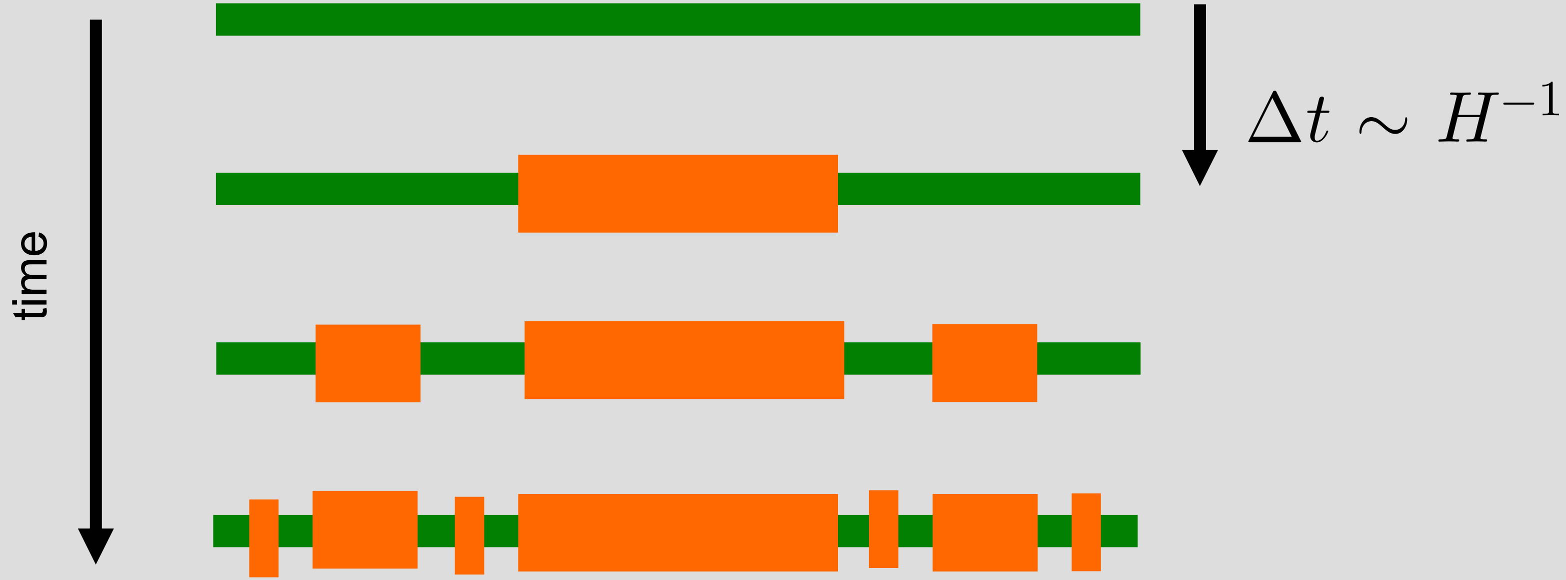
ELF REPRODUCING UNIVERSE

De Sitter inflation : $a(t) = \exp\left(Ht\right)$

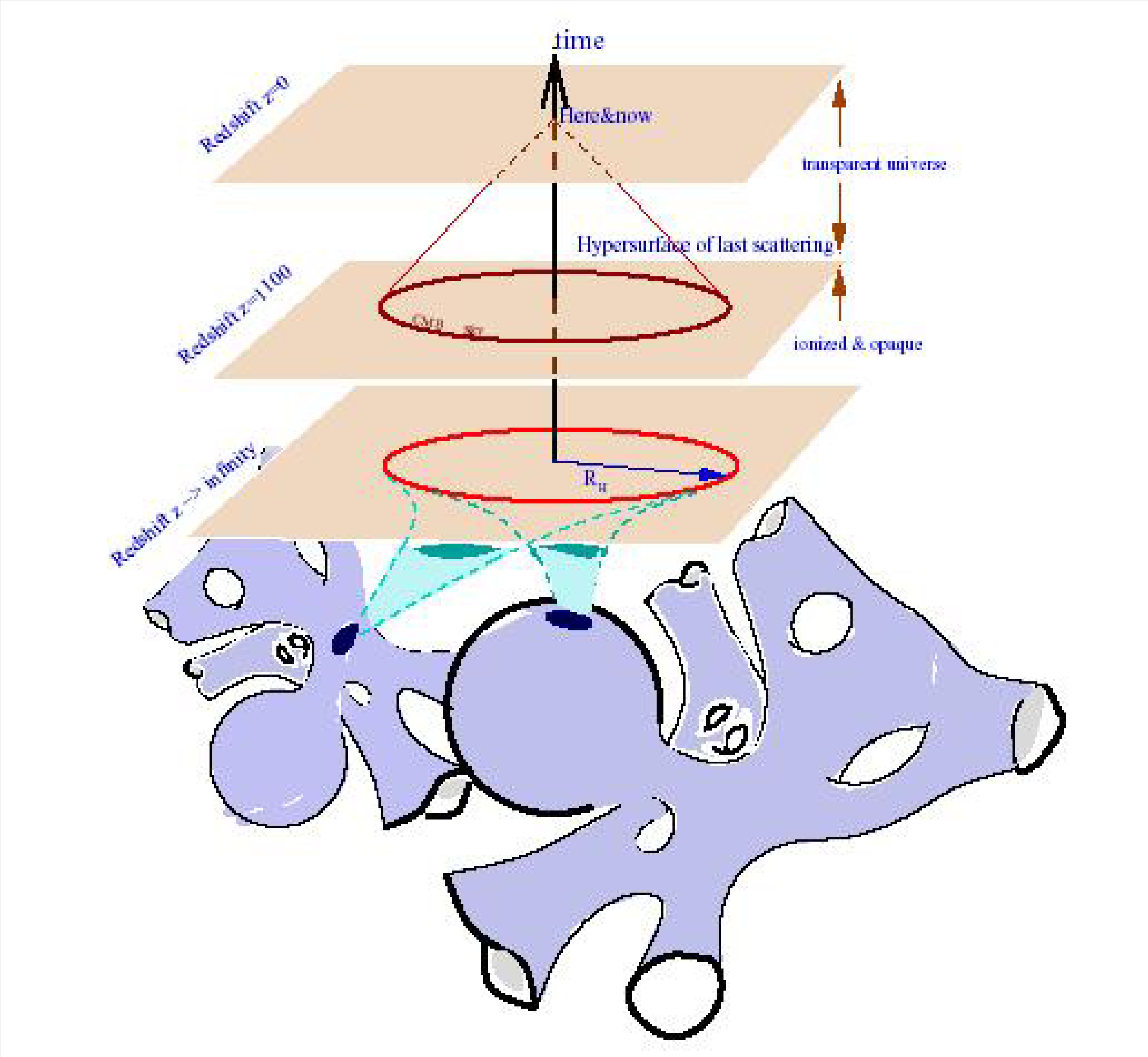
Consider a patch of size H^{-1}

During a time step of H^{-1}

$$V_{t+\Delta t}(\phi > \phi_*) \sim \frac{1}{2}\left(e^{H\Delta t}\right)^3 V_t(\phi > \phi_*) \sim \frac{1}{2} \times 20 \times V_t(\phi > \phi_*$$



A NEW PICTURE OF THE UNIVERSE



PART VI: THE INPUT FROM HIGH ENERGY PHYSICS

Main topics

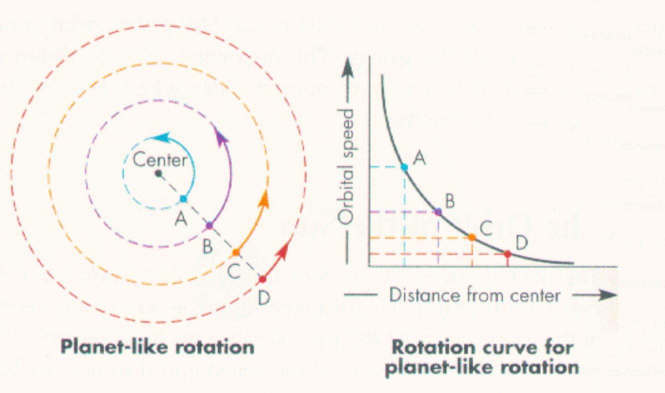
- *Dark matter*
- *Dark energy*

- *String inspired cosmology*
  - *Extra-dimensions*
  - *branes*

ARK MATTER: GALAXY ROTATION CURVE

Keplerian motion

$$\frac{v^2(r)}{r} = \frac{GM(<r)}{r^2}$$



Planet-like rotation

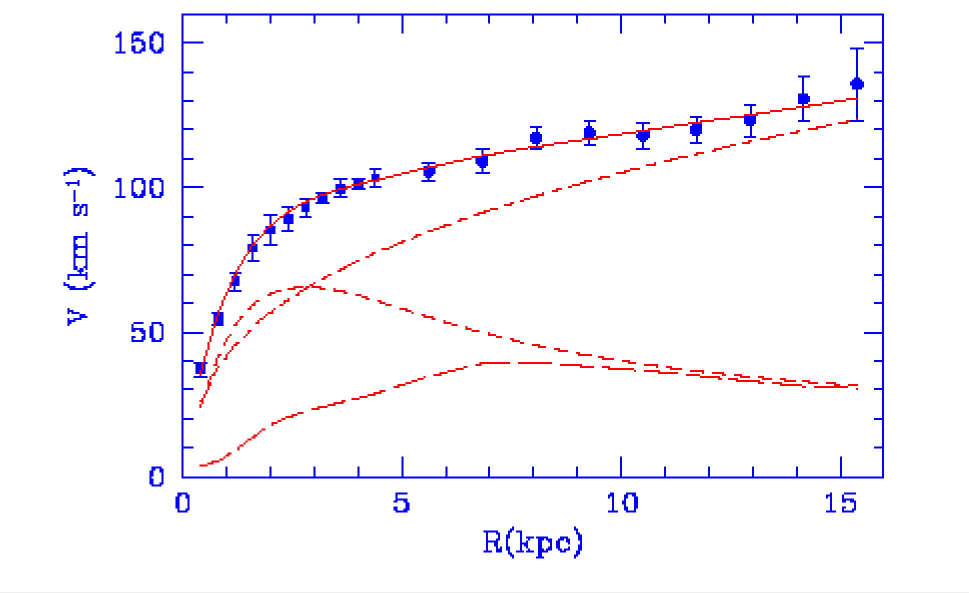
Rotation curve for planet-like rotation

Spiral galaxy

$$v \longrightarrow v_\infty$$

which implies

$$M(<r) \propto r, \qquad \rho \propto 1/r^2$$



Tully-Fisher

There is a scaling law

$$L \propto v_\infty^4$$

Most of the mass of the galaxy (halo) is dark

BARYON OSCILLATIONS

Radiation pressure

Oscillations in the photon fluid

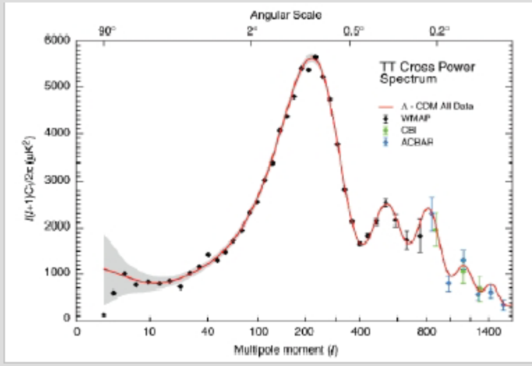
Thomson scattering

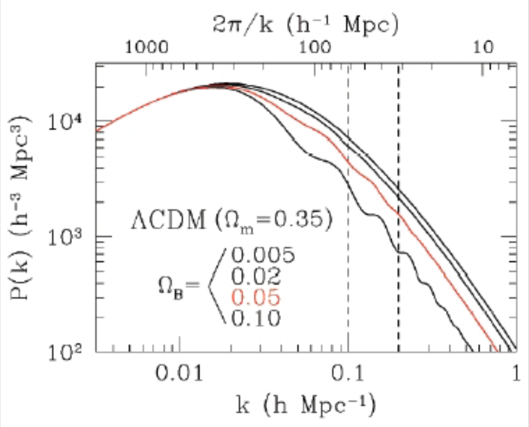
Baryons oscillate with photon
Not the dark matter

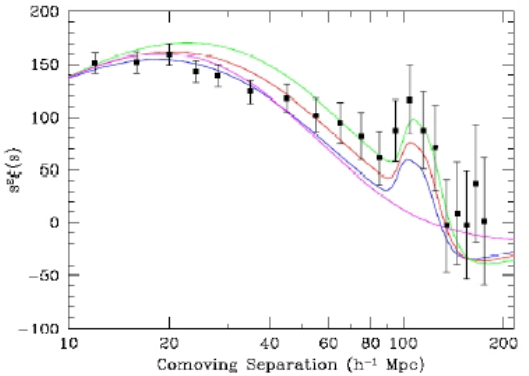
Characteristic scales of the oscillations
sonic horizon

Amplitude is attenuated by a factor Ω_b

Detected in jan. 2005 (SDSS+2dF)







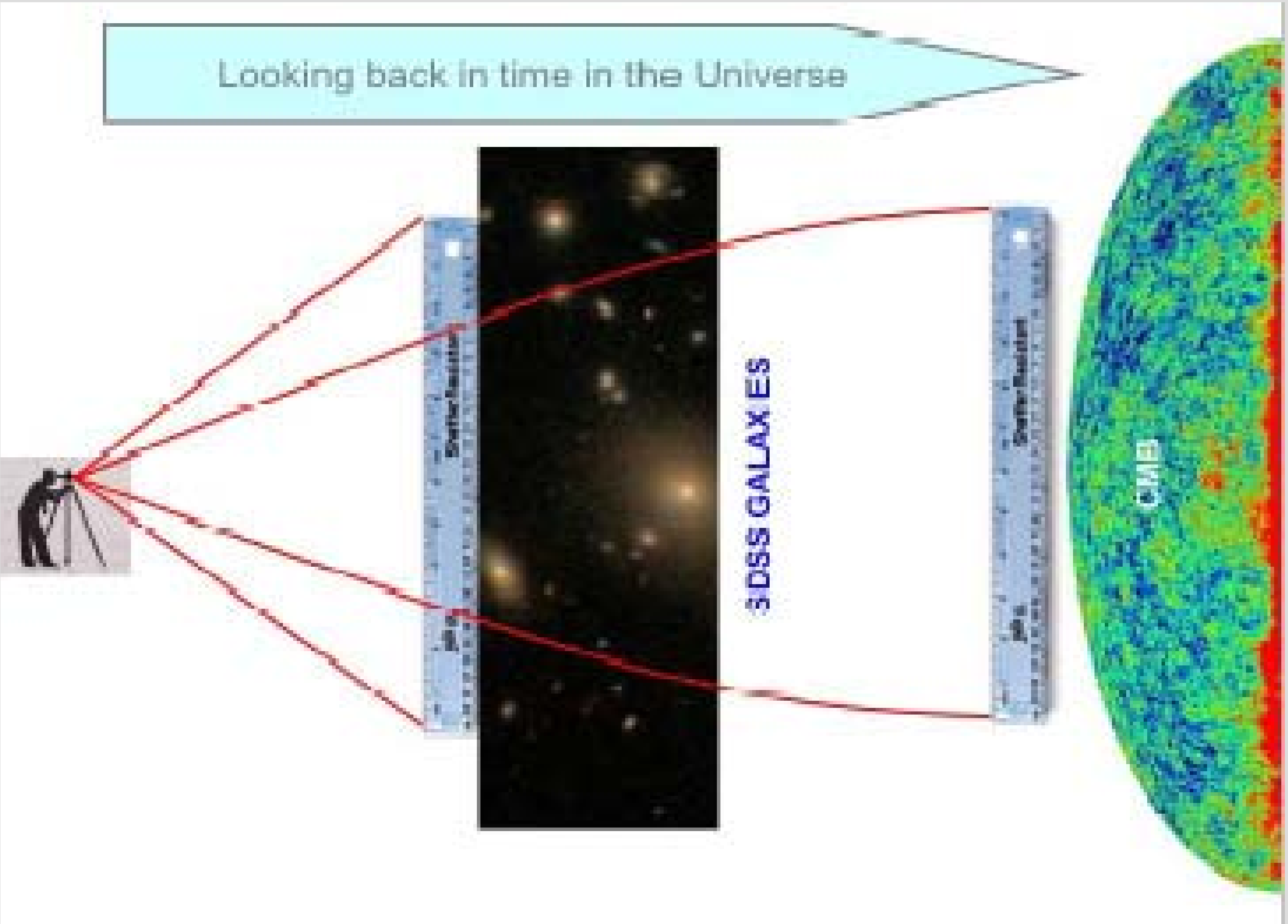
Eisensteinl et al. , astro-ph/0501171; Cole et al., astro-ph/0501174

ARYON OSCILLATIONS

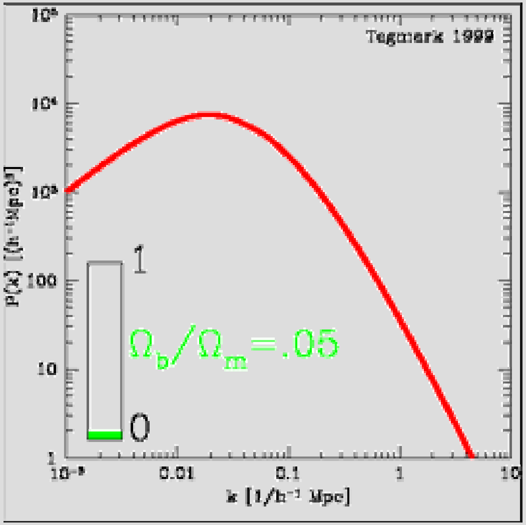
Gives a measure of the angular distance

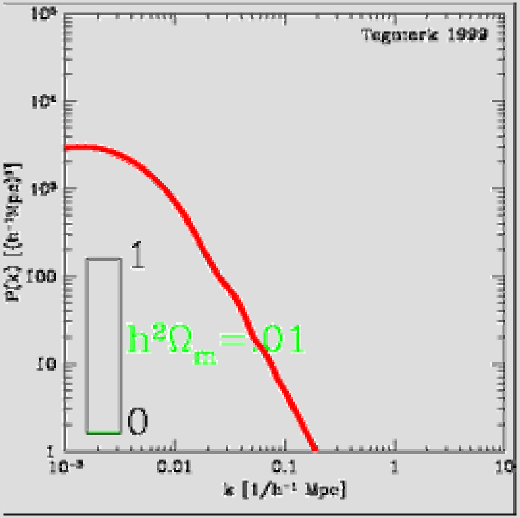
z~0.35

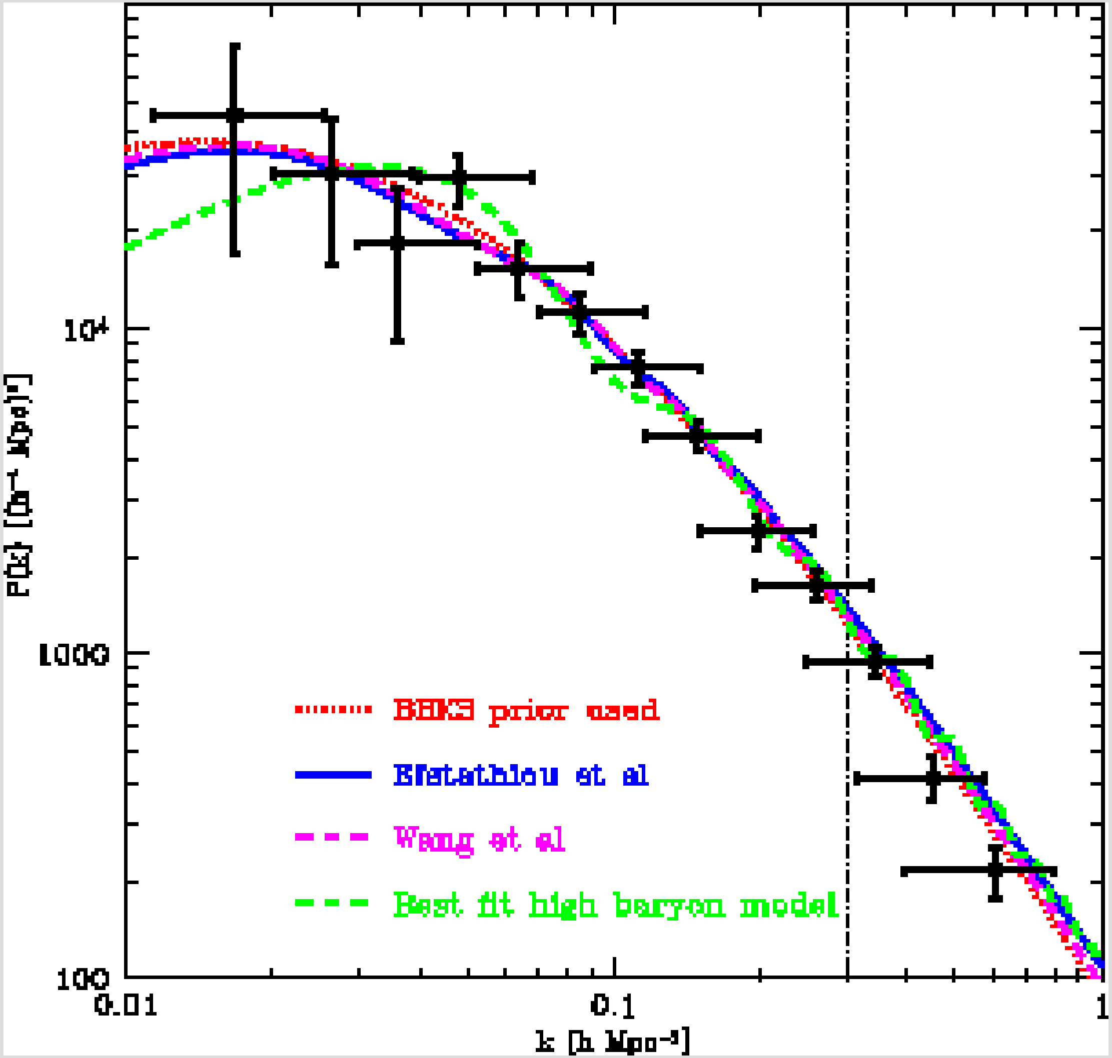
z~1100



ARK MATTER: LSS EVIDENCE







On cosmological scale all the matter CANNOT be baryonic

ARK MATTER: SUMMARY

Evidences

- Galaxy rotation curves
- Galaxy power spectrum
- Dynamics of clusters
- BBN

$$\Omega_{\text{matter}} \sim 23\%, \qquad \Omega_{\text{baryon}} \sim 4\%$$

scaling

Scale	Spiral galaxies	clusters	universe
Baryon/DM	8.5±1.5	7.2±2.0	4.83±0.87

Question

Nature of the matter - WIMP

Caveat

Gravity described by GR on these scales

ARK MATTER: NATURE?

WIMP

Weakly interacting massive particle
Abundance depends on m and Γ

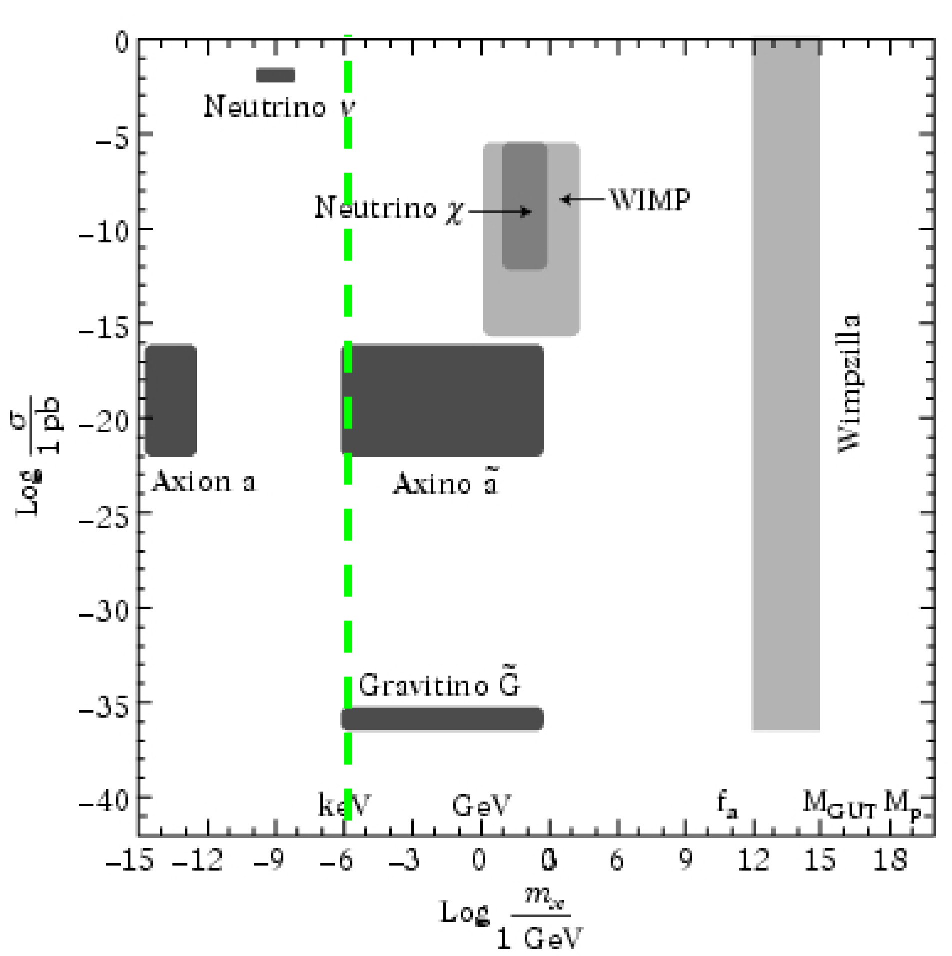
Classification

Hot - cold: mass of the particle at the time of galaxy formation

$$m \sim 1\ \text{keV}$$

Thermal-non termal:
e.g. neutrinos-neutralinos

Theoretical:
existing (neutrino)
motivated (LSP, axion)
exotic (WIMPZILLA)



Modifying Newton law

$$m\mathbf{a}\mu(a/a_0) = \mathbf{F} \qquad \begin{aligned}\mu(x) &= x \quad x \ll 1 \\ &= 1 \quad x \gg 1\end{aligned}$$

Modification in the small acceleration regime

Explain the Tully-Fisher law

New scale

$$\ell_0 = c^2/a_0 \sim 9 \times 10^{26} \ \mathrm{m}$$

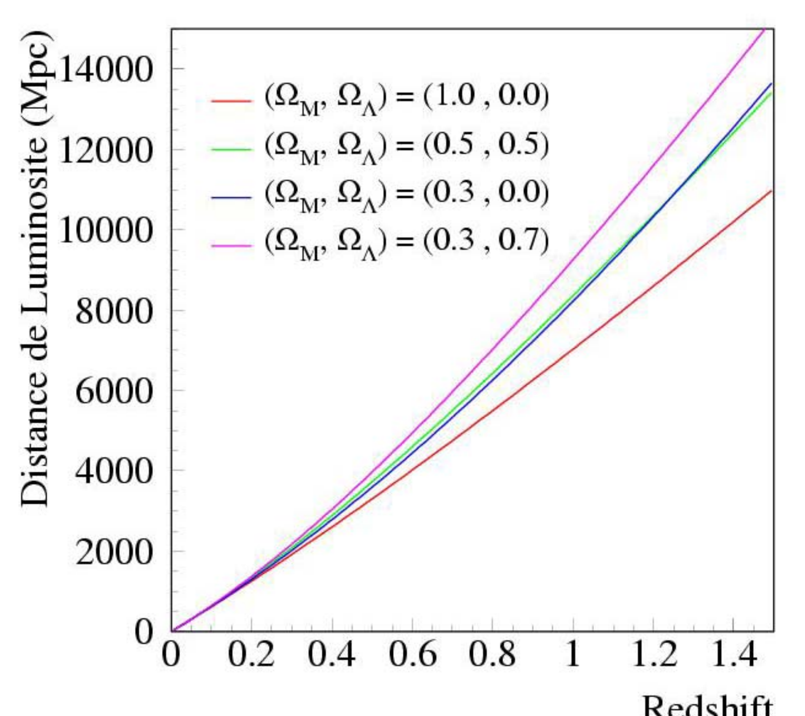
Difficulties

Lacking of a field theory
Behavior in other regimes?

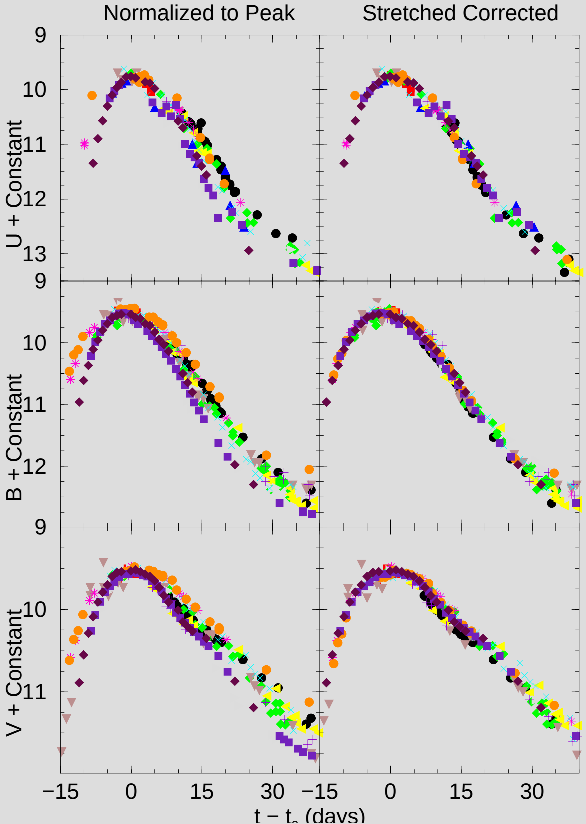
UPERNOVAE: PRINCIPLE

$$\phi_{\rm obs} = \frac{L_{\rm source}(z)}{4\pi D^2_{\rm lum}(z)}$$

$$D_{\rm lum} \propto (1+z)\int_0^z \frac{dz'}{H(z')}$$



Standard candles?



SCP team

SNe Ia exhibit a correlation of
Their light
(*similar to the T-L relation for Cepheids)*

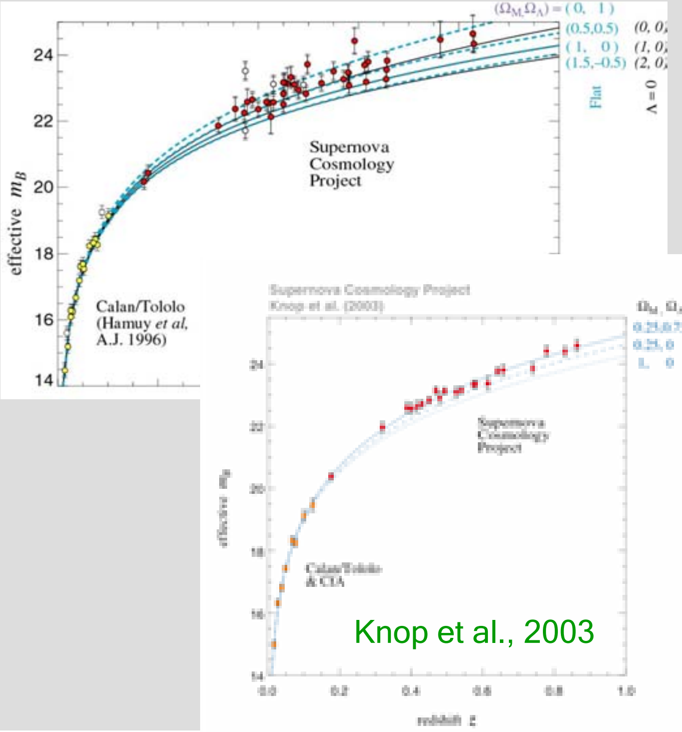
Allow to measure distances with

5-10% accuracy

UPERNOVAE: PROGRESSES

<u>1998:</u> first reliable results

2 independent teams (High-z Team et SCP) obtained results from 42 (SCP) et 10 (HZT) high-redshift SNe and 20-40 low-redshift SNe.

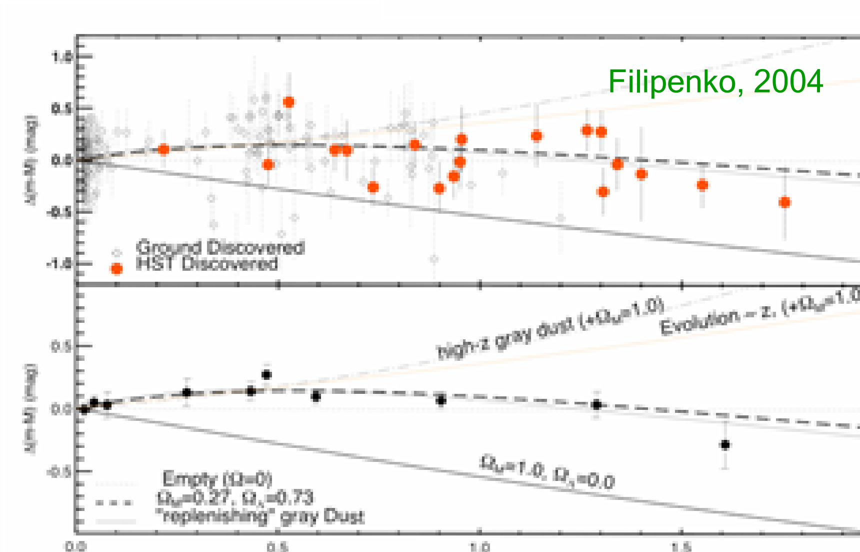


Knop et al., 2003

<u>2003:</u> *SCP +11 SN with a HST photometric follow-up*

Update of the Hubbl diagram.

Search for high-redshift SN (GOOD/ACS survey)



Filipenko, 2004

<u>2004:</u> *HST supernovae*

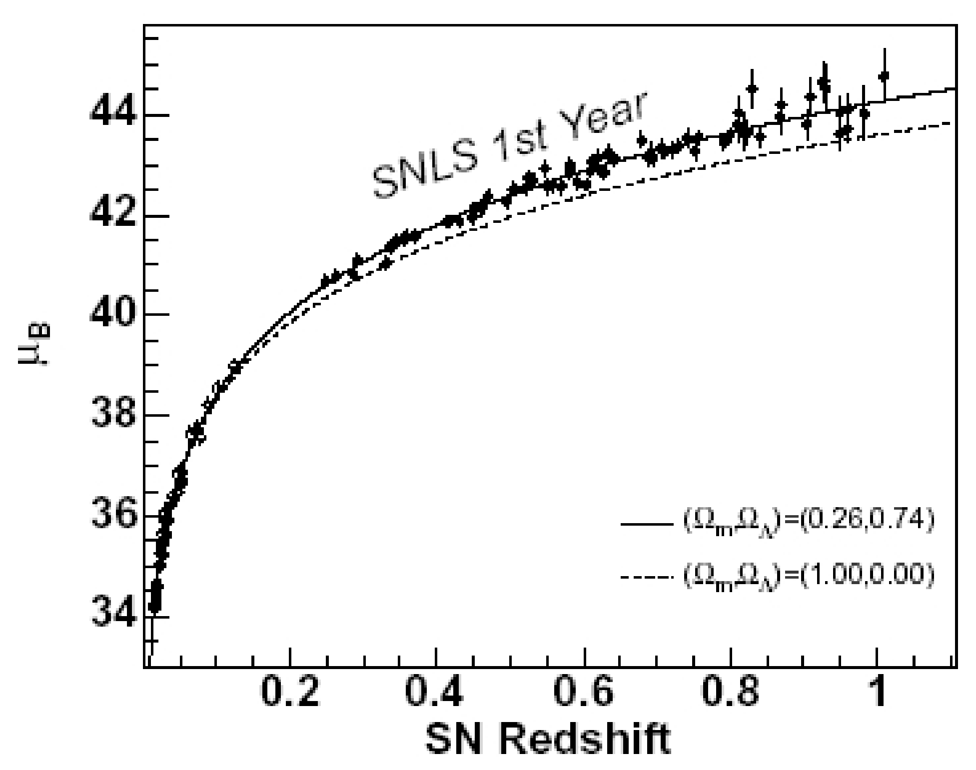
Expansion shifts from deceleration to

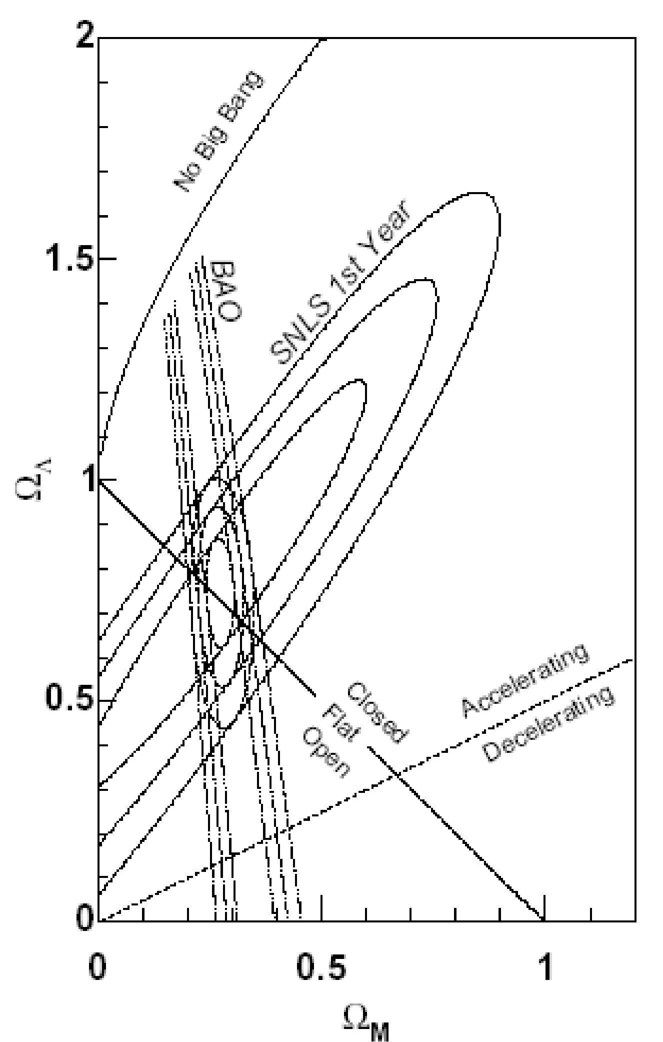
Exclude dust absorption.

UPERNOVAE: SNLS

Astier et al., astro-ph/0510447

Final catalog : **45** close + **71** SNLS SN





For a flat ΛCDM model

$\Omega_M = 0.263$ +/- 0.042 (stat) +/- 0.032 (syst)

HE ACCELERATION OF THE UNIVERSE

Independently of any theory, we can expand the scale factor as

$$a(t) = a_0 \left[1 + H_0(t - t_0) - \tfrac{1}{2} q_0 H_0^2 (t - t_0)^2 + \ldots \right]$$

so that

$$H^2(z)/H_0^2 = 1 + (q_0 + 1)z + \mathcal{O}(z^2)$$

$$q_0 = \Omega_{m0}/2$$

The Hubble diagram gives

- H_0 at small z
- q_0

Supernovae data (1998+) prove that

$$\boxed{q_0 < 0}$$

↔

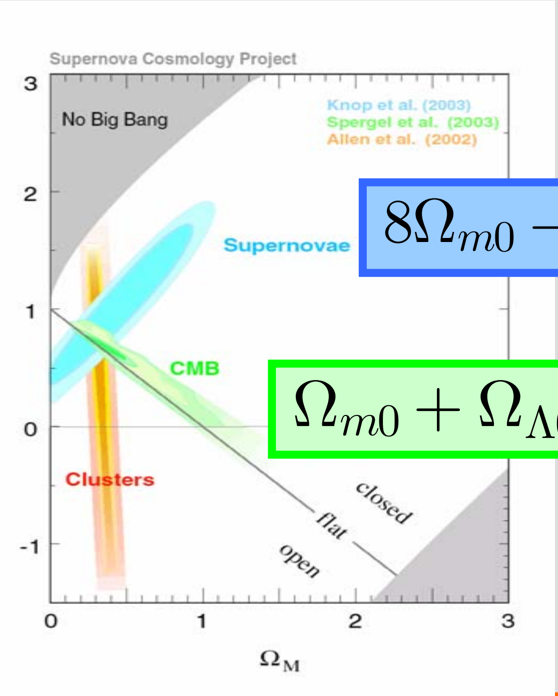
Expansion is now accelerating

HE ΛCDM MODEL

The simplest extension is the introduction of a cosmological constant

- Einstein (1917)
- interpretation as vacuum quantum energy
- constant energy density
- well-defined and predictif model.

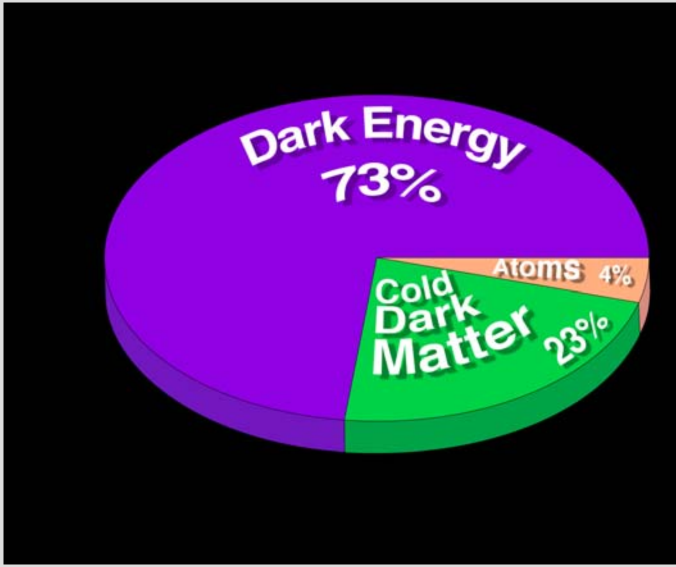
$$\rho_\Lambda = \frac{\Lambda}{8\pi G} = -P_\Lambda$$



$$8\Omega_{m0} - 6\Omega_{\Lambda 0} \simeq -2$$

$$\Omega_{m0} + \Omega_{\Lambda 0} \simeq 1$$

$$\Omega_{m0} \sim 0.3, \quad \Omega_{\Lambda 0} \sim 0.7$$



HE ΛCDM MODEL

Observationnally, OK with all data

Phénomenologically, very simple (1 parameter)

But

$$\left.\begin{array}{l} \rho_{\Lambda,obs} = \frac{\Lambda}{8\pi G} = H_0^2 M_p^2 = 10^{-47}\mathrm{GeV}^4 \\ \rho_{\Lambda,th} = M_{\mathrm{fondamental}}^4 > 10^{12}\mathrm{GeV}^4 \end{array}\right\}$$

Cosmological constant problem

$$\boxed{\rho_\Lambda > 10^{59}\rho_{\Lambda,obs} \;\; !!}$$

Today, no solution
Critical problem of fundamental physics

OSSIBILITIES

The observed acceleration implies that

$$\boxed{(\rho + 3P) < 0}$$

si general relativity and the Copernician principle hold on cosmological scales

One must change one of the 3 assumptions of the model

1- The opernician principle is not valid

2- It exists matter such that $\rho+3P<0$

3- Gravitation is not described by general relativity on large scales

Nature of the dark energy

LASSES OF MODELS

De nombreux modèles existent,

- liste longue et rébarbative!

On ne peut pas tester les modèles un par un !

- définir des grandes classes de modèles
- peut-on extraire des paramétrisations physiquement motivées pour analyser les données?

But :
- donner une idée de la diversité
- avoir une vue des tests permettant de distinguer ces modèles
- définir des stratégies observationnelles
- nécessité d'un modèle au-delà du ΛCDM ?

On suppose le problème de la constante cosmologique
RESOLU

OPERNICIAN PRINCIPLE

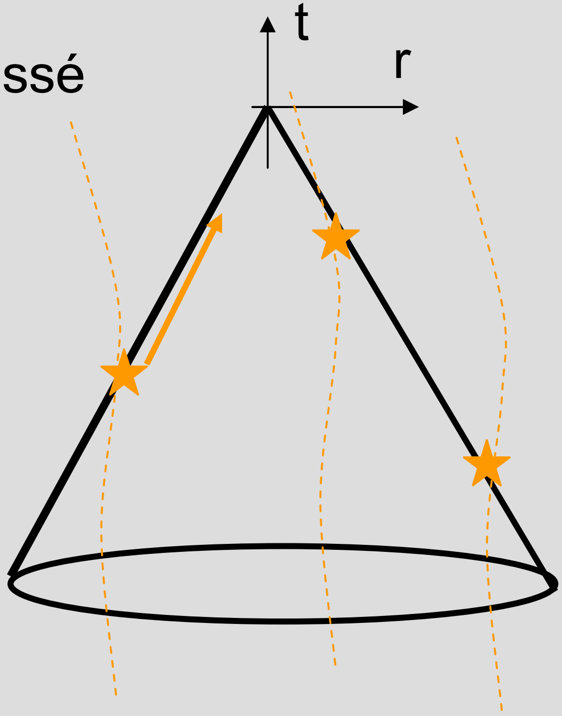
Hypothèses la moins profonde: symétrie de l’espace-temps et non théorie.

Ne touche pas à la relativité / n’invoque pas de nouvelle matière.

Mais la moins étudiée (difficulté technique)

Principe :

on mesure un décalage spectral
dégénérescence sur le cône de lumière passé



Principe copernicien:

implique une relation bi-univoque entre
redshift et temps d’émission

L’accélération observée pourrait être un effet de structure à grande échelle et d’un mauvais choix de symétries

EW FORM OF MATTER

Voie la plus étudiée

Principe : rajouter une nouvelle composante de matière qui

1- domine le contenu de l’univers récemment
2- est telle que $\rho+3P<0$

Questions à résoudre :

1- nature de cette matière
2- problème de coïncidence
3- compatibilité avec le modèle standard de la physique des particules

Archétype : quintessence

QUINTESSENCE

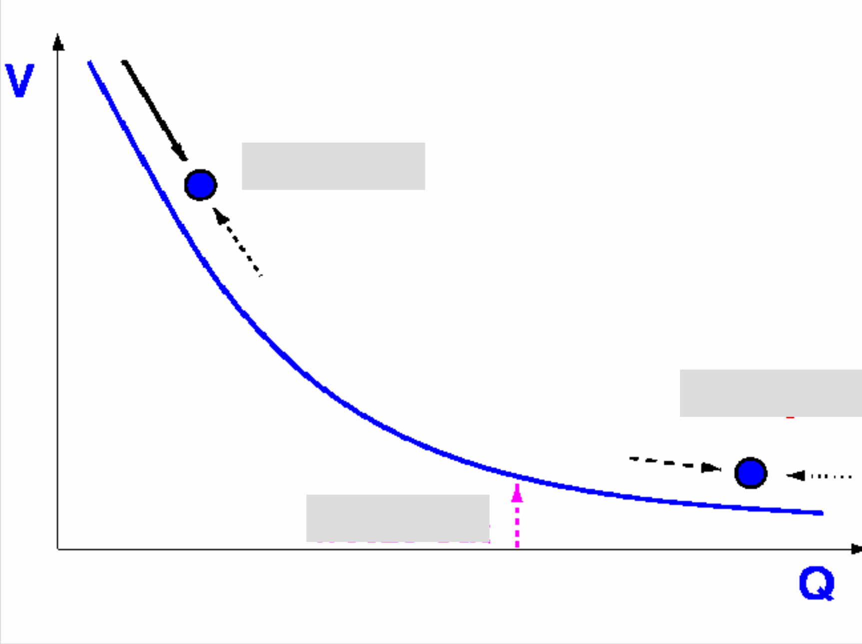
Champ scalaire évoluant
dans un potentiel

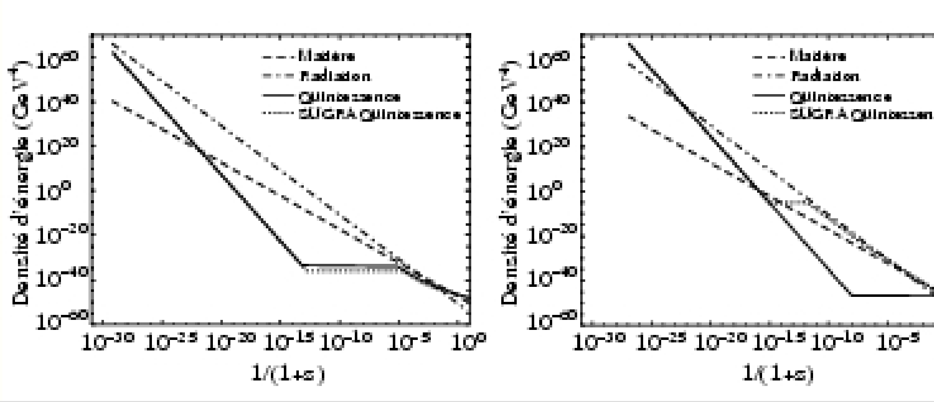
$$\rho = K + V, \quad P = K - V$$

$$-1 \leq \frac{P}{\rho} \leq 1$$

En régime de roulement lent (K<<V)

$$\frac{P}{\rho} = \frac{K-V}{K+V} \simeq -1 + 2\frac{K}{V}$$



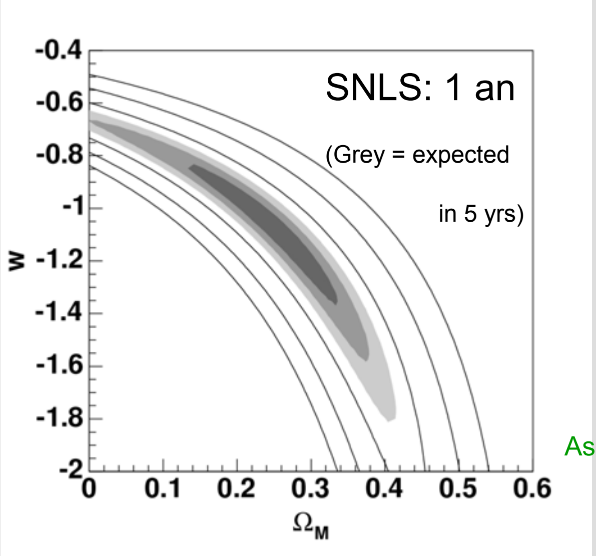


Mécanisme d'attraction

Potentiels justifiables en SUSY

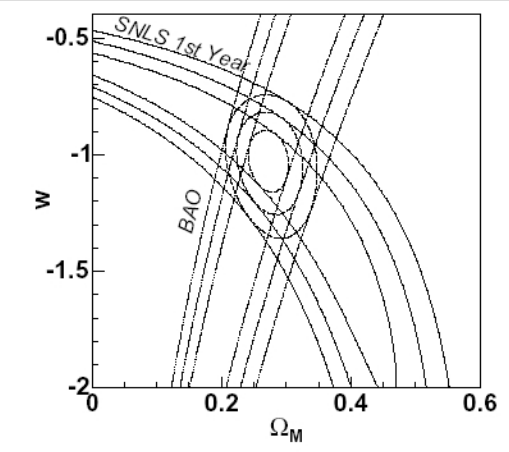
ONSTRAINTS ON A CONSTANT EQUATION OF STATE

SN



Astier et al 2005

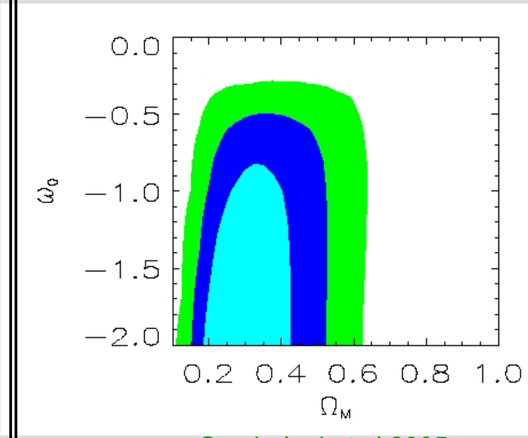
BAO-SN



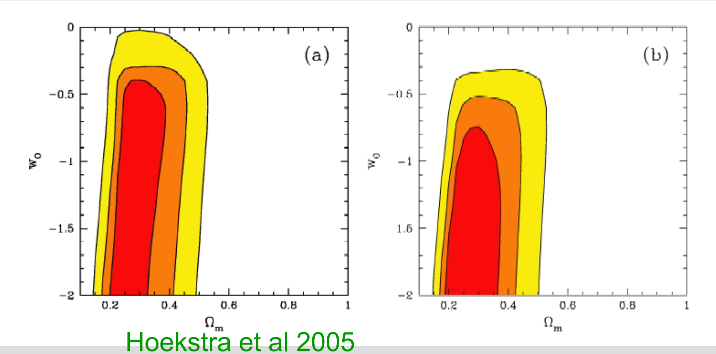
Astier et al 2005

Einsestein et al. 2005

Lensing



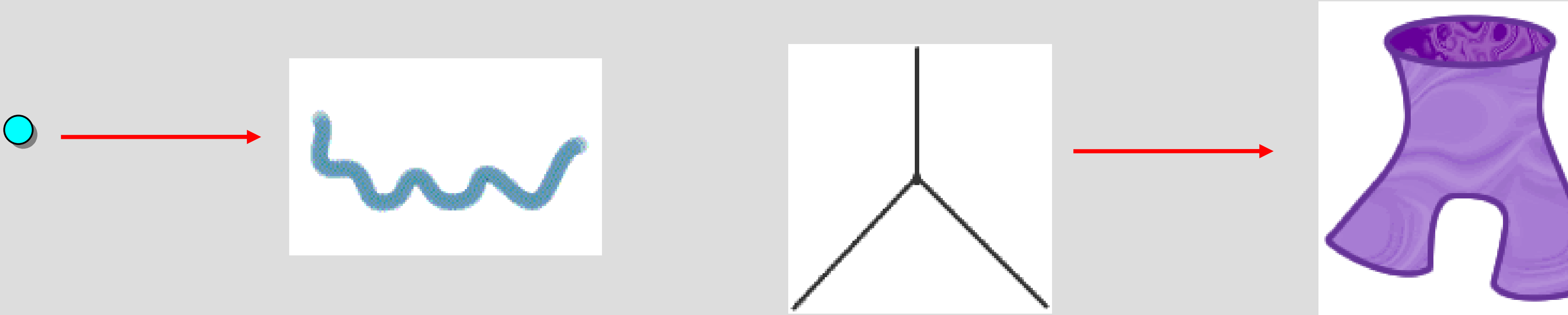
Semboloni et al 2005



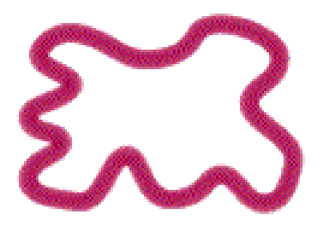
Hoekstra et al 2005

STRING THEORY

- removes the infinities of QFT

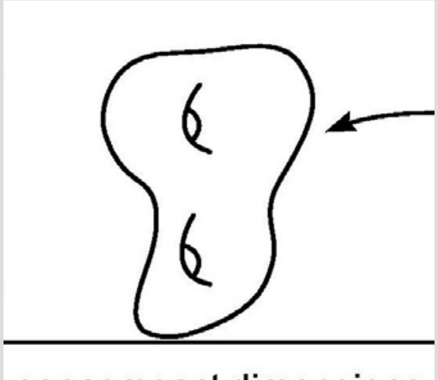


- and includes the graviton



- but there is a price!

  9 space dimensions



TRING INSPIRED COSMOLOGY

Early universe

Extra-dimension dominates at early time / high energy

Models

Pre big-bang models
Braneworld model
Brane gaz

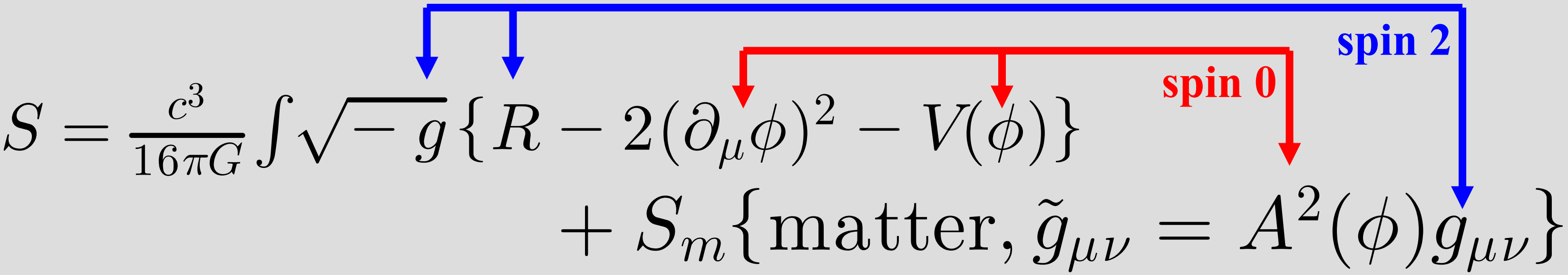
Phenomenology

Is there a natural mechanism for 3 dimensions to grow very large?
Is the observable 3D universe born out of brane-collision?
Is there a signature of extra dimensions in observations?
Dark matter?
Dark energy?

OW ENERGY LIMIT

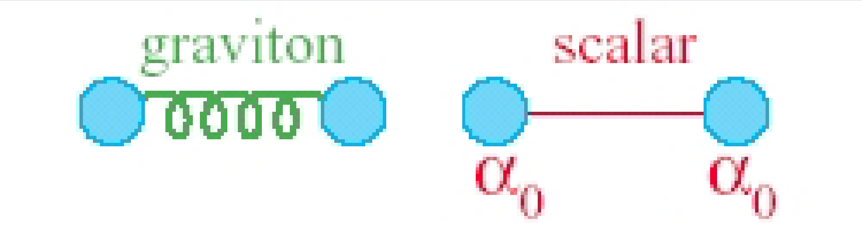
Most general theories of gravity include a scalar field beside the metric

- Mathematically **consistent** (no ghost, no adynamical field)
- Motivated by **superstring**
  - **dilaton** in the graviton supermultiplet,
  - **modulii** after dimensional reduction
- Only consistent massless field theory to satisfy WEP
- Preserve most **symmetries** of general relativity
- Useful extension of GR (simple but general enough)



$$S = \frac{c^3}{16\pi G}\int\sqrt{-g}\{R - 2(\partial_\mu\phi)^2 - V(\phi)\}$$

$$+ S_m\{\text{matter}, \tilde{g}_{\mu\nu} = A^2(\phi)g_{\mu\nu}\}$$

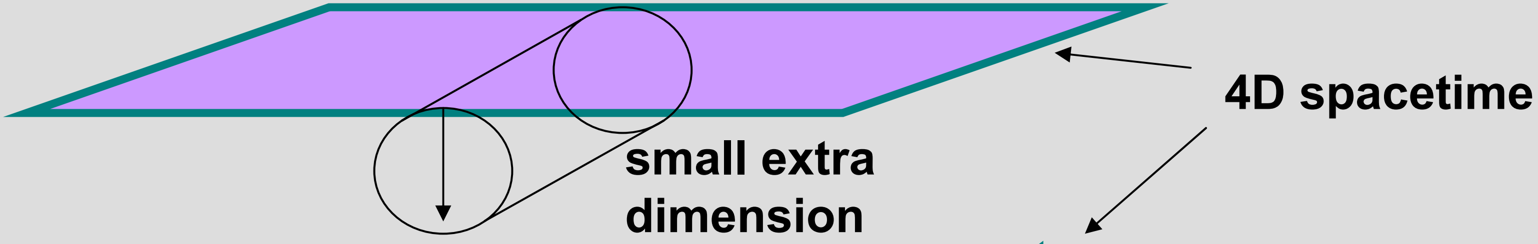


$$\alpha = \mathrm{d}\ln A/\mathrm{d}\phi$$

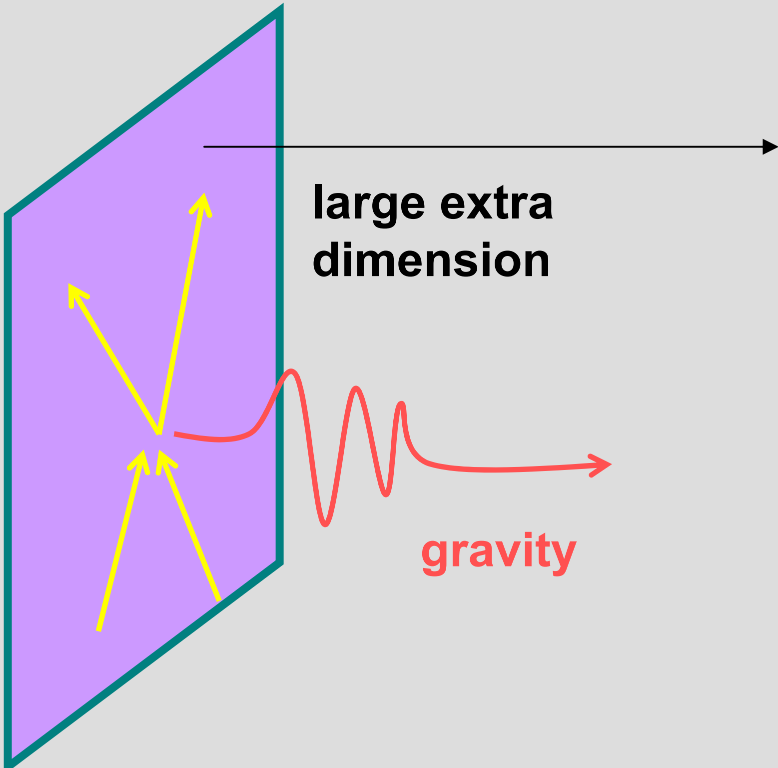
HY DON'T WE SEE EXTRA-DIMENSIONS?

- conventional Kaluza-Klein idea:

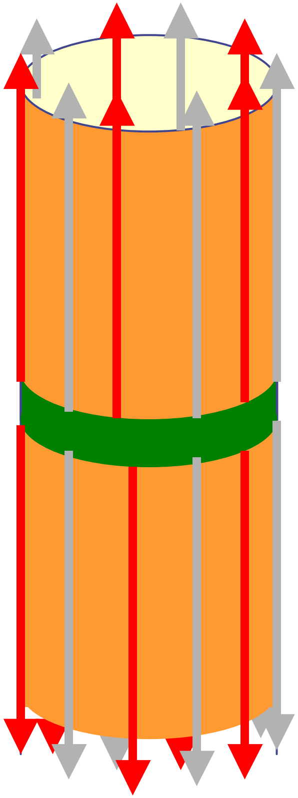
  extra dimensions too small to be seen



- discovery of D-brane
  - *matter fields restricted to lower dimensional brane*
  - *external bulk felt only through gravity*
  - *extra dimension bigger*

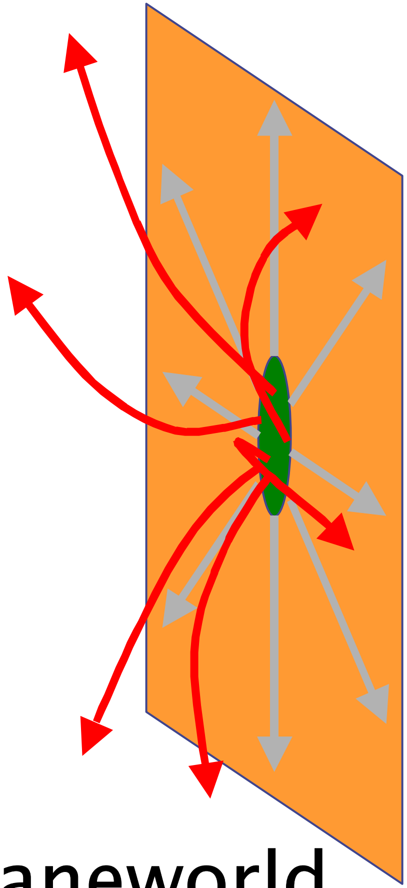


ALUZA-KLEIN VS BRANE



Flux lines of gravity

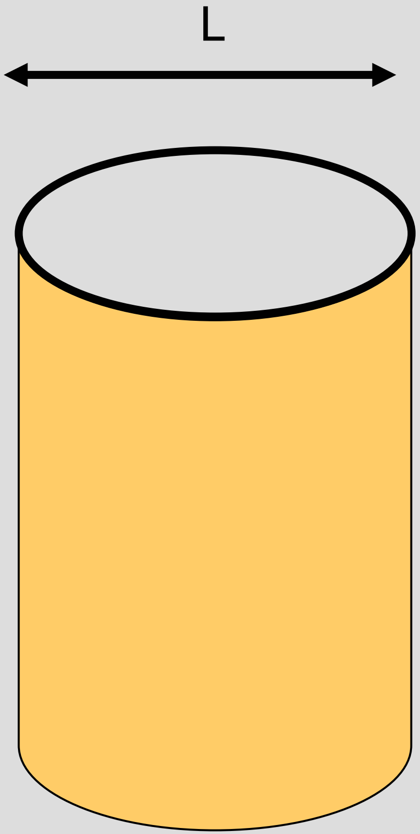
Electric flux line



Kaluza–Klein extra dimensions have SM fields

Braneworld extra dimensions are “dark”

ARGE EXTRA-DIMENSIONS



n extra-dimensions

$$M_4 = (M_{4+n})^{n+2} L^n$$

We can fix n such that M_{4+n} ~ M_{EW}.

$$L \sim 10^{-17+30/n} (\frac{1 \text{ TeV}}{M_{\text{EW}}})^{1+2/n}$$

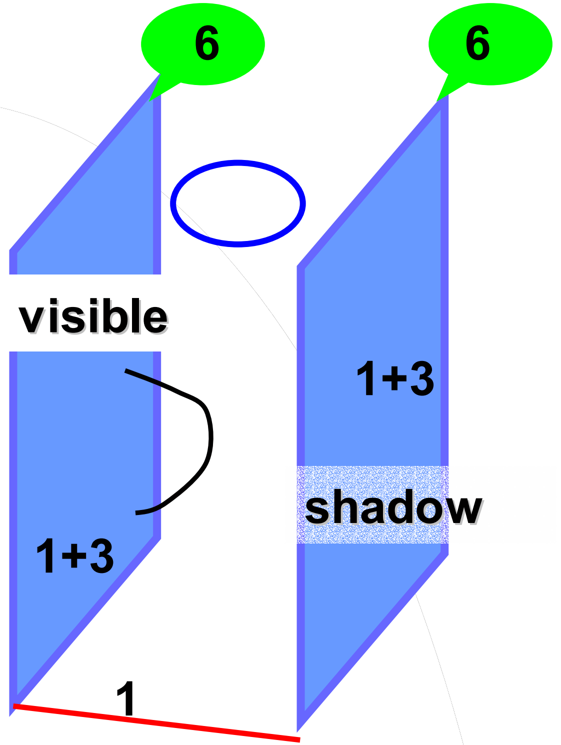
Gravity on small scales.

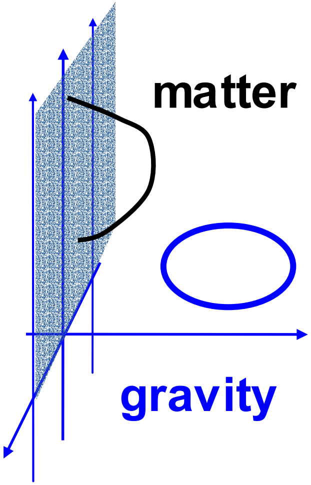
$$V \propto 1/r^{n+1} \quad r < L$$

This imposes L<1 mm so that.

$$n > 2$$

M-THEORY AND HORAVA-WITTEN MODEL





M theory

1 time + 10 space dimensions

$$1+10 \rightarrow 1+3+1+(6)$$

braneworld

large extra dimension

→ effective 5D braneworld

$$M_5 \sim \left(M_4^2 / L\right)^{1/3} << M_4$$

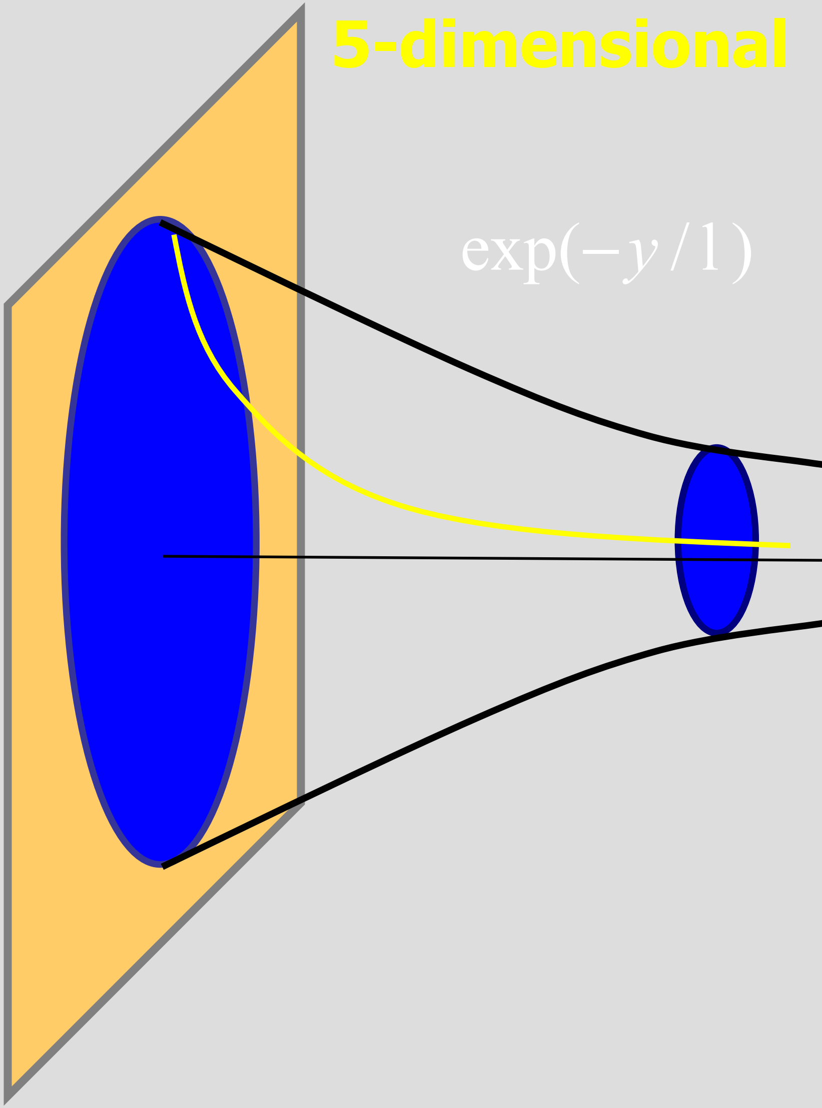
ANDALL-SUNDRUM MODEL

<u>warped geometry</u>

- extra-dimension shrinks toward infinity
- curvature scale: l

4D GR is recovered on large scales

$r > l$ ⇨ $l < 0.1\text{mm}$



DEVIATION FROM GR IN THE EARLY UNIVERSE

Gravity leakage to the fifth dimension

Gravity becomes effectively five-dimensional

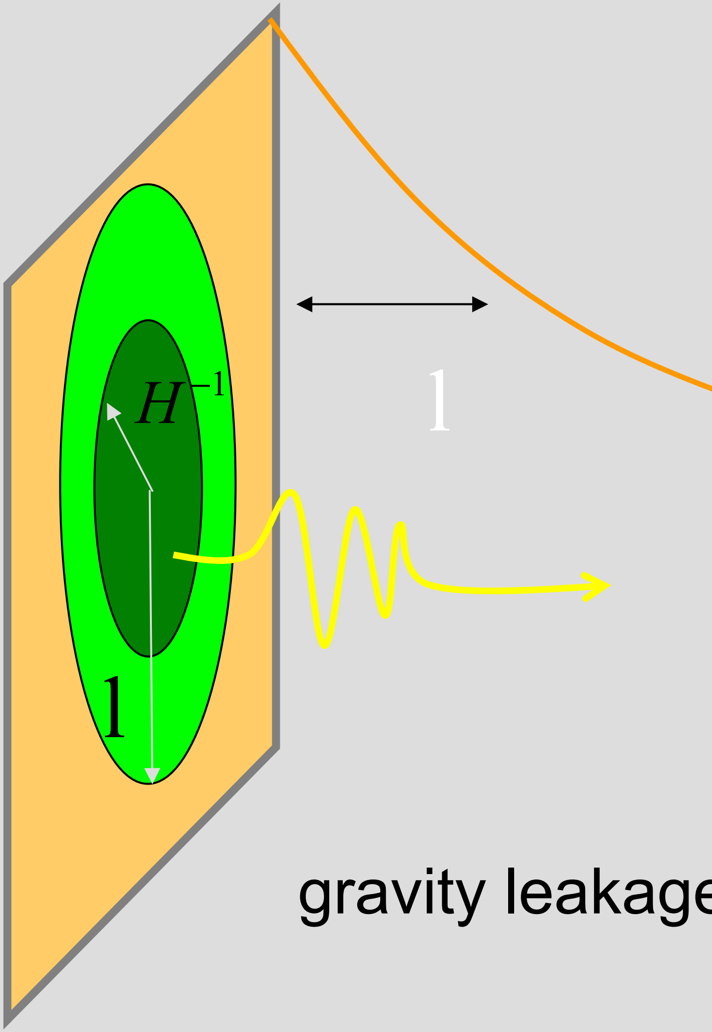
Friedmann equation

$$H^2 = \frac{8\pi G\rho}{3}\left(1+\frac{\rho}{\lambda}\right)+\frac{\Lambda}{3}-\frac{K}{a^2}$$

In the early universe

$$H \propto \rho$$

It will modify the phenomenology for inflation



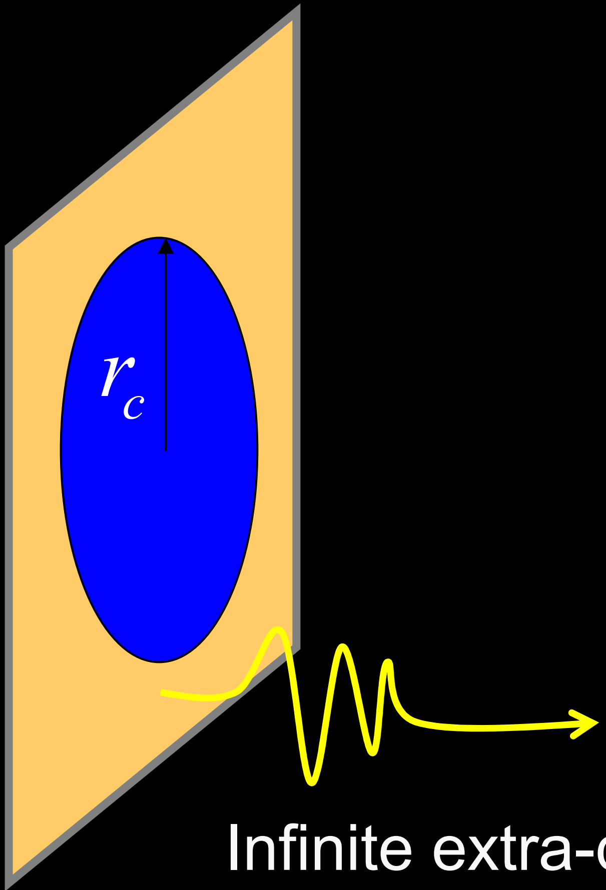
GP MODEL

$$S=\frac{1}{32\pi G r_c}\int d^5x\sqrt{-{}^{(5)}g}\,{}^{(5)}R+\frac{1}{16\pi G}\int d^4x\sqrt{-g}\left(R+L_m\right)$$

- Crossover scale r_c

$r < r_c$ 4D Newtonian gravity

$r > r_c$ 5D Newtonian gravity



gravity leakage

Infinite extra-dimension

EVIATION FROM GR IN THE EARLY UNIVERSE

Friedmann equation

$$H^2 = \left(\frac{1}{2r_c} + \sqrt{\frac{1}{4r_c^2} + \frac{8\pi G\rho}{3}} \right)^2 - \frac{K}{a^2}$$

Modification in the Infra-red (large distance)

May explain the recent acceleration of the universe

FFECT ON THE FUNDAMENTAL CONSTANTS

In string theory, all dimensionless parameters become VEV of some fields: dynamical.

e.g. $M_4^2 = \mathrm{e}^{-2\Phi} V_6 M_I^8$ $\qquad g_{YM}^{-2} = \mathrm{e}^{-\Phi} V_6 M_I^6 \quad (+c_i M_i)$

$G \propto R^{-D}, \qquad g_{YM}^{-2} \propto K_i(D) G R^2$

The low energy limit are scalar-tensor theories (dilaton) at tree level.

Loop corrections: need to be understood better
couplings are not universal Dudas (2000)

$$M_4^2 = \phi M_H^8, \qquad g_{YM}^{-2} = \phi M_H^6, \qquad \phi = V_6 \, \mathrm{e}^{-2\Phi}$$

$$g_{YM}^{-2} = \phi M_H^6 - \frac{b_a}{2}(R M_H^2) + \dots$$

Phenomenologically: couplings of the quintessence field

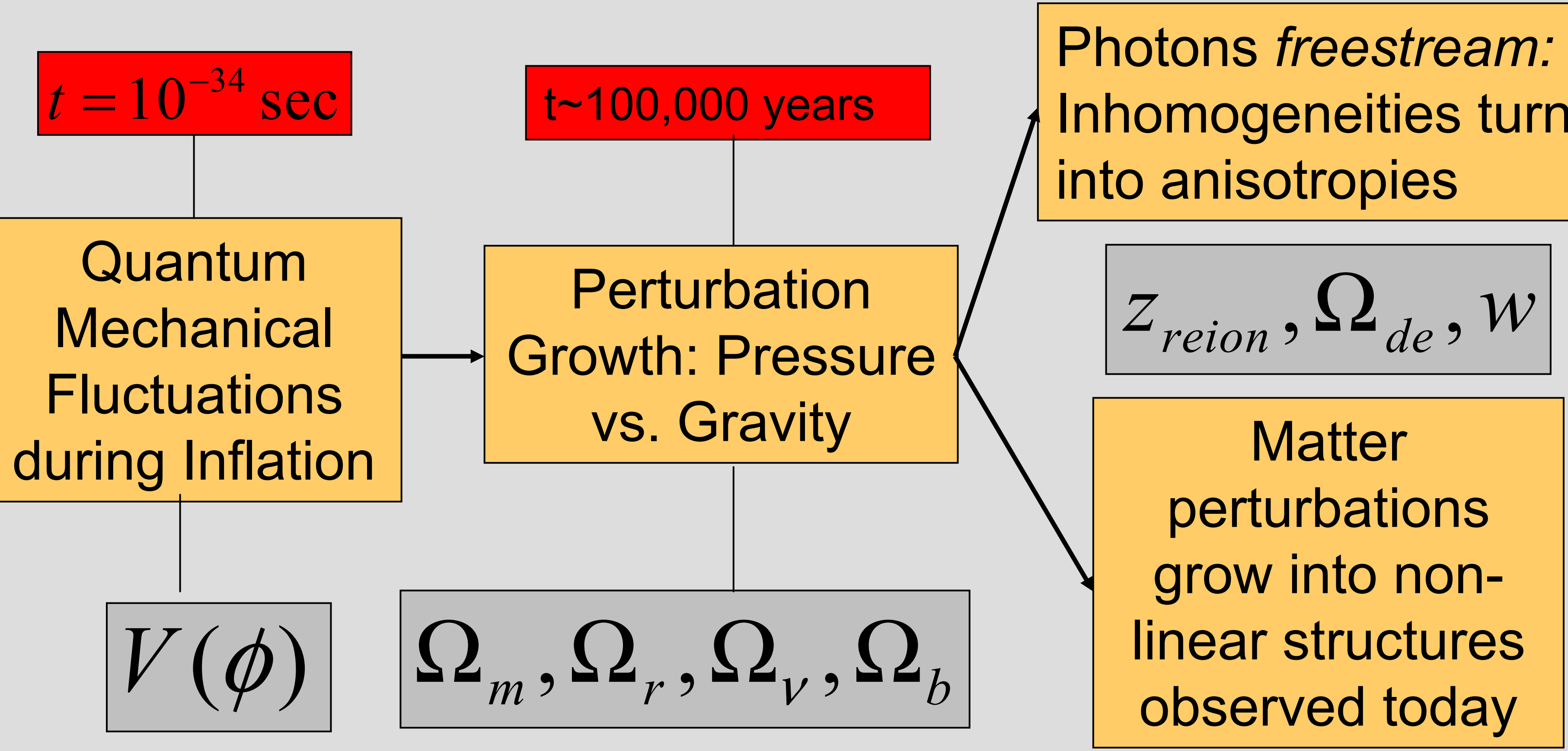
brane models Palma et al PRD 68 (2003) 12351

PART VII: CONCLUSIONS

Main topics

- *Status of the model*
- *Open issues and questions*

COHERENT PICTURE OF STRUCTURE FORMATION



$t = 10^{-34}$ sec

Quantum Mechanical Fluctuations during Inflation

$V(\phi)$

t~100,000 years

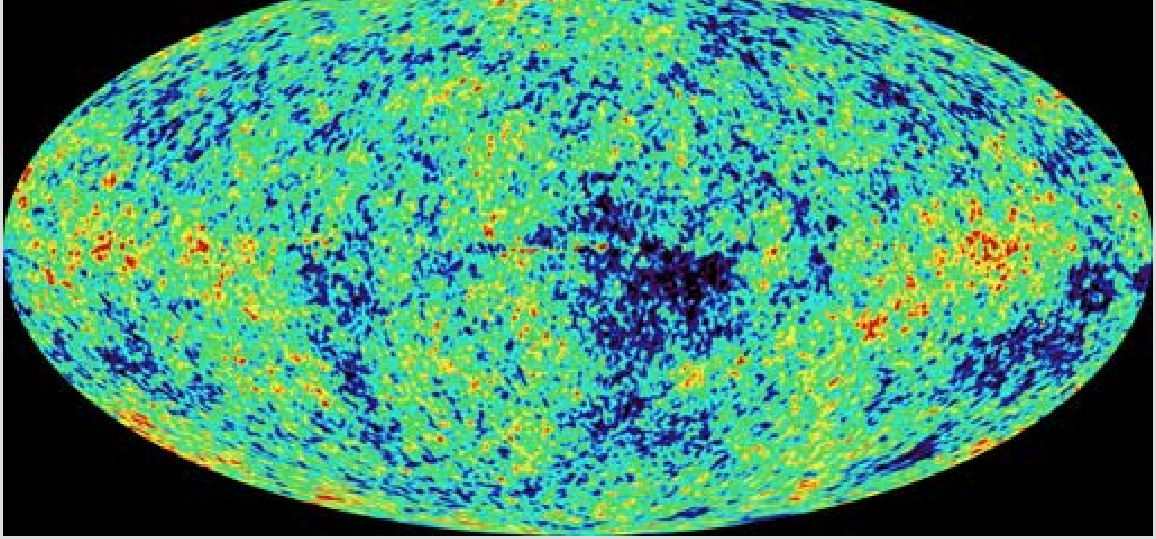
Perturbation Growth: Pressure vs. Gravity

$\Omega_m, \Omega_r, \Omega_\nu, \Omega_b$

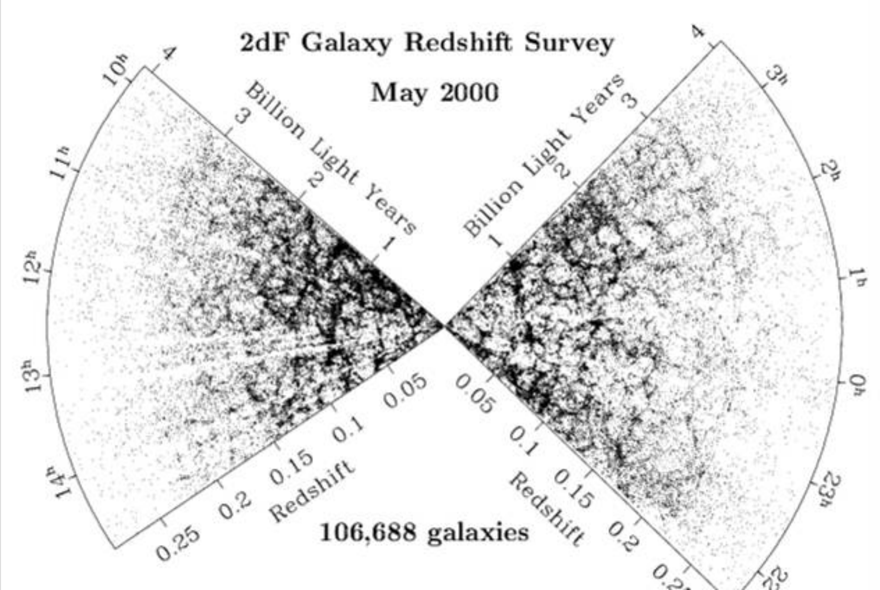
Photons *freestream:* Inhomogeneities turn into anisotropies

$z_{reion}, \Omega_{de}, w$

Matter perturbations grow into non-linear structures observed today t= 300.000 yrs



t=15 Giga yrs



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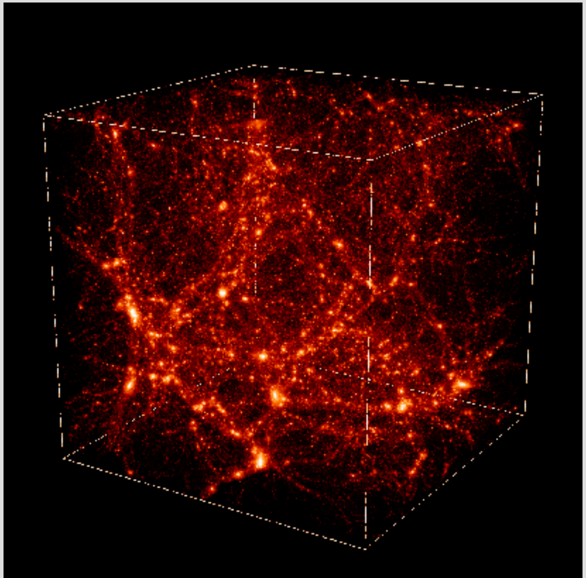


XCELLENT STANDARD MODEL

The standard model is based on

- relativity
- electromagnetism
- weak interaction
- GUT - SUSY
- QFT
- string - quantum gravity

It explains

- the dynamics of the universe
- the origin of LSS

Tests

- numerous and successful
- more test of fundamental physics are needed
- what physics behind the parameters

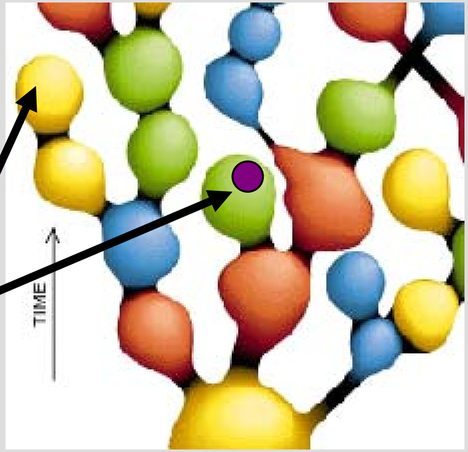
Questions

- dark matter?
- dark energy ?
- inflaton ?
- constants ?

Laboratory

- constraints on extension of the standard models
- Model building

ANDSCAPE PICTURE-THE LIMIT OF EXPLANATION



The universe is « machine » producing space

It produces baby-universes

The observable universe is a small patch of a baby-universe

Each baby-universe can have a different physics
(a different vacuum of string theory)

Observable universe < universe < multiverse

Eternal inflation + string theory gives a new picture of the universe

AN WE EXPLAIN THE VALUE OF THE FUNDAMENTAL CTES?

low many numbers:

Standard model of Part. Phys.: 18 + 3 (c,G,h) + 7 (neutrinos) + …
MSSM : 124 parameters

These parameters are arbitrary (different values will let the theoretical rchitecture safe).

e.g. a=2/137 will let part. Phys safe but will affect chemistry, biology…

ine tuning among these parameters

e.g. triple alpha…

wo approaches:

1-existence of an attraction mechanism

e.g. attraction of scalar-tensor theories toward GR

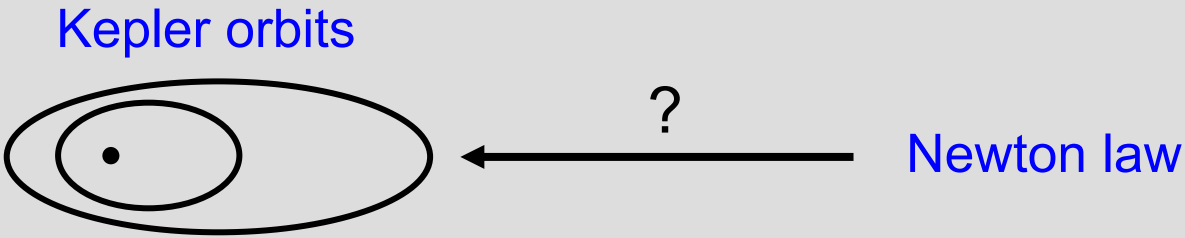
2-anthropic approach

TRING AND CONSTANTS

1- fundamental object = string. Characterized by 1 constant (λ)
2- 4 interactions are unified: 1 coupling constant (g_s)
3- Witten (87): all parameters are dynamical variables

But

Theory is defined in D= 9 + 1
One needs to determine the vacuum corresponding to our universe



Kepler orbits

?

Newton law

Equations depend one 1 parameter but solutions on much more.

D=3+1+1: one more parameter (radius of compactification)
D=3+6+1: several hundreds to describe topology and size!
Idea: exploring the landscape of vacua.

HE ANTHROPIC WAY

Selectionist approach:

goal is not to derived the value of the fundamental parameters but Show that are necessary conditions for some physical phenomena to exist

$$P \Rightarrow C \Longleftrightarrow !C \Rightarrow !P$$

not a finalist approach

does not assume that P has to exist!

P = existence of human life (anthropic principle)

existence of some life (biotic principle)

existence of carbon (carbonic principle)

But to apply this principle

1- one needs to conceive that other (part of the) Universe may not have the same physics

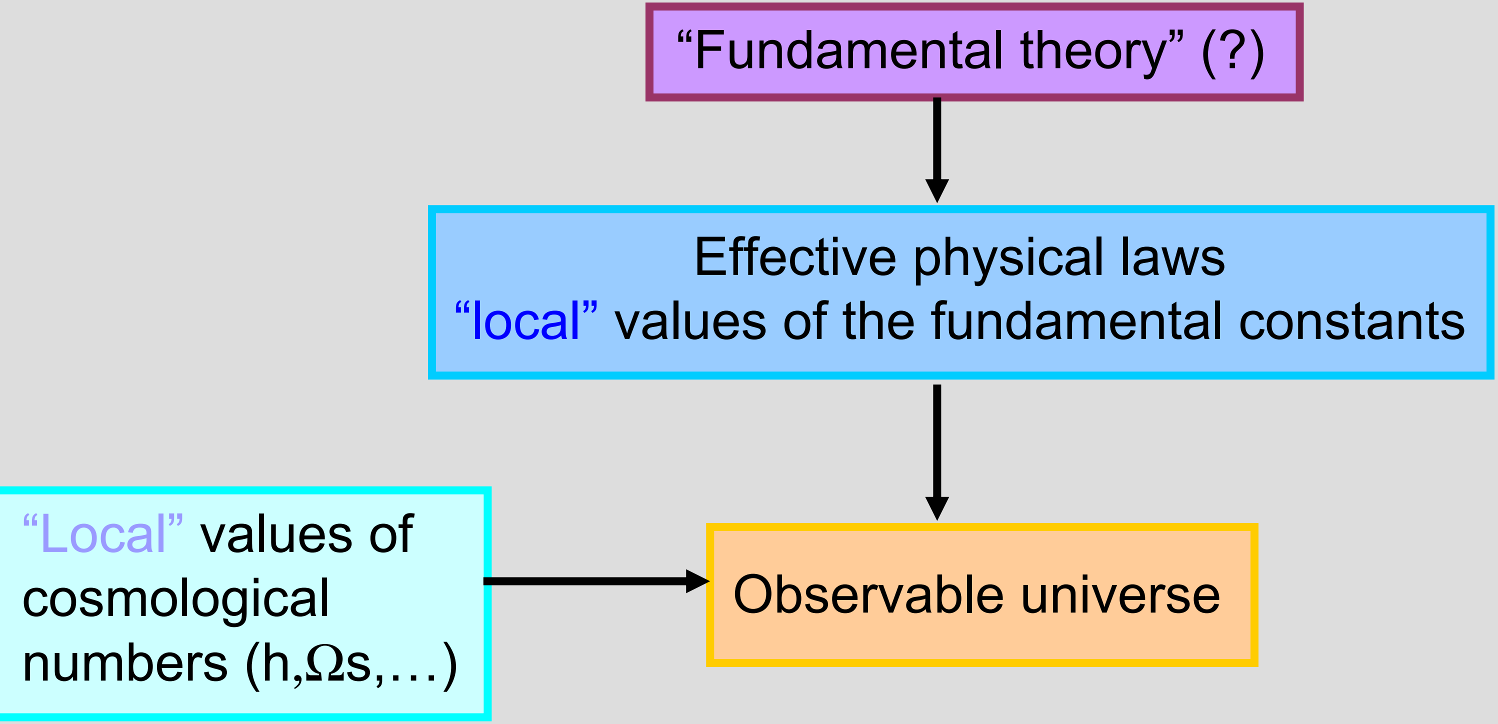
2- find a mechanism to explore the set of possible universes

(not trivial: does our universe belong to this set? Fine tuning for discretisation?)

Ideas:

Eternal inflation, landscape…

AN WE EXPLAIN THE VALUE OF THE FUNDAMENTAL CTES?



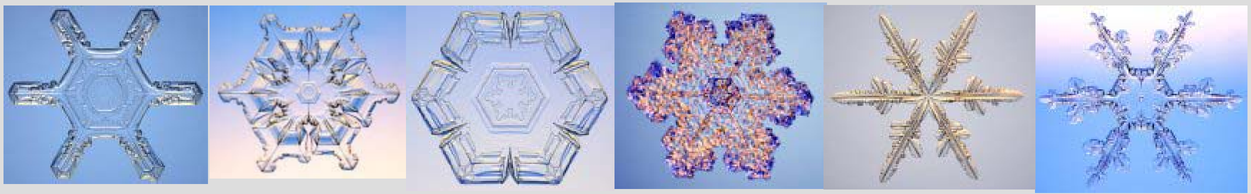
"Fundamental theory" (?)

Effective physical laws
"local" values of the fundamental constants

"Local" values of cosmological numbers (h,Ωs,…)

Observable universe

Which numerical coincidences are consequences of the structure of the fundamental theory and which are accidents?



..AND I'LL STOP HERE

Thank you!