Exercises for the lectures on neutrino physics

1. Derive in the two-neutrino picture:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

the appearance and disappearance formulas:

$$P(\nu_e \to \nu_\mu; t = L/c) = \left| \langle \nu_\mu(0) | \nu_e(L/c) \rangle \right|^2 = \sin^2(2\theta) \sin^2 \left[\frac{1.27 \Delta m^2 \left[eV^2 \right] L \left[km \right]}{E_\nu \left[GeV \right]} \right],$$

$$P(\nu_e \to \nu_e; t = L/c) = \left| \langle \nu_e(0) | \nu_e(L/c) \rangle \right|^2 = 1 - \sin^2(2\theta) \sin^2 \left[\frac{1.27 \Delta m^2 \left[eV^2 \right] L \left[km \right]}{E_\nu \left[GeV \right]} \right]$$

with: $\Delta m^2 = m_2^2 - m_1^2$ and $t = L/c$.

What is the oscillation length $L = L_0$ from minimum to maximum ?

2. We start from the two-flavor neutrino oscillation formula for the mixing of the muon and tau neutrinos:

$$P(\nu_\mu \to \nu_\mu; t = L/c) = 1 - \sin^2(2\theta) \sin^2 \left[\frac{1.27 \Delta m^2 \left[eV^2 \right] L \left[km \right]}{E_\nu \left[GeV \right]} \right]$$

We have the experiments of Super-K, which derive from atmospheric neutrinos θ_{23} ~ 41.5° and Δm_{23}^2 ~ 2.4 · 10^{-3} [eV^2]. For the atmospheric neutrinos, L is about 10 km for the ones from above and 13 000 km for the neutrinos from below (other side of the earth).

The long baseline experiment at KEK (K2K) has L = 250 km, and MINOS (Fermi-Lab to Soudan mine) and OPERA (CERN -Gran Sasso) have L = 730 km.

Calculate $P\left(\nu_\mu \to \nu_\mu; L\right)$ for the following values of the neutrino energies and distances:

a) E =0.1GeV; L =10, 250, 730, 13000 km
b) E =2 GeV; L =10, 250, 730, 13000 km

c) For OPERA and a neutrino mean energy of 17 GeV, what would be the optimal distance to place the detector?