Introduction to Free-Electron Lasers

Richard P. Walker, Diamond Light Source, U.K.

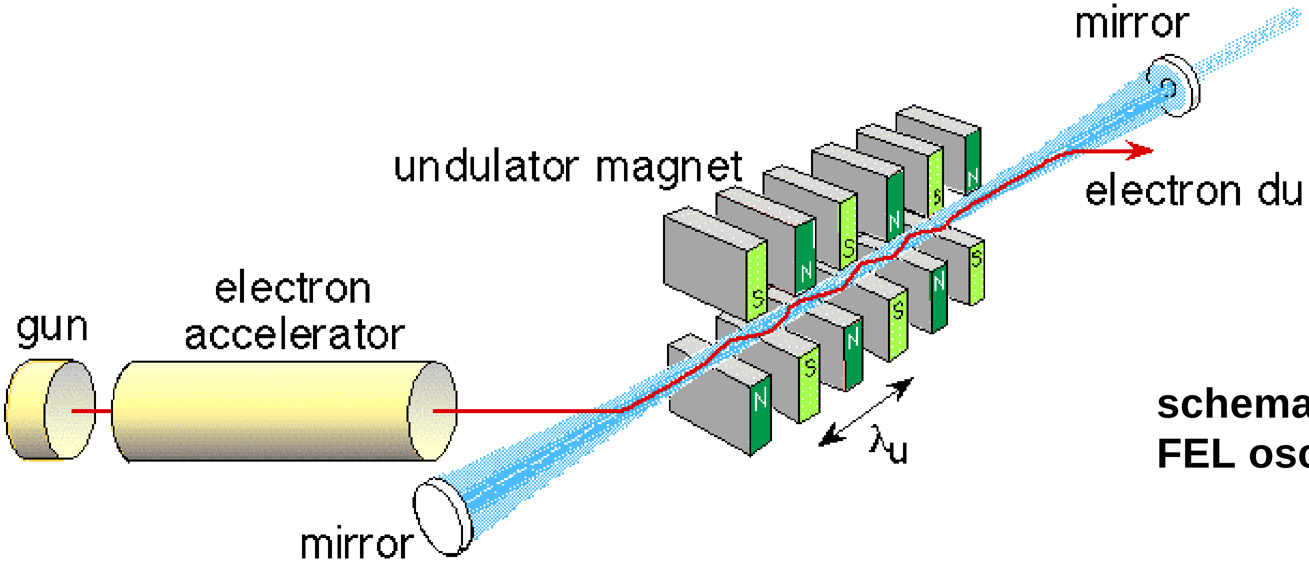
- Introduction
- Historical Background
- Basic FEL Physics
- Low-Gain FELs
- High-Gain FELs
- Technical Challenges for Short Wavelength SASE FELs
- Harmonics, Seeding and Short Pulse Generation

Scope of the Lectures

- Introductory
- Basic concepts and phenomena
- Very little maths
- Historical development
- Different FEL types
- Applications
- Technology
- World scene
- Future directions

So, what is a Free-Electron Laser ?

- a device which amplifies short-wavelength radiation by stimulated emission when the radiation and a relativistic electron beam propagate together through an "undulator" or "wiggler" magnet:

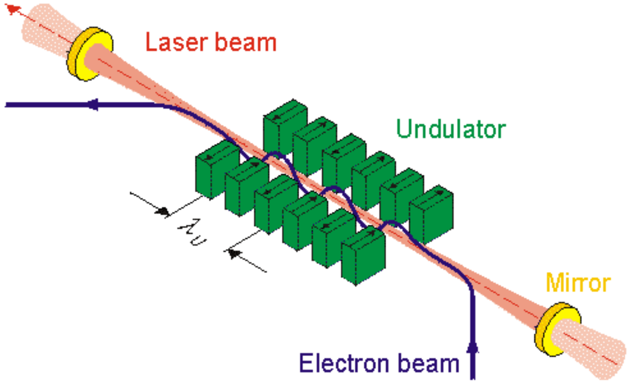


schematic of an FEL oscillator

Not a coventional laser ! - electrons are 'free' in the sense that they are not bound to atoms as in conventional lasers

(but not completey free since they are under the influence of magnetic forces which cause them to radiate)

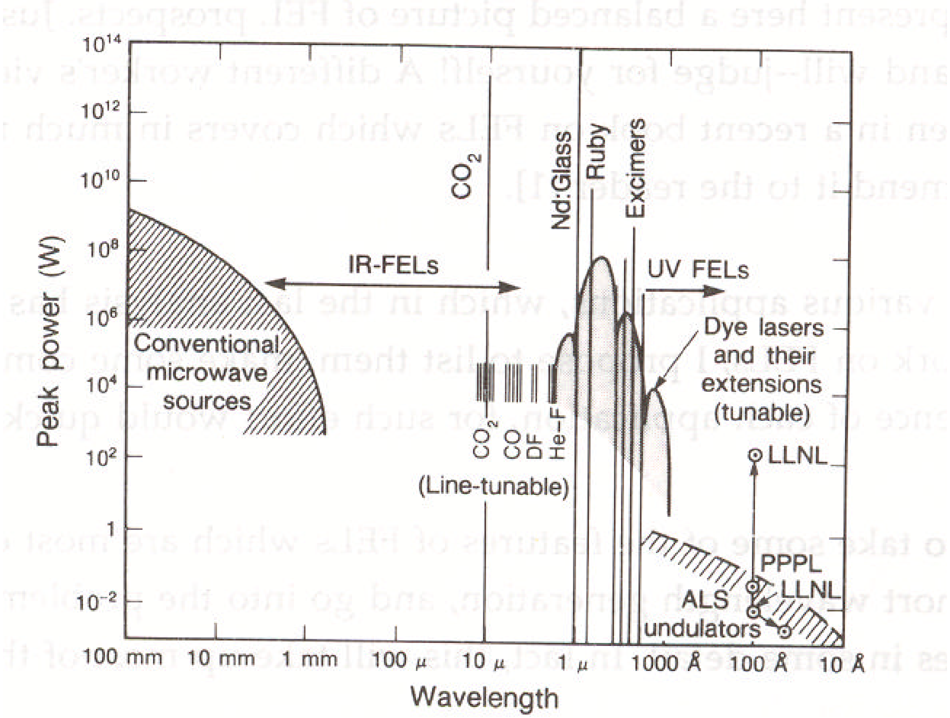
Main features of FELs



Unlike conventional lasers:

- the wavelength of the radiation depends on the electron beam energy and magnetic field strength and hence is **continuously tuneable**
- FELs are capable in principle of being extended to very **short wavelengths**. FELs have operated from the mm-wave regime through infra-red and visible and into the vacuum ultra-violet (100 nm) and various projects aim at 1Angstrom.
- no lasing medium, hence no breakdown problems; possibility of obtaining **high peak and average power levels**
- **flexible time structure** of the radiation: pulse length and repetition frequency is determined by that of the electron beam and hence can be manipulated relatively easily

Role of the FEL vs. Other Light Sources



The main role for the FEL lies in the

Curent Status:

- far-infrared (FIR) (10 μm - 1 mm) → mature technology; several user facilities
- vacuum ultra-violet (VUV) (200 - 10 nm) → Current R&D
- X-ray regions (10 nm - 0.1 nm) → Proposed future R&D

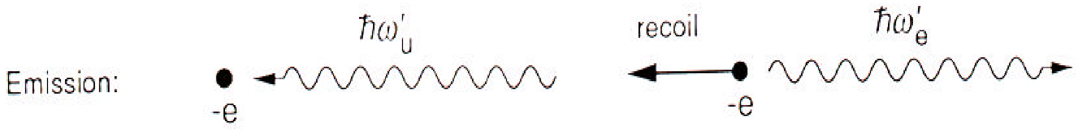
Applications of the FEL

- Physics/Chemistry
  - IR : solid state physics, semiconductors, surface chemsitry etc.
  - VUV: electronic excitations, photochemistry etc.
  - Xray: atomic physics, structure determination etc.
- Biology
  - microscopy, DNA studies, cell response
- Medicine
  - surgery, ablation, photo-therapy (IR)
- Industrial
  - materials processing, microfabrication, photochemistry etc. (IR,UV)
- High power microwave applications
  - power beaming to satellites, plasma heating
  - remote atmospheric sensing etc.
- Accelerators
  - Inverse FEL, Two-beam accelerator
- Nuclear physics
  - gamma ray production by Compton backscattering
- Military (SDI/"Star Wars")

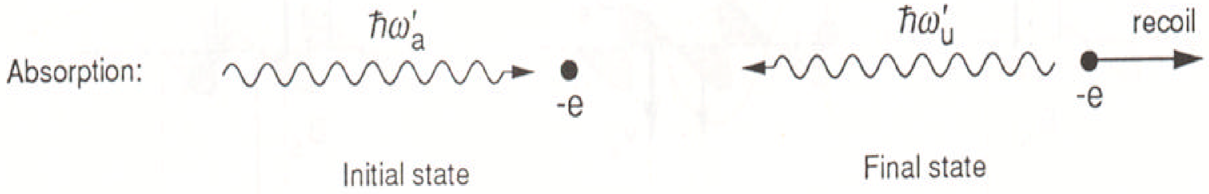
First Description of the FEL:
Stimulated Compton Scattering

The undulator magnetic field is seen by the relativistic electron as an electromagnetic wave

The electrons can either -
i) scatter an undulator "photon" in the forward direction and loose momentum (Emission) :

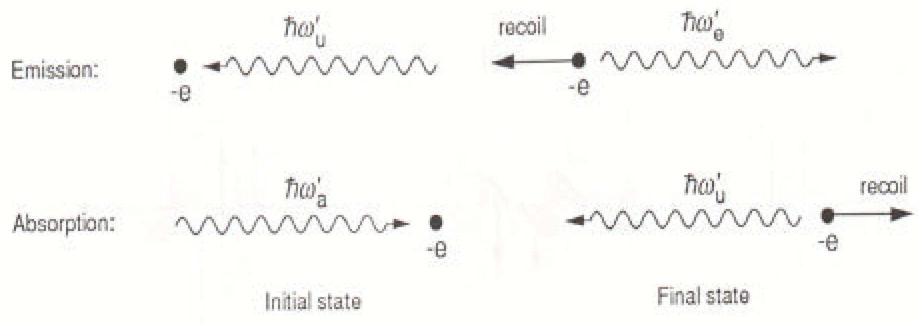


or,
ii) scatter a laser photon in the backward direction and gain momentum (Absorption) :



J.M.J. Madey, J. Appl. Phys., 42 (1971) 1906.

Stimulated Compton scattering

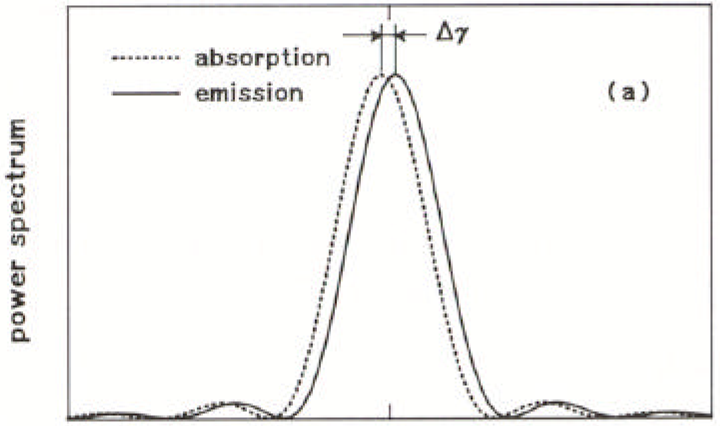


Because of electron recoil:

$$\hbar w'_e < \hbar w'_u \quad \text{and} \quad \hbar w'_a > \hbar w'_u$$

i.e. emission and absorption of a photon of a given frequency requires slightly different "undulator photon" energies, and hence different electron energies.

The probability curves for emission/absorption are therefore slightly shifted in energy:

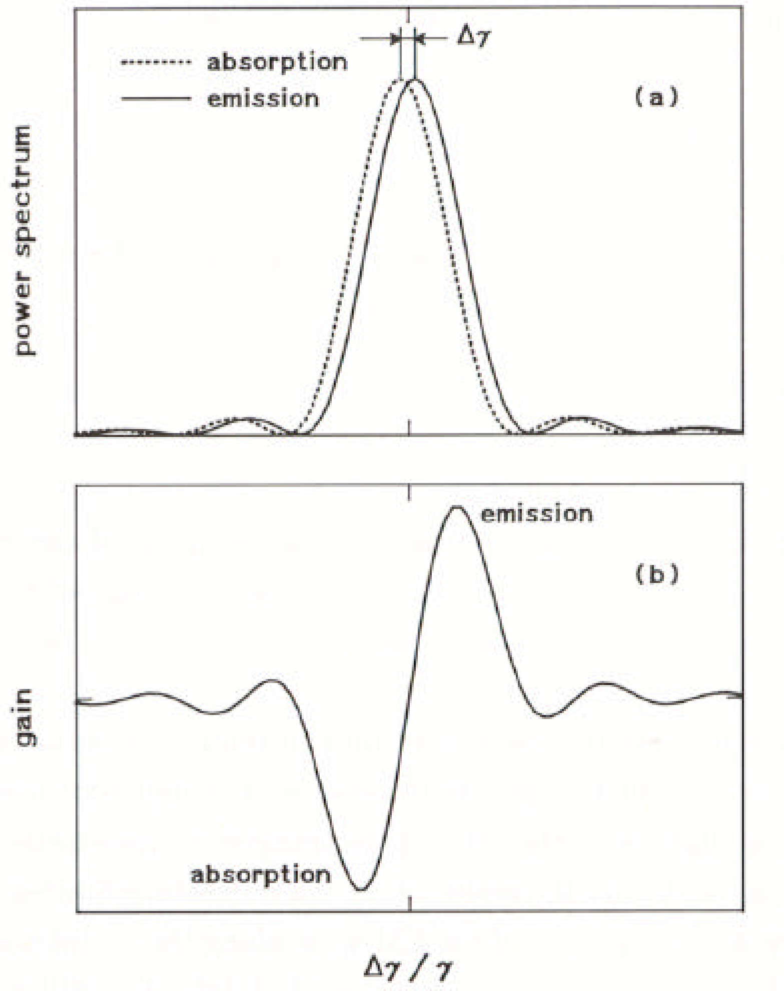


Stimulated Compton scattering

Thus, the "Gain Curve"
i.e. rate of (emission - absorption) is the derivative of the spontaneous emission curve :

"Madey's Theorem"

- a useful general result that allows the influence on the gain to be determined from the effect on the spontaneous emission spectrum (which is easier to calculate, and measure).



Stimulated Compton scattering

Madey's work is closely related to a proposal for Stimulated Compton scattering of a relativistic electron beam from microwave radiation

(Pantell et al., 1968):

itself following work going back to Kapitza and Dirac (1933)

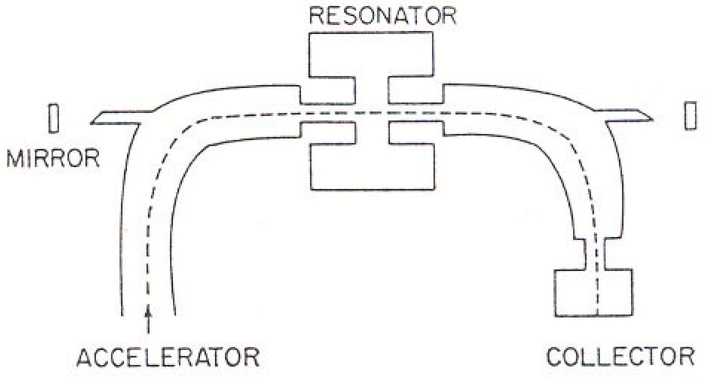


Fig. 1. Physical configuration for stimulated Compton scattering at infrared wavelengths.

e.g. E = 17.8 MeV, λ_0 = 10 cm, λ = 20 μm

"*The advantages of a Compton laser are that it is voltage tunable over a wide range and may provide intense, coherent radiation in portions of the spectrum where other sources are not readily available*"

The main difference of Madey's proposal with respect to earlier work is the use of a static magnetic field, rather than an electromagnetic one.

R.H. Pantell et al., IEEE J. Quantum Electr. QE-4 (1968) 905.

But is it a "Laser" ?

The first analysis of the FEL (Madey, 1971) was made using quantum theory, and the physical principles of FEL operation were considered different to those of earlier devices.

It was noted however that $\hbar$ cancelled out of the final equations and many doubts were expressed whether it was a 'true' laser ...

Later, a fully classical picture was developed* (Hopf et al., Colson, 1976):

> ***"the quantum theory of a free-electron laser is extremely tedious, and is neither desirable nor necessary"   Hopf et al., 1976***

The physical picture is of electrons **bunching** on the scale of the radiation wavelength and so emitting radiation coherently.

Slightly later a connection was made with earlier theoretical work showing that the FEL did indeed operate according to the same principles as earlier devices (Kroll et al., 1978)and so it eventually became clear that the FEL was essentially the latest in a long series of electron beam devices that generate coherent radiation.

* *but also separately in R.B. Palmer, J. Appl. Phys. 43 (1972 3014.*

Bunching

For electrons having different longitudinal positions the Electric field of the emitted radiation depends on the phase with respect to the radiation wavelength: $f = 2p\, z / l$

Electric field: $E = E_o \sum_{k=1}^{N_e} \exp(i f_k) = E_o\, B$

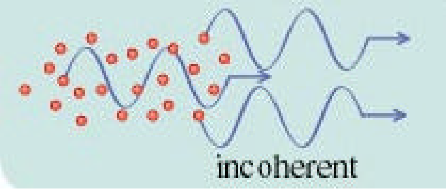
Intensity: $I = I_o \left| B^2 \right|$

Bunching factor

1) **uniform distribution**: $B = 0, \quad I = 0$

2) **random distribution:**

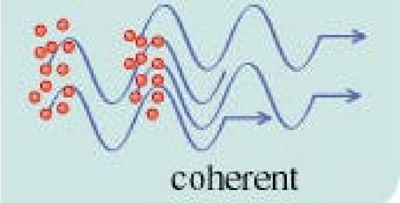
$\langle B \rangle = 0, \quad \langle |B^2| \rangle = N_e, \quad I = I_o N_e$



usual case:
spontaneous emission,
synchrotron radiation etc.
Intensity ~ N_e

3) **electrons all in phase:**

$\langle B \rangle = N_e, \quad \langle |B^2| \rangle = N_e^2, \quad I = I_o N_e^2$

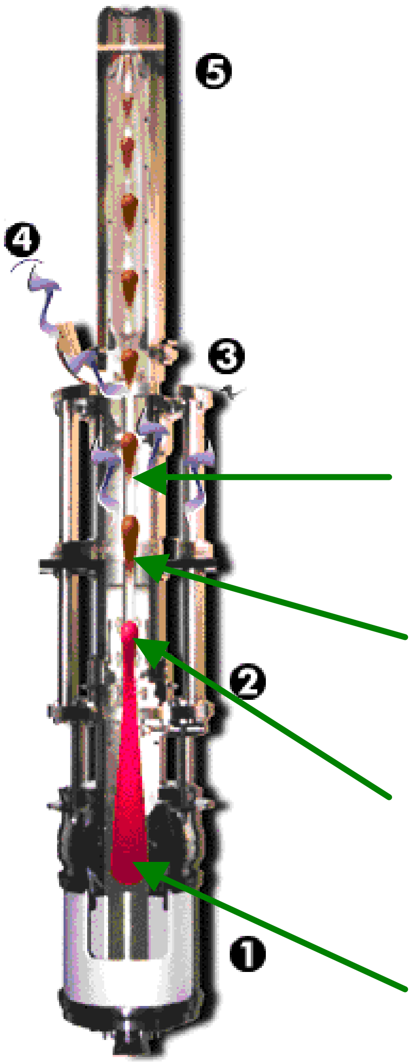


coherent emission:
Intensity ~ N_e^2

Historical Background to the FEL - the Klystron

In the first microwave devices (triode) a bunched beam was produced by direct modulation of the beam intensity.

In 1937 the klystron was invented capable of much higher`frequency operation, using a new technique of <u>velocity modulation</u>:



output cavity - the bunched beam delivers power to the electromagnetic field

drift region - velocity modulation converts to density modulation

input cavity - r.f. voltage produces a velocity modulation

d.c. electron gun

The Travelling Wave Tube

(Kompfner, 1947; Pierce, 1950)

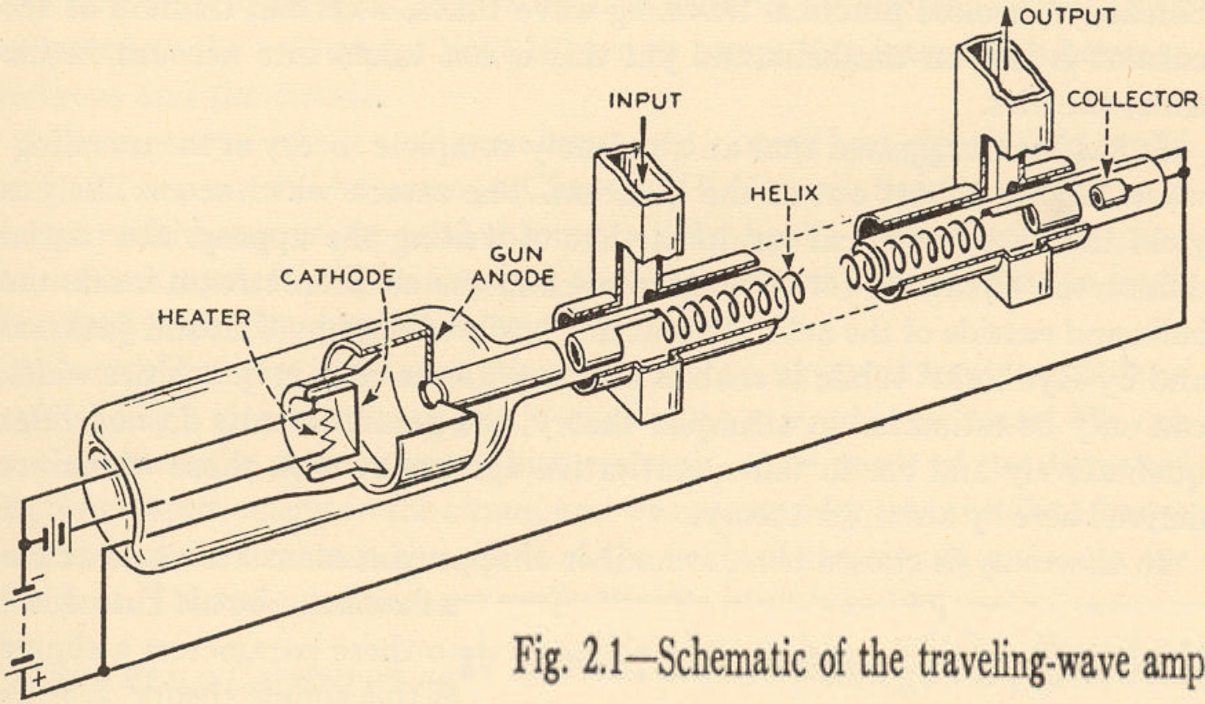
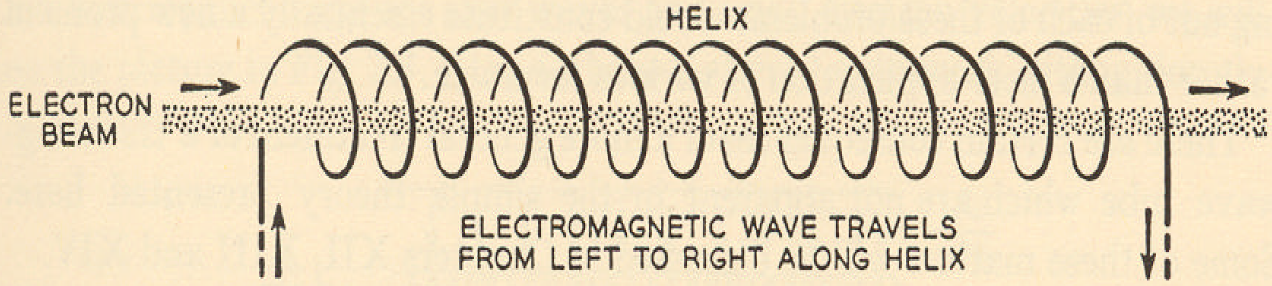


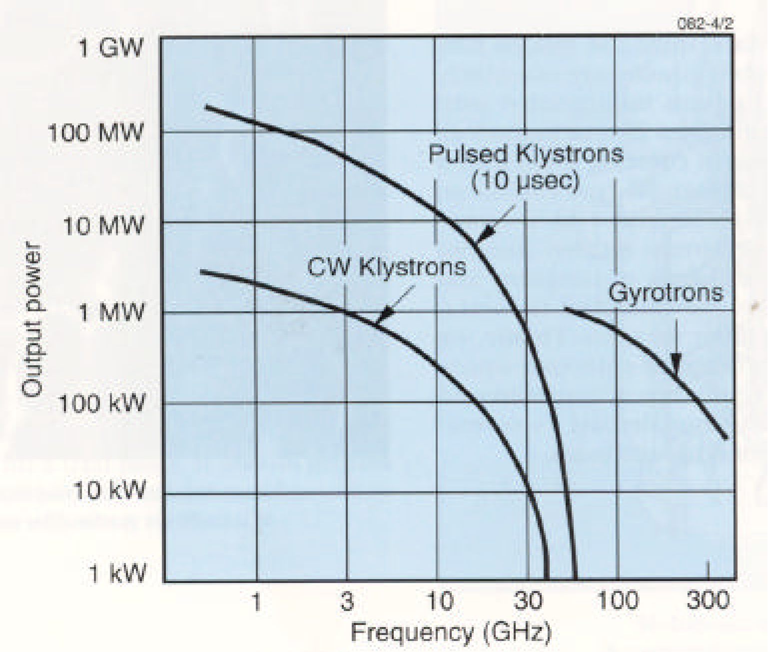
Fig. 2.1—Schematic of the traveling-wave amplifier.

the helix slows down the electromagnetic wave allowing synchronism with the electron beam

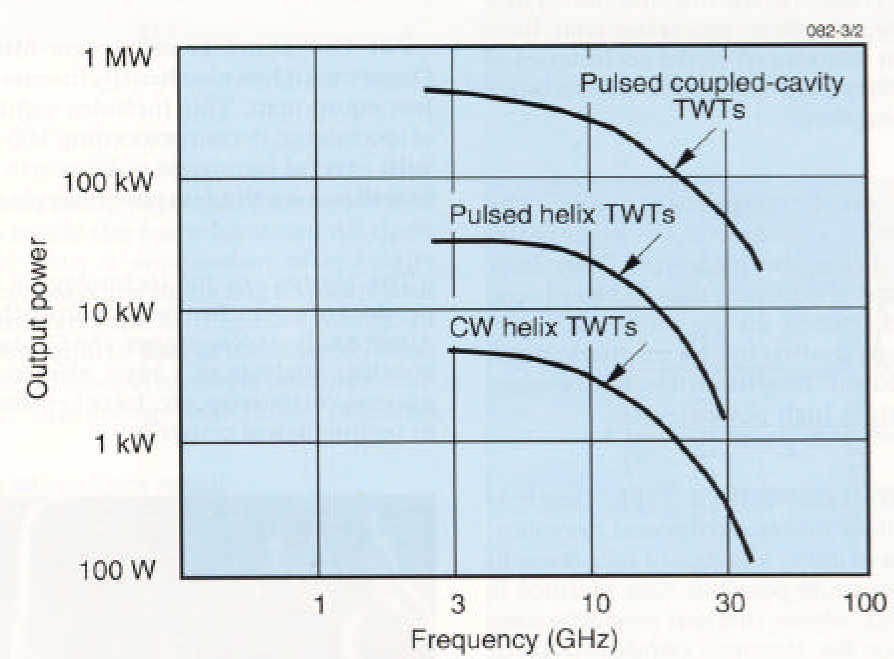


In these devices, the electromagnetic radiation is either contained in cavities (klystron, magnetron) or propagated along a loaded waveguide (TWT) to slow down the radiation to allow synchronism between radiation and electron beams ("slow-wave" structures).

The minimum wavelength is limited by the difficulty of fabricating small cavity and waveguide structures:



High-power Klystrons and Gyrotrons



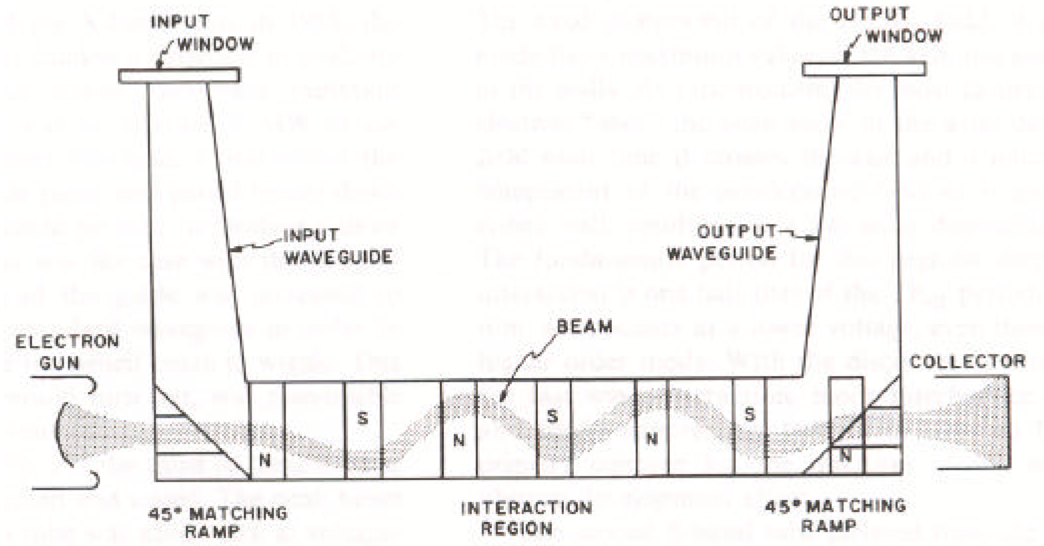
Traveling-wave tubes (TWTs)

The Ubitron

Undulating **B**eam **I**n**t**e**r**acti**on**

Invented in 1957 (Phillips).

A "fast wave" structure with a new interaction mechanism - undulation of the electron beam.



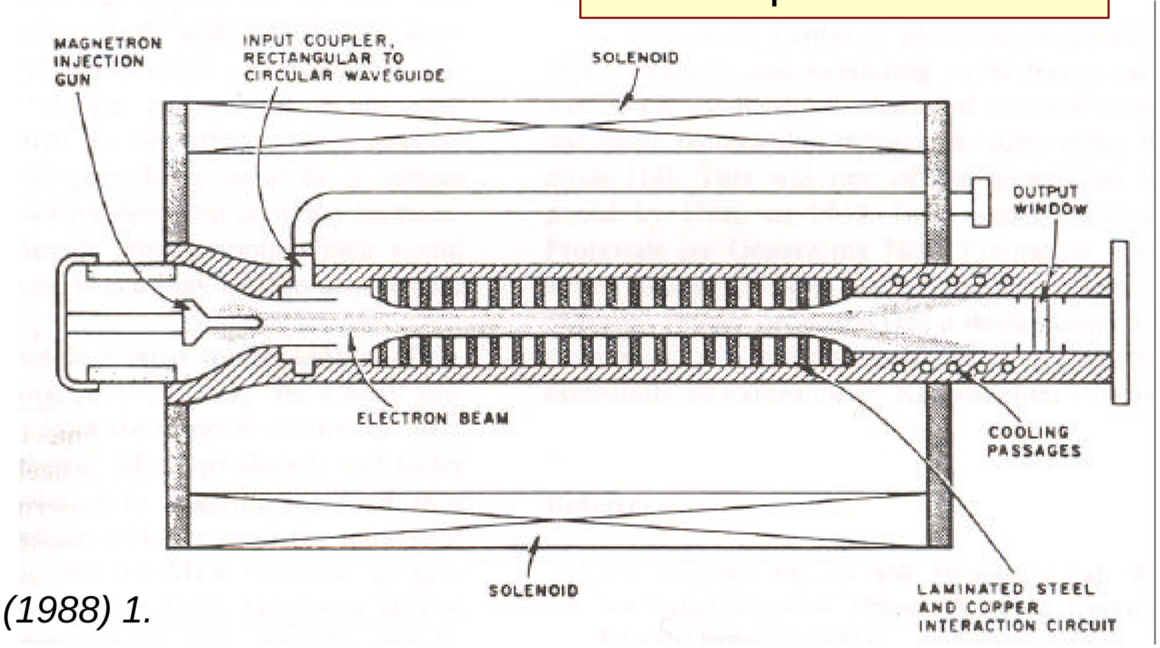
both a microwave tube and a non-relativistic FEL amplifier.

The first S-band Ubitron
(3 GHz, λ ~ 10 cm)

V-band Ubitron,
54 GHz, λ ~ 5 mm

70 kV beam energy

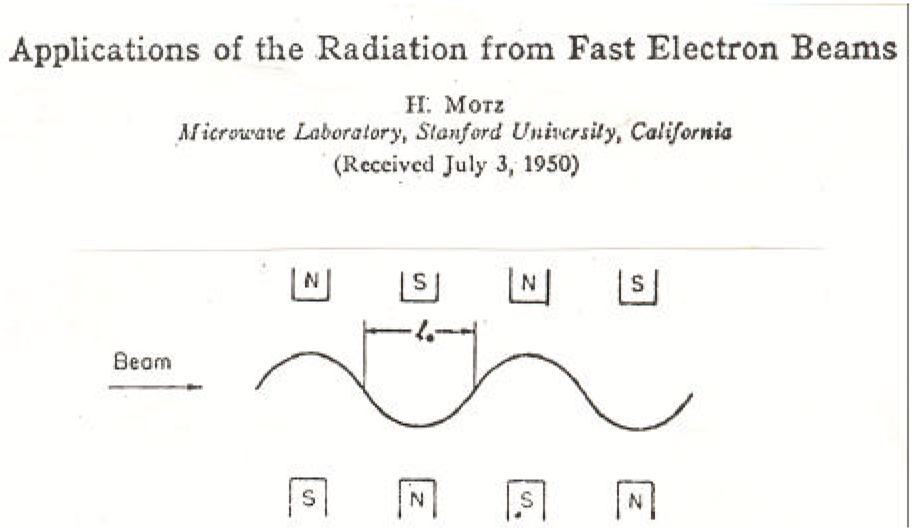
150 kW output power



R.M. Phillips, Nucl. Instr. Meth. A272 (1988) 1.

The Undulator

Invented in 1951 by Motz as a means of producing coherent millimetre waves from a pre-bunched electron beam.



Applications of the Radiation from Fast Electron Beams

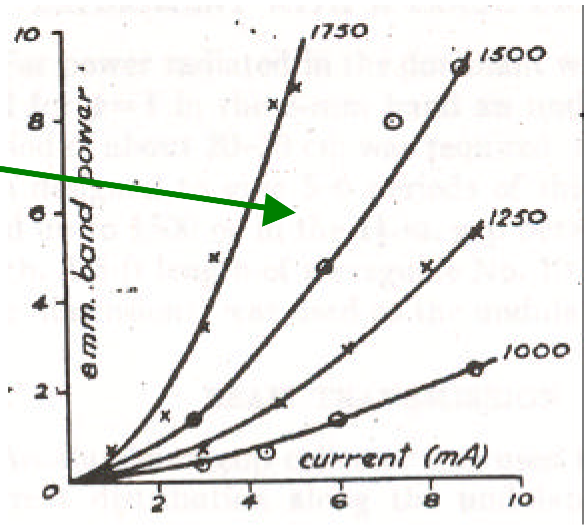
H. Motz
Microwave Laboratory, Stanford University, California
(Received July 3, 1950)

H. Motz ,J. Appl. Phys. 22 (1951) 527.

Later experiments (1961) with a 3-5 MeV electron beam showed semi-coherent emission of 6-8 mm band radiation with

$$\text{Intensity} \sim (\text{current})^{1.7}$$

indicating that the bunch length was about 10 mm.



H. Motz and D. Walsh, J. Appl. Phys. 33 (1962) 978.

Visible radiation was also generated using a 100 MeV electron beam (Motz, 1953)

The Undulator Amplifier

It was also shown (Motz, 1959) using the same analysis as for a TWT, that the undulator could be used to amplify a radiation beam - "***fast wave amplification***".

Essentially this was a relativistic FEL amplifier; the feeding back of the generated radiation to produce bunching at any wavelength was not however considered.

This could have been the start of the FEL development, but as Motz pointed out:

"*in the relativistic range amplification from undulating electrons leads to small gains .. radiation from pre-bunched electrons seems a more hopeful approach..*"

H. Motz and M. Nakamura, Symposium on Millimeter Waves, Brooklyn, 1959.

Historical Background to the FEL

In conclusion,

although the FEL follows on naturally from earlier work on stimulated compton scattering, it also has roots in both the Undulator Amplifier (a relativistic FEL amplifier) and Ubitron (a non-relativistic FEL amplifier) - both fast-wave structures - which themselves had close similarities with the Travelling Wave Tube (slow-wave structure).

FEL Operating Regimes

A. Low-gain (Compton)

single electron interaction; G~ z^3; gain per pass small (G<< 1)
usually are **oscillators**

B. High-gain (Compton)

collective interaction or “instability”; space-charge effects negligible; G ~ exp(z), G >> 1,

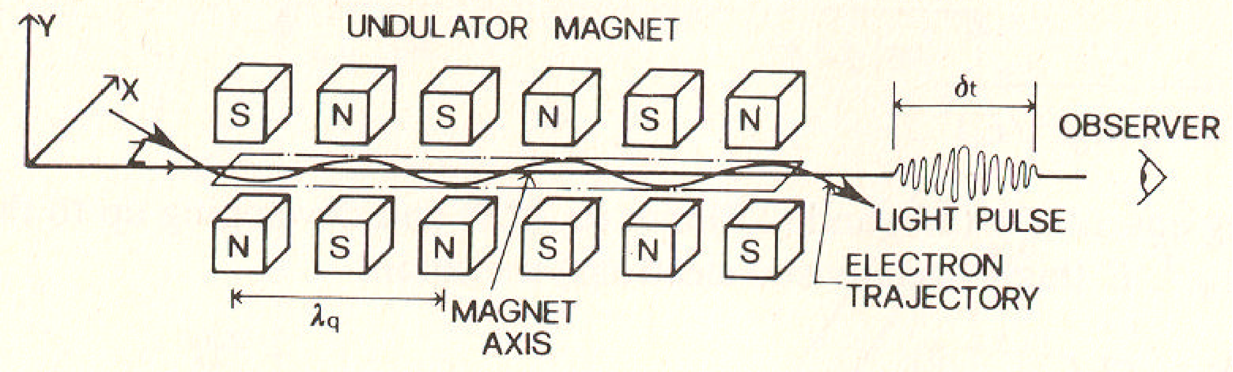
can be **amplifiers**, or build up from noise (“super-radiant” or “Self Amplified Spontaneous Emission”, **SASE**)

C. High-gain (Raman)

“three-wave” device; electron beam is sufficiently dense, and low energy, that the collective interaction with space-charge (plasma) oscillations waves is dominant

can be **amplifiers**, **oscillators**, or **SASE**

Electron Motion in an Undulator



vertical sinusoidal
field component:

$$B_y = B_o \cos(kz), \quad k = 2p / l_o$$

equation of motion
from $\underline{F} = e(\underline{E} + \underline{v} \wedge \underline{B})$

$$\ddot{x} = -\frac{e}{g\, m} \dot{z} B_y$$

$$\left\{ g = \frac{E}{mc^2} = \frac{E[MeV]}{0.511} \right\}$$

integrating gives
the velocity:

$$\dot{x} = -\frac{eB_o}{g\, m} \frac{\cos(kz)}{k} \qquad b_x = \frac{\dot{x}}{c} = \frac{K}{g} \cos(kz)$$

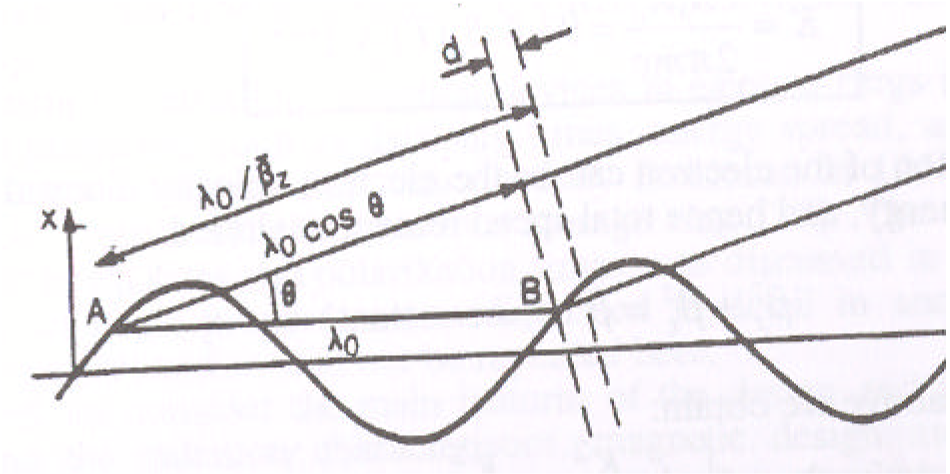
where: $$K = \frac{e B_o\, l_o}{2p\, mc} = 0.93 B_o[T]\, l_o[cm]$$ (dimensionless)

since $b_x^2 + b_z^2 = b^2$ $(= \text{constant})$

$$\bar{b}_z \cong b\left(1 - \frac{K^2}{4g^2}\right) = 1 - \frac{1}{2g^2} - \frac{K^2}{4g^2}$$

NB] the <u>average velocity</u> along z is reduced due to the undulating motion

Interference Condition



distance between wavefronts emitted from points A and B

$$d = \frac{\lambda_o}{\bar{\beta}_z} - \lambda_o \cos\theta$$

constructive interference if

$$d = n\,\lambda$$

using the previous result

$$\bar{\beta}_z = 1 - \frac{1}{2\gamma^2} - \frac{K^2}{4\gamma^2}$$

$$\boxed{\lambda = \frac{1}{n}\frac{\lambda_o}{2\gamma^2}\left(1 + \frac{K^2}{2} + \gamma^2\theta^2\right)}$$

in the ideal case, on-axis, n = 1, 3, 5 etc.

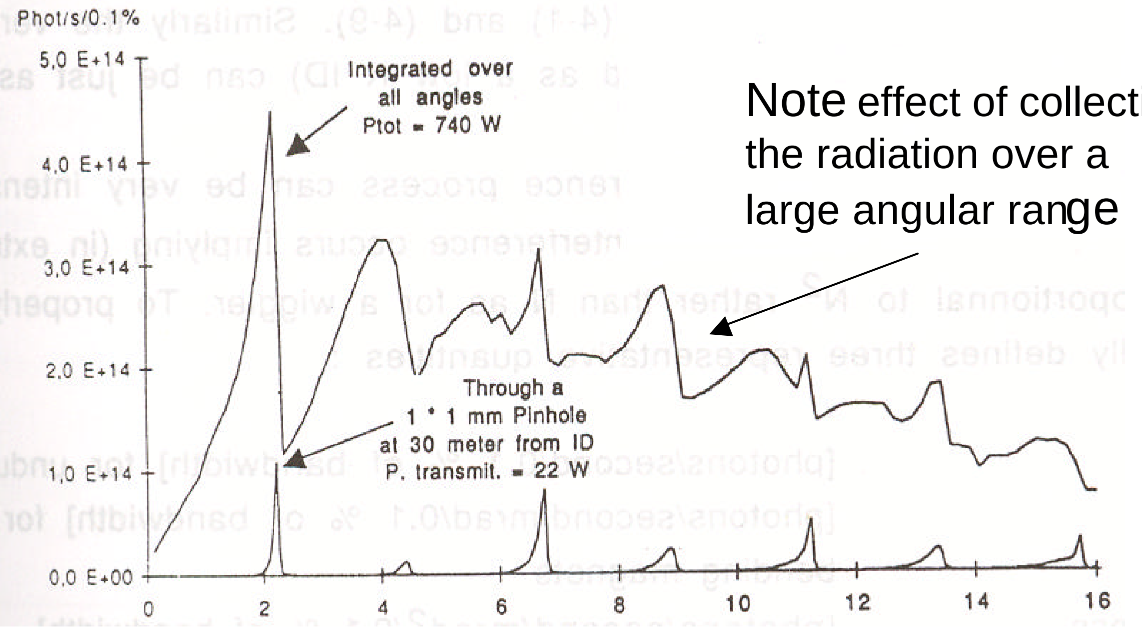
Spontaneous (Undulator) Radiation

$$\boxed{l = \frac{1}{n}\frac{l_o}{2g^2}\left(1+\frac{K^2}{2}+g^2q^2\right)}$$

The spontaneous radiation emitted in an undulator consists of a series of harmonics of a fundamental, which has a wavelength much smaller than the magnet period, l_o (γ >> 1)

$$\left\{g = \frac{E}{mc^2} = \frac{E[MeV]}{0.511}\right\}$$

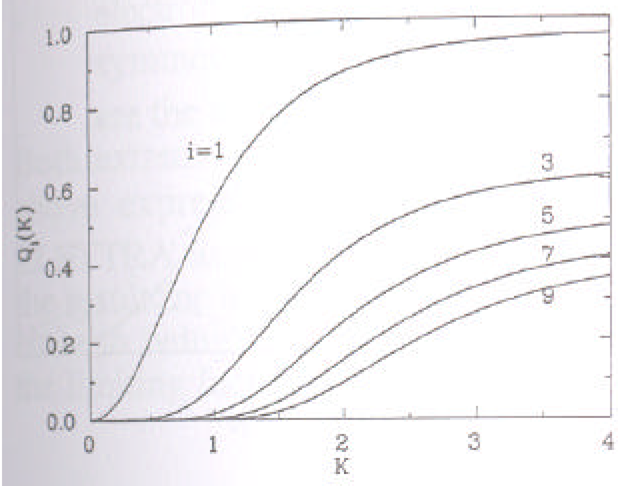
Typical undulator radiation spectra



photon energy

Note effect of collecting the radiation over a large angular range

Number of harmonics in the spectrum increases rapidly with K (~ K^3):



The Helical Undulator

$$\underline{B} = B_o\left(\cos(k_o z)\hat{x} + \sin(k_o z)\hat{y}\right) \quad k_o = 2p/l_o$$

$$\underline{b} = -\frac{K}{g}\left(\cos(k_o z)\hat{x} + \sin(k_o z)\hat{y}\right)$$

$$b_x^2 + b_y^2 + b_z^2 = b^2 \quad (= \text{constant})$$

$$\bar{b}_z = 1 - \frac{1}{2g^2} - \frac{K^2}{2g^2}$$

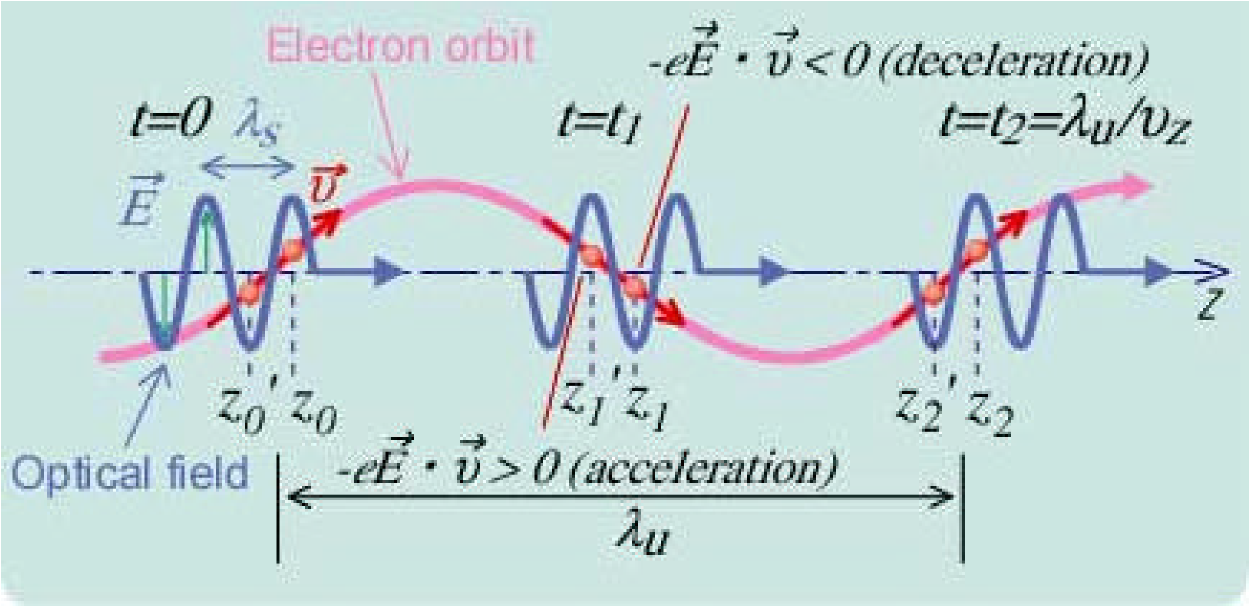
$$\boxed{l = \frac{l_o}{2g^2}\left(1 + K^2 + g^2 q^2\right)}$$

NB] Higher symmetry
results in only one
harmonic on-axis

Resonance Condition

$$\boxed{l = \frac{l_o}{2g^2}\left(1+\frac{K^2}{2}\right)}$$

The same equation gives also the condition for **synchronism** ("resonance condition") between an electron and an external radiation beam :
the electrons slip by one radiation wavelength for each magnet period.



Since:

$$\dot{g} = \frac{e}{mc}\underline{b}\bullet\underline{E}$$

the transverse motion allows an energy exchange between the electron and radiation beams.

A systematic energy exchange can therefore take place, which depends on the initial phase of the electrons with respect to the radiation, resulting in an <u>energy modulation</u>, and hence <u>density modulation</u> (bunching) on the scale of the radiation wavelength.

The FEL Interaction

consider for simplicity a helical magnet:

$$\underline{B} = B_o\left(\cos(k_o z)\hat{x} + \sin(k_o z)\hat{y}\right) \quad k_o = 2p / l_o$$

$$\underline{b} = -\frac{K}{g}\left(\cos(k_o z)\hat{x} + \sin(k_o z)\hat{y}\right)$$

$$\underline{E} = E_o\left(\sin(kz - wt + f_o)\hat{x} + \cos(kz - wt + f_o)\hat{y}\right) \quad k = 2p / l = w/c$$

$$\dot{g} = \frac{e}{mc}\underline{b} \bullet \underline{E}$$

$$\dot{g} = -\frac{eE_o K}{gmc}\sin\Phi \qquad \Phi = (k + k_o)z - wt + f_o$$

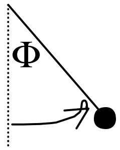
$\Phi = f_o$ when $(k + k_o)z = wt$

i.e. $g = g_r$ with $l = \frac{l_o}{2g_r^2}(1 + K^2)$ same as the interference condition.

The FEL Interaction

for small deviations from resonance define $\eta = (\gamma - \gamma_r)/\gamma_r$

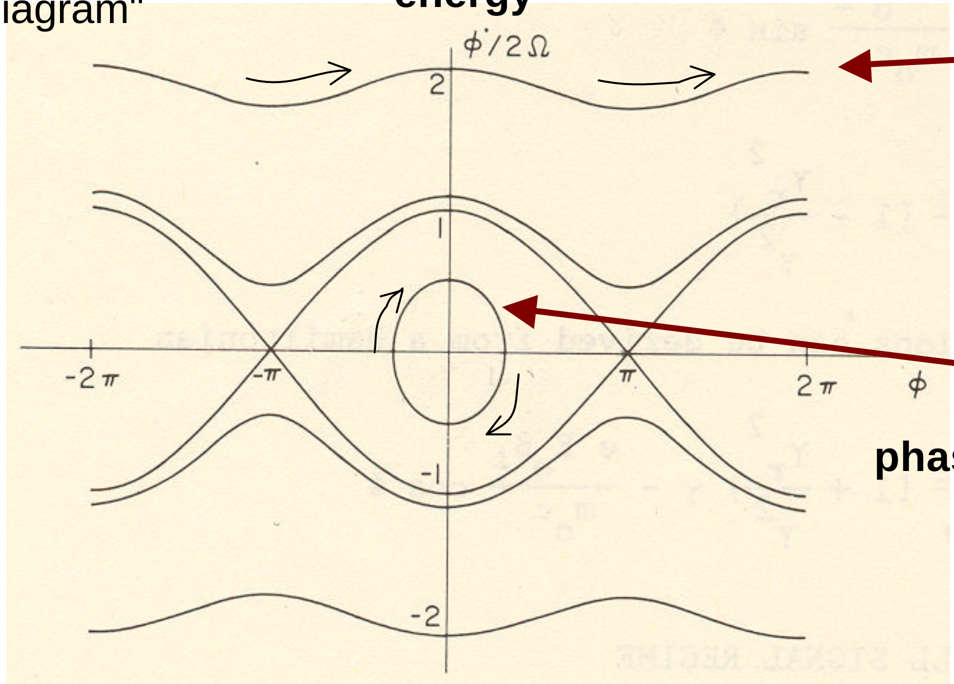
$$\left.\begin{aligned} \dot{\eta} &= -\frac{eE_oK}{\gamma_r^2 mc}\sin\Phi \\ \dot{\Phi} &= \frac{4\pi c}{\lambda_o}\eta \end{aligned}\right\} \ddot{\Phi} = -\Omega^2 \sin\Phi$$



= the motion of a simple pendulum; same as synchrotron motion in a storage ring

"phase space diagram"

energy



phase

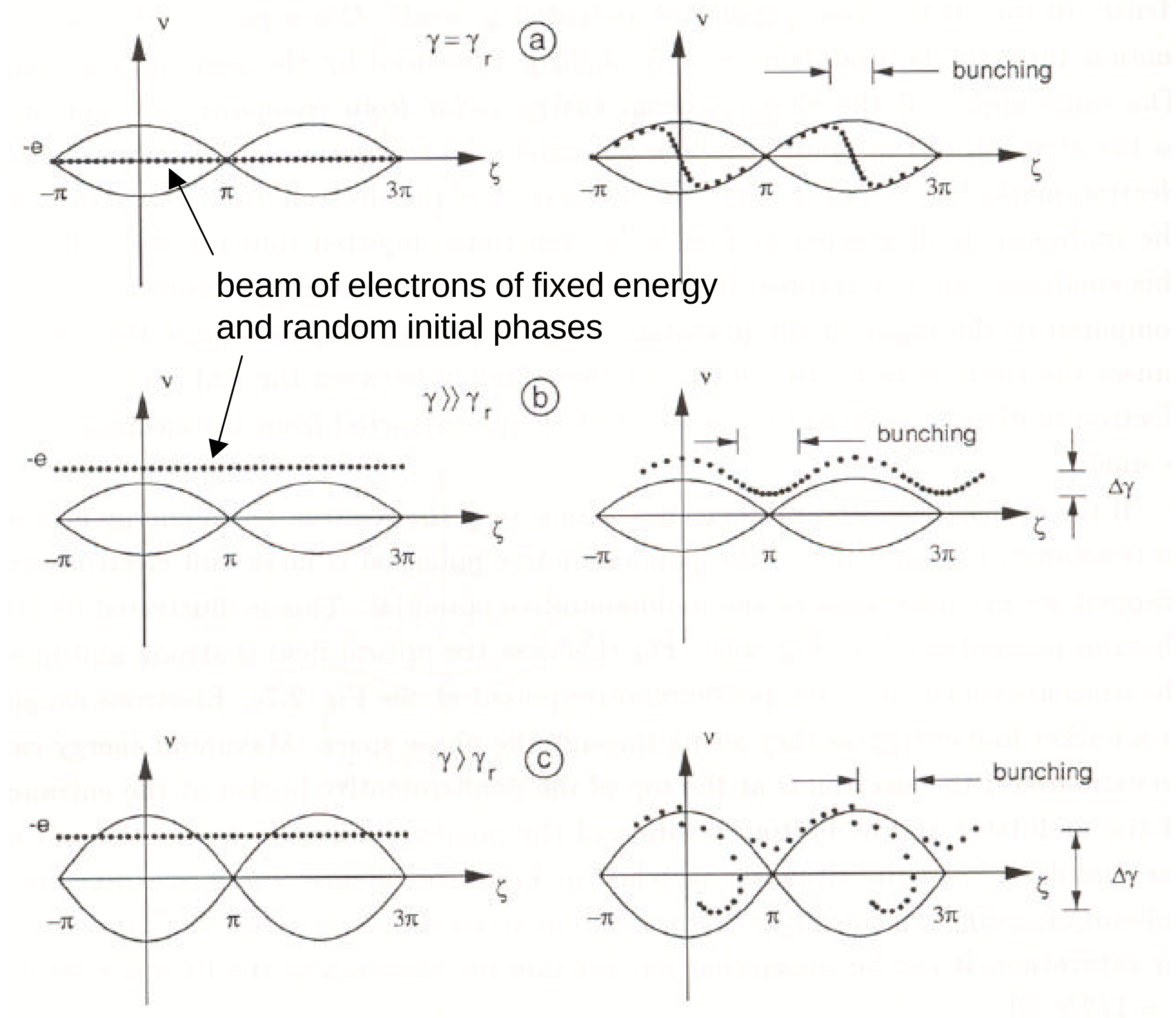
for large Φ,electrons on open trajectories

for small Φ, electrons are "trapped" on closed trajectories (simple harmonic motion)

Motion in phase space

undulator entrance

undulator exit



beam of electrons of fixed energy
and random initial phases

On-resonance :

- **bunching**
- **no net energy exchange**

Far from resonance :

- **no electrons trapped**
- **some bunching**
- **small energy transfer**

Close to resonance :

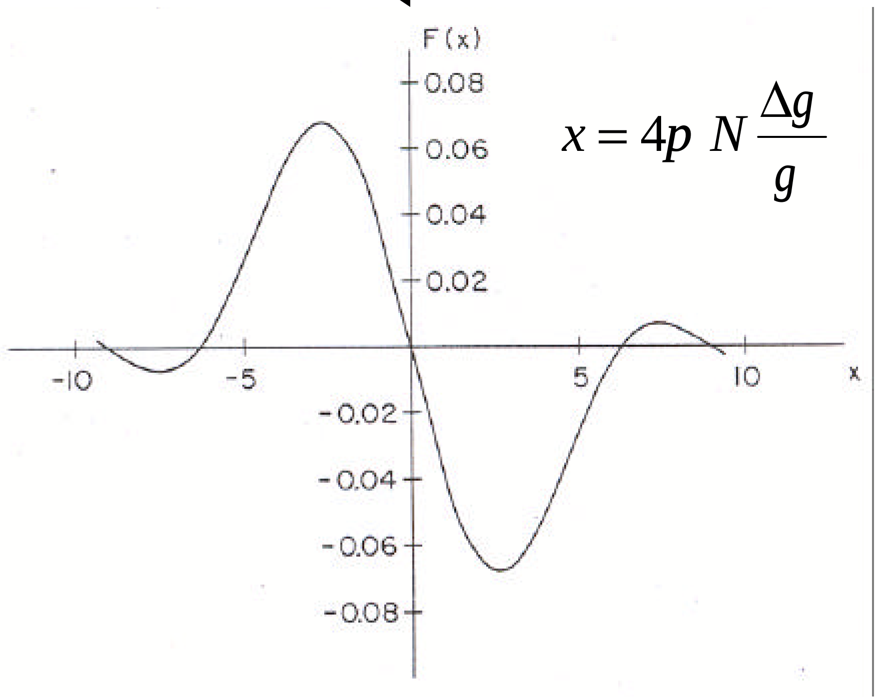
- **many electrons trapped**
- **strong bunching**
- **large induced energy spread**

Small Signal Gain

Averaging the energy loss/gain over all phases,and dividing by the radiation beam intensity results in the following expression for the Gain per pass:

$$G = -\frac{32\sqrt{2}\pi^2 \lambda^{3/2} \lambda_o^{1/2}}{\Sigma} \frac{K^2}{(1+K^2)^{3/2}} \frac{I_{peak}}{I_A} N^3 F(x)$$

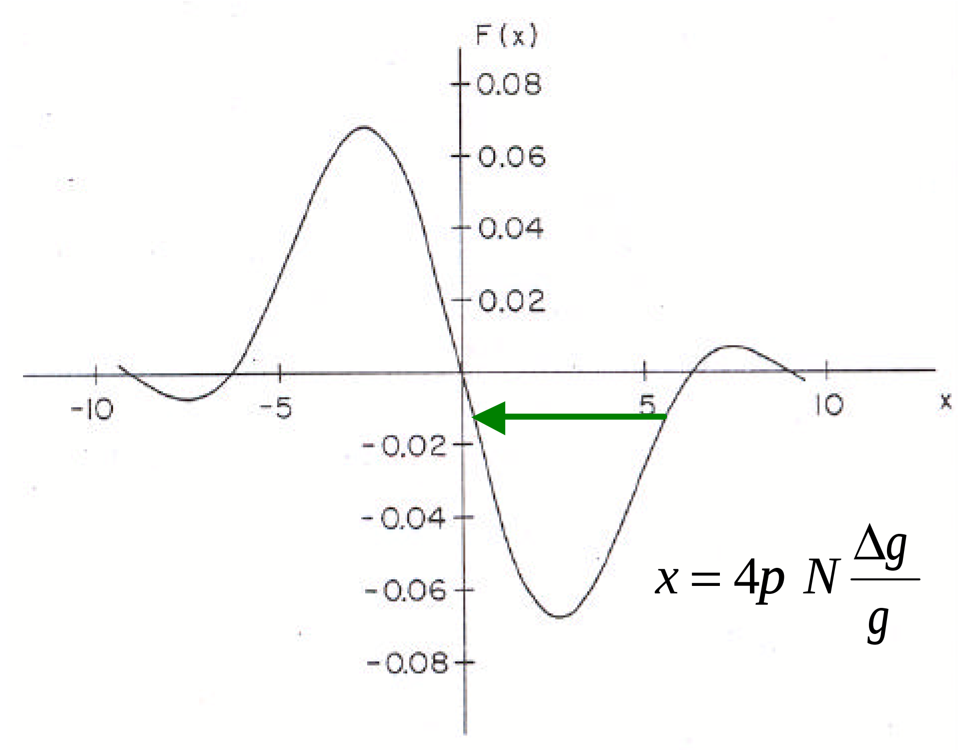
- depends linearly on peak current
- decreases with decreasing wavelength



$$x = 4\pi N \frac{\Delta\gamma}{\gamma}$$

NB] F(x) < 0 means gain !

The Gain Curve



$$x = 4p\ N \frac{\Delta g}{g}$$

- the maximum energy transfer from an electron to the radiation is

$$(\Delta g)_{\max} / g \approx 1/2N$$

- the maximum power that can be extracted is therefore:

$$P_{laser} = (1/2N)\, I_b\, E$$

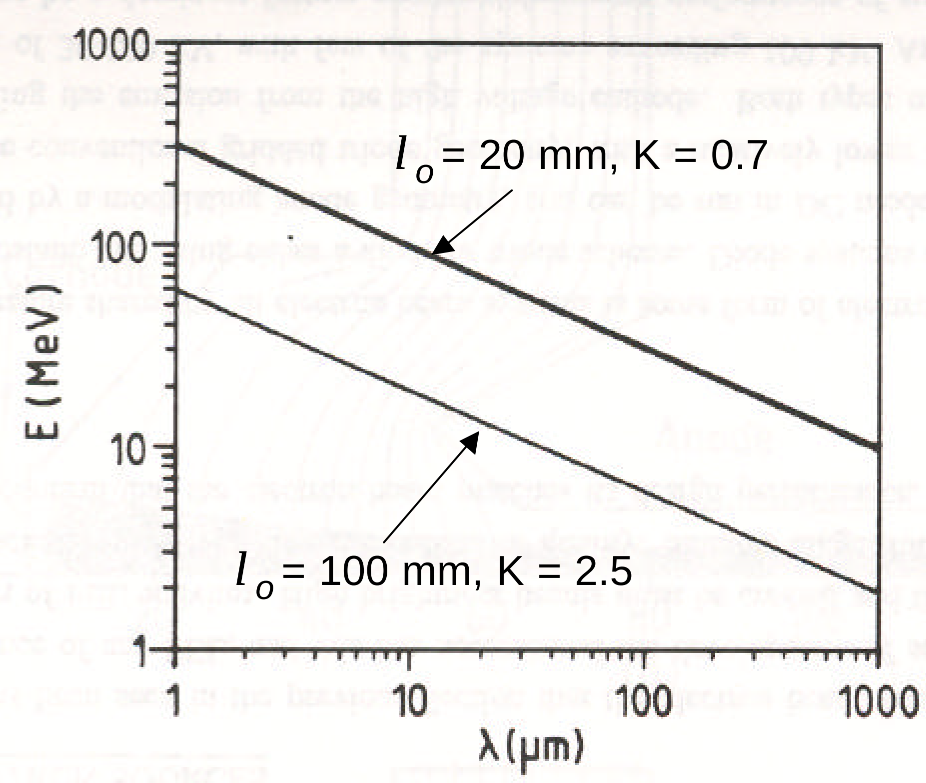
- the energy spread of the electrons must be less than this:

$$s_g / g < 1/2N$$

Electron beam energy and FEL wavelength

$$l = \frac{l_o}{2g^2}\left(1+\frac{K^2}{2}\right) \qquad K = 0.93 B_o[T]\, l_o[cm]$$

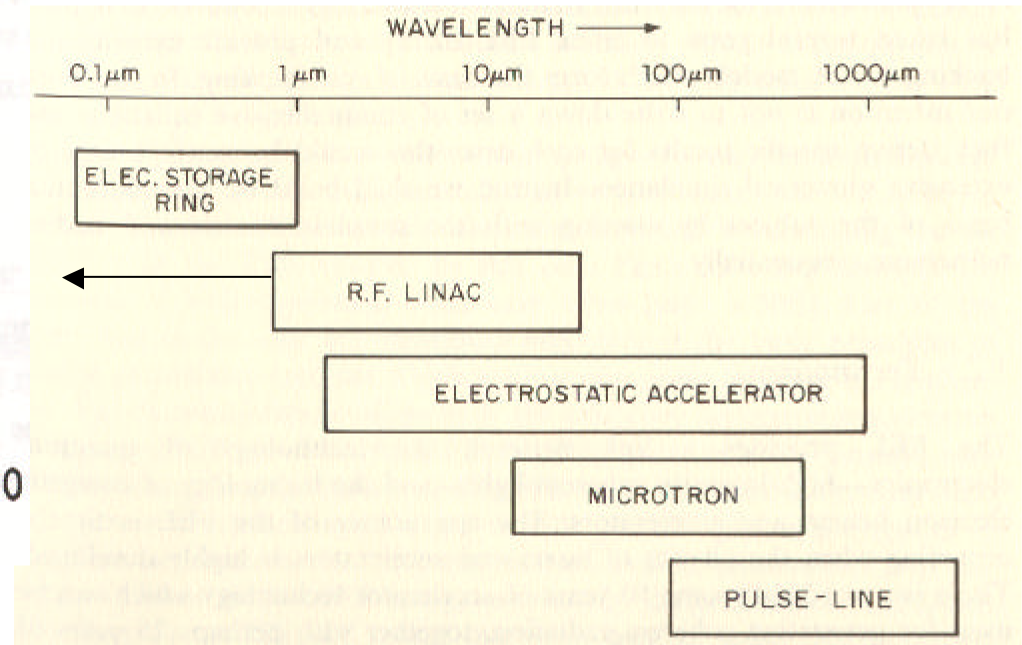
Due to magnet technology reasons, as the period reduces, so does the field amplitude, and hence *K*



l_o = 20 mm, K = 0.7

l_o = 100 mm, K = 2.5

choice of electron source for different radiation wavelengths :



Electron Beam Quality

1) **small energy spread** $s_g / g < 1/2N$

2) **small transverse sizes**

- for good overlap with the photon beam $s_{x,y} < \Sigma$

- because focussing effects in the undulator cause position offsets to turn into angular` offsets (and v.v.)

3) **small angular divergence**:

electrons travelling at an angle θ to the axis have effectively a lower velocity:

$$\frac{\Delta g}{g} = \frac{g^2 q^2}{2(1+K^2/2)}$$

and so to stay in resonance:

$$\left\langle q^2 \right\rangle^{1/2} \leq \frac{(1+K^2/2)^{1/2}}{g\sqrt{N}}$$

4) **high peak current**

Electron Beam Quality

Requirements on angular deviation and energy spread can be derived from the effect on the spontaneous radiation spectrum (Madey theorem):

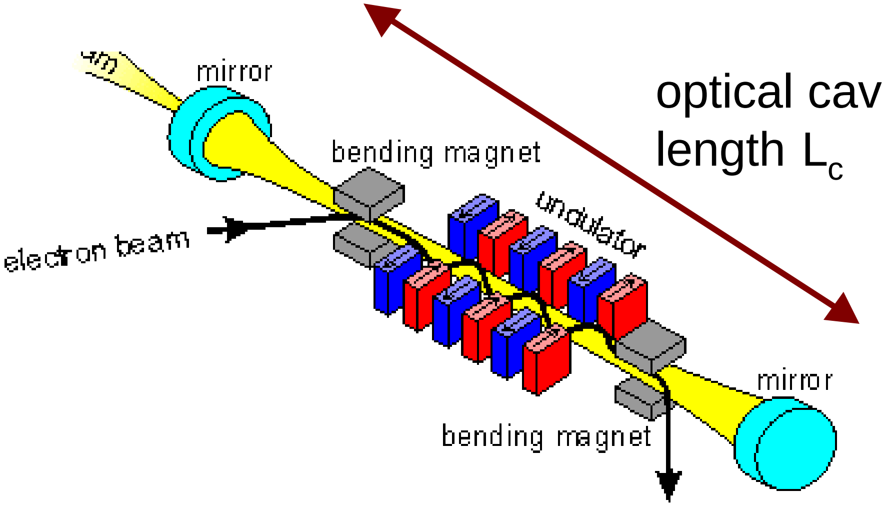
given: $$l = \frac{1}{n}\frac{l_o}{2g^2}\left(1+\frac{K^2}{2}+g^2q^2\right)$$

it follows: $\frac{\Delta l}{l} = 2\frac{\Delta g}{g}$ and $\frac{\Delta l}{l} = \frac{g^2q^2}{1+K^2/2}$

comparing to the natural linewidth $\frac{\Delta l}{l} = \frac{1}{N}$

gives as before $\frac{\Delta g}{g} < \frac{1}{2N}$ and $\left(\Delta q^2\right)^{1/2} < \frac{(1+K^2/2)^{1/2}}{g\sqrt{N}}$

Other Requirements



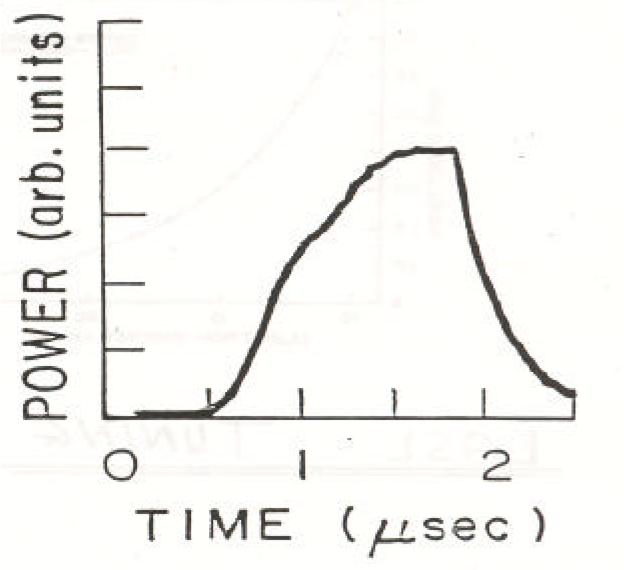
optical cavity length L_c

1) synchronism with the electron beam requires:

$$\frac{2L_c}{c} = \frac{1}{f_{rep.}}$$

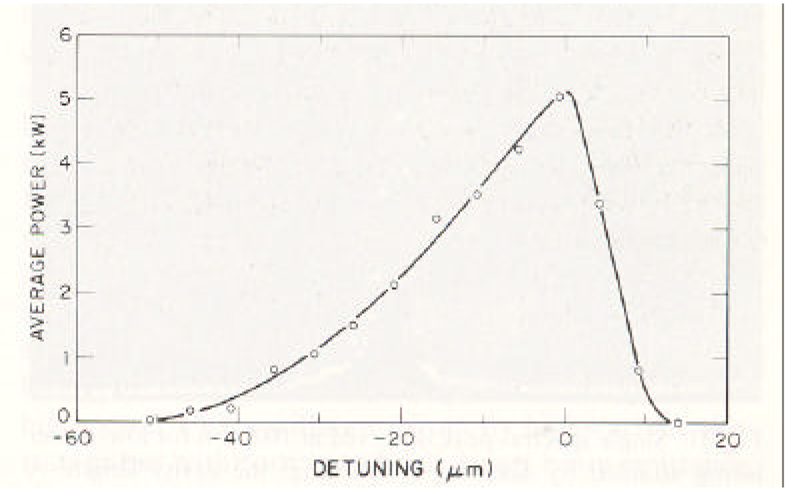
e.g. L_c = 6.9 m (LANL)
$f_{rep.}$ = 214 MHz

2) sufficiently long macropulse to allow build-up to saturation:



Other Requirements

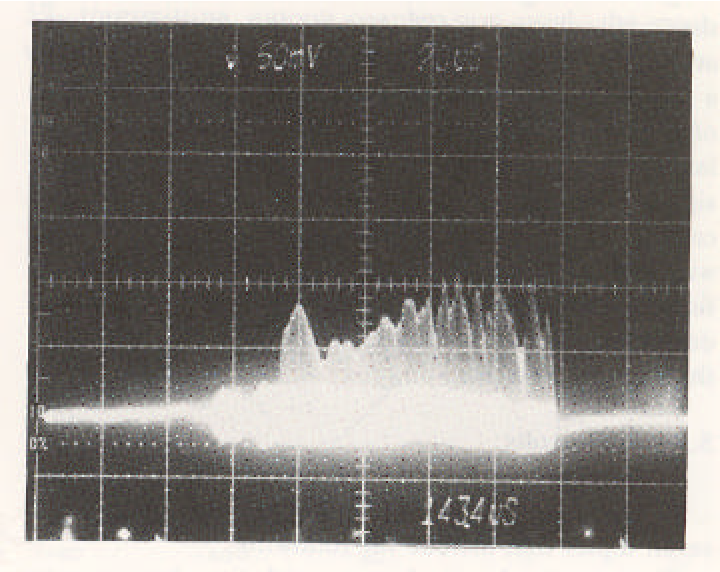
3) correct cavity length



Cavity length detuning curve for the LANL FEL

4) stable electron beam

LANL FEL expts. (1984)
λ = 10 μm
E = 21 MeV
I_{peak} = 40 A
G = 25 %

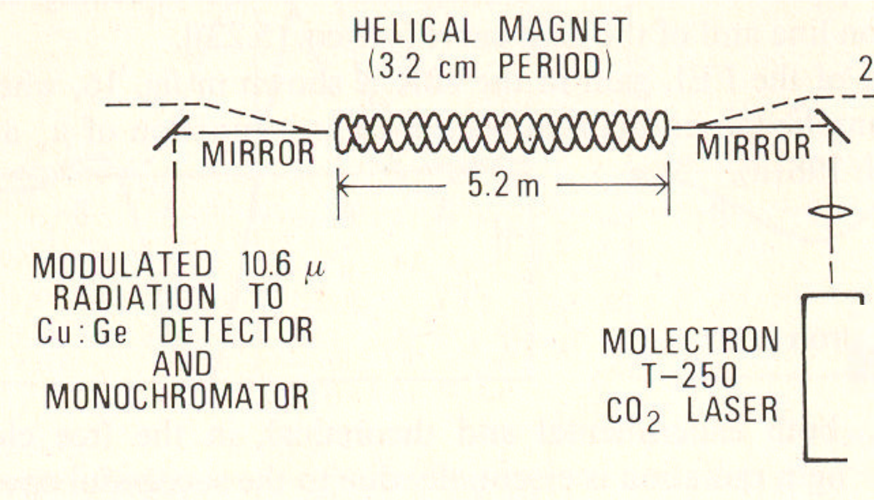


Fluctuations in laser intensity in the LANL FEL:

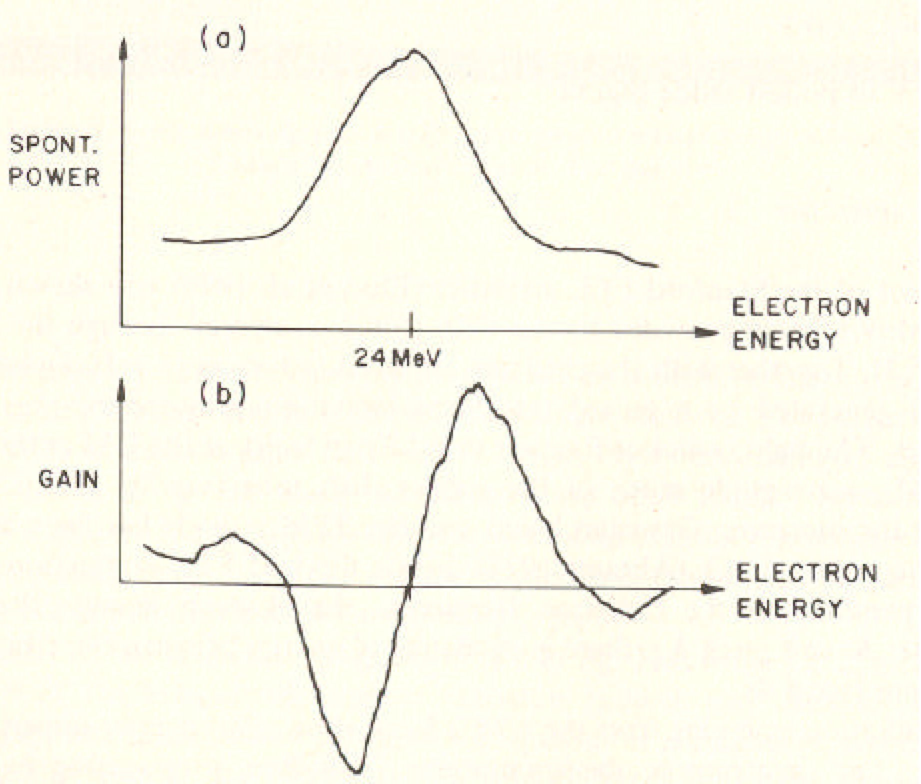
"Rocky Mountain" effect

due to gun and accelerator variations.

The First FEL Amplifier, Stanford 1976



NB] Gain curve = derivative of spontaneous radiation spectrum



E = 24 MeV

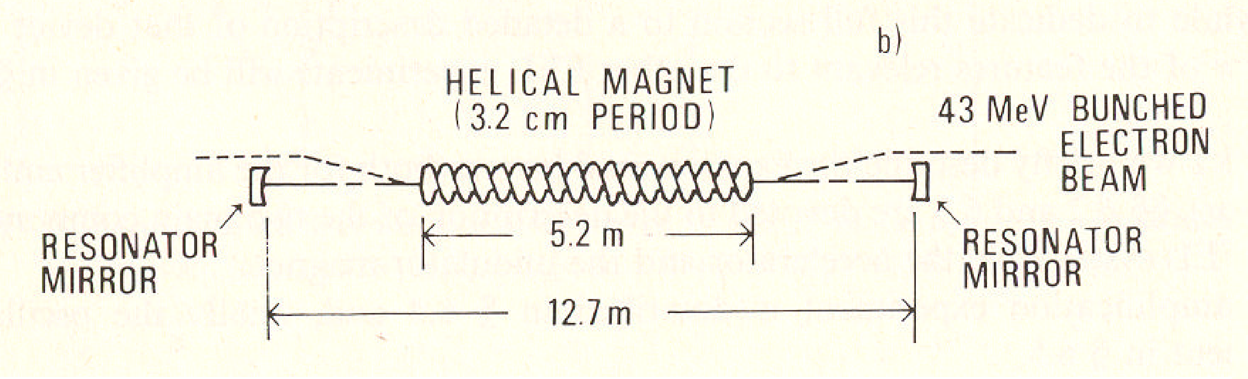
I_{peak} = 70 mA

λ = 10.6 μm

Peak gain = 7 % per pass

L. Elias et al., Phys. Rev. Lett. 36 (1976) 717.

The First FEL Oscillator, Stanford 1977



E = 43 MeV

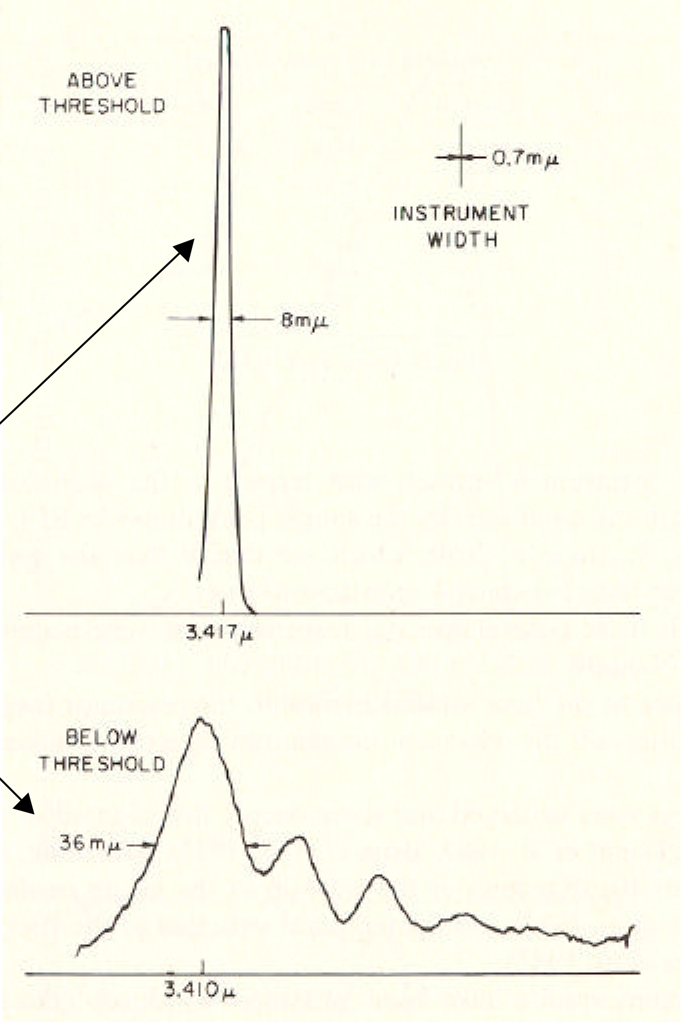
I_{peak} = 2.6 A

λ = 3.4 μm

Average power = 0.36 W

Peak power = 7 kW

Increase in
power 10^8



D. Deacon et al., Phys. Rev. Lett. 38 (1977) 892.

The subsequent years