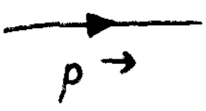
Nonscalar particles have

- additional spin information
- a more complicated propagator



$$= \frac{i}{p^2 - m^2 + i\varepsilon} \left(\underbrace{p^\mu \gamma_\mu}_{\not{p}} + m \right)$$

γ_μ are fixed objects
all is well provided
$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2 g^{\mu\nu}$
$\Rightarrow \gamma_\mu$ are 4×4 matrices:
Dirac matrices

If $p^2 = m^2$ we can find 4-component objects u, v:

$$\not{p} + m = \sum_{\pm s} u(p,s) \bar{u}(p,s) \quad p^0 > 0$$

$$= -\sum_{\pm s} v(p,s) \bar{v}(p,s) \quad p^0 < 0$$

with $p^\mu s_\mu = 0$
$s^\mu s_\mu = -1$

Fermi! $\rightarrow$ $-$

2 spin states → spin-1/2 particles : fermions
2 representations → fermions and anti-fermions

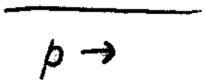
Truncation as before but now:
(all energies $p^0 > 0$)

incoming fermion : $u(p,s)$
" anti-fermion : $\bar{v}(p,s)$

outgoing fermion : $\bar{u}(p,s)$
" anti-fermion : $v(p,s)$

AND $-$ signs for
- closed fermion loops
- interchange of any 2 Dirac particles

Another possibility:



$$\frac{i}{p^2-m^2+i\varepsilon}\left(-g^{\mu\nu}+\frac{p^\mu p^\nu}{m^2}\right)$$

vertices also pick up Lorentz indices

For $p^2=m^2$ there are polarization vectors ε_i such that

$$-g^{\mu\nu}+\frac{p^\mu p^\nu}{m^2}=\sum_{i=1}^{3}\varepsilon_i^\mu\varepsilon_i^{\nu *} \qquad \text{with } \varepsilon_i^\mu p_\mu=0 \quad \varepsilon_i^\mu\varepsilon_{i\mu}=-1$$

3 spin states → spin-1 object : vector boson

Truncation as before, but

incoming vector leg : ε^μ

outgoing vector leg : $\varepsilon^{*\,\nu}$

Possible choices for ε:

$p^\mu = (E, 0, 0, p)$ $\qquad E^2=p^2+m^2$

$\varepsilon_L^\mu = \left(\frac{p}{m}, 0, 0, \frac{E}{m}\right)$ longitudinal polarization

$$\left.\begin{aligned}\varepsilon_1^\mu &= (0, 1, 0, 0)\\ \varepsilon_2^\mu &= (0, 0, 1, 0)\end{aligned}\right\}\text{ linear transverse polarization}$$

OR $$\left.\begin{aligned}\varepsilon_+^\mu &= \frac{1}{\sqrt{2}}(0, 1, i, 0)\\ \varepsilon_-^\mu &= \frac{1}{\sqrt{2}}(0, 1, -i, 0)\end{aligned}\right\}\text{ circular polarization (helicity)}$$

Massless spin-1 particles

If $m/E \ll 1$:

- $k = \frac{1}{2}(E+p)$

$$p^\mu = (E,0,0,p) = k(1,0,0,1) + \frac{m^2}{4k}(1,0,0,-1)$$

$$\varepsilon_L^\mu = \left(\frac{E}{m},0,0,\frac{p}{m}\right) = \frac{k}{m}(1,0,0,1) - \frac{m}{4k}(1,0,0,-1)$$

$$\varepsilon_L^\mu \approx \frac{1}{m} p^\mu$$

- $\varepsilon_L^\mu \to \infty$

$\approx J_\mu \varepsilon_L^\mu \to \infty$ emission of long. pol. particle

$\sigma \to \infty$ unitarity

- UNLESS

  vertices of theory carefully chosen such that

  $J_\mu k^\mu = 0$ for all processes

- back from momentum language to space language:

$$k^\mu J_\mu = 0 \longrightarrow \frac{\partial}{\partial x_\mu} J_\mu = 0$$ current conservation!

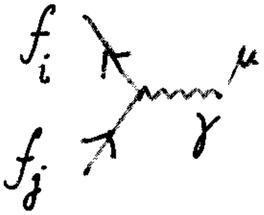
$m_{\text{photon}} = 0 \Rightarrow$ e.m. charge/current conserved

$m_{\text{gluon}} = 0 \Rightarrow$ colour charge/current conserved

Simple example: Quantum Electrodynamics

Ingredients: $\begin{cases} \text{spin-}1/2 \text{ particles } f_i \text{ (electrons, muons, quarks....)} \\ \text{photon } \gamma \end{cases}$

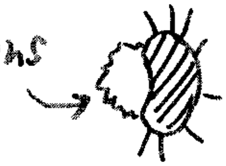
"Only possible" interaction vertex



Feynman Rule: $iq\gamma^\mu$

+ condition: $m_{f_i} = m_{f_j} \Rightarrow f_i = f_j$

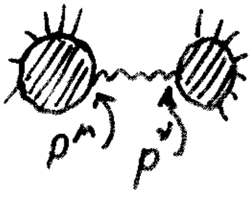
Can prove e.m. current conservation

- for on-shell external photons 
- for off-shell, internal photons 

even for finite m_γ!

$\Rightarrow$ propagator also simpler:

$$\frac{i}{p^2 - m_\gamma^2 + i\varepsilon}\left(-g^{\mu\nu} + \frac{p^\mu p^\nu}{m_\gamma^2}\right)$$



$\Downarrow$ drop $p^\mu p^\nu$

$$\frac{i}{p^2 - m_\gamma^2 + i\varepsilon}(-g^{\mu\nu})$$

$\Downarrow$ $m_\gamma^2 \to 0$

$$\frac{-i\, g^{\mu\nu}}{p^2 + i\varepsilon}$$

Almost-massless spin-1 ?

$m_W, m_Z \neq 0$ but may still have $\frac{m}{E} \to 0$ as $E \to \infty$

Now need not strictly $J_\mu k^\mu = 0$ but

$J_\mu k^\mu = \mathcal{O}(m)$ (quite often will be $\mathcal{O}(m^2)$)

$\Rightarrow$ very tight constraints on allowed vertices

A note on powers of GeV

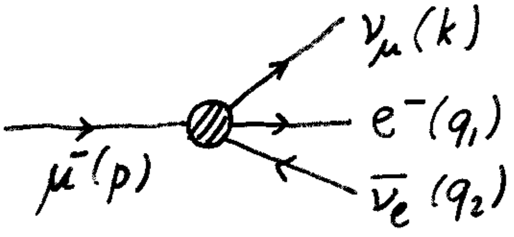
$$\underset{\substack{\downarrow \\ \text{GeV}}}{d\Gamma} \sim \underset{\substack{\downarrow \\ \text{GeV}^{-1}}}{\frac{1}{m}} \langle |M(1\to n)|^2 \rangle \underset{\substack{\downarrow \\ \text{GeV}^{2n-4}}}{dV(1\to n)} \quad \Rightarrow M(1\to n) \sim \text{GeV}^{3-n}$$

$$\underset{\substack{\downarrow \\ \text{GeV}^{-2} \\ \approx m^2}}{d\sigma} \sim \underset{\substack{\downarrow \\ \text{GeV}^{-2}}}{\frac{1}{S}} \langle |M(2\to n)|^2 \rangle \underset{\substack{\downarrow \\ \text{GeV}^{2n-4}}}{dV(2\to n)} \quad \Rightarrow M(2\to n) \sim \text{GeV}^{2-n}$$

$\Rightarrow$ an amplitude with k legs must have units GeV^{4-k}

$\Rightarrow$ dimensionality of coupling constant important !

Fundamental electroweak process: μ decay



Simple model: Fermi theory ('40 s)

$$\mathcal{M} = \frac{G_F}{\sqrt{2}}\,\bar{u}(k)(1+\gamma^5)\gamma^\mu u(p)\;\bar{u}(q_1)(1+\gamma^5)\gamma_\mu v(q_2)$$

$\gamma^5 \sim \varepsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma$: P violation!

$$\left.\begin{aligned} u, \bar{u}, v &\sim \text{GeV}^{1/2} \\ \mathcal{M}(2\to 2) &\sim \text{GeV}^0 \end{aligned}\right\} G_F \sim \text{GeV}^{-2}$$

$$\Gamma_{tot} = \frac{1}{192\pi^3}\, m_\mu^5\, G_F^2$$

obvious

necessary $(m_e \simeq 0)$

the hard-work part

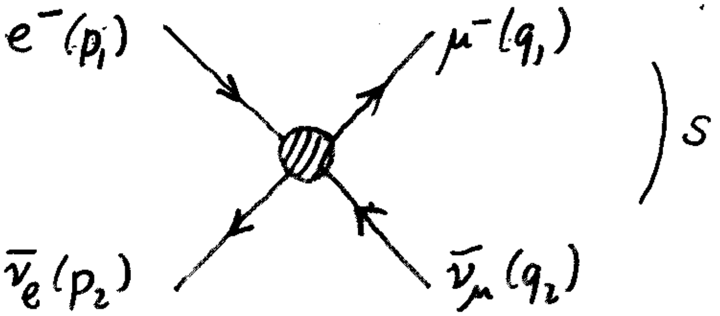
Experiment:

$m_\mu = 0.105658357(5)$ GeV

$\tau_\mu = 1/\Gamma = 2.19703(4)\,\mu$sec

$$\Rightarrow G_F = 1.16637(1)\; 10^{-5}\ \text{GeV}^{-2}$$

If the Fermi model is correct it should also describe other processes:



$$\mathcal{M} \sim \frac{G_F}{\sqrt{2}}\ \bar{v}(p_2)\,(1+\gamma^5)\gamma^\mu u(p_1)\cdot \bar{u}(q_1)\,(1+\gamma^5)\gamma_\mu v(q_2)$$

at "high" energies, $s \gg m_e^2, m_\mu^2$:

$$\sigma \sim \underbrace{K}_{\text{hard-work factor}} \cdot \underbrace{s}_{\text{necessary}} \cdot \underbrace{G_F^2}_{\text{obvious}}$$

- $\sigma(e\bar{\nu}_e \to \mu\bar{\nu}_\mu)$ has right units, but $\sigma \propto s$!
- violates unitarity at high enough s
- Fermi model is wrong
  ("low-energy effective theory")