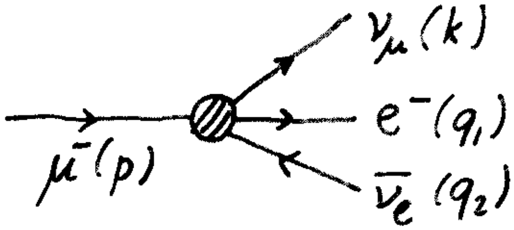
Fundamental electroweak process: μ decay



Simple model: Fermi theory ('40 s)

$$\mathcal{M} = \frac{G_F}{\sqrt{2}}\, \bar{u}(k)\,(1+\gamma^5)\gamma^\mu u(p)\;\; \bar{u}(q_1)\,(1+\gamma^5)\gamma_\mu v(q_2)$$

$\gamma^5 \sim \varepsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma$: P violation!

$$\left.\begin{array}{l} u, \bar{u}, v \sim \mathrm{GeV}^{1/2} \\ \mathcal{M}(2\to 2) \sim \mathrm{GeV}^0 \end{array}\right\}\; G_F \sim \mathrm{GeV}^{-2}$$

$$\Gamma_{tot} = \frac{1}{192\pi^3}\, m_\mu^5\, G_F^2$$

G_F^2 ← obvious

m_μ^5 ← necessary ($m_e \simeq 0$)

$\frac{1}{192\pi^3}$ ← the hard-work part

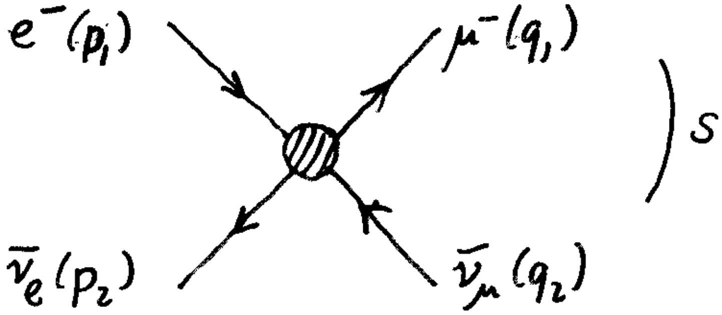
Experiment:

$$m_\mu = 0.105658357(5)\ \mathrm{GeV}$$

$$\tau_\mu = 1/\Gamma = 2.19703(4)\,\mu\mathrm{sec}$$

$$\Rightarrow G_F = 1.16637(1)\ 10^{-5}\ \mathrm{GeV}^{-2}$$

If the Fermi model is correct it should also describe other processes:



$$\mathcal{M} \sim \frac{G_F}{\sqrt{2}} \, \bar{v}(p_2)\,(1+\gamma^5)\gamma^\mu u(p_1) \cdot \bar{u}(q_1)\,(1+\gamma^5)\gamma_\mu v(q_2)$$

at "high" energies, $s \gg m_e^2, m_\mu^2$:

$$\sigma \sim K \cdot s \cdot G_F^2$$

obvious

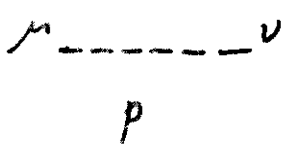
necessary

hard-work factor

- $\sigma(e\bar{\nu}_e \to \mu\bar{\nu}_\mu)$ has right units, but $\sigma \propto s$!
- violates unitarity at high enough s
- Fermi model is wrong
  ("low-energy effective theory")

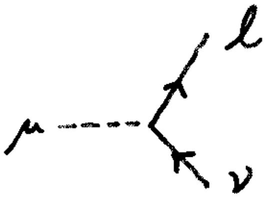
W to the rescue!

To solve unitarity problem in $e^-\bar{\nu}_e \to \mu^- \bar{\nu}_\mu$,
introduce a new particle, W:



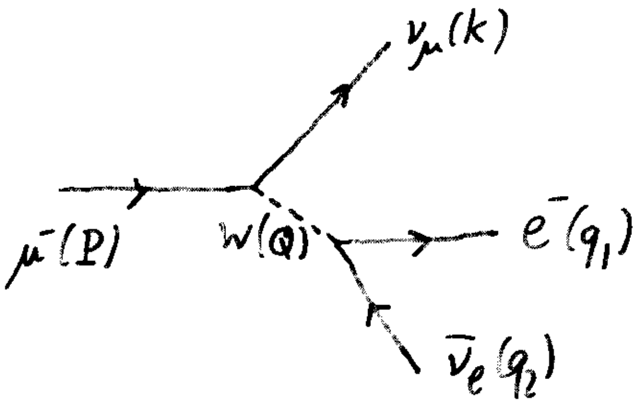
$$= \frac{-i}{p^2 - m_W^2 + i\varepsilon}\left(-g^{\mu\nu} + \frac{p^\mu p^\nu}{m_W^2}\right)$$

massive spin-1



$$= -i g_W (1+\gamma^5)\gamma^\mu$$

"like QED"



$$\boxed{g_W = \frac{e}{\sqrt{8}\,\sin\theta_W}}$$

definition of θ_W as ratio of couplings (justified later)

$$\mathcal{M} = i\ \bar{u}(k)\,(1+\gamma^5)\gamma^\mu\, u(P)\cdot g_W$$

$$\times \frac{1}{Q^2 - m_W^2 + i\varepsilon}\left(g_{\mu\nu} - \frac{Q^\mu Q^\nu}{m_W^2}\right)$$

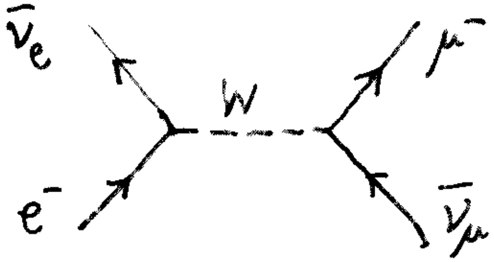
$$\times\ \bar{u}(q_1)\,(1+\gamma^5)\gamma^\nu\, v(q_2)\cdot g_W$$

$$\frac{Q^\mu Q^\nu}{m_W^2} \sim \frac{m_\mu m_e}{m_W^2} \sim 0$$

in μ decay : $Q^2 \lesssim \mathcal{O}(m_\mu^2) \ll m_W^2$ (assumed)

$$\Rightarrow \quad \frac{g_W^2}{Q^2 - m_W^2} \simeq -\frac{g_W^2}{m_W^2} \Longrightarrow \boxed{\frac{g_W^2}{m_W^2} = \frac{G_F}{\sqrt{2}}}$$

back to $e^- \bar{\nu}_e \to \mu^- \nu_\mu$:



Replace

Fermi model $\longrightarrow$ W model

$$G_F \longrightarrow \sqrt{2}\,\frac{g_W^2}{s - m_W^2 + i m_W \Gamma_W}$$

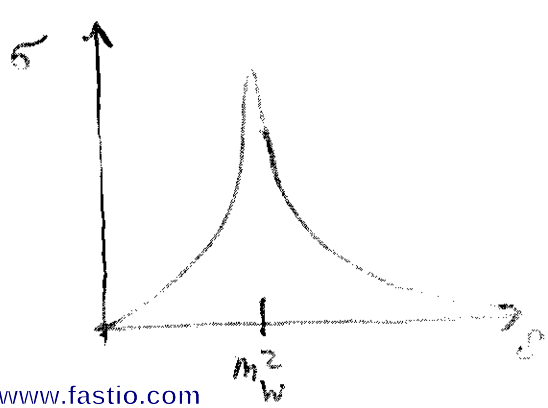
So

$$\sigma = k \cdot s \cdot G_F^2 \longrightarrow 2k \cdot s \cdot \frac{g_W^4}{(s-m_W^2)^2 + m_W^2 \Gamma_W^2}$$

now : $s \ll m_W^2$: no difference with Fermi

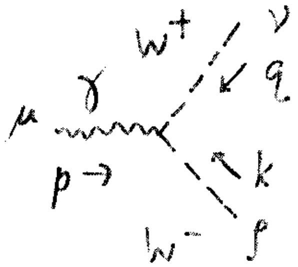
$s \approx m_W^2$: $\sigma \sim \dfrac{2k\, g_W^4}{\Gamma_W^2}$ unitarity bound

$s \gg m_W^2$: $\sigma \sim \dfrac{2k\, g_W^4}{s}$ 



The W is charged ($W^- \to e^- \bar{\nu}_e$!)

$\Rightarrow$ should couple to the photon:



$$= i e \underbrace{\left[(q-k)^\mu g^{\nu\rho} + (k-p)^\nu g^{\rho\mu} + (p-q)^\rho g^{\mu\nu} \right]}$$

$\equiv Y^{\mu\nu\rho}$: the only vertex that works (i.e. conserves unitarity)

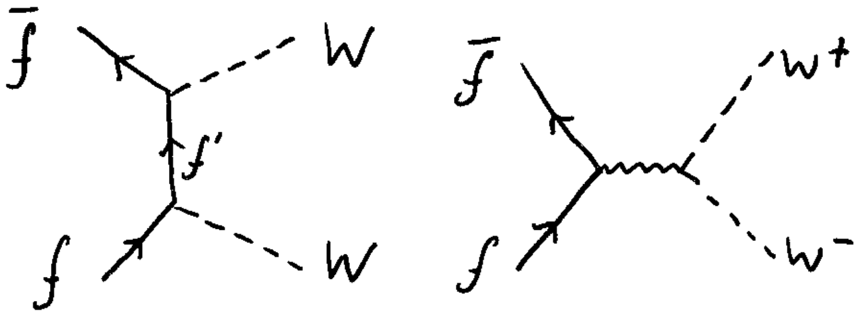
the "Yang-Mills vertex"

Model development strategy

1) process violates unitarity for this model
2) extend model by new particle and/or vertex
3) process o.k. now
4) go to new process
5) go to 1)

.... Hopefully, this ends somewhere!

new process: $f\bar{f} \to W^+W^-$



take 1 W long. pol., other trans. pol: $\varepsilon_1 \sim \frac{p}{m} = \mathcal{O}\left(\frac{E}{m}\right)$

$\varepsilon_2 \sim \mathcal{O}(1)$

1

$\sqrt{E}$

$\frac{1}{E}$

$\sqrt{E}$ $\quad E^{-2}$ $\quad 1$

$\downarrow$

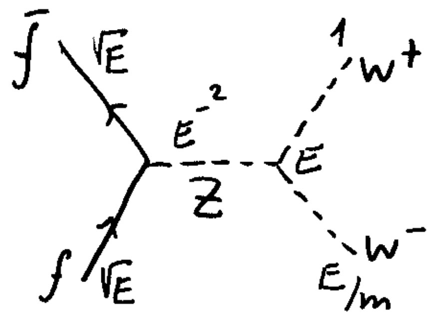
$\uparrow$ $\quad E/m$

$\sqrt{E}$ $\quad \frac{E}{m}$ $\quad L$ $\qquad \sqrt{E}$ $\quad E$ $\quad L$

$\Rightarrow \mathcal{M}$ now $\propto \frac{E}{m}$: problem!

solution: introduce new particle, Z

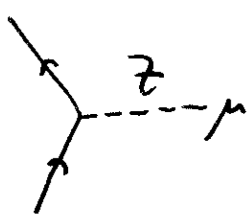
E^{-2}



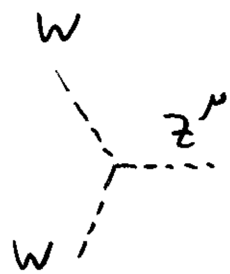
$$Z = \text{spin-1} \Rightarrow \text{propagator} \sim \frac{i}{p^2 - m_Z^2}\Big(-g^{\mu\nu} + \overset{\curvearrowright \text{drop}}{\frac{\widetilde{p^\mu p^\nu}}{m_Z^2}}\Big) \sim \mathcal{O}\Big(\frac{1}{E^2}\Big)$$

new diagrams also $\frac{E}{m}$: possibility of cancellation!

- indeed: new vertex



$\sim i(v + a\gamma^5)\gamma^\mu$



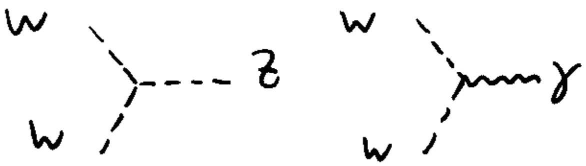
$\sim Y^{\mu\nu\rho}$

makes unitarity o.k. at $E/m \to \infty$

- <u>no</u> info on m_Z (since $m/E \to 0$)

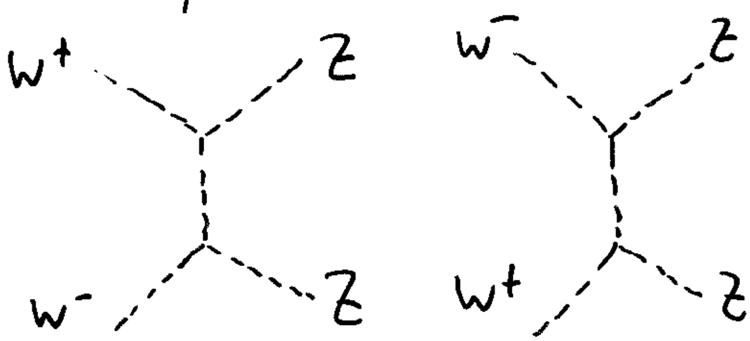
- here is where $\sin\theta_W$ comes in

We now have

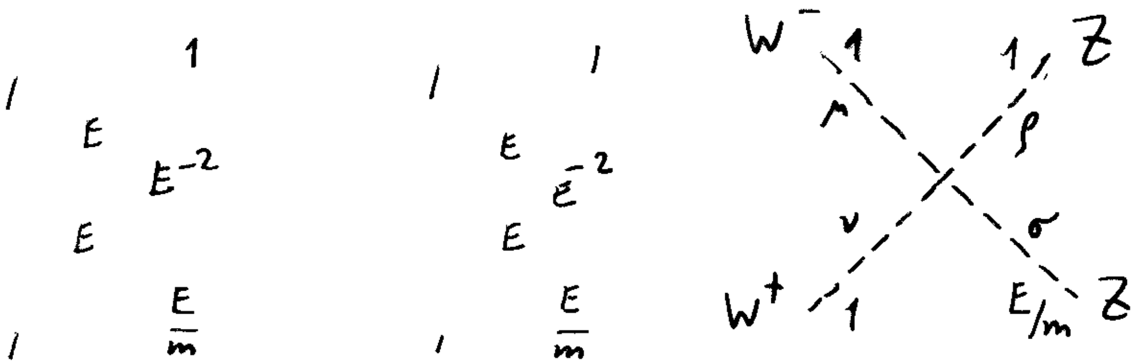


so can consider, e.g. $W^+W^- \to W^+W^-$, $W^+W^- \to ZZ$, $W^+W^- \to \gamma Z$ ...

Example



Now go to $s \gg m_W^2, m_Z^2$ and take 1 boson longitudinal



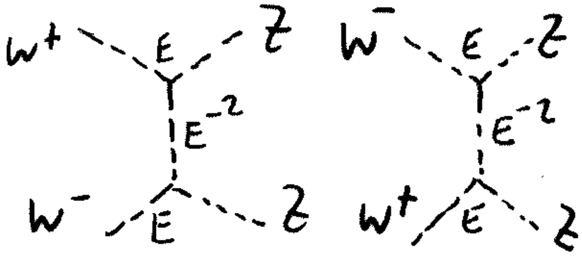
- $\mathcal{M} \propto E/m$: problem!

solution: introduce new <u>vertex</u>

- cancellation possible: vertex that works is

$$\Upsilon^{\mu\nu\rho\sigma} \propto g_w^2 \left(2 g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho} \right)$$

- analogous results for $W^+W^-Z\gamma$, $W^+W^-\gamma\gamma$ vertices new process : $W^+W^- \to ZZ$ with <u>all</u> long. polarized!

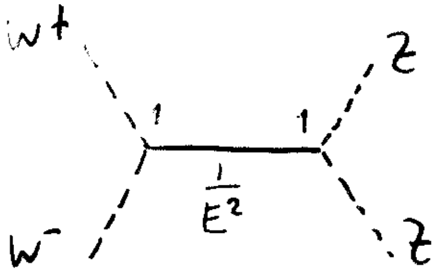


$$\propto \frac{E^4}{m_Z^2 m_W^2}$$

$$\propto \frac{E^4}{m_W^2 m_Z^2}$$

partial cancellation down to $\propto \dfrac{E^2}{m^2_{W,Z}}$: problem!

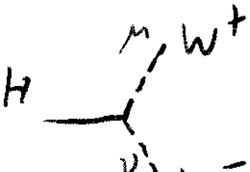
Solution: new <u>particle</u> H, scalar prop., neutral



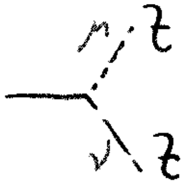
$$\propto \frac{E^2}{m^2_{W,Z}}$$

cancellation possible: consider also $W^+W^- \to W^+W^-$ etc. etc.

$\Rightarrow$ unitarity o.k. provided



$\propto m_W g^{\mu\nu}$



$\propto m_Z g^{\mu\nu}$

<u>AND</u>

$$\frac{m_W}{m_Z} = \cos\theta_W$$

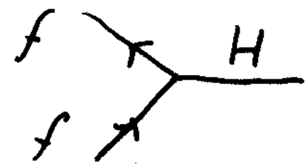
introduction of Higgs $\Rightarrow$ relates ratio of <u>masses</u> to ratio of <u>couplings</u>

6.8.4

6.8.3

Reconsider: $f\bar{f} \to W_L^+ W_L^-$, keep $m_f \neq 0$:

also need



$\propto m_f$

$\Rightarrow$ • Higgs couples to particles $\propto$ mass

• assuming that $\exists$ Higgs: • relation masses — couplings for W, Z
  • no relation for f
  • ≥ 1 Higgses: less predicative

m_H? • many "aesthetic" arguments (see A. Pich?)

• most robust bound: if $m_H \to \infty$, $W^+W^- \to ZZ$ violates unitarity $\longrightarrow$ upper limit on m_H:

$m_H \lesssim 1000$ GeV

• The future: LHC

  • find H : SM possibly o.k. end of particle physics?

OR

  • find no H / something else:
    Fundamental insights in masses etc.

New Fundamental Concepts!