Fundamental Concepts of Particle Physics

summer
student
lectures
CERN
2001

classical Newton	classical Relativity	$\big\downarrow \hbar$
classical quantum mechanics	quantum field theory	

$\xrightarrow{\quad c \quad}$

fund. natural constants as yardsticks

$$c = 3 \cdot 10^{8} \ \mathrm{m/sec}$$

$$\hbar = 10^{-34} \ \mathrm{kg\,m^2/sec}$$

Fundamental division of physicist's world

		speed	
		slow	fast
action	large		
	small		

Fundamental units

length : L
time : T
energy : E
or mass : M

choose such that

$c = 1 \; L/T$

$\hbar = 1 \; E{\cdot}T = 1 \; M{\cdot}L^2/T$

1 freedom left : choose $E = 1$ GeV $\sim 1.6\,10^{-10}$ J

energy of 1 GeV $\sim 1.6\;10^{-10}$ J

mass of 1 GeV $\sim 1.8\;10^{-27}$ kg

length of 1 GeV^{-1} $\sim 0.2\;10^{-15}$m $= 0.2$ fm

time of 1 GeV^{-1} $\sim 0.7\;10^{-24}$ sec

... typical of el. particles

Fundamental dimensionless numbers

Example : electromagnetic coupling constant

- Coulomb: $|\vec{F}| = \frac{1}{4\pi}\, e \cdot e \, \frac{1}{|\vec{r}|^2}$

- e^2 has dimension $kg \cdot m^3/sec^2 = (kg\, m^2/sec) \cdot (m/sec)$

- $\alpha = \frac{e^2}{4\pi \hbar c}$ is dimensionless

  → same value in <u>any</u> units system

- $\alpha \simeq \frac{1}{137.036\ldots}$ is fund. dim.less number

  why this value ?...

Fundamental theorist's business:

Explain magnitude of fund. dim. less numbers

Categories of explainability in particle physics

- Well-understood ($\geq$ in principle)

$$R_{had} = \frac{\sigma(e^+e^- \to had)}{\sigma(e^+e^- \to \mu^+\mu^-)} \quad , \quad \frac{m_Z}{m_W} \quad , \ldots$$

- Almost understood ($\leq$ "techical details")

$$m_n / m_p \quad , \quad \frac{\sigma(pp \to \ldots)}{\sigma(ee \to \mu\mu)} \quad , \ldots$$

- Not a clue

$$\alpha_{QED} \quad , \quad \frac{m_{top}}{m_e} \quad , \quad N_{fam} \quad , \ldots$$

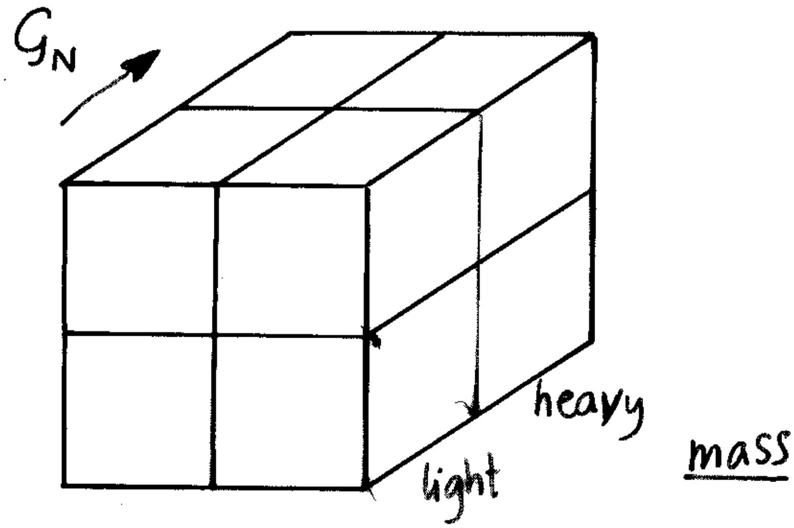
- "Aesthetic" arguments

$$\frac{m_H}{m_W}$$

- "Under specific model assumptions"

$$\frac{\alpha_{EW}}{\alpha_{QED}} \quad , \quad \frac{\alpha_{QCD}}{\alpha_{QED}} \quad , \quad \frac{m_b}{m_\tau} \quad , \ldots$$

Another fund. division?



Newton: $|\vec{F}| = G_N\, m \cdot m \frac{1}{|\vec{r}|^2}$

- $G_N = 6.6\ 10^{-11}\ m^3/kg \cdot sec^2$
- could take over rôle of E or M fund. unit
- $\Rightarrow$ "truly fundamental numbers":

$$\text{mass} = (\hbar c / G_N)^{1/2} = 1.2\ 10^{19}\ \text{GeV} \qquad = m_{PL}$$

$$\text{length} = 10^{-33}\ \text{cm} \qquad = \ell_{PL}$$

- particle physics not yet truly fundamental
- try to explain fund. dim. less $\frac{m_{PL}}{m_e} \approx 10^{22}$

Fundamental theoretical principle

Copernican principle:
"Your system of coordinates and units is nothing special" ← empirical!

- physics independent of system choice
- physics description simplest in terms of "invariant" or "covariant" objects
- recipe to move from one s.c. to another
- .... then, choose a system

- "invariance" of s.c. — sounds reasonable
  — must be tested!

Example 1 Special Relativity

- physics independent of chosen (inertial) system
- space time point

$$a^\mu = (ct, x, y, z) = (a^0, a^1, a^2, a^3) = a$$

<u>not</u> invariant under translations

- space time vector

$$(a + \Delta a)^\mu - a^\mu = \Delta a^\mu = (c\Delta t, \Delta x, \Delta y, \Delta z)$$

inv. under translations, <u>not</u> under rotations, boosts

- Einstein postulate:
the <u>real</u> invariant distance is

$$(\Delta^0)^2 - (\Delta^1)^2 - (\Delta^2)^2 - (\Delta^3)^2 = \sum_{\mu,\nu=0}^{3} g_{\mu\nu} \Delta^\mu \Delta^\nu = g_{\mu\nu} \Delta^\mu \Delta^\nu = \Delta \cdot \Delta = \Delta^\mu \Delta_\mu = \Delta^2$$

$$g_{\mu\nu} = \mathrm{diag}(+1, -1, -1, -1)$$

- Physics descriptions invariant/covariant under all transformations that leave all $g_{\mu\nu}\Delta^\mu\Delta^\nu$ invariant:

translations and Lorentz transforms

Lorentz transforms: $x^\mu \rightarrow \Lambda^\mu{}_\nu x^\nu = \sum_{\nu=0}^{3} \Lambda^\mu{}_\nu x^\nu = \tilde{x}^\mu$

matrix

$$g_{\mu\nu}\tilde{x}^\mu \tilde{x}^\nu = g_{\alpha\beta} x^\alpha x^\beta$$

$$\Rightarrow g_{\mu\nu} \Lambda^\mu{}_\alpha \Lambda^\nu{}_\beta = g_{\alpha\beta}$$

Solutions:

- 3 rotations $\mathcal{R}$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & 0 & \sin\theta \\ 0 & 0 & 1 & 0 \\ 0 & -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

- 3 boosts $\mathcal{B}$

$$\begin{pmatrix} \cosh\alpha & \sinh\alpha & 0 & 0 \\ \sinh\alpha & \cosh\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- space reflection: parity, P

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

"mirror symmetry"

- time reflection: time reversal, T

$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Lorentz transforms are a <u>group</u>
- $\mathcal{R}$ and $\mathcal{B}$ continuously connected to identity
- P and T not " " " " (in 3+1 dim!)

.... and a good thing, too!