7) Standard Model and CP violation

Strong interaction — gluons

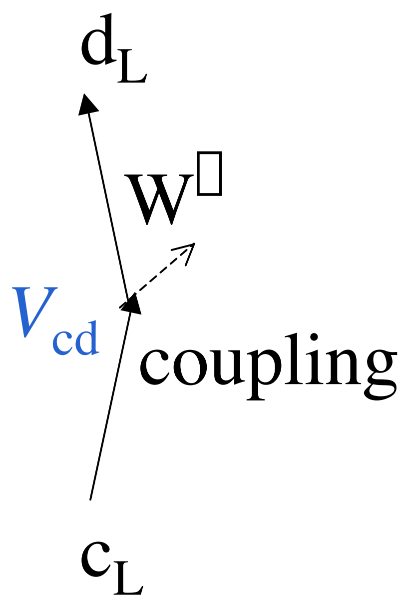
Electromagnetic interaction — photons

Weak interaction — neutral current; charged current: $W^{\pm}$

Up type quark spinor field	Down type quark spinor field
$Q = 2/3$	$Q = -1/3$

$$\boldsymbol{U} = \begin{pmatrix} u \\ c \\ t \end{pmatrix} \qquad \boldsymbol{D} = \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

example



there are $3 \times 3 = 9$ V's

“simplified” way...

1) Since CPT is respected, $\not{CP}$ is like $\not{T}$

2) T transformation is like making complex conjugation:

$$e^{-iEt} \rightarrow \mathrm{T} \rightarrow e^{iEt}$$

3) T transformation to the Hamiltonian operator H

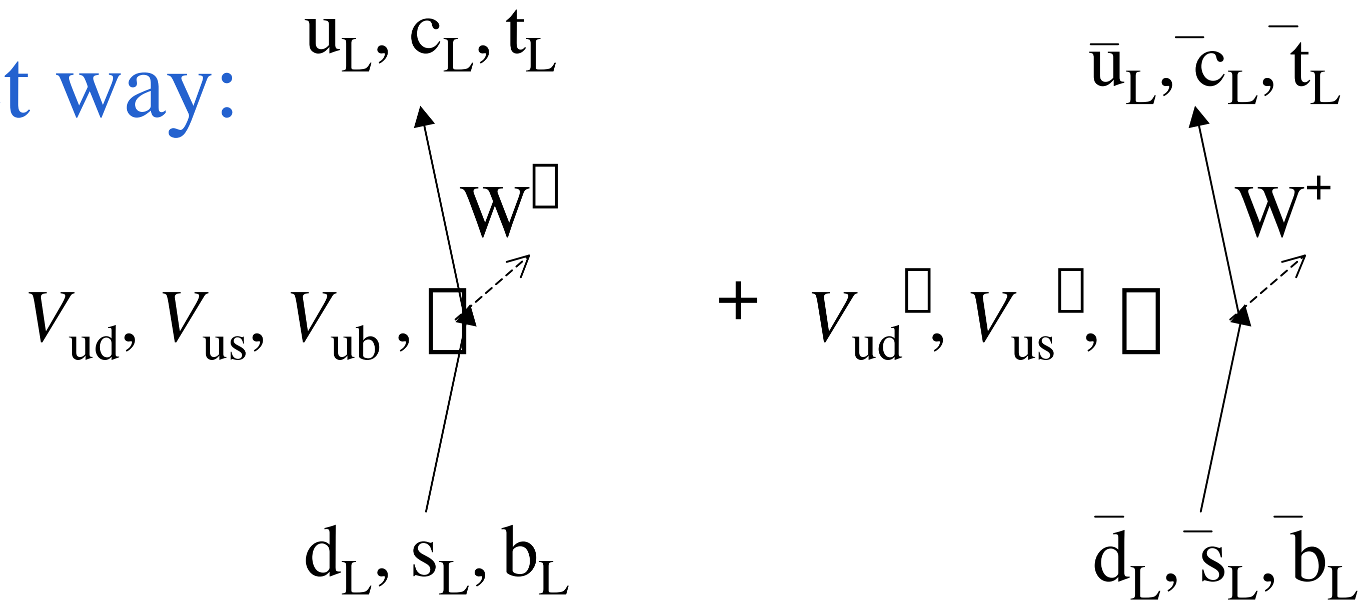
$$H \rightarrow \mathrm{T} \rightarrow H^{*}$$

if $H \neq H^{*}$, $\not{T}$ i.e. $\not{CP}$

Complex coupling $V^{*} \neq V$ generates CP violation,

$\boldsymbol{V} = \{V_{ij}\}$: Unitary matrix

Correct way:



$$L \propto V_{ij}\, \bar{U}_i\, \gamma^\mu (1-\gamma_5)\, D_j\, W_\mu^\dagger + V_{ij}^*\, \bar{D}_i\, \gamma^\mu (1-\gamma_5)\, U_j\, W_\mu$$

CP conjugation

$$L_{CP} \propto V_{ij}\, \bar{D}_i\, \gamma^\mu (1-\gamma_5)\, U_j\, W_\mu + V_{ij}^*\, \bar{U}_i\, \gamma^\mu (1-\gamma_5)\, D_j\, W_\mu^\dagger$$

If $V_{ij}^* = V_{ij} \Rightarrow L = L_{CP}$: i.e. CP conservation

Let us look at now: $V_{ij}\,\bar{U}_i\,\gamma^{\mu}(1-\gamma_5)\,D_j$

One family

$\boldsymbol{V}$ 1 free phase
1 free modula $= |V|\,e^{i\phi}$

$$|V|\,e^{i\phi}\,\bar{u}\,\gamma^{\mu}(1-\gamma_5)\,d \longrightarrow |V|\,\bar{u}\,\gamma^{\mu}(1-\gamma_5)\,d$$

Changing u quark phase: $u \rightarrow u\,e^{i\phi}$

Unitarity: $\boldsymbol{V}^{\dagger}V = \boldsymbol{V}\boldsymbol{V}^{\dagger} = \boldsymbol{E}$ (one constraint)

$$|V|^2 = 1$$

0 free phase
0 free modula

NO ~~CP~~

Two families $\boldsymbol{V} = \begin{pmatrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{pmatrix}$ 4 free phase

4 free moduli (or rotation angles)

1) The phase of V_{ij} can be absorbed by adjusting the phase differences between i- and j- quark

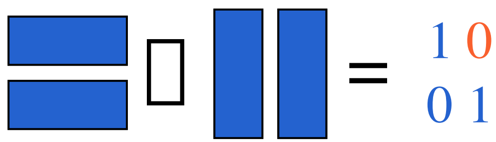
4 quarks = 3 phase differences

4 − 3 = 1 phase left

2) Unitarity $\boldsymbol{V}^{\dagger}\boldsymbol{V} = \boldsymbol{V}\boldsymbol{V}^{\dagger} = \boldsymbol{E}$: four constraints:

1 off-diagonal constraint for the phase

1 − 1 = 0 phase left



three constraint for the rest

4 − 3 = 1 rotation angle left

V is real, i.e. no $\not{CP}$.

Explicit demonstration

$$|V_{ud}|e^{i\phi_{ud}}\,\bar{u}\,\gamma^{\mu}(1-\gamma_5)\,d + |V_{us}|e^{i\phi_{us}}\,\bar{u}\,\gamma^{\mu}(1-\gamma_5)\,s$$
$$+\,|V_{cd}|e^{i\phi_{cd}}\,\bar{c}\,\gamma^{\mu}(1-\gamma_5)\,d + |V_{cs}|e^{i\phi_{cs}}\,\bar{c}\,\gamma^{\mu}(1-\gamma_5)\,s$$

$$u \to u\,e^{i\phi_{ud}}$$

$$|V_{ud}|\,\bar{u}\,\gamma^{\mu}(1-\gamma_5)\,d + |V_{us}|e^{i(\phi_{us}-\phi_{ud})}\,\bar{u}\,\gamma^{\mu}(1-\gamma_5)\,s$$
$$+\,|V_{cd}|e^{i\phi_{cd}}\,\bar{c}\,\gamma^{\mu}(1-\gamma_5)\,d + |V_{cs}|e^{i\phi_{cs}}\,\bar{c}\,\gamma^{\mu}(1-\gamma_5)\,s$$

$$s \to s\,e^{-i(\phi_{us}-\phi_{ud})},\quad c \to c\,e^{i(\phi_{cs}-\phi_{us}+\phi_{ud})}$$

$$|V_{ud}|\,\bar{u}\,\gamma^{\mu}(1-\gamma_5)\,d + |V_{us}|\,\bar{u}\,\gamma^{\mu}(1-\gamma_5)\,s$$
$$+\,|V_{cd}|e^{i\delta}\,\bar{c}\,\gamma^{\mu}(1-\gamma_5)\,d + |V_{cs}|\,\bar{c}\,\gamma^{\mu}(1-\gamma_5)\,s$$

Out of four quark, three quark phases can be adjusted:

4 free phase → 1 free phase

Unitarity: $\boldsymbol{V}^{\dagger}\boldsymbol{V} = \boldsymbol{V}\boldsymbol{V}^{\dagger} = \boldsymbol{E}$ (4 constraints) $\begin{pmatrix} V_{ud}^{*} & V_{cd}^{*} \\ V_{us}^{*} & V_{cs}^{*} \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{pmatrix}$

$$V_{ud}^{*}\, V_{us} + V_{cd}^{*}\, V_{cs} = 0 \;\square\; |V_{ud}|\,|V_{us}| + |V_{cd}|\,|V_{cs}|\, e^{\square i \square} = 0$$

0 free phase: $\square = \square$

$$|V_{ud}|\,|V_{cd}| \;\square\; |V_{us}|\,|V_{cs}| = 0$$

$$|V_{ud}|^2 + |V_{cd}|^2 = 1,\; |V_{us}|^2 + |V_{cs}|^2 = 1$$

1 free modula or rotation angle

$$|V_{11}| = \cos\square,\; |V_{22}| = \cos\square,\; |V_{12}| = \sin\square,\; |V_{21}| = \sin\square$$

$$\boldsymbol{V} = \begin{pmatrix} \cos\square & \sin\square \\ \square\sin\square & \cos\square \end{pmatrix}$$

One rotation angle without phase: □ NO ~~CP~~

(Cabibbo angle)

Three families

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

9 free phase

9 free moduli (or rotation angles)

Out of six quark, five quark phases can be adjusted:

9 free phase → 4 free phase

1 0 0
0 1 0
0 0 1

Out of nine unitarity constraints, three are for the phases

4 free phase → **1 free phase**

the rest (six) are for the rotation angles

9 free rotation angles → 3 free rotation angles

Three rotation angles with one phase:

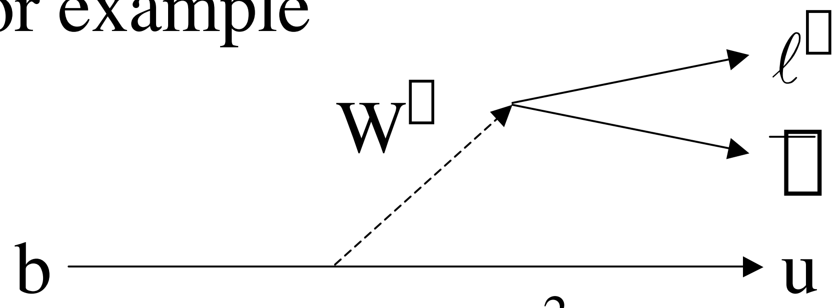
→ **$\not{CP}$ can be generated**

Electroweak theory with 3 families can naturally accommodate CP violation in the charged current induced interactions through the complex Cabibbo-Kobayashi-Maskawa quark mixing matrix $\boldsymbol{V}$, with 4 parameters.

To be more qualitative □ determination of the CKM matrix

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ V_{td} & V_{ts} & |V_{tb}| \end{pmatrix}$$

From the "tree" level quark decays
for example



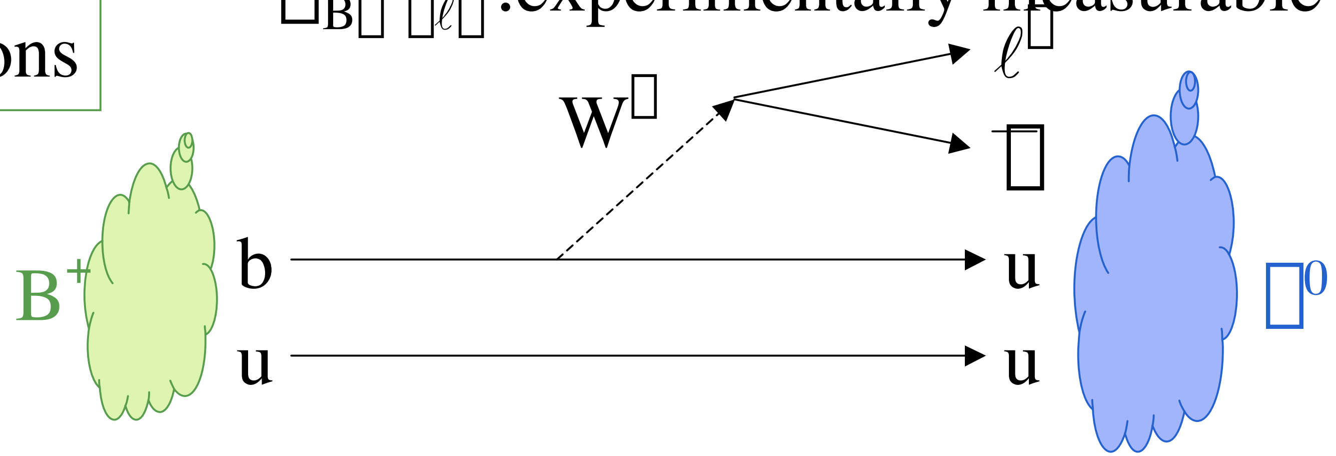
□$_{b□ u\ell□}$ = F ($|V_{ub}|^2$) known function

What you observe is "hadrons".

difficult to relate!

non-purturbative
strong interactions

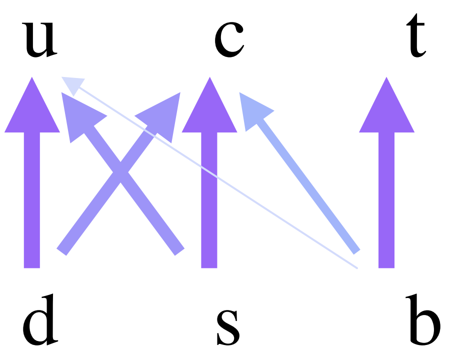
□$_{B□ □\ell□}$:experimentally measurable



The "tree" level measurements give...

$$|\boldsymbol{V}| = \begin{pmatrix} 0.9734 \pm 0.0008 & 0.2196 \pm 0.0023 & 0.0036 \pm 0.0007 \\ 0.224 \pm 0.016 & 0.996 \pm 0.13 & 0.0412 \pm 0.0020 \\ \square & \square & 0.99 \pm 0.29 \end{pmatrix}$$

This 2×2 sub-matrix (first two generation) is almost unitary.



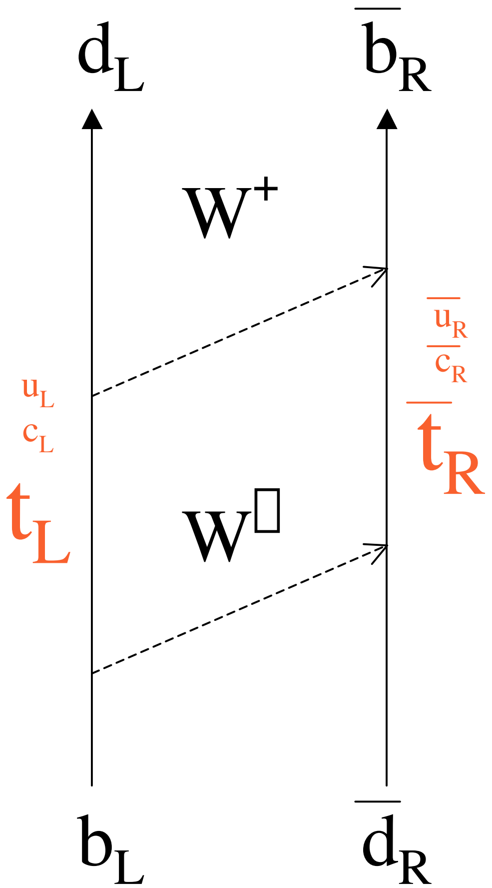
Errors are all theoretical, except $|V_{tb}|$.

Access to the top quark through virtual process

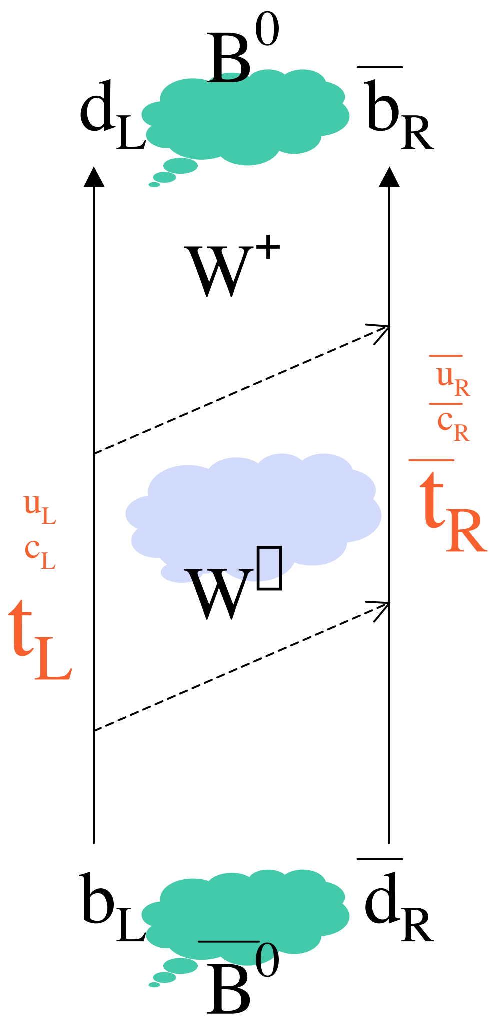
$(b\bar{d})$□ $(\bar{b}d)$ oscillation frequency is given by the "box" diagrams

$□ = f(|V_{td}|^2|V_{tb}|^2)$: known function

What we measure is $\bar{B}^0$□ B^0 oscillation frequency



Again non-purturbative strong interactions make them difficult to relate!

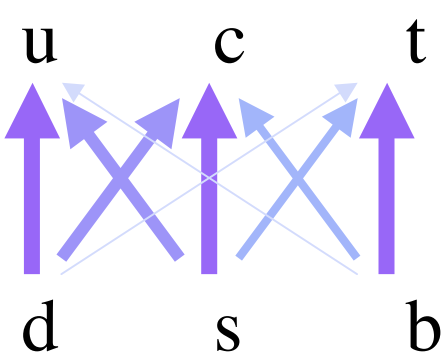


measurement with "trees",
measurements with "box"
and assuming unitarity

$$|V| = \begin{pmatrix} 0.9734 \pm 0.0008 & 0.2196 \pm 0.0023 & 0.0036 \pm 0.0007 \\ 0.224 \pm 0.016 & 0.996 \pm 0.13 & 0.0412 \pm 0.0020 \\ 0.0083 \pm 0.0016 & ? & \sim 1 \end{pmatrix}$$

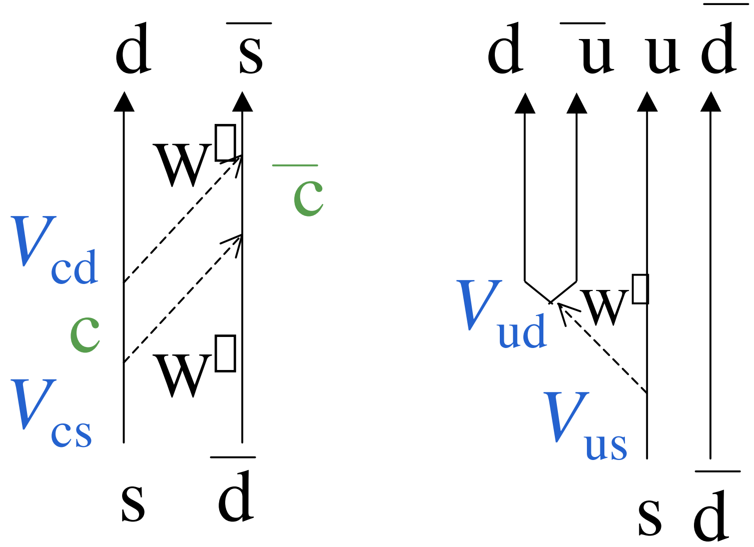
Waiting for measurements by CDF and D0

Interesting pattern:



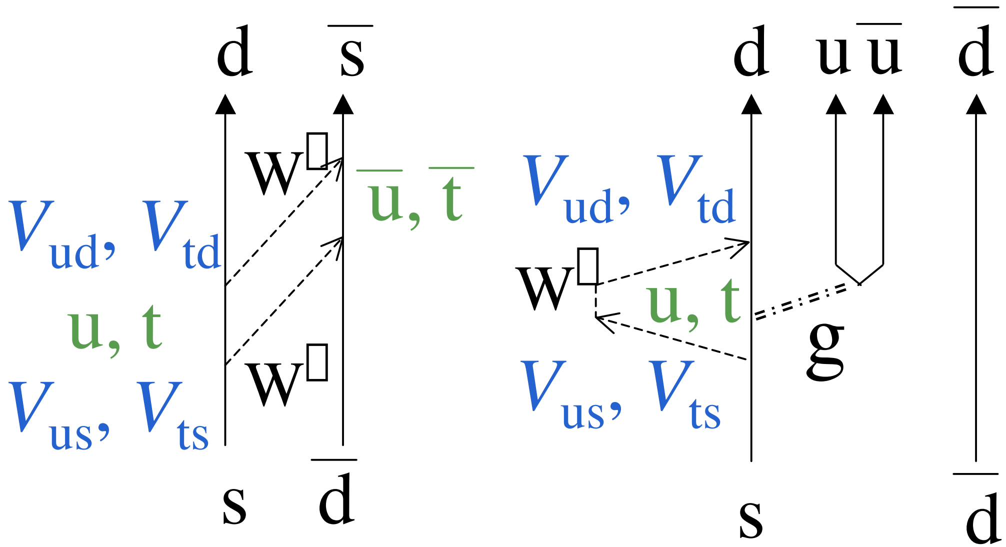
Dominant processes
in K^0 system involve:

V_{ud}	V_{us}	•
V_{cd}	V_{cs}	•
•	•	•



oscillations decays

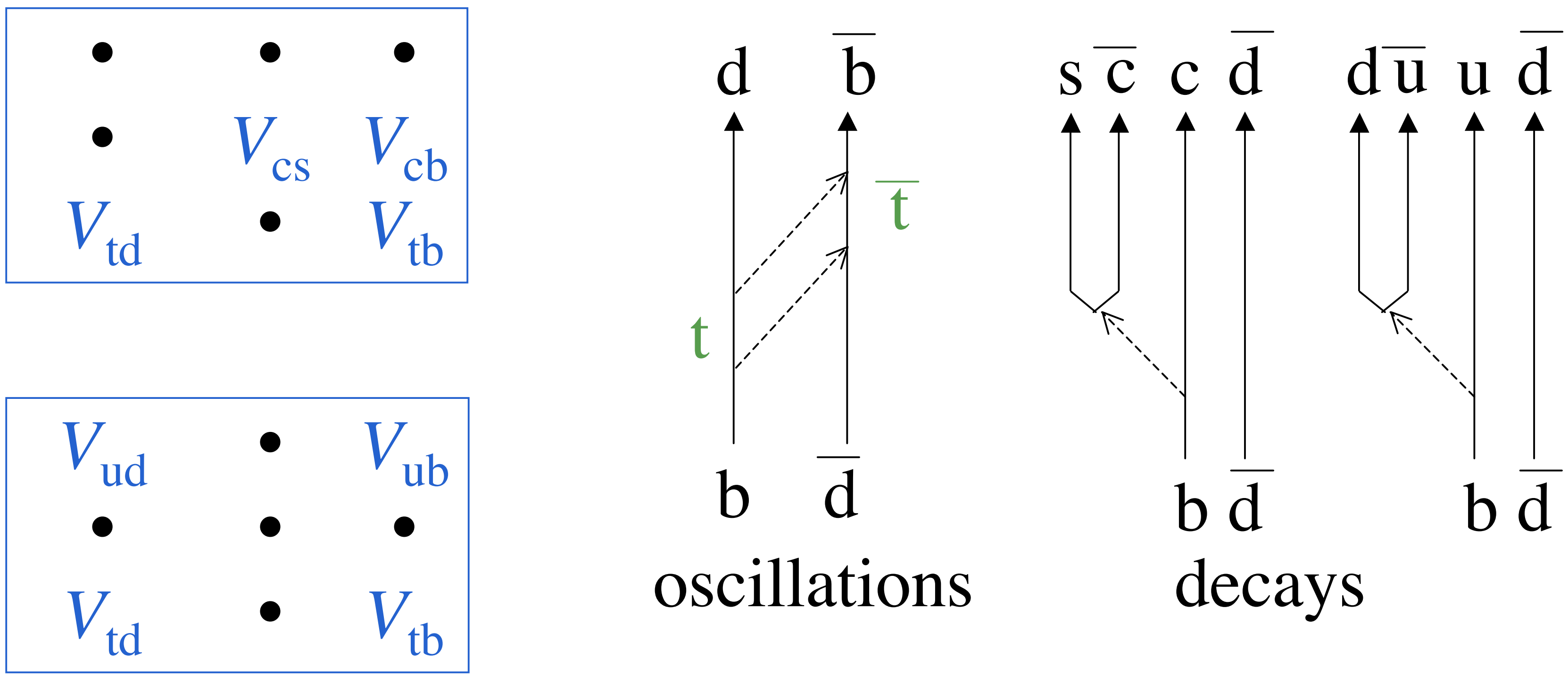
Almost unitary part of V ⇒ No CP violation.



Much smaller contributions to oscillations and decay: all three families of quarks.

CP violation: $CP(K_L) \neq -1$, $|\varepsilon_{+-}| \neq |\varepsilon_{00}|$, but small effect!

Dominant processes in B^0 system involve:



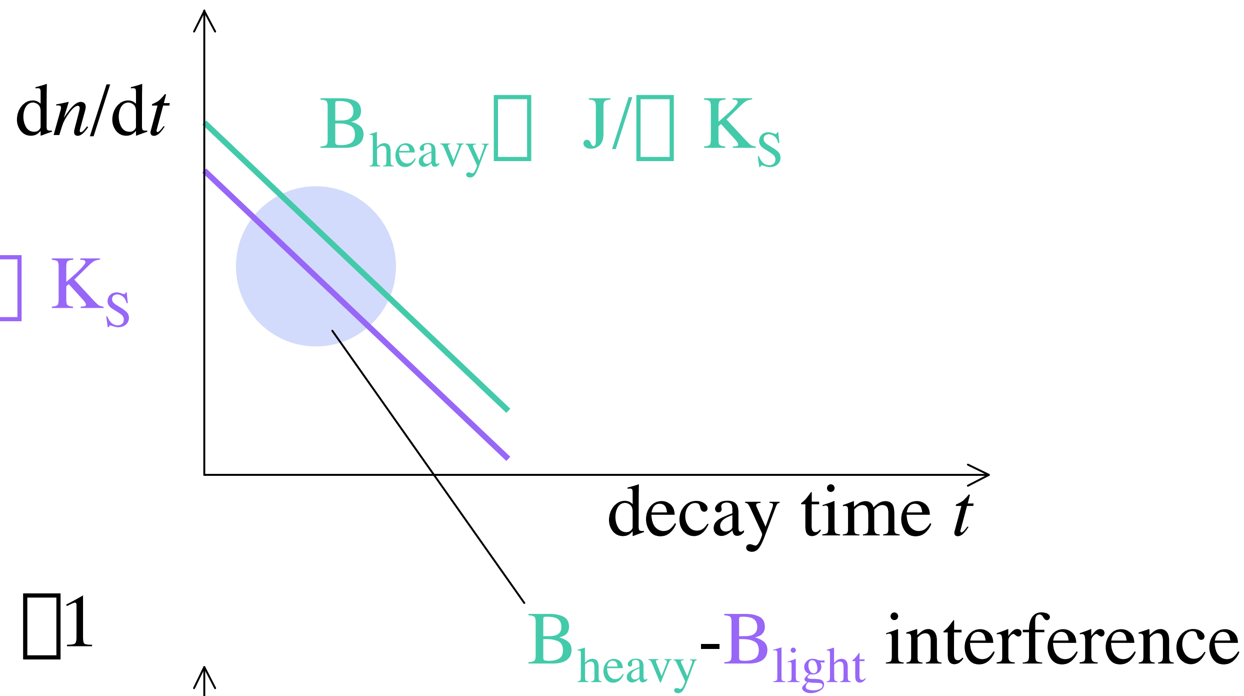
Non unitary part of ***V***

CP violation is expected to be large.

$B^0 = (d\bar{b})$, $\bar{B}^0 = (\bar{d}b)$

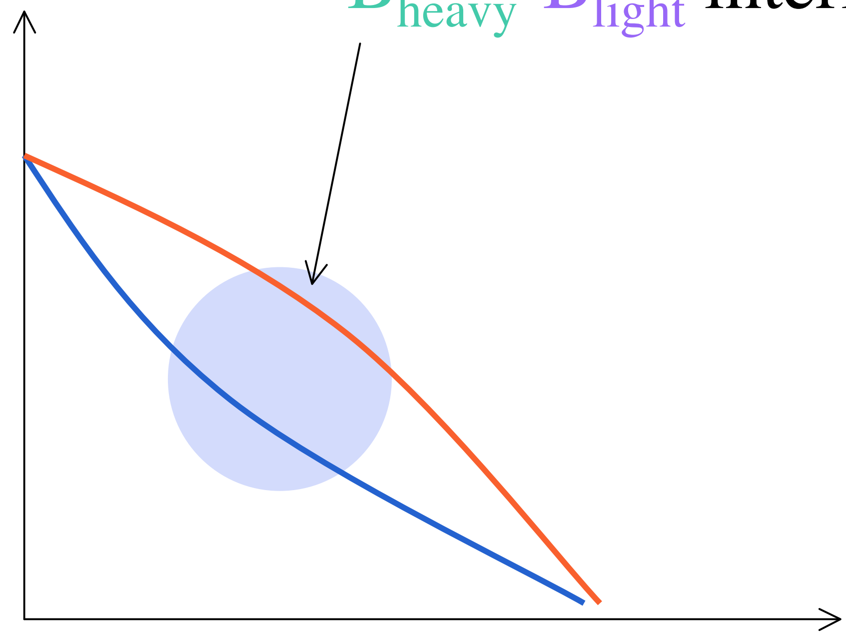
□□ □0

□m_B □ 100□m_K

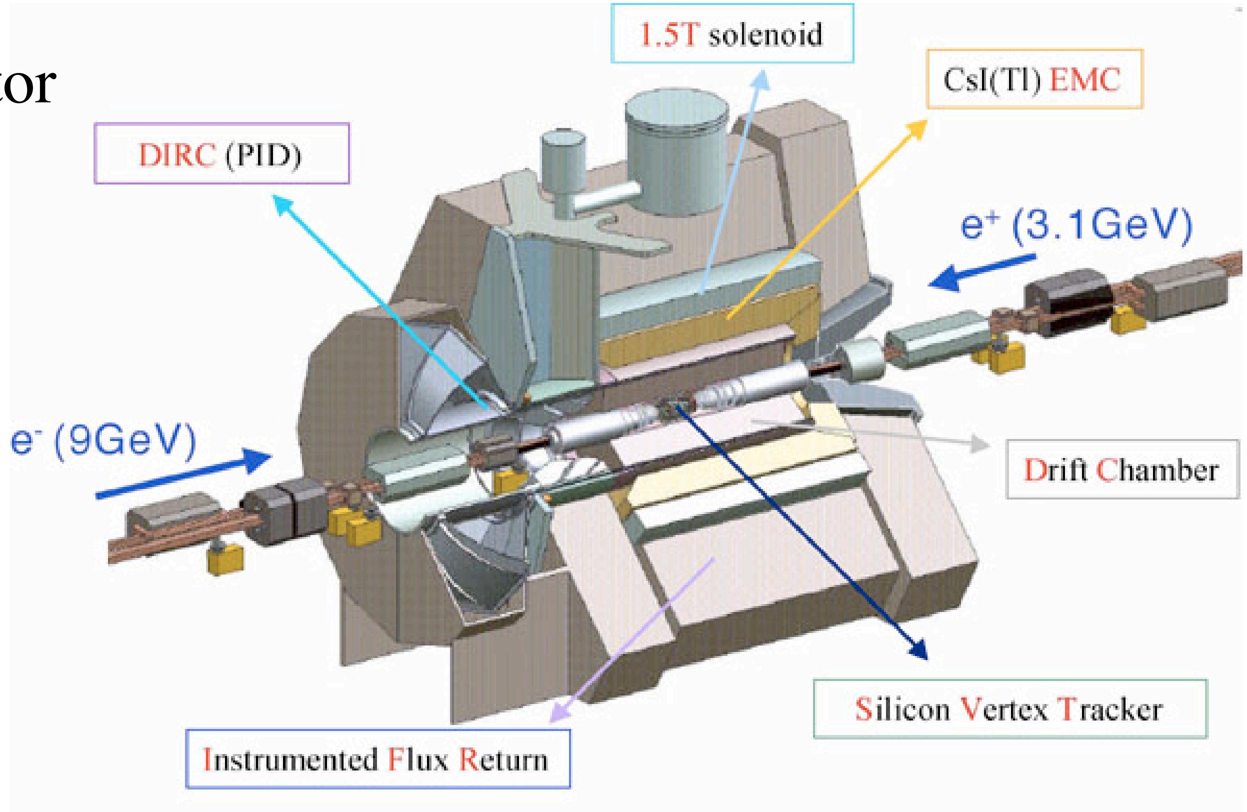


B□ J/□ K_S decays

CP (J/□ K_S) = □1



BABAR detector



in the lab. frame

$e^{□}$ → ← e^+

↓

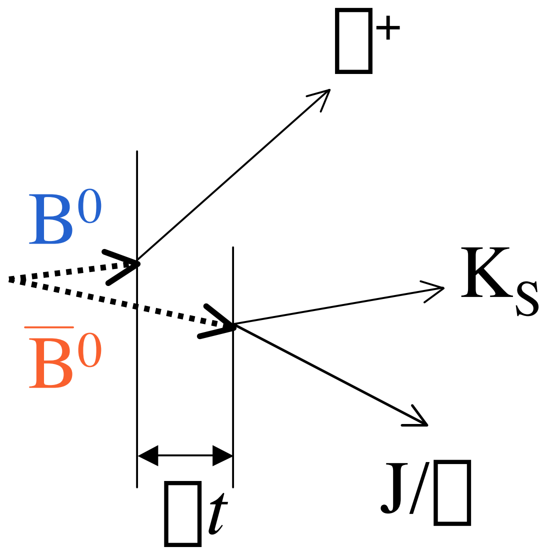
(4S)

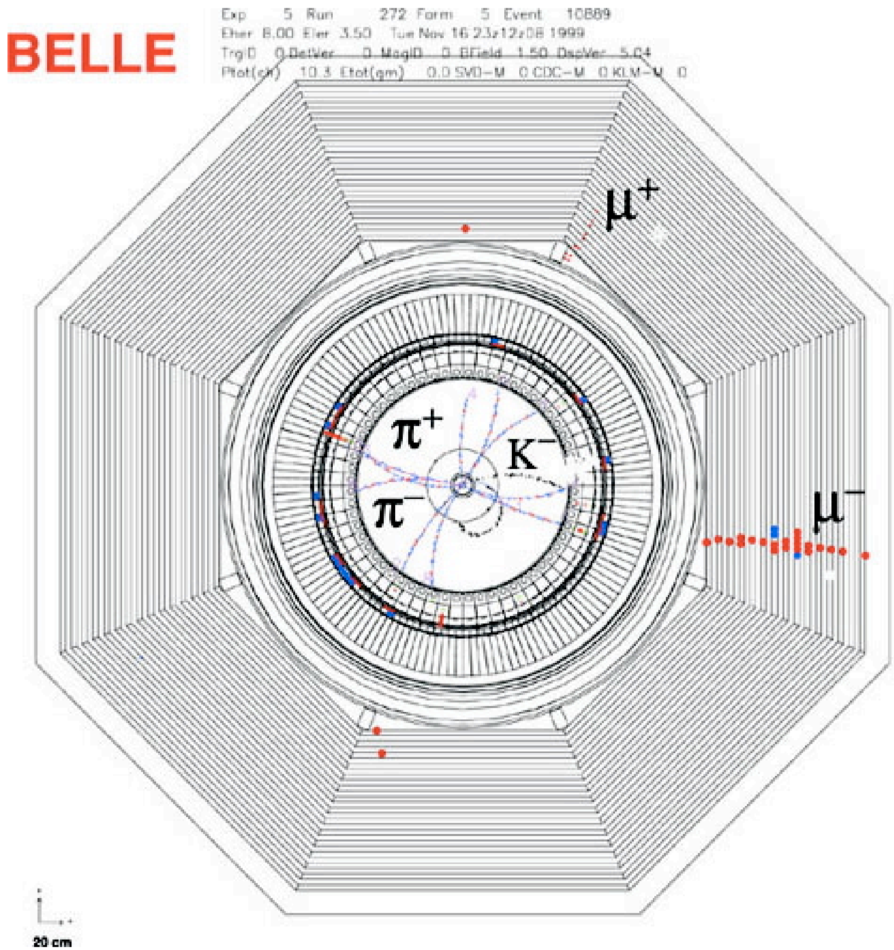
⇛ boost

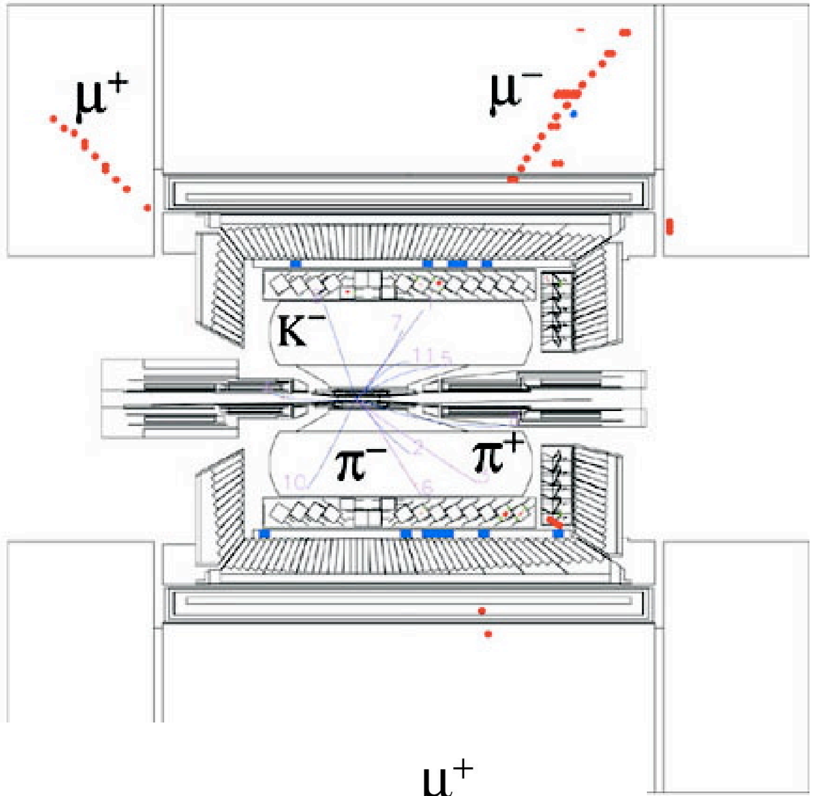
boosted

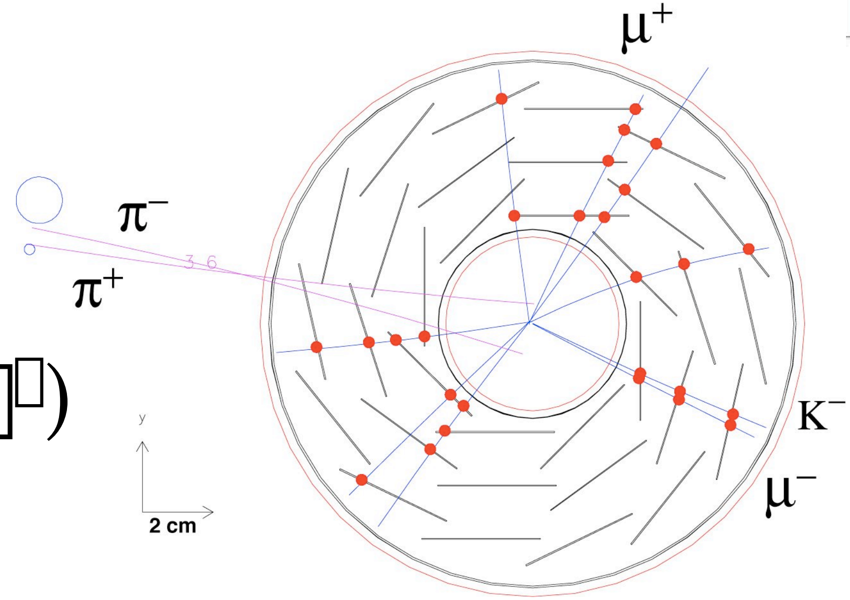
$\bar{B}^0 B^0$ system

(remains always $\bar{B}^0 B^0$)







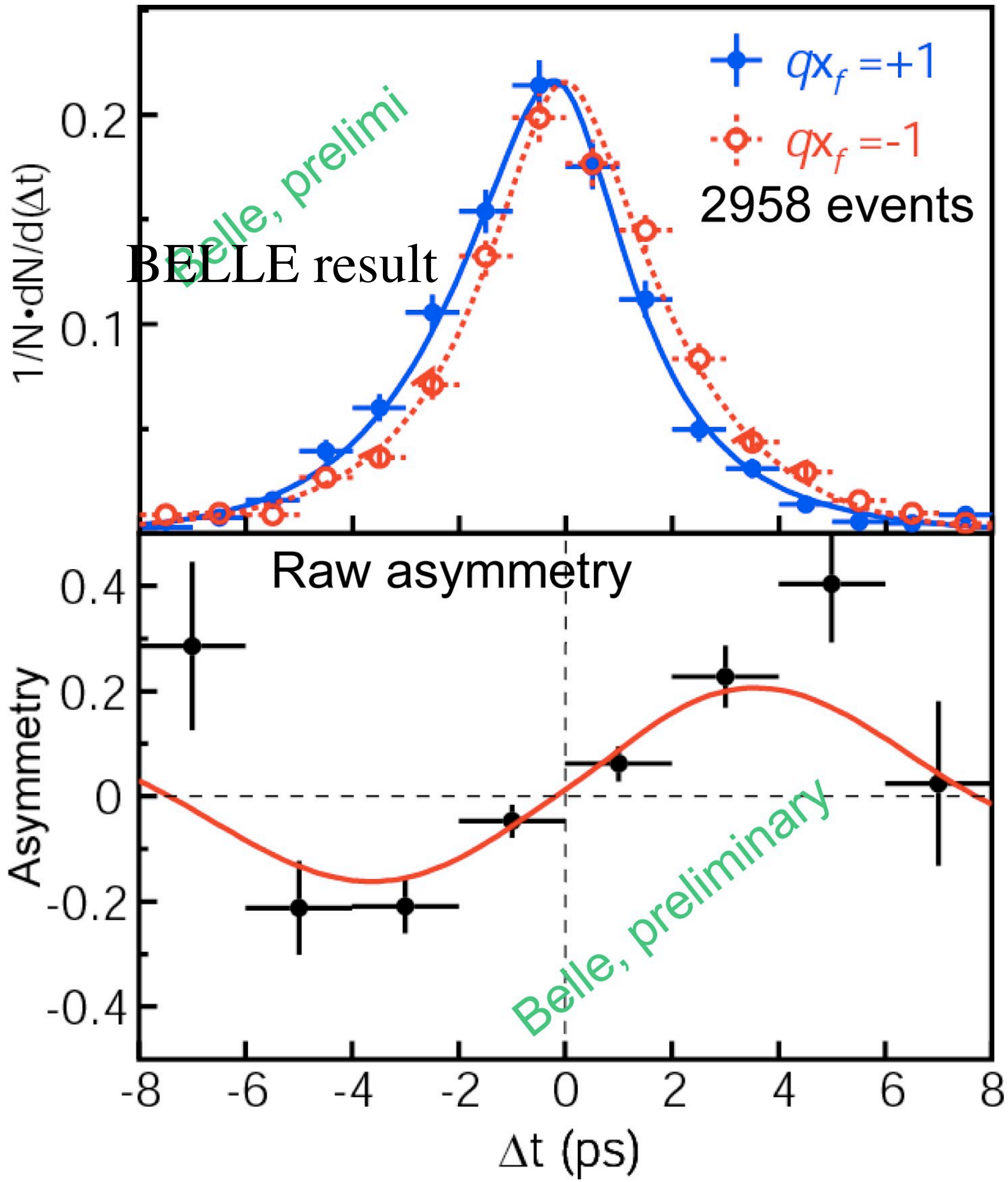


$B^0 \to J/\psi\,(\mu^+\mu^-)\ K_S\,(\pi^+\pi^-)$

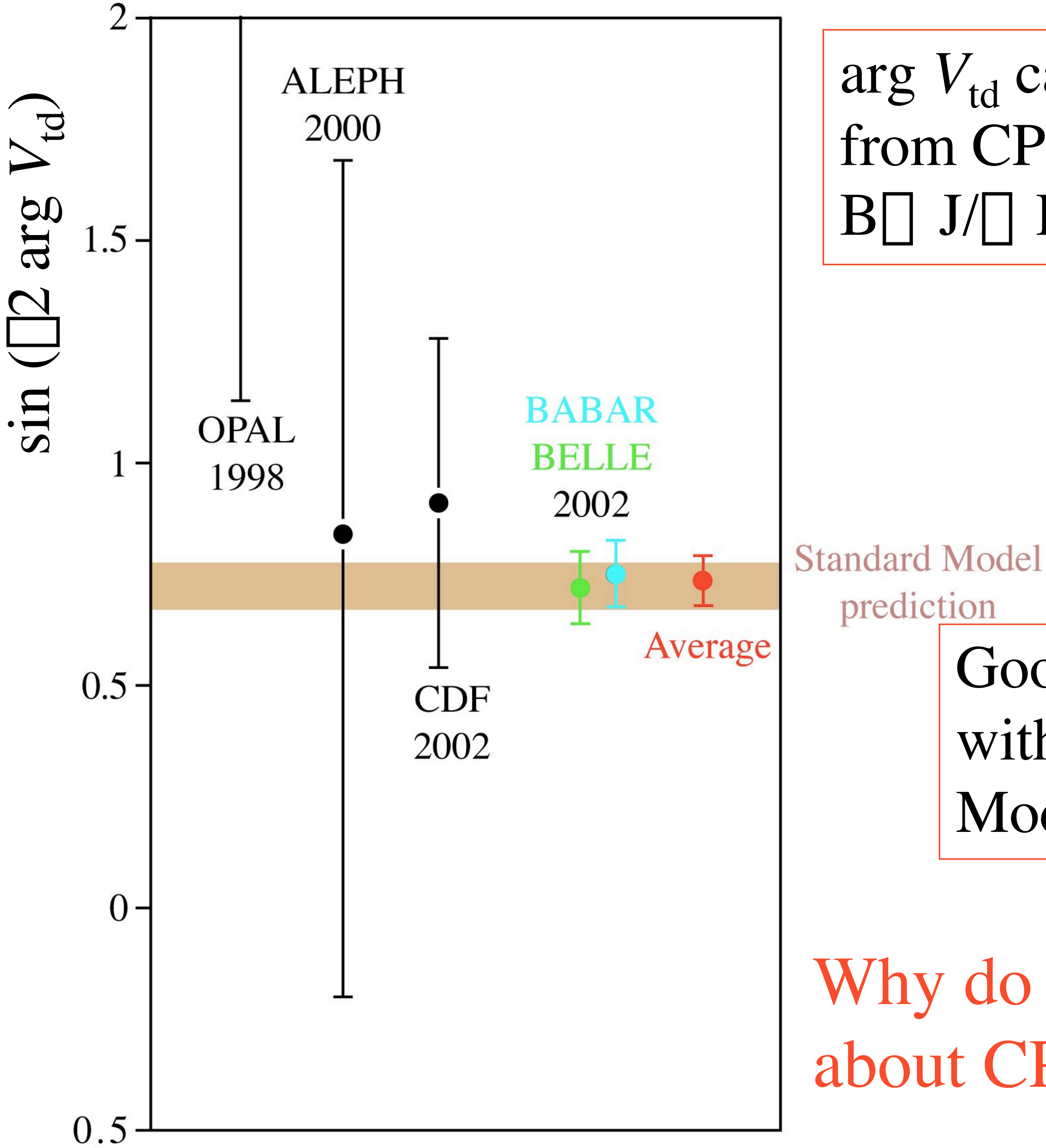
$\overline{B}^0 \to K^- + X$

B^0 □ J/□ K_S

$\overline{B}^0$ □ J/□ K_S



CP violation!



arg V_{td} can be extracted from CP asymmetry in $B \to J/\psi K_S$ decays

Good agreement with the Standard Model prediction.

Why do we still worry about CP violation?