To summarize:

It is not excluded that our matter fields are bound to 3-brane in extra dimensions.

- Apparent energy non-conservation
(until now: above some threshold)
Missing energy (into bulk).
Possibly no suppression compared to conventional processes
- Apparent non-conservation of electric charge

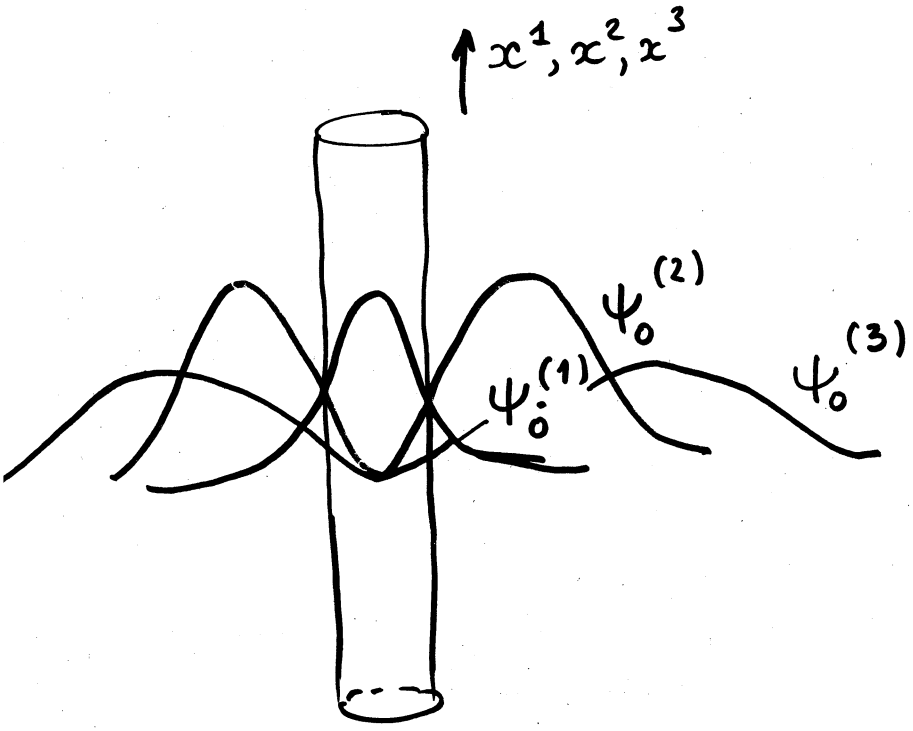
Lots of novel possibilities for model building.

Example: three 4-dim. fermion generations out of a single fundamental (higher-dimensional) generation

Three types of zero fermion modes in background of a single defect.

E.G.: FLUX-3 VORTEX (COSMIC STRING) IN 6-DIM'S.

THREE ZERO MODES (BY INDEX THEOREM) FOR FERMIONS



DIFFERENT SHAPES OF ZERO MODES.

DIFFERENT ANGULAR DEPENDENCE IN TRANSVERSE SPACE

4 DIM'S

$$\psi_0^{(1)} = f^{(1)}(\rho) \qquad \text{1ST GENERATION}$$

$$\psi_0^{(2)} = e^{i\theta} f^{(2)}(\rho) \qquad \text{2ND}$$

$$\psi_0^{(3)} = e^{2i\theta} f^{(3)}(\rho) \qquad \text{3D}$$

SMALL MIXING BETWEEN GENERATIONS

$\Updownarrow$ IN TRANSVERSE SPACE

(APPROXIMATE) SYMMETRY UNDER ROTATIONS, CONSERVATION OF ANGULAR MOMENTUM

- DIFFERENT MASSES OF FERMIONS

$\Updownarrow$

DIFFERENT RADIAL SHAPES OF ZERO MODES
$\Rightarrow$ DIFFERENT OVERLAPS WITH HIGGS MODE LOCALIZED ON VORTEX

- K-K COPIES OF W, Z:

$$\tilde{W}, \tilde{Z} \propto e^{-i\theta} W(\rho)$$

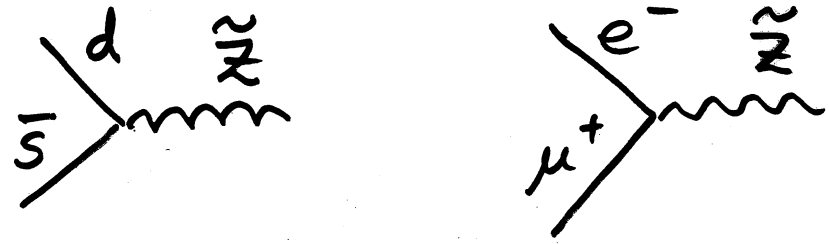
$\Downarrow$

Dominant interactions: angular momentum conserved

$$\bar{\psi}^{(1)} \tilde{W}, \tilde{Z} \psi^{(2)} \leftarrow e^{i\theta} f^{(2)}(\rho) \quad , \quad \bar{\psi}^{(2)} \tilde{W}, \tilde{Z} \psi^{(3)}$$

$$f^{(1)}(\rho) \qquad e^{-i\theta} W(\rho)$$

FCNC, LEPTON # VIOLATION



ANGULAR MOMENTUM (GENERATION #) CONSERVED (TO LEADING ORDER)

$\Downarrow$

$K_L \to \mu e$ LARGEST

RARE DECAYS SENSITIVE TO EXTRA DIMENSIONS

MANY MORE possibilities

FOR MODEL BUILDING, FANTASY...

A NOTE ON SUPERSYMMETRY:

IN SUSY THEORIES WITH EXTRA DIMENSIONS, BRANE MAY RESPECT 4-dim. SUSY (BPS-BRANE) OR NOT RESPECT (non-BPS BRANE).

UNTIL NOW: NO GRAVITY

GRAVITY PHENOMENOLOGICALLY INTERESTING FOR HIGH ENERGY PHYSICS, IFF FUNDAMENTAL GRAVITY SCALE IS LOW.

RECALL: 4-dim. couplings are effective
THEY ARE RELATED TO FUNDAMENTAL couplings IN A PECULIAR WAY THAT INVOLVES SIZE OF EXTRA DIMENSIONS.

4-dim's: GAUGE HIERARCHY PROBLEM; WHY $M_{WEAK} \ll M_{Planck}$?

EXCELLENT SOLUTION:
GRAND UNIFICATION + SUSY.

EXTRA DIM'S: possible TWIST

UNIFICATION OF couplings, GUTs PROBLEMATIC $\longrightarrow$ $M_{GRAVITY} \sim M_{WEAK}$

WHILE M_{Pl} = EFFECTIVE 4dim. parameter

IF SO, EXPECT QUANTUM GRAVITY EFFECTS AT $E \sim TeV$

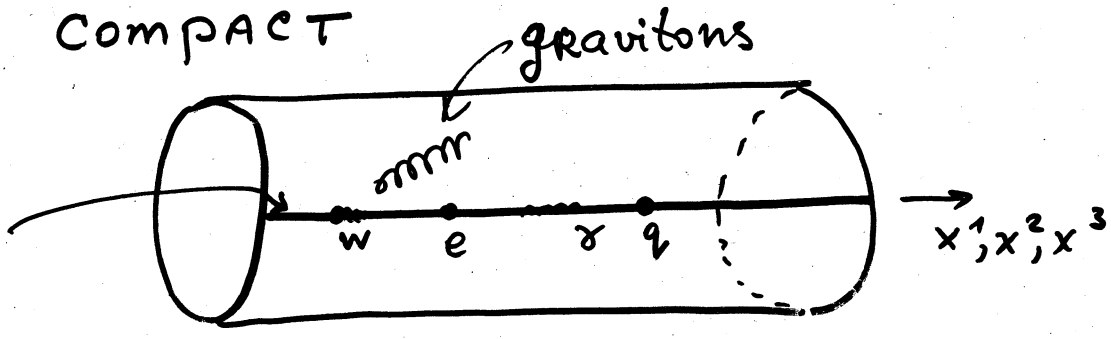
IS THIS SCENARIO POSSIBLE AT ALL?
WHAT IS THE PRICE TO PAY?
WHAT ARE POSSIBLE SIGNATURES?
NEED CONCRETE EXAMPLES.

EXAMPLE #1: LARGE EXTRA DIMENSIONS, ADD.

Matter on brane, gravity in Bulk

BRANE WITHOUT TENSION (MASS) not so great!

BULK COMPACT



KALUZA - KLEIN PICTURE FOR GRAVITONS.

THEY HAVE 4-dim. masses

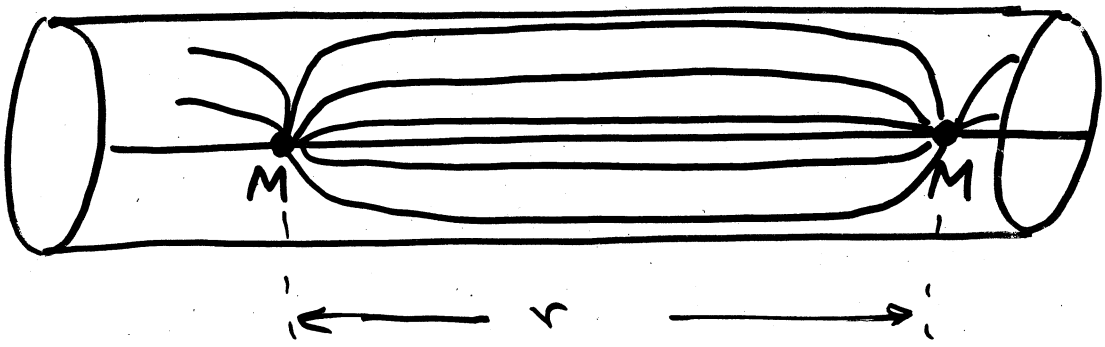
$$m^2_{\{n\}} = \frac{\sum_{i=1}^{N} n_i^2}{(2\pi R)^2}$$

N EXTRA DIM'S; ALL OF SAME SIZE R for simplicity.

Long-distance 4-dim. gravity due to graviton zero mode

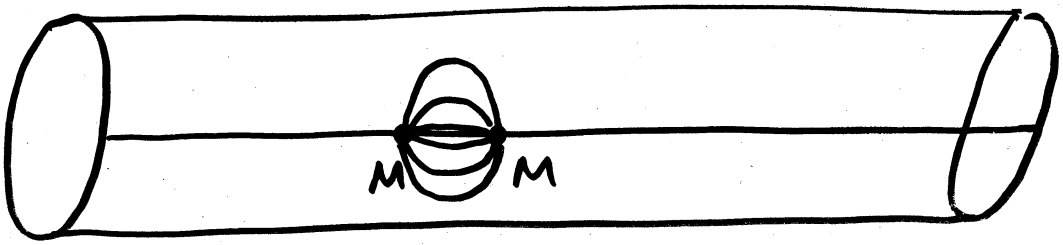
$$n_1 = n_2 = \ldots = n_N = 0 \iff m^2 = 0$$

GRAVITY DEVIATES FROM 4-dim. Newton's law AT $r \lesssim R$:



4-dim. PICTURE OF LINES OF FORCE FOR $r \gg R$

$V(r) \propto \frac{1}{r}$



$D \equiv (4+N)$ -dim. PICTURE FOR $r \ll R$; $V(r) \propto \frac{1}{r^{N+1}}$

(more detail later)

EFFECTIVE 4-dim. Planck mass:

FUNDAMENTAL ACTION FOR GRAVITY

$$S_G = \frac{1}{16\pi G_D} \int R^{(D)} \sqrt{g^{(D)}}\, d^4x\, d^N y$$

LARGE DISTANCES: ONLY HOMOGENEOUS MODE RELEVANT

$$\Rightarrow \quad g_{\mu\nu} = g_{\mu\nu}(x)$$

Plug THIS into S_G $\Rightarrow$ OBTAIN effective 4-dim. action relevant at $r \gg R$

$$S_{eff} = \frac{V_N}{16\pi G_D} \int \sqrt{g^{(4)}}\, R^{(4)}\, d^4x$$

$$\boxed{G_4 = \frac{G_D}{V_N}} \quad ; \quad V_N = (2\pi R)^N \quad \text{if all sizes are equal.}$$

$$G_4 = \frac{1}{M_{Pl}^2} \qquad G_D = \frac{1}{M^{2+N}}$$

↑ FUNDAMENTAL GRAVITY SCALE

N.B: $\frac{G_4}{r} \sim \frac{G_D}{r^{N+1}}$ @ $r \sim R$

$$\boxed{M_{Pl} = M \cdot (MR)^{N/2}}$$

NEGLECTING 2's AND π's

CHOOSE $M \sim$ TeV $\Rightarrow$ 4-dim. gravity weak

BECAUSE VOLUME OF EXTRA DIMENSIONS LARGE.

HOW LARGE IS R?

$$R \sim \frac{1}{M}\left(\frac{M_{Pl}}{M}\right)^{\frac{2}{N}} \sim 10^{-17}\,\text{cm} \cdot 10^{\frac{32}{N}}$$

$M_{Pl} = 10^{19}$ GeV

$M = 10^{3}$ GeV

- $N=1 \Rightarrow R \sim 10^{15}$ cm : UNACCEPTABLE
- $N=2 \Rightarrow R \sim$ mm Interesting:

NEWTON'S LAW CHECKED DOWN TO 0.2 mm

MOTIVATION for measuring gravity at shorter distances.

GRAIN OF SOLT: ASTROPHYSICS at $N=2 \Rightarrow M > 30$ TeV,

$R \lesssim 10^{-4}$ cm $\simeq \mu$m Very difficult, BUT NOT impossible

- $N=3 \Rightarrow R \sim 10^{-6}$ cm

  Hardly possible to reach in gravity measurements

IN ANY CASE, R QUITE LARGE

E.g. $N=6$ $(D=10) \Rightarrow R \sim 10^{-12}$ cm $\sim (10\ \text{MeV})^{-1}$

NB: IF EXTRA DIMENSIONS HAVE UNEQUAL SIZES $R_1, \dots, R_N$, THEN

$$V_N = \frac{1}{M^N} \frac{M_{Pl}^2}{M^2}$$

$$\parallel$$

$$R_1 \cdot R_2 \cdot \dots \cdot R_N$$

R_1 may BE LARGE, $R_2, \dots, R_N$ SMALL

$\Downarrow$

DEVIATION FROM 4Dim NEWTON'S LAW AT $r \sim R_1$, possibly $r \sim$ mm

ASTROPHYSICS/COSMOLOGY DOES NOT FORBID.

TWIST IN HIERARCHY PROBLEM:
WHY R IS SO LARGE?

NO COMPELLING ANSWER.

NEW PHENOMENOLOGICALLY INTERESTING PARTICES: Kaluza-Klein GRAVITONS.

MASS SPLITTING $\sim \frac{1}{R}$

VERY SMALL: $N=2 \Rightarrow \frac{1}{R} \sim \frac{1}{mm} \sim 10^{-4}\,eV \sim {}^{\circ}K$

$\vdots$

$N=6 \Rightarrow \frac{1}{R} \sim 10\,MeV$

$\mathcal{N}(E)$: NUMBER OF GRAVITONS WITH $m < E$

$$m^2 = \frac{\sum_{i=1}^{N} n_i}{R^2} < E^2$$

$\mathcal{N}(E)$ = # OF POINTS WITH INTEGER COORDINATES $n_1, \ldots, n_N$ (IN N DIM'S) INSIDE A SPHERE OF RADIUS (RE)

$= (ER)^N \times$ (VOLUME OF UNIT N-dim. BALL).

$$\boxed{\mathcal{N}(E) = (E \cdot R)^N}$$

NEGLECTING 2'S AND π'S

LARGE # OF K-K GRAVITON SPIECES.

E.G. $E = GeV$, $M = TeV$, $N=4 \Rightarrow \mathcal{N}(E) \sim 10^{20}$

NB: $R^N = \frac{1}{M^N} \cdot \frac{M_{Pe}^2}{M^2} \Rightarrow \mathcal{N}(E) = \left(\frac{E}{M}\right)^N \cdot \frac{M_{Pe}^2}{M^2}$

THE LARGER N, THE SMALLER $\mathcal{N}(E)$ FOR $E \ll M$

INTERACTION WITH MATTER

FUNDAMENTAL ACTION

Neglect brane thickness ↓

$$S = \frac{1}{16\pi G_D}\int R^{(D)}\sqrt{g^{(D)}}\,d^4x\,d^Ny + \int \mathcal{L}_{MATTER}\left(\begin{matrix}\text{matter}\\ \text{fields}\end{matrix}, g_{\mu\nu}\right) d^4x\,\delta^N(y)\,d^Ny$$

$g_{\mu\nu}$: metric on BRANE

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}(x,y)$$

$$h(x,y) = \sum_{\{\vec{n}\}} e^{i\vec{n}\vec{y}/R}\, h_n(x)$$

OMITTING INDICES, IGNORING TENSOR STRUCTURE

$$\Downarrow$$

$$S_{eff} = \frac{V_N}{16\pi G_D}\sum_{\{\vec{n}\}}\int d^4x\left[\partial_\mu h_n(x)\right]^2 + \text{higher order terms}$$

$$+\sum_{\{n\}}\int d^4x\, T^{\mu\nu} h_{n,\mu\nu}$$

$$\frac{1}{16\pi G_4} = M_{Pl}^2$$

Canonically normalize: $h_n = \frac{1}{M_{Pl}}\hat{h}_n$

$$\Downarrow$$

$$S_{eff} = \sum_{\{n\}}\int d^4x\,(\partial_\mu \hat{h}_n)^2 + \frac{1}{M_{Pl}}\int d^4x\, T_{\mu\nu}\hat{h}_n^{\mu\nu}$$

EACH K-K graviton interacts with our matter at 4-dim. gravitational strength (i.e. suppressed by $\frac{1}{M_{Pl}}$). SINGLE K-K graviton PRODUCTION POSSIBLE

Q: WHY GRAVITY IS NOT M_{Pl} SUPPRESSED AT $r \ll R$?

POTENTIAL BETWEEN TWO UNIT MASSES:

$$V(r) = -G_4 \sum_{\{n\}} \frac{e^{-m_{\{n\}} \cdot r}}{r} \qquad m_{\{n\}} = \frac{1}{R}\sqrt{\vec{n}^2}$$

$r \ll R$: MANY KK GRAVITONS CONTRIBUTE

$$\Downarrow$$

$$V(r) = -G_4 \int d^N n \, \frac{e^{-\frac{r}{R}\sqrt{\vec{n}^2}}}{r}$$

$$= -S_{N-1}\, G_4 \int n^{N-1} dn \, \frac{e^{-\frac{r}{R}n}}{r}$$

area of $(N-1)$-dim. sphere $\Rightarrow$ 2's and π's

$$= -G_4 \cdot \frac{1}{r} \cdot \underline{\underline{\left(\frac{R}{r}\right)^N}}$$

CRUCIAL FACTOR: NUMBER OF K-K graviton SPIECES WITH $m \lesssim \frac{1}{r}$

$G_D = G_4 \cdot R^N$

$$V(r) = -G_D \frac{1}{r^{N+1}}, \quad D = (N+4)\text{-dim.}$$

NEWTON'S LAW

A: BECAUSE THERE ARE PLENTY OF GRAVITON SPIECES CONTRIBUTING.

SUBTLETY OF K-K DECOMPOSITION

ORIGINAL FIELDS:

h_{AB} , $A,B = 0,1,\ldots D-1$ TAKE $D=(N+1)$ VALUES

OF THESE:

$h_{\mu\nu}$ are 4-dim. tensors $\mu = 0,1,2,3$
(K-K gravitons proper)

$h_{\mu i}$ are 4-vectors $i = 4,\ldots, N+3$
Do not interact with our matter

h_{ij} are scalars
One at each KK level
INTERACTS WITH T^{μ}_{μ} at gravitational strength.

Dangerous: $\vec{n}=0 \Rightarrow$ MASSLESS SCALAR $\equiv$ BRANS-DICKE FIELD.
INTERACTS WITH T^{μ}_{μ}, SPOILS BENDING OF LIGHT.

THIS IS MODULUS CORRESPONDING TO VOLUME OF EXTRA DIMENSIONS.

⇓

NEED MECHANISM TO FIX (STABILIZE) SIZE OF EXTRA DIMENSIONS

THERE ARE MANY MECHANISMS. SCALARS OFTEN LIGHT, $m_{min} \sim \frac{1}{R}$; K-K spectrum UNCHANGED AT $n \gg 1$.

WE'LL DISCUSS POSSIBLE MANIFESTATIONS LATER.

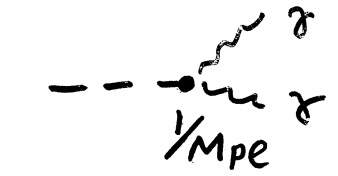
BACK TO K-K gravitons

- DECAY TO OUR MATTER,
  E.G. $G \to \gamma\gamma$, $G \to e^+e^-$, etc.

  SUPPRESSED BY M_{Pl}^{-1}:

$$\Gamma \sim \frac{1}{M_{Pl}^2} M_G^3$$

↑ ON DIMENSIONAL GROUNDS



Even for $M_G \sim$ TeV LIFETIME AGAINST DECAY INTO OUR MATTER LARGE

$$\tau \sim \frac{1}{M_G}\left(\frac{M_{Pl}}{M_G}\right)^2 \sim 10^{-27}\,s \cdot 10^{32} \sim 10^5\,s$$

MUCH LONGER FOR LIGHTER GRAVITONS.

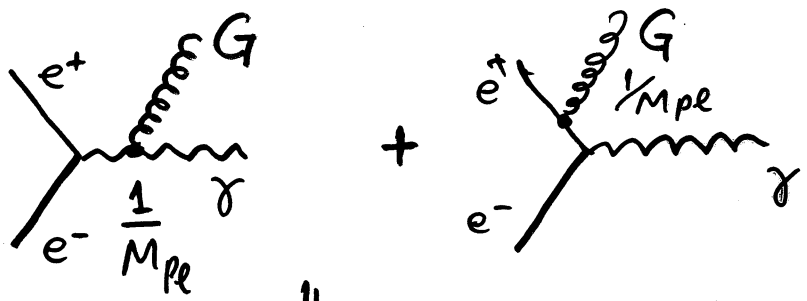
- DECAY OF K-K gravitons into BULK/OTHER BRANES MATTER (IF ANY) AND K-K gravitons THEMSELVES IS FASTER, MODEL DEPENDENT. THIS IS INVISIBLE MODE ANYWAY.
- INTERACTION OF K-K GRAVITONS WITH OUR MATTER IS VERY WEAK, SUPPRESSED BY M_{Pl}^{-1}.

⇓

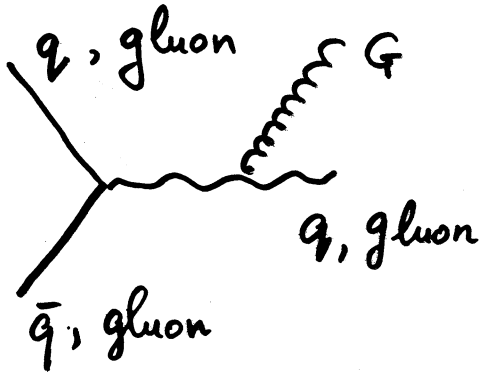
K-K GRAVITONS ARE INVISIBLE, DO NOT LEAVE TRACKS in DETECTORS

COLLIDERS :

K-K GRAVITON = MISSING ENERGY



$$e^+e^- \to \gamma + \not{E}$$



: $pp \to \text{jet} + \not{E}_T$

- EFFECTIVELY CONTINUOUS SPECTRUM of KK gravitons (mass splitting less than MeV)

⇓

BOTH ENERGY AND ANGLE OF PHOTON (jet) ARBITRARY

- MORE HEAVY GRAVITONS THAN LIGHT

⇓

LOWER ENERGY PHOTONS (jETS) ENHANCED.

CROSS SECTIONS RAPIDLY GROW WITH ENERGY.

TWO WAYS TO CALCULATE CROSS SECTION

1: 4-dim. viewpoint

EACH TYPE OF K-K gravitons created at 4-dim. gravitational strength,

$$\sigma \sim \frac{\alpha}{M_{Pl}^2} \leftarrow \text{gauge coupling}$$

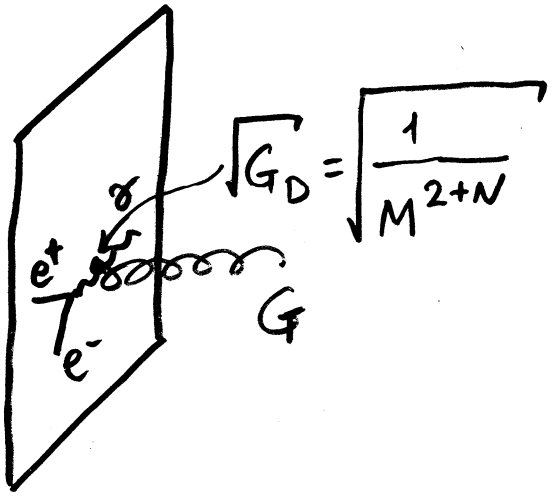
BUT ALL GRAVITONS WITH $m < E$ CAN BE EMITTED:

$$\sigma_{tot} \sim \frac{\alpha}{M_{Pl}^2} N(E) \sim \frac{\alpha}{M_{Pl}^2} (RE)^N = \boxed{\frac{\alpha}{M^{2+N}} E^N}$$

↑ TeV scale!

- Large (saturates unitarity) at $E \sim M$ (cf: D-dim. gravity strongly coupled at $E \sim M$). TRUST RESULT AT $E < M$
- D-DIM. BEHAVIOR

2: D-Dim. viewpoint. EXTRA Dim's EFFECTIVELY INFINITELY LARGE



$$\sigma_{tot} \propto \frac{\alpha}{M^{2+N}} E^N$$

LHC: $pp \to \text{jet} + \not{E}_T$

BACKGROUND $pp \to \text{jet} + Z$, $Z \to \nu\bar{\nu}$

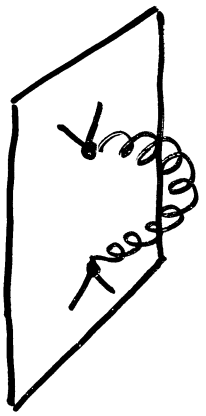
Fig.

Sensitivity of LHC: $M = 8 \longrightarrow 4.5$ TeV

FOR $N = 2 \longrightarrow 6$

CONTACT INTERACTIONS $\Longleftrightarrow$ VIRTUAL GRAVITON EXCHANGE.

D-DIM. CALCULATION



$$\mathcal{L}_{eff} = \frac{1}{M^{N+2}} T_{\mu\nu}(p)\, T^{\mu\nu}(-p) \times \int_0^\infty \frac{d^N q_y}{p^2 + q_y^2}$$

ULTRAVIOLET DIVERGENT!

Need UV completion (UV behavior of FUNDAMENTAL THEORY) TO CALCULATE.

CAN ONLY GUESS...

Generally:

$$\left(T_{\mu\nu} T^{\mu\nu} - \frac{1}{N+2} T_\lambda^\lambda T_\mu^\mu\right) \cdot \frac{1}{\Lambda^4}$$

$\Lambda \sim M$? OR SOMEWHAT LOWER? OR SOMEWHAT LARGER?

– HARDLY

Standard story

...

INTERESTING AT LHC:

$pp \to \gamma\gamma$, LARGE $E_{T,\gamma}$, $M_{\gamma\gamma}$

SENSITIVITY SIMILAR TO DIRECT PRODUCTION

(32a)

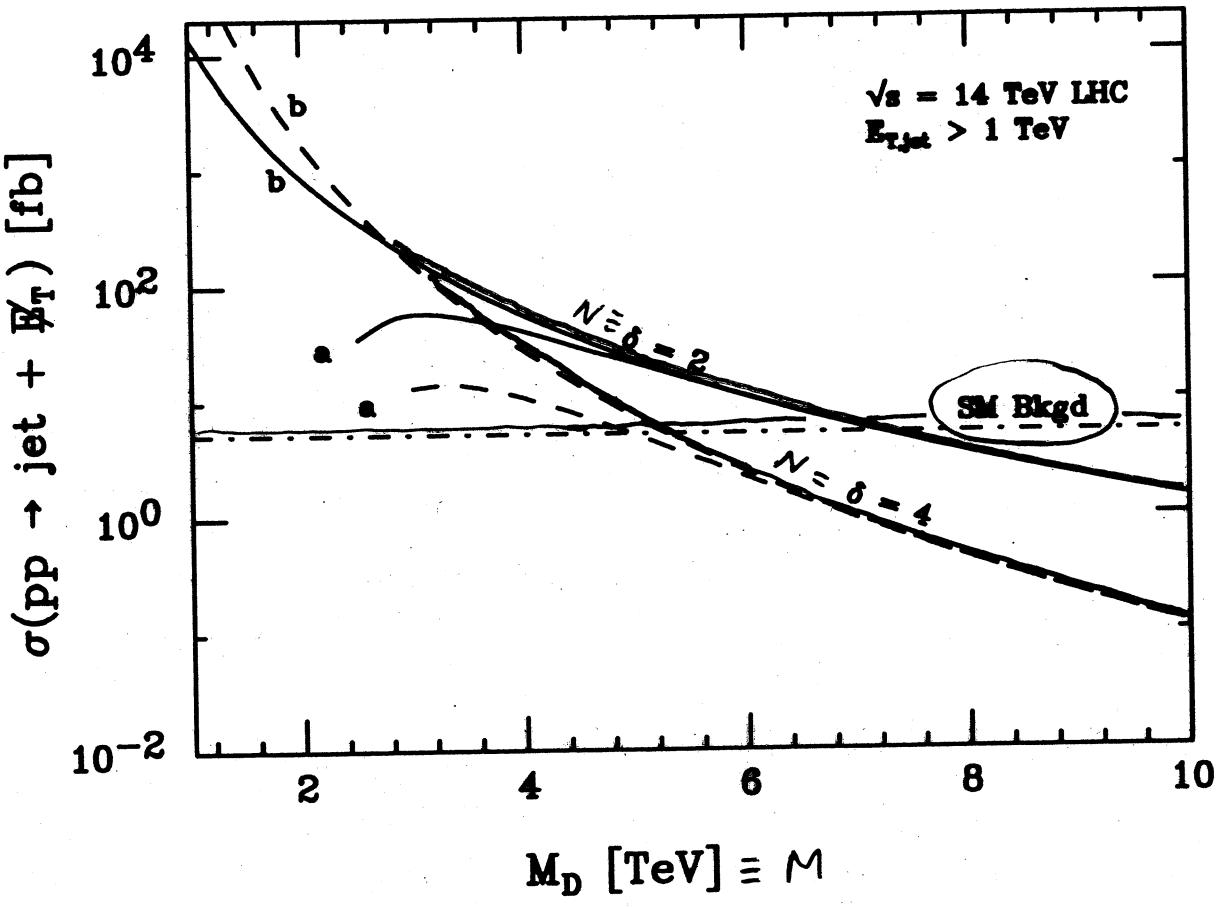


Figure 4: The total jet + nothing cross-section versus M_D at the LHC integrated for all $E_{T\,\mathrm{jet}} > 1$ TeV with the requirement that $|\eta_{\mathrm{jet}}| < 3.0$. The Standard Model background is the dash-dotted line, and the signal is plotted as solid and dashed lines for $\delta = 2$ and 4 extra dimensions. The a (b) lines are constructed by integrating the cross-section over $\hat{s} < M_D^2$ (all $\hat{s}$).

GIUDICE, RATTAZZI, WELLS

GRAVISCALARS:

Higgs - graviscalar mixing

EXAMPLE OF DECAY INTO BULK.

4 dim's: SCALAR PART OF METRIC PERTURBATION h^λ_λ NOT DYNAMICAL

NO LONGER TRUE IN EXTRA dim's.

OUR HIGGS + GRAVITY: ALLOWED TERM

$$\int d^4x \sqrt{g^{(4)}} \cdot \xi R^{(4)} \cdot \varphi^+\varphi$$

ξ: free dimensionless parameter; φ: our Higgs field

Lowest order:

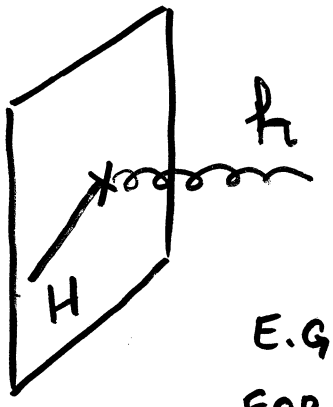
$$R^{(4)} = \Box^{(4)} h^\lambda_\lambda + \dots$$

$$\varphi = \begin{pmatrix} 0 \\ v+H \end{pmatrix}$$

$$\Downarrow$$

MIXING $\quad \xi v \, \partial_\mu h \cdot \partial_\mu H$

Higgs converts into scalar graviton and escapes FROM OUR BRANE $\Rightarrow$ INVISIBLE Higgs DECAY



$$\Gamma \sim \xi^2 \frac{v^2}{M^{2+N}} \cdot m_H^{1+N}$$

NOT SO small for $M \sim$ TeV

E.G. $N=4$, $Br(H \to h) \gtrsim 10\%$

FOR $m_H < 160$ GeV (BELOW WW THRESHOLD).