Plan of Talks

1. The Standard Model and (a Little) Beyond
2. Neutrinos (Mainly) from Heaven
3. The Numbers and What They Tell Us
4. Flavor Models
5. Leptogenesis

Plan of Talk II

Neutrinos (Mainly) from Heaven

1. Vacuum oscillations
   - Atmospheric neutrinos (AN)
   - Reactor neutrinos (RN)
   - Solar neutrinos (SN)
2. The MSW effect
   - Solar neutrinos (SN)

Vacuum Oscillations

Pontecorvo, 1957

Flavor Transitions (I)

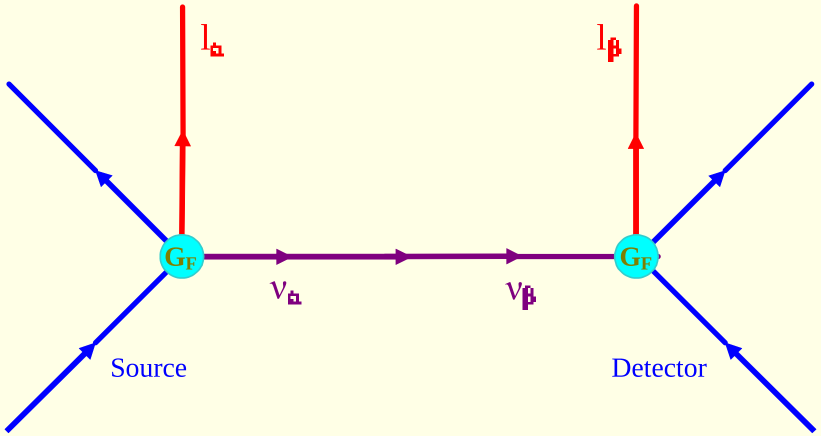
- Flavor basis (production and detection): $\nu_e, \ \nu_\mu, \ \nu_\tau$
- Mass basis (free propagation in space-time): $\nu_1, \ \nu_2, \ \nu_3$

- In general, flavor eigenstates $\neq$ mass eigenstates
- $U(\nu_1, \nu_2, \nu_3)^T = (\nu_e, \nu_\mu, \nu_\tau)^T$

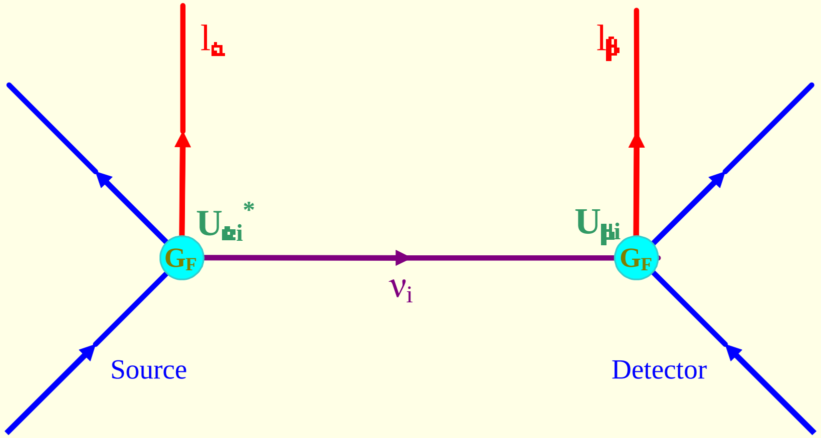
$$\Downarrow$$

- Flavor is not conserved during propagation in space-time
- ν_α is produced but $\nu_{\beta\neq\alpha}$ might be detected $(\alpha, \beta =$ flavors)

Flavor Transitions (II)



$$= \sum_i$$



Flavor Transitions (III)

The probability $P_{\alpha\beta}$ of producing neutrinos of flavor α and detecting neutrinos of flavor β is calculable in terms of

- The neutrino energy E
- The distance between source and detector L
- The mass-squared differences $\Delta m^2_{ij} \equiv m^2_i - m^2_j$
  - ($P_{\alpha\beta}$ is independent of the absolute mass scale)
- U parameters (mixing angles and phase)
  - ($P_{\alpha\beta}$ is independent of the Majorana phases)

Oscillations

$$|\nu_\alpha\rangle = U^*_{\alpha i}|\nu_i\rangle \qquad |\nu_\alpha(t)\rangle = \sum_i U^*_{\alpha i}|\nu_i(t)\rangle$$

$$|\nu_i(t)\rangle = e^{-iE_i t}|\nu_i(0) \qquad E_i = \sqrt{p^2+m_i^2} \simeq p + \frac{m_i^2}{2E}$$

$$\begin{aligned} P_{\alpha\beta} &= |\langle\nu_\beta|\nu_\alpha(t)\rangle|^2 \\ &= \sum_i |\langle\nu_\beta|\nu_i\rangle\langle\nu_i|\nu_\alpha(t)\rangle|^2 \\ &= \delta_{\alpha\beta} - 4\sum_{j>i} \mathcal{R}e(U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j}) \sin^2\left[(\Delta m^2_{ij}L)/(4E)\right] \\ &\qquad +2\sum_{j>i} \mathcal{I}m(U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j}) \sin\left[(\Delta m^2_{ij}L)/(2E)\right] \end{aligned}$$

Two Generations

- A single mixing angle: $U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$
- A single mass-squared difference: $\Delta m^2 = m_2^2 - m_1^2$

$$\Downarrow$$

$$\boxed{P_{\alpha\beta} = \sin^2 2\theta \;\; \sin^2\left(\frac{\Delta m^2 L}{4E}\right)}$$

L/E must be right

- Experimental parameters: $E,\ L$
- Theory parameters: $\Delta m^2,\ \theta$
- $P_{\alpha\beta} = \sin^2 2\theta\ \sin^2 \left[1.27\ \frac{\Delta m^2}{\mathrm{eV}^2} \frac{L/E}{\mathrm{m/MeV}}\right]$

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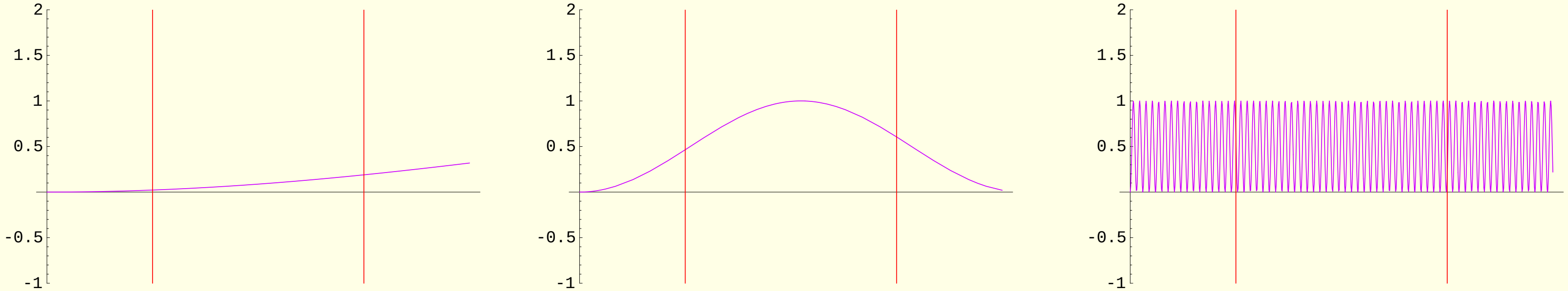


$\Delta m^2 L/E \ll 1$ — $P_{\alpha\beta} \to 0$

$\Delta m^2 L/E \sim 1$ — $P_{\alpha\beta}$ sensitive to Δm^2

$\Delta m^2 L/E \gg 1$ — $P_{\alpha\beta} \to \frac{1}{2}\sin^2 2\theta$

Exploring θ and Δm^2

To allow observation of neutrino oscillation,

- Nature has to be generous: $\sin^2 2\theta \not\ll 1$
- To probe small Δm^2 we need large L/E
- In particular, to probe $\Delta m^2 \sim 10^{-11}\ eV^2$ with $E \sim MeV$ neutrinos, we need the reactor at $L \sim 10^8$ km

Source	E[MeV]	L[km]		Δm^2[eV^2]
SN	1	10^8	$\Longrightarrow$	$10^{-11} - 10^{-9}$
RN	1	10^2	$\Longrightarrow$	$10^{-5} - 10^{-3}$
AN	10^3	10^{1-4}	$\Longrightarrow$	$10^{-4} - 1$

The MSW Effect

Wolfenstein (1978); Mikheev and Smirnov (1985)

Matter Effects

- In vacuum, in mass basis (ν_1, ν_2): $H = p + \begin{pmatrix} \frac{m_1^2}{2E} & 0 \\ 0 & \frac{m_2^2}{2E} \end{pmatrix}$

- In vacuum, in interaction basis (ν_e, ν_a):

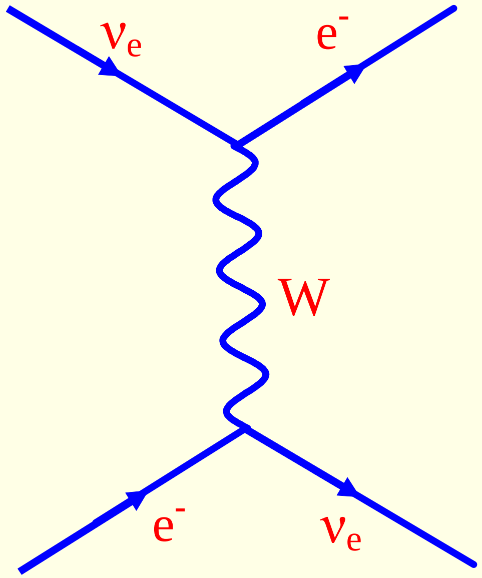
$$H = p + \frac{m_1^2+m_2^2}{4E} + \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta \end{pmatrix}$$

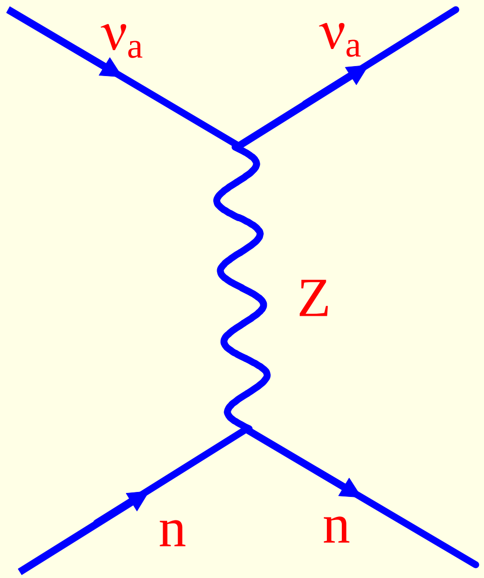
- In matter (e, p, n), in interaction basis (ν_e, ν_a):

$$H = p + V_a + \frac{m_1^2+m_2^2}{4E} + \begin{pmatrix} (V_e - V_a) - \frac{\Delta m^2}{4E}\cos 2\theta & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta \end{pmatrix}$$

- All active neutrinos have NC interactions, but only ν_e has CC interactions with matter: $V_e - V_a = \sqrt{2} G_F n_e$

CC $\leftrightarrow$ NC





Charged Current Interactions

ν_e only

Neutral Current Interactions

$\nu_a, \quad a = e, \mu, \tau$

$(i)\ \theta_m \neq \theta$

$$H \sim \begin{pmatrix} \sqrt{2}G_F n_e - \frac{\Delta m^2}{4E}\cos 2\theta & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta \end{pmatrix}$$

$$\Downarrow$$

- The mixing angle relating (ν_e, ν_a) to (ν_1^m, ν_2^m) depends on the matter density:

$$\tan 2\theta_m = \frac{\Delta m^2 \sin 2\theta}{\Delta m^2 \cos 2\theta - 2\sqrt{2}G_F n_e E}$$

- Example: $\sqrt{2}G_F n_e \gg \frac{\Delta m^2}{2E} \implies \theta_m \to \pi/2$

$\implies \nu_e$ is very close to the heavier mass eigenstate ν_2^m

(ii) $\theta_m = \theta_m(t)$

For a neutrino propagating in varying density $n_e(x)$

- The mixing angle changes: $\theta_m = \theta_m(n_e(x))$
- $\tan 2\theta_m(x) = \frac{\Delta m^2 \sin 2\theta}{\Delta m^2 \cos 2\theta - 2\sqrt{2} G_F n_e(x) E}$
- As $n_e(x) \downarrow$: $\theta_m \downarrow$
- In particular,
  - At $n_e \gg n_e^R$: $\theta_m \approx \pi/2$
  - At $n_e^R = \frac{\Delta m^2 \cos 2\theta}{2\sqrt{2} G_F E}$: $\theta_m^R = \pi/4$
  - At $n_e = 0$ (vacuum): $\theta_m = \theta$

$$\Downarrow$$

ν_2^m propagating in $n_e \downarrow$ is mostly ν_e above n_e^R, and mostly ν_a below n_e^R

$(iii)\ \nu_1^m \leftrightarrow \nu_2^m$ transitions

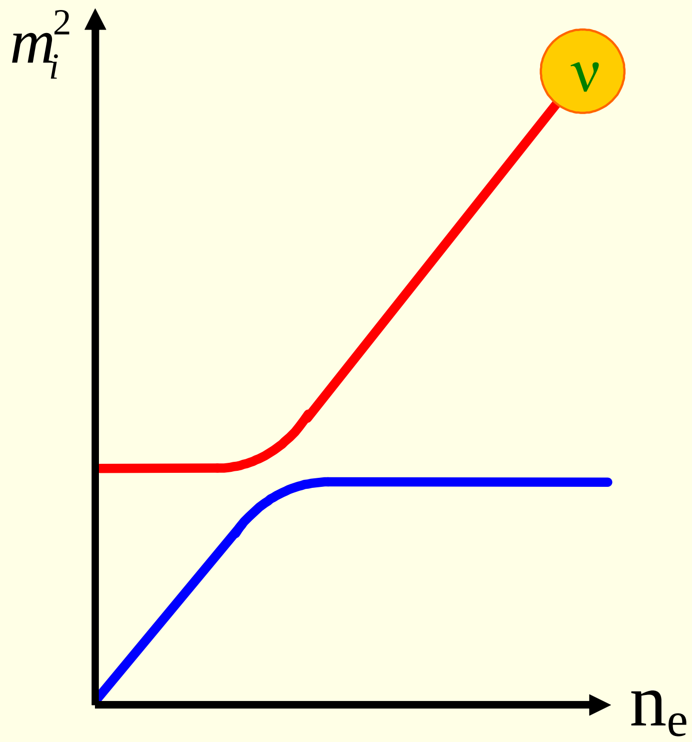
For varying density, $H = H(t)$,

- $e^{-iH(t)t} \neq e^{-i\int H(t')dt'}$
- Instantaneous mass eigenstates $\neq$ eigenstates of time evolution
- The transitions $\nu_{1m} \leftrightarrow \nu_{2m}$ occur

For slowly varying density, $\dot{H}t \ll H$,

- $e^{-i\int H(t')dt'} = e^{-i(Ht+\dot{H}t^2+\cdots)} \approx e^{-iHt}$
- The transitions $\nu_{1m} \leftrightarrow \nu_{2m}$ can be neglected
- The adiabatic condition: $\boxed{\frac{1}{n}\frac{dn}{dx} \ll \frac{\Delta m^2}{E}\frac{\sin^2 2\theta}{\cos 2\theta}}$

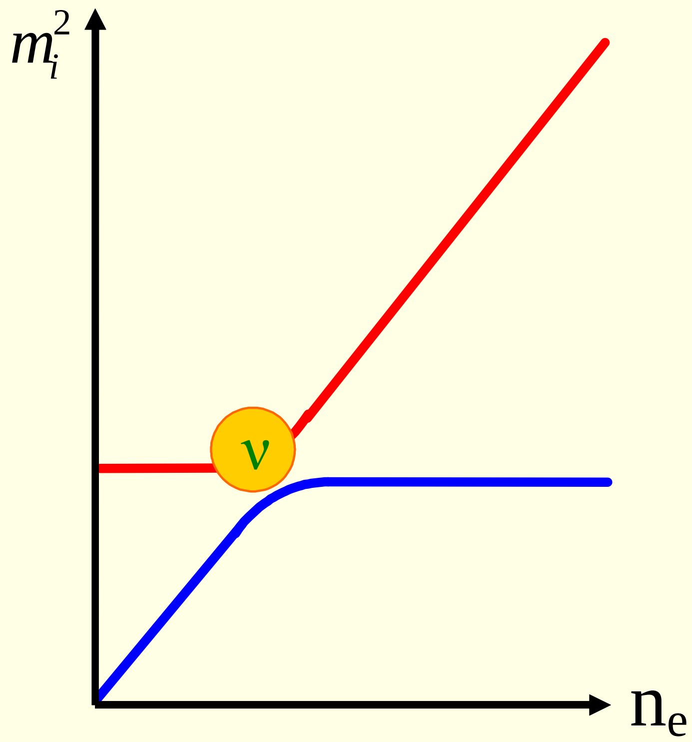
$$E \gg \frac{\Delta m^2}{G_F \;\; n_e^{\text{prod}}}$$



Production with $n_e^{\text{prod}} \gg n_e^R$

$$\nu = \nu_2^m(\theta_m = \pi/2)$$

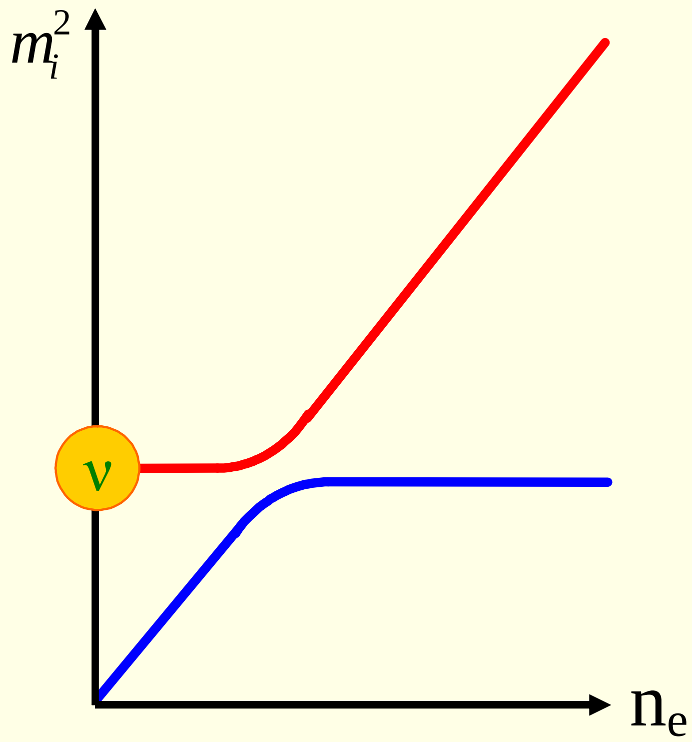
$$E \gg \frac{\Delta m^2}{G_F \;\; n_e^{\text{prod}}}$$



Adiabatic $\left(E \ll \frac{\Delta m^2}{\frac{1}{n}\frac{dn}{dx}} \frac{\sin^2 2\theta}{\cos 2\theta}\right)$ propagation at $n_e \sim n_e^R$

$$\nu = \nu_2^m(\theta_m = \pi/4)$$

$$E \gg \frac{\Delta m^2}{G_F \; n_e^{\text{prod}}}$$



Approaching the surface of the Sun

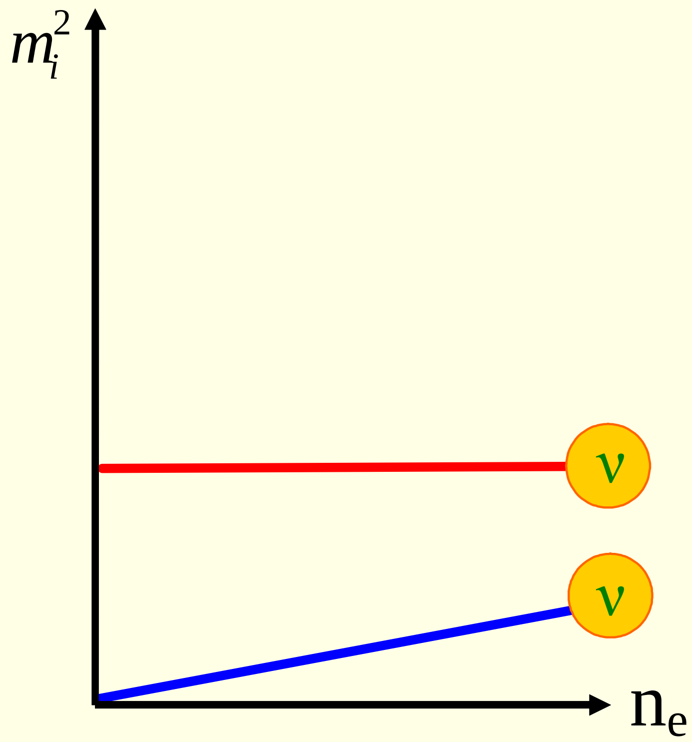
$$\nu = \nu_2^m(\theta_m = \theta) = \nu_2 \implies P_{ee} = |\langle \nu_e | \nu_2 \rangle|^2 = \sin^2 \theta$$

$$\frac{\Delta m^2}{\frac{1}{n}\frac{dn}{dx}}\frac{\sin^2 2\theta}{\cos 2\theta} \gg E \gg \frac{\Delta m^2}{G_F \; n_e^{\rm prod}}$$

$$\boxed{P_{ee} = |\langle \nu_e | \nu_2 \rangle|^2 = \sin^2\theta}$$

1. High sensitivity to θ;
2. The only way to probe small angles ($\sin^2\theta \gtrsim 10^{-4}$ for $\Delta m^2 \sim 10^{-4}\ eV^2$)
3. $P_{ee} < \frac{1}{2}$ is possible
   In constrast to averaged vacuum oscillations,
   $P_{ee} = 1 - \frac{1}{2}\sin^2 2\theta > \frac{1}{2}$

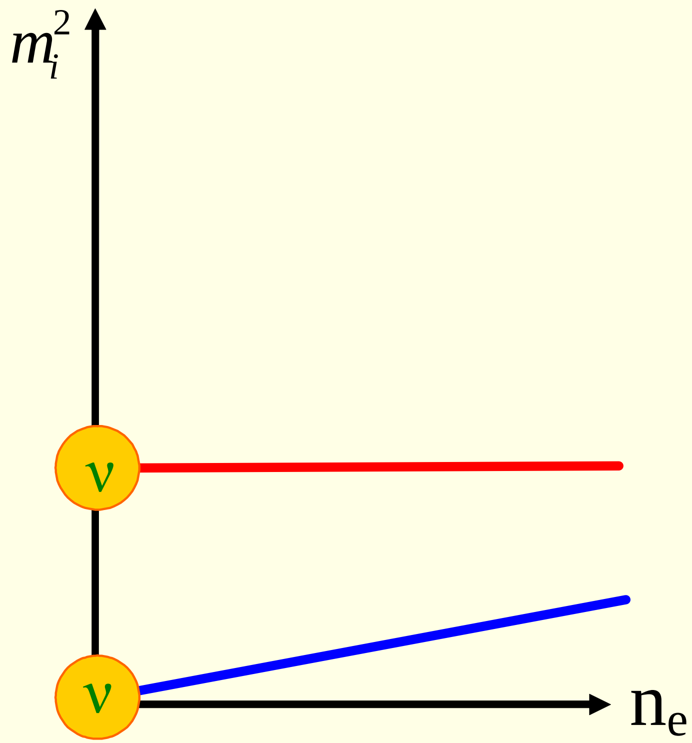
$$E \ll \frac{\Delta m^2 \cos 2\theta}{G_F \;\; n_e^{\text{prod}}}$$



Production with $n_e^{\text{prod}} \ll n_e^R$

$$\nu = \sin\theta \;\nu_2^m + \cos\theta \;\nu_1^m$$

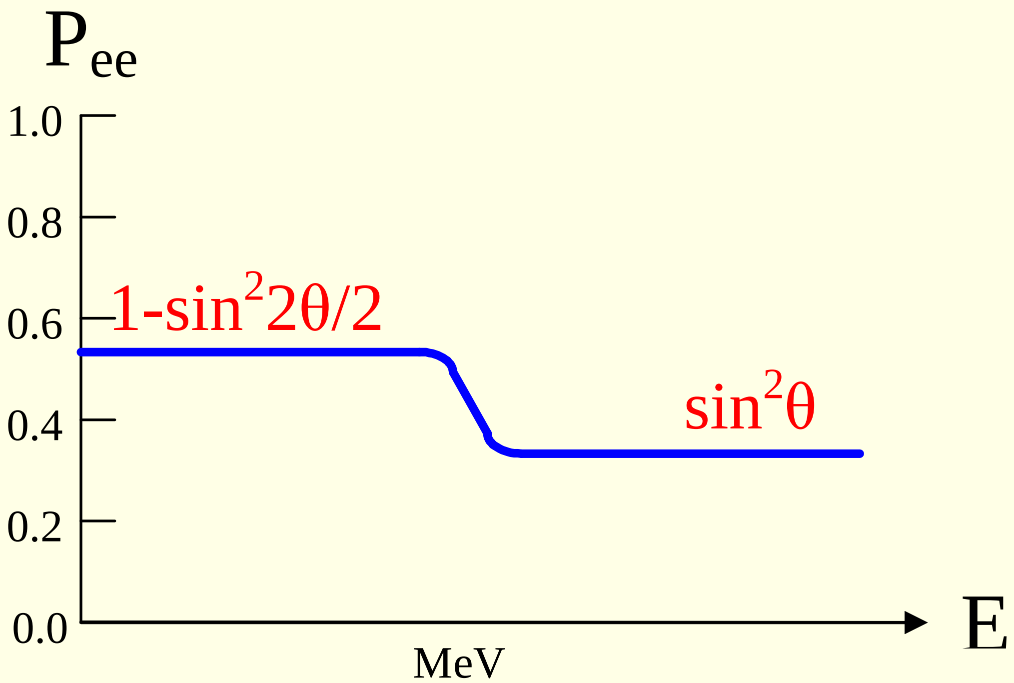
$$E \ll \frac{\Delta m^2 \cos 2\theta}{G_F \; n_e^{\text{prod}}}$$



Approaching the surface of the Sun

$$\nu = \sin\theta \; \nu_2 + \cos\theta \; \nu_1 = \nu_e \implies P_{ee}(R_\odot) = 1 \implies P_{ee}(\text{Earth}) = 1 - \tfrac{1}{2}\sin^2 2\theta$$

MSW in the Sun, Qualitatively



MSW in the Sun, Quantitatively

- The Sun is a source of MeV ν_e's
  - To have resonance: $n_e^{\text{prod}} > n_e^R = \frac{\Delta m^2 \cos 2\theta}{2\sqrt{2} G_F E}$
    $\Longrightarrow$ To probe Δm^2 up to $\sim 10^{-5}\ eV^2$, we need
    $n_e^{\text{prod}} \sim 4 \times 10^{-25}\ \text{cm}^{-3}$
  - To have adiabatic propagation: $\frac{\Delta m^2}{E} \frac{\sin^2 2\theta}{\cos 2\theta} \left| \frac{d \ln n_e}{dx} \right|_{\text{res}}^{-1} \gg 1$
    $\Longrightarrow$ To probe Δm^2 down to $\sim 10^{-9}\ eV^2$, we need
    $r_0 \sim 3 \times 10^9$ cm $[n_e(x) \approx 2 n_0 \exp(-x/r_0)]$

Source	$n_0[\text{cm}^{-3}]$	$r_0[\text{cm}]$		$\Delta m^2[\text{eV}^2]$
SN	6×10^{-25}	7×10^9	$\Longrightarrow$	$10^{-9} - 10^{-5}$

Summary: What can we see?

Source	Effect	$\Delta m^2[\mathrm{eV}^2]$
SN	VO	$10^{-11} - 10^{-9}$
SN	MSW	$10^{-9} - 10^{-5}$
KN	VO	$10^{-5} - 10^{-3}$
AN	VO	$10^{-4} - 1$

• If $\theta \not\ll 1$, we should be able to discover neutrino masses in the entire theoretically interesting range: $10^{-11}\ eV^2 < \Delta m^2 < eV^2$
• If $10^{-2} \lesssim \theta \ll 1$ we could still discover it via the adiabatic MSW effect for $\Delta m^2 \sim 10^{-5}\ eV^2$

Next
What do we see?

$\nu_1^m \leftrightarrow \nu_2^m$ transitions

$$\begin{pmatrix} \nu_e \\ \nu_a \end{pmatrix} = \begin{pmatrix} \cos\theta_m & \sin\theta_m \\ -\sin\theta_m & \cos\theta_m \end{pmatrix} \begin{pmatrix} \nu_1^m \\ \nu_2^m \end{pmatrix}$$

$$\frac{\partial}{\partial t} \begin{pmatrix} \nu_e \\ \nu_a \end{pmatrix} = \dot{U}(\theta_m) \begin{pmatrix} \nu_1^m \\ \nu_2^m \end{pmatrix} + U(\theta_m) \begin{pmatrix} \dot{\nu}_1^m \\ \dot{\nu}_2^m \end{pmatrix}$$

$$i \begin{pmatrix} \dot{\nu}_1^m \\ \dot{\nu}_2^m \end{pmatrix} = \frac{1}{4E} \begin{pmatrix} (m_1^m)^2 - (m_2^m)^2 & -4iE\dot{\theta}_m \\ 4iE\dot{\theta}_m & (m_2^m)^2 - (m_1^m)^2 \end{pmatrix} \begin{pmatrix} \nu_1^m \\ \nu_2^m \end{pmatrix}$$

$$\dot{\theta}_m(t) = \frac{\sqrt{2} G_F E \Delta m^2 \sin 2\theta \ \dot{n}_e}{[(m_2^m)^2 - (m_1^m)^2]^2}$$