Selected Topics in the Theory of Heavy Ion Collisions

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Aim: understand data from these machines

- Alternating Gradient Synchrotron (AGS) at Brookhaven BNL
  - variety of beams, since 80's

$$\sqrt{s_{NN}}\,\Big|^{AGS}_{Au+Au} \simeq 2-5\,GeV$$

- CERN SPS fixed target experiments
  - variety of beams, Pb-beams since 1994

$$\sqrt{s_{NN}}\,\Big|^{SPS}_{Pb+Pb} < 17\,GeV$$

- Relativistic Heavy Ion Collider (RHIC) at Brookhaven BNL
  - start in 2000, so far p+p, Au+Au and d+Au

$$\sqrt{s_{NN}}\,\Big|^{RHIC}_{Au+Au} \leq 200\,GeV$$

- Large Hadron Collider (LHC) at CERN
  - start in 2007 with p+p, in 2008 with Pb+Pb

$$\sqrt{s_{NN}}\,\Big|^{LHC}_{Pb+Pb} = 5.5\,TeV$$

- total cross section $\sigma^{Pb+Pb}_{total} = 8\,barn = 10^{-24}\,cm^2$
- maximal luminosity $L^{Pb+Pb}_{max} \sim 10^{27}\,cm^{-2}\,s^{-1}$

8000 collisions per second !

What is measured when at the LHC ?

$\sigma_{total}^{Pb+Pb} = 8\ barn = 10^{-24}\ cm^2$ $\quad$ $L_{max}^{Pb+Pb} \sim 10^{27}\ cm^{-2}\ s^{-1}$

When ?	How much data ?	Data on what ?
after $15\ min \sim 10^3\ s$	$L_{integrated} = 1\ \mu b^{-1}$	• event multiplicity • low-pt hadronic spectra • particle ratios
after 1 month $\sim 10^6\ s$ (1 LHC Pb+Pb year)	$L_{integrated}^{realistic,reduced} = 10 - 100\ \mu b^{-1}$ $L_{integrated}^{optimal} = 1\ nb^{-1}$	• abundant high-pt processes such as jets ..... • rare hadronic and leptonic processes ....
after 2-3 LHC years		p+Pb collisions

Strategy for these lectures:

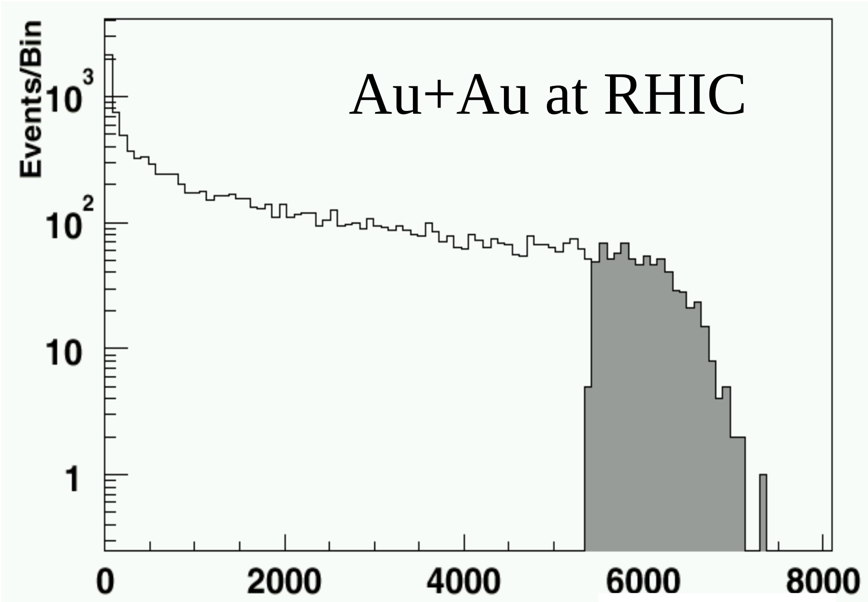
- explain basic theory for the data accessible at the LHC
- explain it in the order in which the data will be obtained
- give motivation for measurements by explaining measurements (not before)
- point out where theory is incomplete
- use whereever possible real RHIC data as *training site* for the LHC

Lecture 1:
Measuring Extensions and Orientations in the Bulk
First Signs of Collectivity

a. Event Multiplicity and Multiplicity Distribution
- Glauber Theory
- Centrality Classes

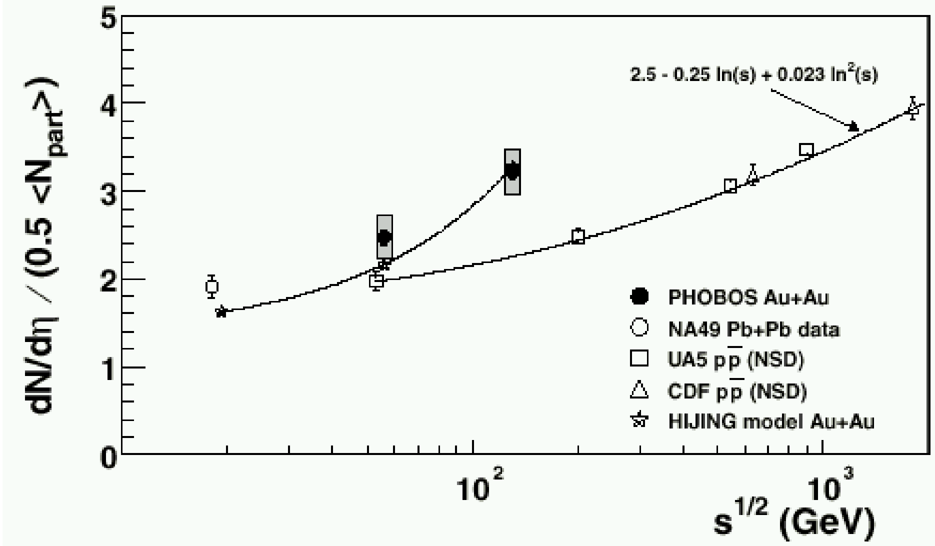
b. Particle Production with respect to the Reaction Plane
- how to measure it (despite multiplicity fluctuations)
- elliptic flow as a signal of collective behaviour

The Day 1 Observable at RHIC and LHC



signal proportional to multiplicity

PHOBOS, Phys. Rev. Lett. 85 (2000) 3100

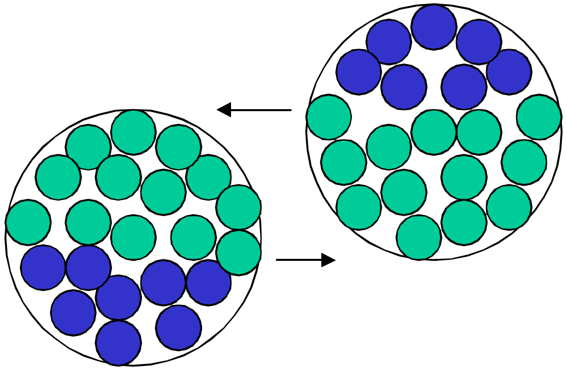


Our starting point: to search for collective effects,
establish baseline in which collective effects are absent

Baseline here: multiplicity distribution in inelastic A+A collisions
is incoherent composition of individual n+n collisions
i.e. extrapolate p+p --> p+A --> A+A without collective effects

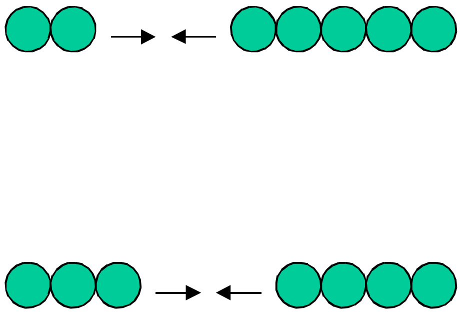
→ **Glauber theory**

What are the correct Variables for A+B Collisions ?



Spectator Nucleons

Participating Nucleons



$N_{part} = 7$

$N_{coll.} = 10$

$N_{quarks\ +gluons} = ?$

$N_{inelastic} = 1$

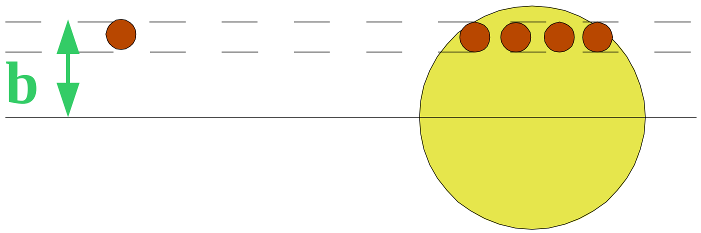
In calculating N_{part} or N_{coll} σ taken to be nucleon-nucleon inelastic cross-section. A priori no reason for this choice other than that it seems to give a useful parameterization.

$\sigma_{inel} \sim (R_1+R_2)^2 \sim (A_1^{1/3} + A_2^{1/3})^2 \sim A^{2/3}$

$N_{part} \sim A^{2/3}(A_1^{1/3} + A_2^{1/3}) \sim A$

$N_{coll} \sim A^{2/3}(A_1^{1/3} * A_2^{1/3}) \sim A^{4/3}$

Glauber Theory for n+A and A+B Collisions



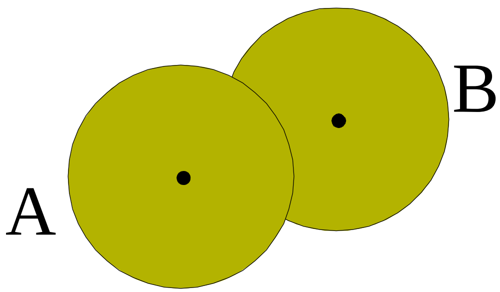
nuclear density: $\rho(b,z)$

$$\int dz\, db\, \rho(b,z) = 1$$

nuclear profile function: $T_A(b) = \int_{-\infty}^{+\infty} dz\, \rho(b,z)$

average number of n-n collisions at b: $\overline{N}^A_{coll}(b) = A\, T_A(b)\, \sigma^{inel}_{NN} = N^A_{part}(b) - 1$

inelastic n-A cross section: $\sigma^{inel}_A = \int d\vec{b}\,(1 - \underbrace{[1 - T_A(b)\,\sigma^{inel}_{NN}]^A}_{P_0(\vec{b})})$

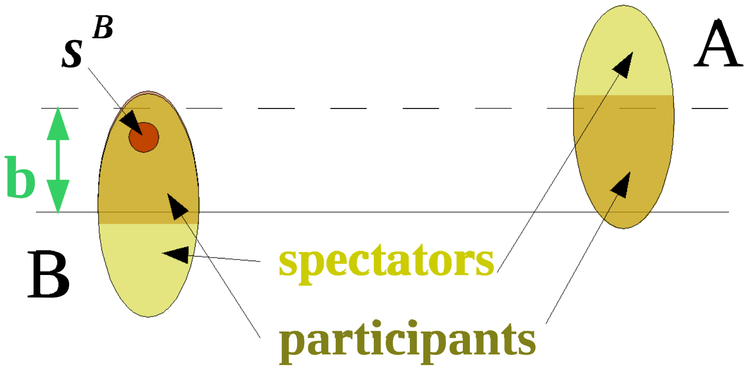


b-dependent nuclear overlap function:

$$T_{AB}(\vec{b}) = \int d\vec{s}\, T_A(\vec{s})\, T_B(\vec{b} - \vec{s})$$

Glauber theory for A+B is more involved ...

Glauber Theory for A+B Collisions



probability that nucleon at s^B in B is wounded by A in configuration $\{s_i^A\}$:

$$p(s^B,\{s_i^A\}) = 1 - \prod_{i=1}^{A} [1 - \sigma(s^B - s_i^A)]$$

norm: $\int ds\, \sigma(s) = \sigma_{NN}^{inel}$

Probability of finding w_B wounded nucleons in nucleus B:

$$P(w_b, B, b) = \int \left(\prod_{i=1}^{A} \prod_{j=1}^{B} ds_i^A ds_j^B T_A(s_i^A) T_B(s_j^B - b) \right) p(s_1^B, \{s_i^A\}) \ldots$$
$$\ldots p(s_{w_B}^B, \{s_i^A\}) [1 - p(s_{w_B+1}^B, \{s_i^A\})] \ldots [1 - p(s_B^B, \{s_i^A\})]$$

inelastic cross section: $\sigma_{AB}(b) = \sum_{w_B=1}^{B} P(w_B, B, b) \approx 1 - [1 - T_{AB}(b) \sigma_{NN}^{inel}]^{AB}$

number of collisions: $\bar{N}_{coll}^{AB}(b) = A\, B\, T_{AB}(b) \sigma_{NN}^{inel}$

Problem 1: derive these expressions,
use e.g. A. Bialas et al.,
Nucl. Phys. B111 (1976) 461

number of participants:
(= no of wounded nucleons)

$$\bar{N}_{part}^{AB}(b) = \frac{A \sigma_B^{inel}(b)}{\sigma_{AB}^{inel}(b)} + \frac{B \sigma_A^{inel}(b)}{\sigma_{AB}^{inel}(b)} \neq \bar{N}_{coll}^{AB}(b) + 1$$

Multiplicity in the Wounded Nucleon Model

model ansatz for average multiplicity $\bar{n}_{AB}(b)$:
(x=0 is wounded nucleon model)

$$\bar{n}_{AB}(b) = \left(\frac{1-x}{2} \bar{N}^{AB}_{part}(b) + x\, \bar{N}^{AB}_{coll}(b) \right) \bar{n}_{nn}$$

probability distribution around mean at fixed b:

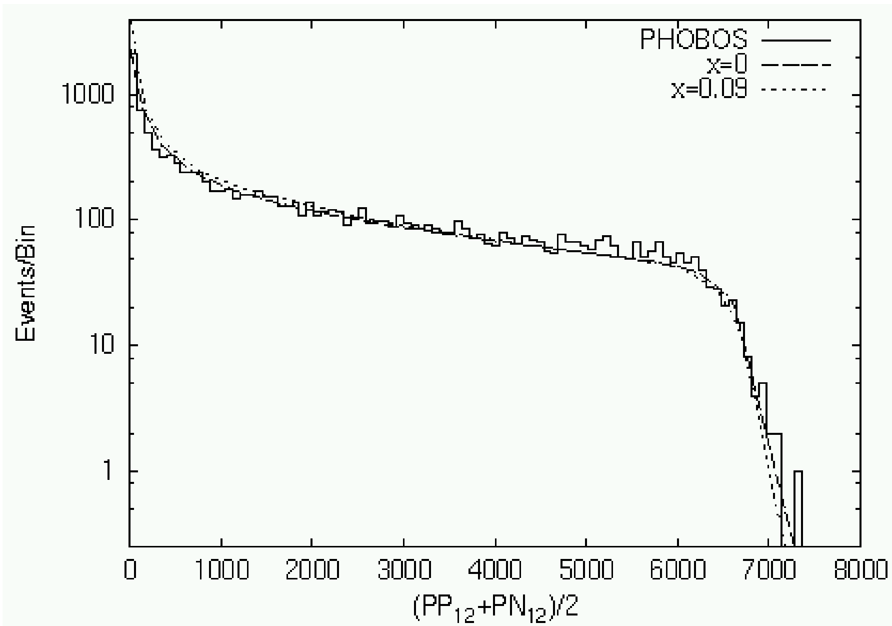
$$P(n,b) = \frac{1}{\sqrt{2\pi\, d\, \bar{n}_{AB}(b)}} \exp\left(- \frac{[n - \bar{n}_{AB}(b)]^2}{2\, d\, \bar{n}_{AB}(b)} \right)$$

dispersion is model dependent fit parameter

multiplicity distribution:

$$\frac{dN_{events}}{dn} = \int db\, P(n,b) \underbrace{\left[1 - \left(1 - \sigma_{NN} T_{AB}(b) \right)^{AB} \right]}_{1 - P_0(b)}$$

- shape determined by geometry
- shape insensitive to details of particle production or collective effects

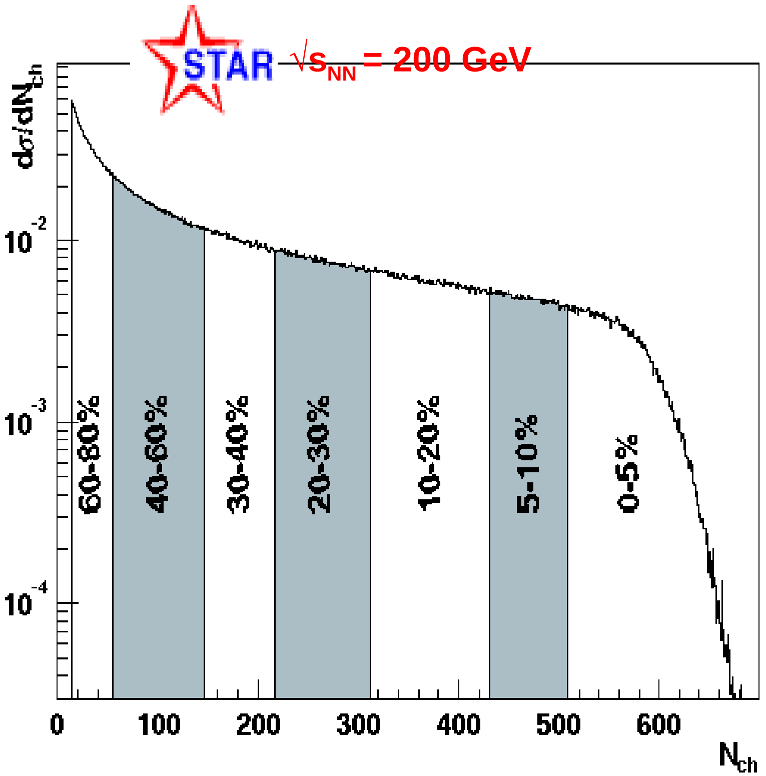


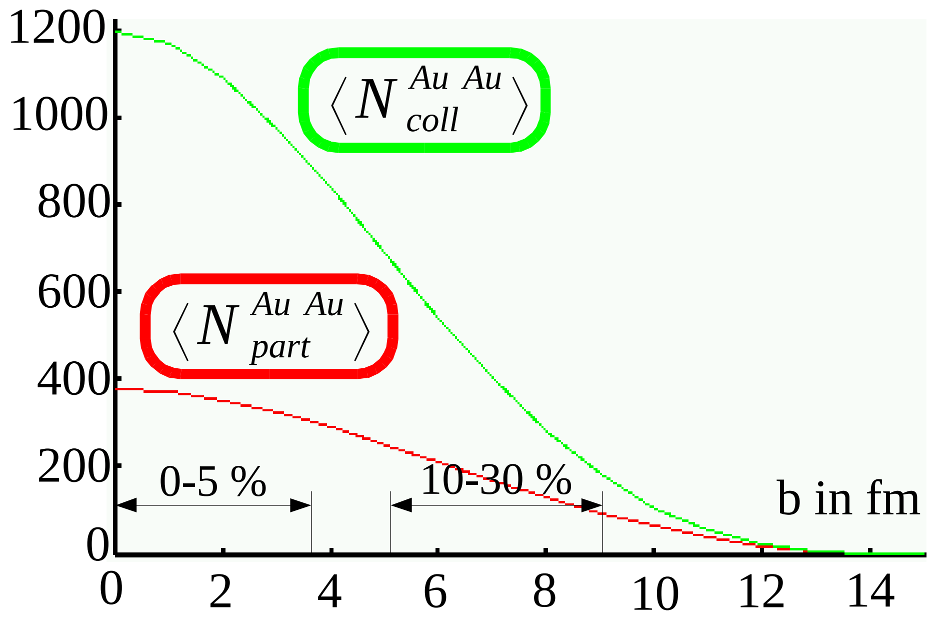
Multiplicity as a Centrality Measure

- It is customary to express centrality in terms of $\langle N_{part}^{Au\,Au} \rangle_{n>n_0} = \dfrac{\int_{n_0} dn \int db\, P(n,b)\left[1-P_0(b)\right] N_{part}(b)}{\int_{n_0} dn \int db\, P(n,b)\left[1-P_0(b)\right]}$

- Centrality class = percentage of the minimum bias cross section

- Centrality class specifies range of impact parameters

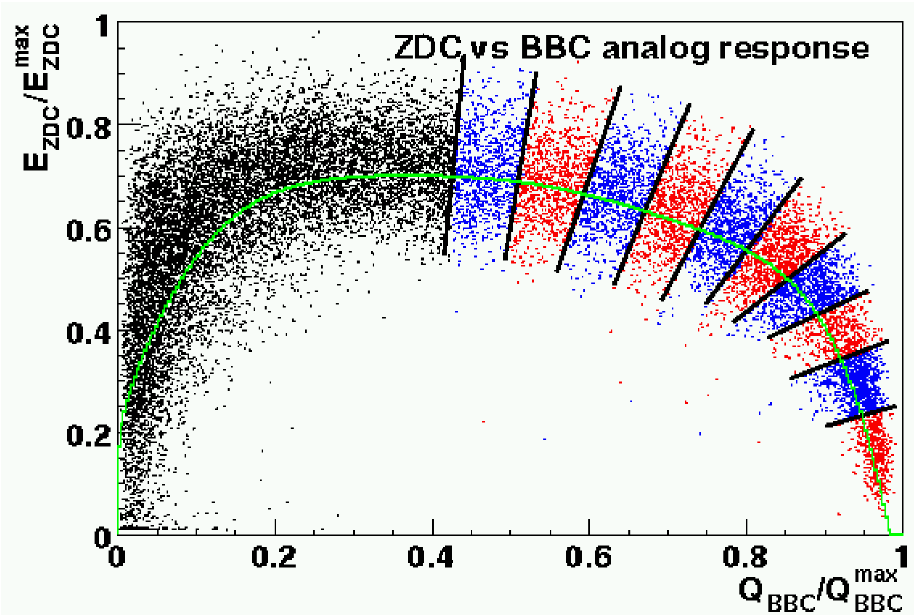




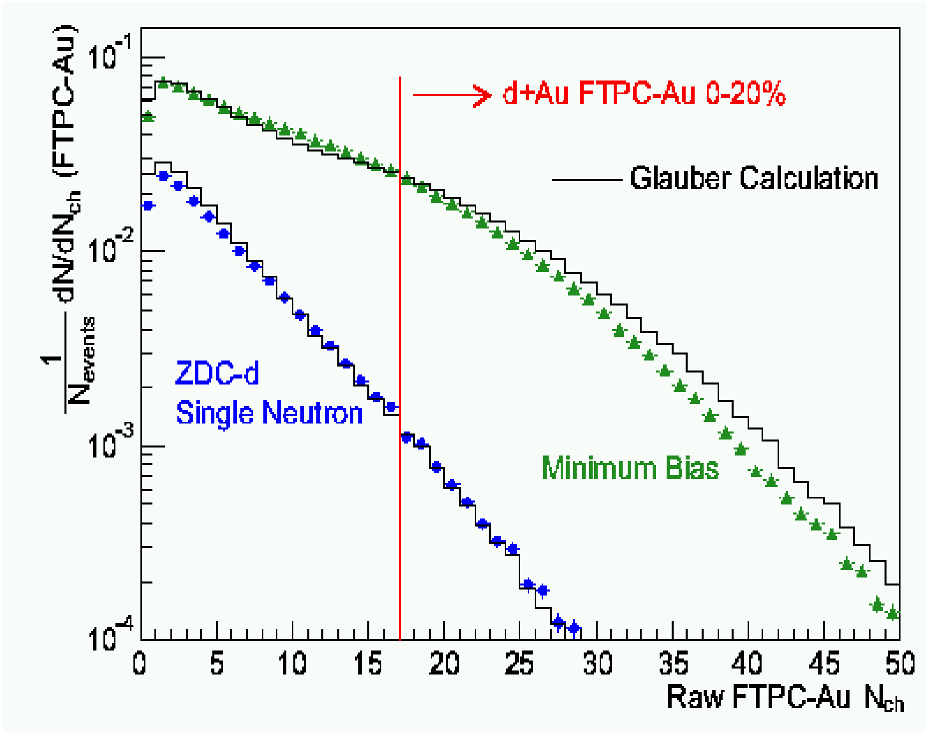
Cross-Checking Centrality Measurements

Energy E_F of spectators is deposited in Zero Degree Calorimeter (ZDC)

$$E_F = \left(A - N_{part}(b)/2\right)\sqrt{s}/2$$

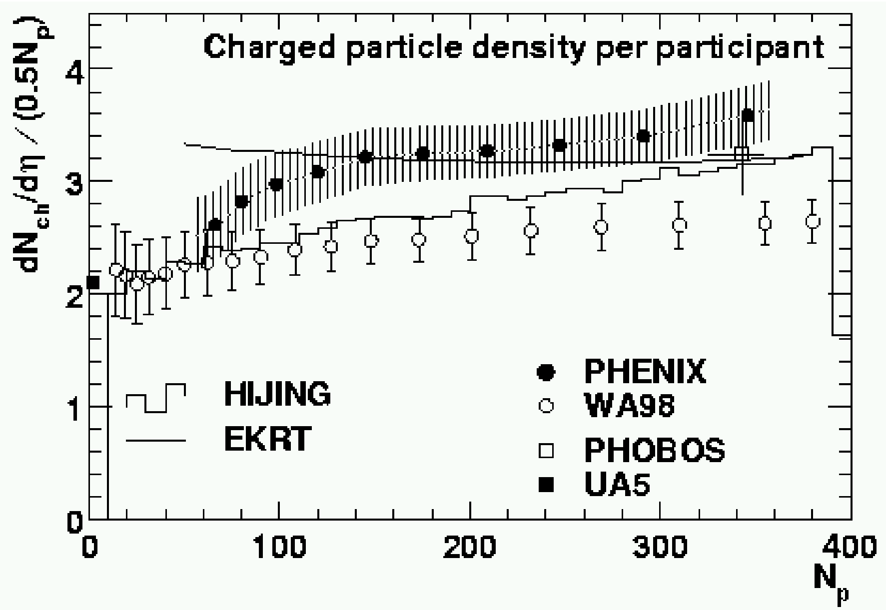


Testing Glauber in d+Au and in p+Au (+ n forward)

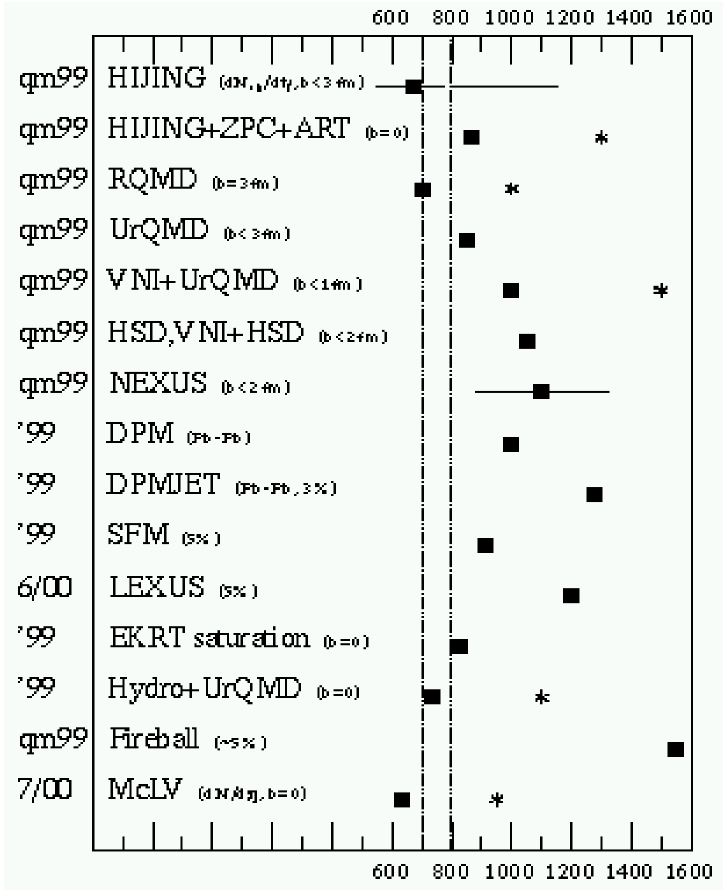


Multiplicity in A+B is NOT understood

- Clear deviations from multiplicity of wounded nucleon model

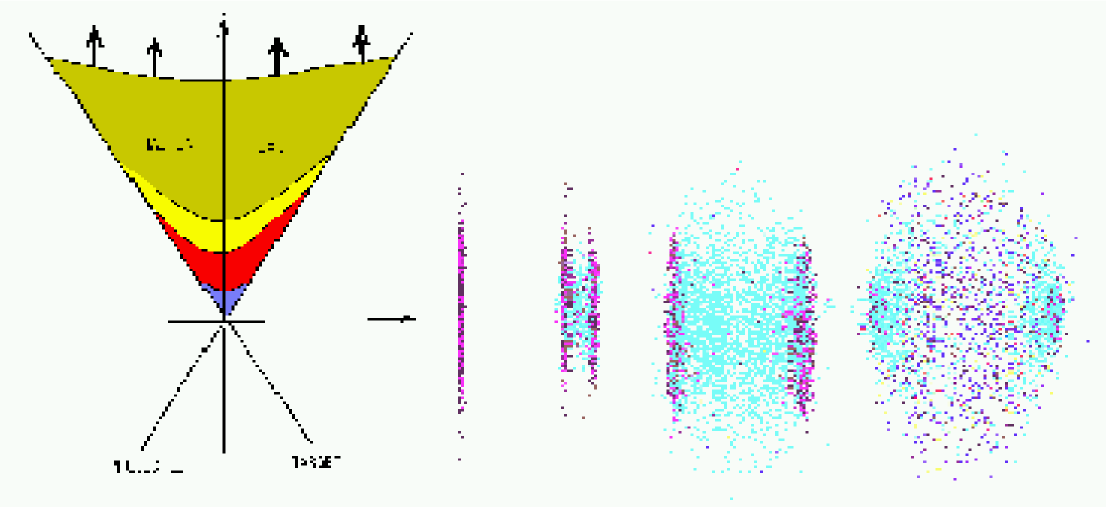


- Total charged multiplicity: models vs. truth



In preparing for HI at LHC, one uses agnostic estimate: $\frac{dN_{ch}}{d\eta}\Big|_{\eta=0} = 2000 - 8000$

Bjorken's Estimate of the Energy Density



- Multiplicity (or transverse energy) determines basic properties of produced matter

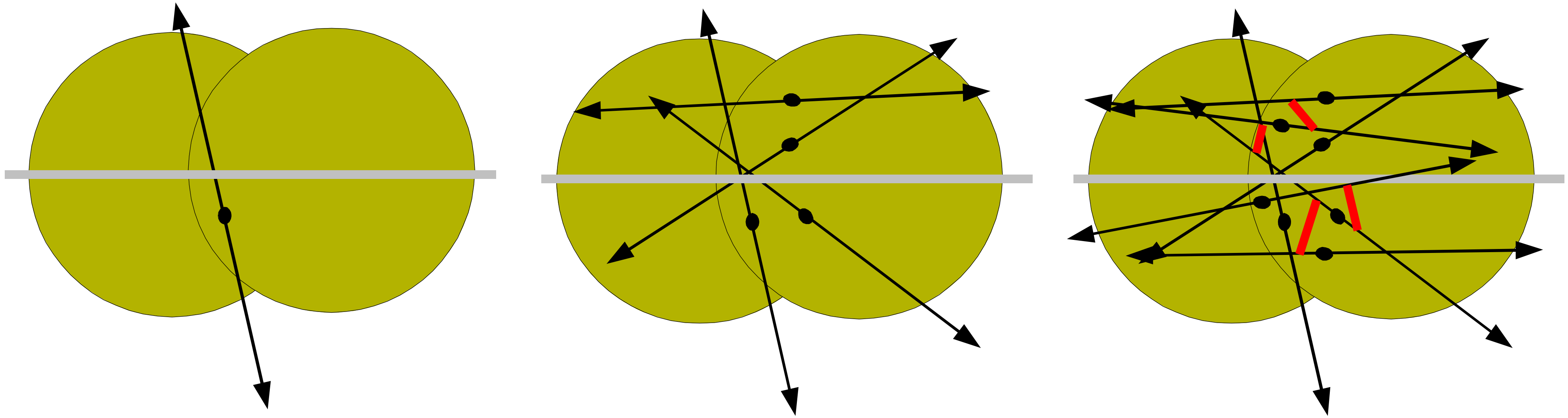
$$\epsilon(\tau_0) = \frac{1}{\pi R^2} \frac{1}{\tau_0} \frac{dE_T}{dy}$$

$$\frac{dE_T}{dy} \approx \frac{dN}{dy} \langle E_T \rangle$$

- This estimate is based on geometry, thermalization is <u>not</u> assumed. Numerically:

$$\epsilon^{SPS}(\tau_0 \simeq 1\, fm/c) = 3 - 4\, GeV/fm^3$$

Particle Production with respect to Reaction Plane



- single 2 ->2 process
- maximal asymmetry
- NOT correlated to the reaction plane

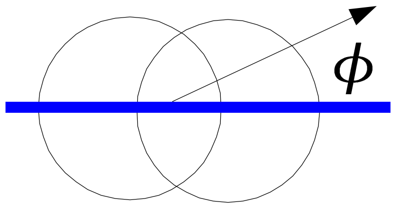
- many 2->2 or 2->n processes
- reduced asymmetry $\propto \frac{1}{\sqrt{N}}$ cancels in the limit
- NOT correlated to the reaction plane

- final state interactions
- asymmetry may not be caused only by finite multiplicity fluctuations
- collective component is correlated to reaction plane

The azimuthal asymmetry of particle production has a collective and a random component. Disentangling the two requires a statistical analysis of finite multiplicity fluctuations →

Particle production w.r.t. reaction plane: Elliptic Flow

- Want to measure particle production as function of angle w.r.t. reaction plane



$$v_n(D) = \langle e^{in\phi} \rangle_D$$

But reaction plane is unknown ...

- Have to measure particle correlations:

$$\langle e^{in(\phi_1-\phi_2)} \rangle_{D_1 x D_2} = v_n(D_1) v_n(D_2) + \underbrace{\langle e^{in(\phi_1-\phi_2)} \rangle^{corr}_{D_1 x D_2}}_{.\sim O\left(\frac{1}{N}\right)}$$

"Non-flow effects"

But this requires signals $v_n > \frac{1}{\sqrt{N}}$

- Improve measurement with higher cumulants:

Problem 2: derive this cumulant improvement, see Borghini et al. Phys. Rev. C63 054906 (2001)

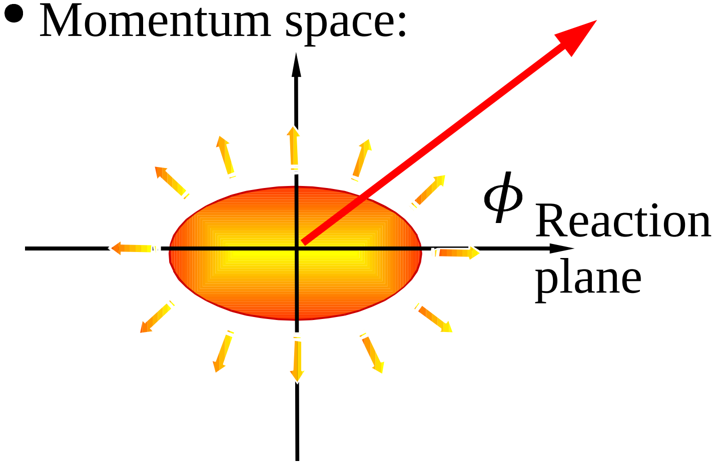
$$\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)} \rangle - \langle e^{in(\phi_1-\phi_3)} \rangle \langle e^{in(\phi_2-\phi_4)} \rangle - \langle e^{in(\phi_1-\phi_4)} \rangle \langle e^{in(\phi_2-\phi_3)} \rangle = -v_n^4 + O\left(\frac{1}{N^3}\right)$$

This requires signals $v_n > \frac{1}{N^{3/4}}$

Elliptic Flow

$$E\frac{dN}{d^3p}=\frac{1}{2\pi}\frac{dN}{p_t\,dp_t\,d\eta}\left[1+2\,v_2(p_t)\cos\left(2\left(\phi-\psi_{reaction\ plane}\right)\right)\right]$$

- Momentum space:



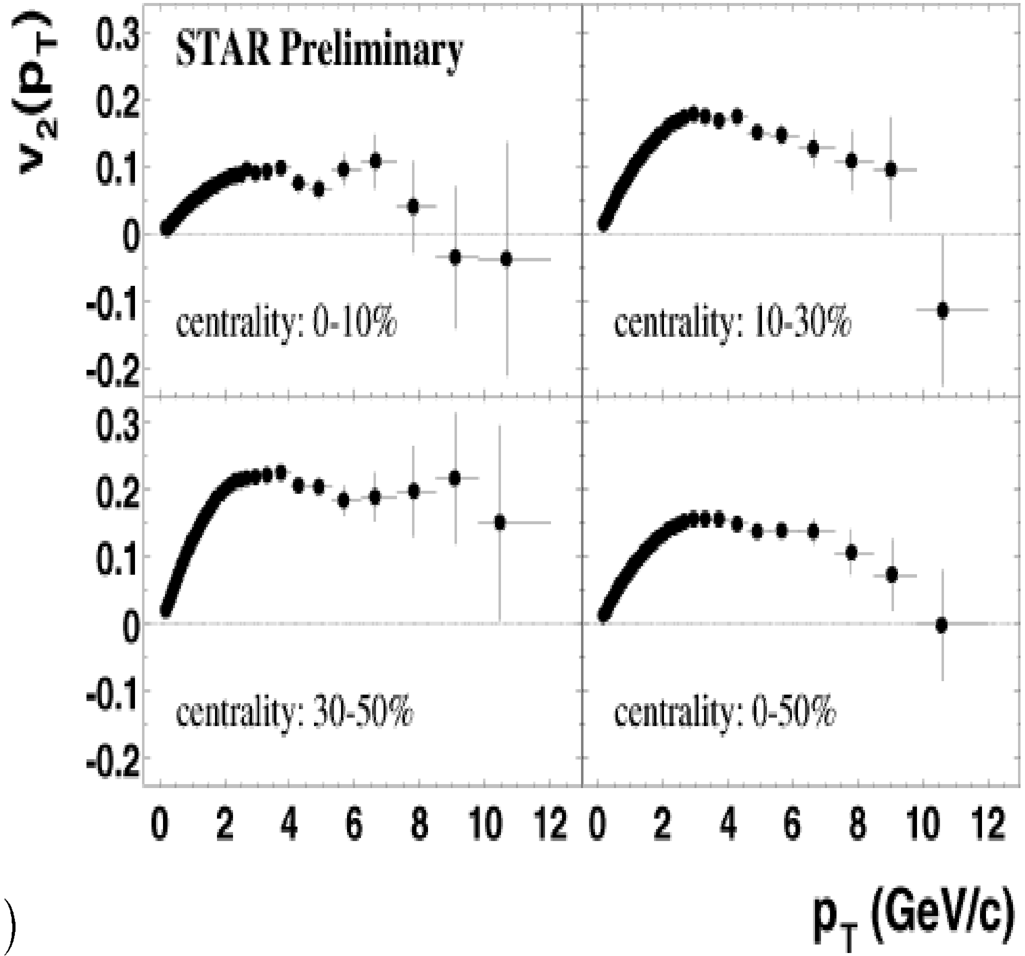
- Signal $v_2 \approx 0.2$
  implies 40 % more particles
  emitted in the reaction plane

- 'Non-flow' background
  for 2nd order cumulants

$$N\sim 100\rightarrow\frac{1}{\sqrt{N}}\sim 0.1\sim O(v_2)$$

  for 4th order cumulants

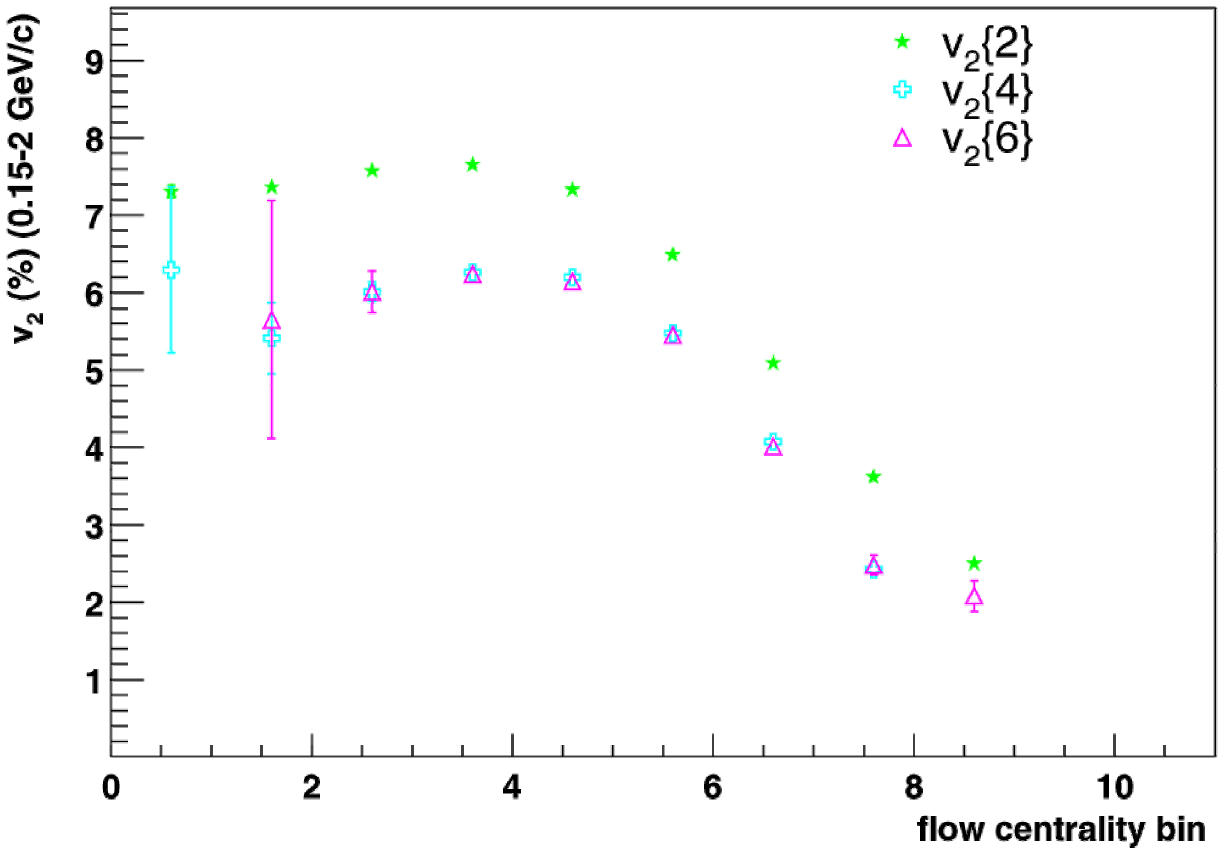
$$\frac{1}{N^{3/4}}\sim 0.03\ll v_2$$



→ non-flow effects should disappear if going from 2nd to 4th order cumulants

pt-integrated Elliptic Flow

STAR Coll, Phys. Rev. C66 (2002) 034904

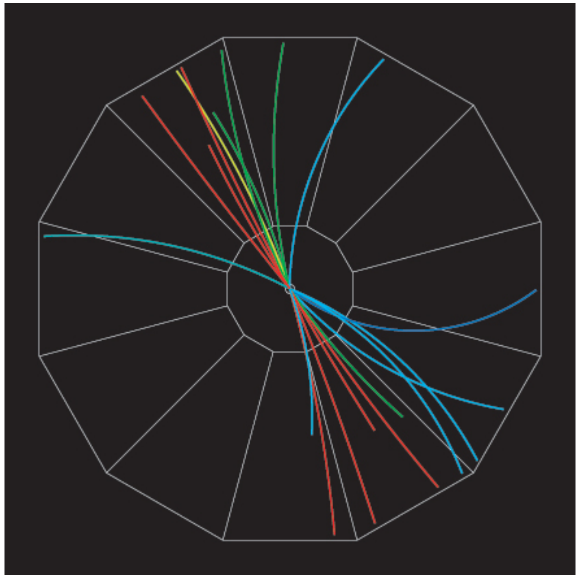


Elliptic Flow is stable if reconstructed from higher cumulants

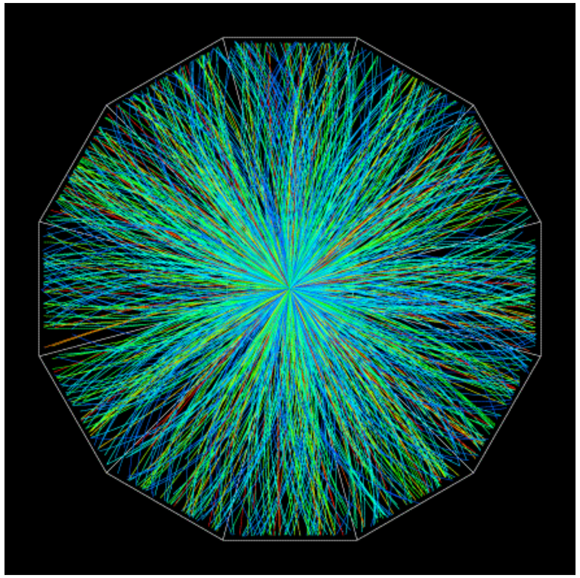
→ collective effect, which cannot be mimicked by multiplicity fluctuations in the reaction plane!

1st Conclusion about Elliptic Flow

p+p @ RHIC



Au+Au @ RHIC



- compared to the reaction plane, this is rotationally symmetric
- azimuthal asymmetry comes from non-flow effects (here: momentum conservation)

- compared to the reaction plane, this is rotationally asymmetric for semi-central collisions
- azimuthal asymmetry is much larger than non-flow effects allow

To understand the size of v2, let us study a theoretical baseline: the zero mean free path limit of final state interactions

→ Hydrodynamics

The Hydrodynamic Model of A+A Collisions

- Main assumption: matter is thermalized fluid, described by

  energy momentum tensor $T^{\mu\nu}(x) = \big(e(x) + p(x)\big) u^{\mu}(x) u^{\nu}(x) - p(x) g^{\mu\nu}$

  flow field in global rest frame: $u^{\mu}(x) = \gamma(x)\big(1, \vec{v}(x)\big), \quad \gamma(x) = 1/\sqrt{1-\vec{v}^2(x)}$

  conserved charge currents: $j_i^{\mu}(x) = n_i(x) u^{\mu}(x), \quad i = 1, \ldots M$

  (e.g. baryon #, strangeness,...)

- Hydrodynamic equations of motion

  local energy momentum conservation: $\partial_{\mu} T^{\mu\nu}(x) = 0$ (4 eqs.)

  local charge conservation: $\partial_{\mu} j_i^{\mu}(x) = 0$ (M eqs.)

  M+5 undetermined fields: 3 flows $\vec{v}(x)$, energy density $e(x)$, pressure $p(x)$, M charges $n_i(x)$

  set of equations closed by: $p(e, n)$ equation of state (EOS)

  results from lattice QCD and/or from (a large class of) hadronic models enter here

Two-dimensional Bjorken Hydrodynamics

- Main assumption: init. conditions for thdyn. fields do not depend on $\eta = \frac{1}{2}\ln\left(\frac{t+z}{t-z}\right)$

  longitudinal flow has 'Hubble form' $v_z = z/t$

  Bjorken scaling: hydrodyn. eqs. preserve Hubble form

- Parametrization of Bjorken longitudinally boost-invariant flow field

$$u^{\mu} = \cosh(y_T)\left(\cosh(\eta), v_x, v_y, \sinh(\eta)\right)$$

$$t = \tau\cosh(\eta), \quad z = \tau\sinh(\eta)$$

$$\tau = \sqrt{t^2 - z^2}, \quad \eta = \mathrm{artanh}(z/t)$$

at mid-rapidity $\quad v_r(\tau, r, \eta = 0) \equiv tanh(y_T(\tau, r)), \quad v_r = \sqrt{v_x^2 + v_y^2}$

at other rapidities $\quad v_r(\tau, r, \eta) = \dfrac{v_r(\tau, r, \eta = 0)}{\cosh\eta}$

- Equations of motion (η -direction is trivial)

$$T^{\tau\tau}_{,\tau} + \left(v_x T^{\tau\tau}\right)_{,x} + \left(v_y T^{\tau\tau}\right)_{,y} = -\frac{1}{\tau}\left(T^{\tau\tau} + p\right) - \left(p\, v_x\right)_{,x} - \left(p\, v_y\right)_{,y}$$

$$T^{\tau x}_{,\tau} + \left(v_x T^{\tau x}\right)_{,x} + \left(v_y T^{\tau x}\right)_{,y} = -\frac{1}{\tau} T^{\tau x} - p_{,x}$$

$$T^{\tau y}_{,\tau} + \left(v_x T^{\tau y}\right)_{,x} + \left(v_y T^{\tau y}\right)_{,y} = -\frac{1}{\tau} T^{\tau y} - p_{,y}$$

$$\frac{1}{\tau^2} p_{,\eta} = 0$$

$$j^{\tau}_{,\tau} + \left(v_x j^{\tau}\right)_{,x} + \left(v_y j^{\tau}\right)_{,y} = -\frac{1}{\tau} j^{\tau}$$

Problem 3: derive these equations of, motion, check e.g. Kolb et al. Phys. Rev. C 62 (2000) 054909

Input for Hydrodynamic Simulations

- Initialization: a typical ansatz is

$$\epsilon_{init}(x) \propto \left(\frac{1-a}{2} \overline{N}^{AB}_{part}(b,x) + a\,\overline{N}^{AB}_{coll}(b,x) \right)$$

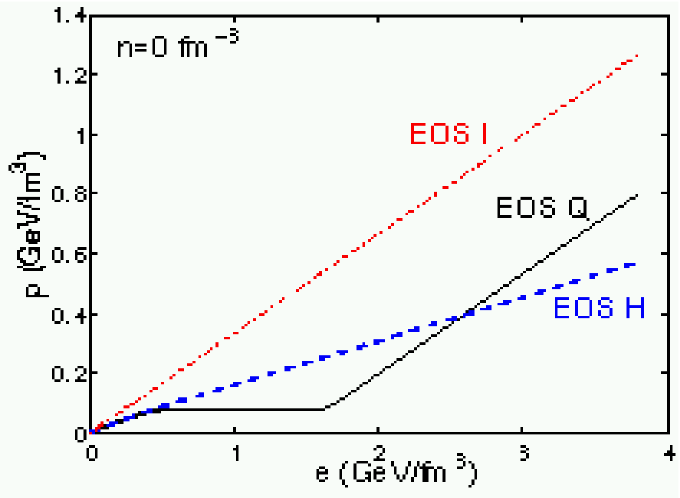
- Equation of State (EOS): $p(e,n)$

Velocity of sound: $c_s^2 = \frac{\partial p}{\partial e}$

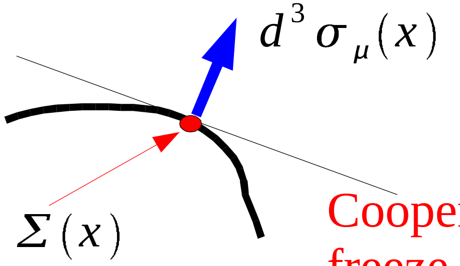
Expectation: $c_s^2 \approx 0.15$ soft EOS (hadron resonance gas)

$c_s^2 = 1/3$ stiff EOS

Input from (many) models and from lattice QCD



- Freeze-out: local temperature $T(x) = T_{fo}$ defines space-time hypersurface element $\Sigma(x)$



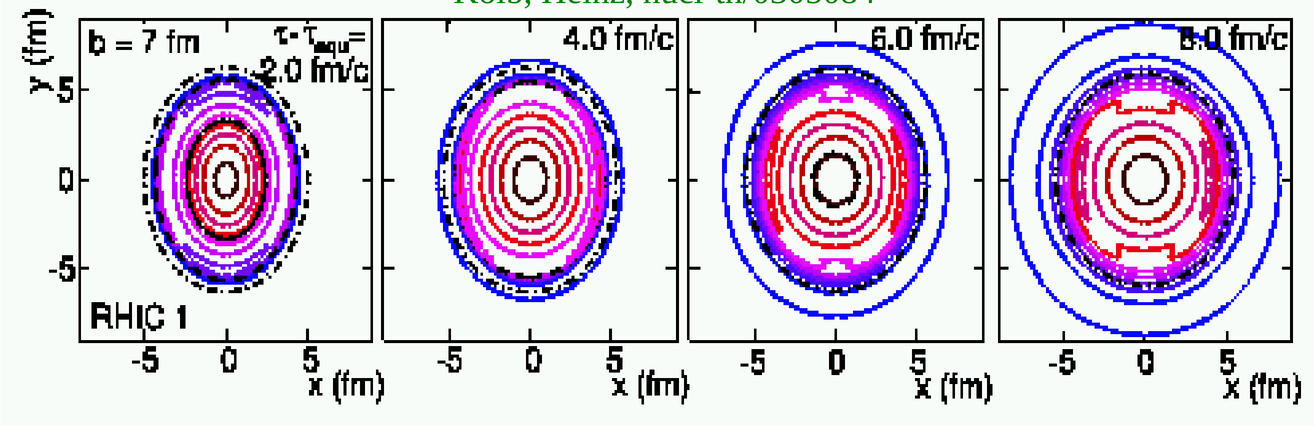
spectrum: $$E\frac{dN_i}{d^3p} = \frac{g_i}{(2\pi)^3}\int_\Sigma p\cdot d^3\sigma(x)\, f_i(p\cdot u(x), x)$$

$$f_i(E,x) = \frac{1}{\exp\left[(E-\mu_i(x))/T(x)\right] \pm 1}$$

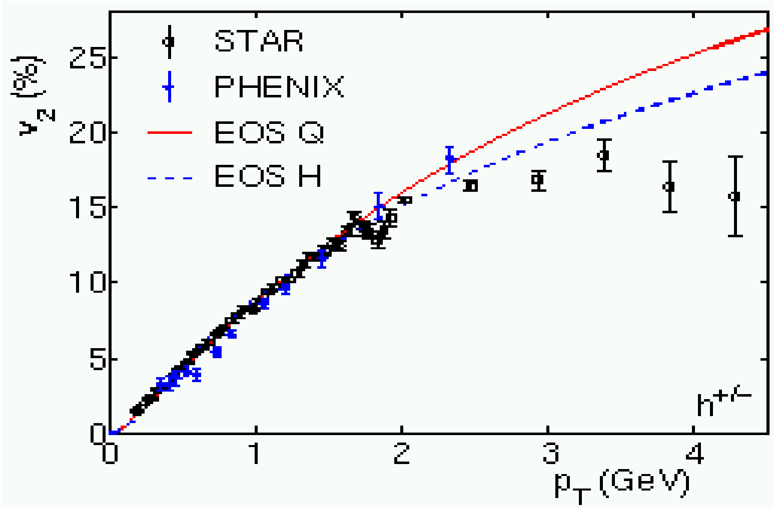
Hydrodynamic Simulations account for Elliptic Flow

- Space-time evolution of 2-dim hydro in transverse plane

Kolb, Heinz, nucl-th/0305084

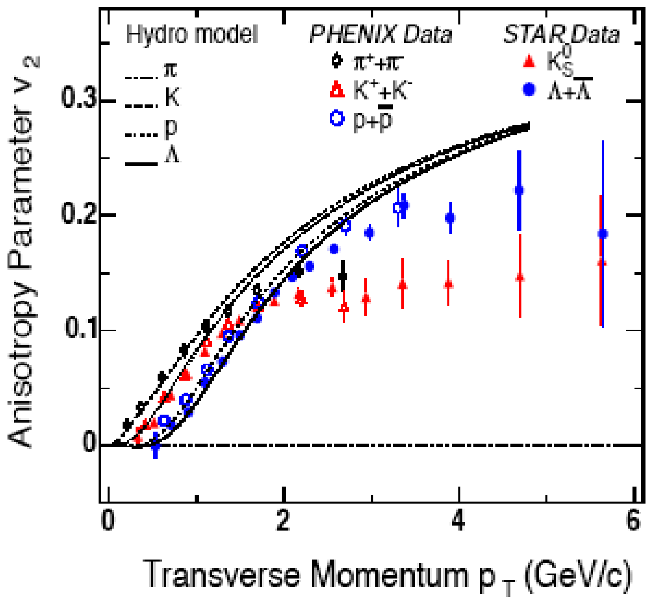


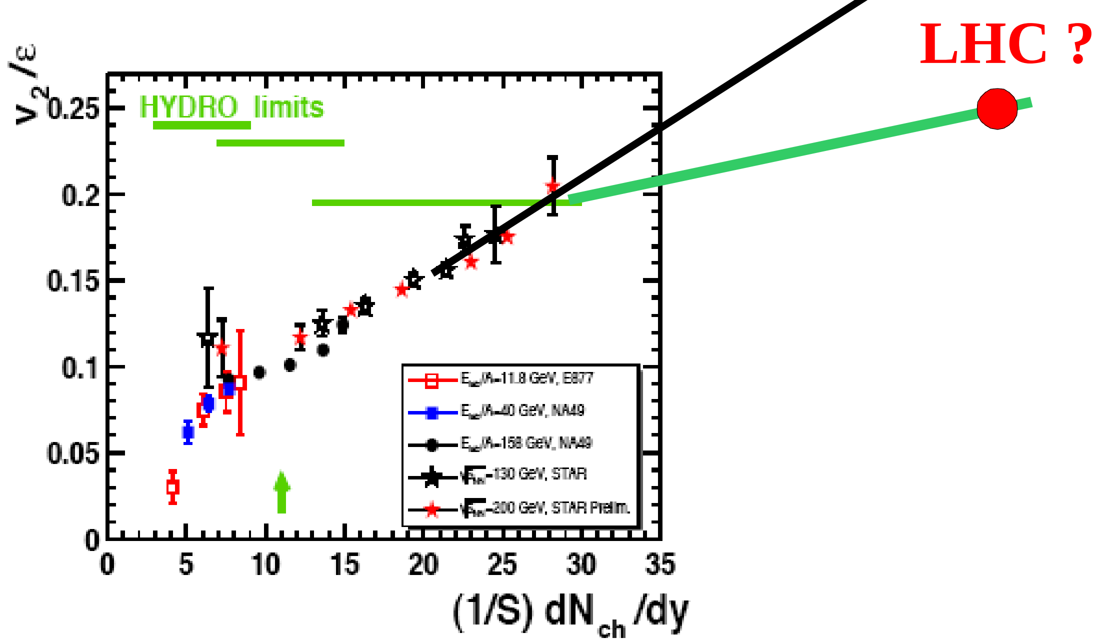
- Zero mean free path approximation does not apply for $p_T > 2\ GeV$



Caveats ...

• Hydro limit reached at upper RHIC energy only

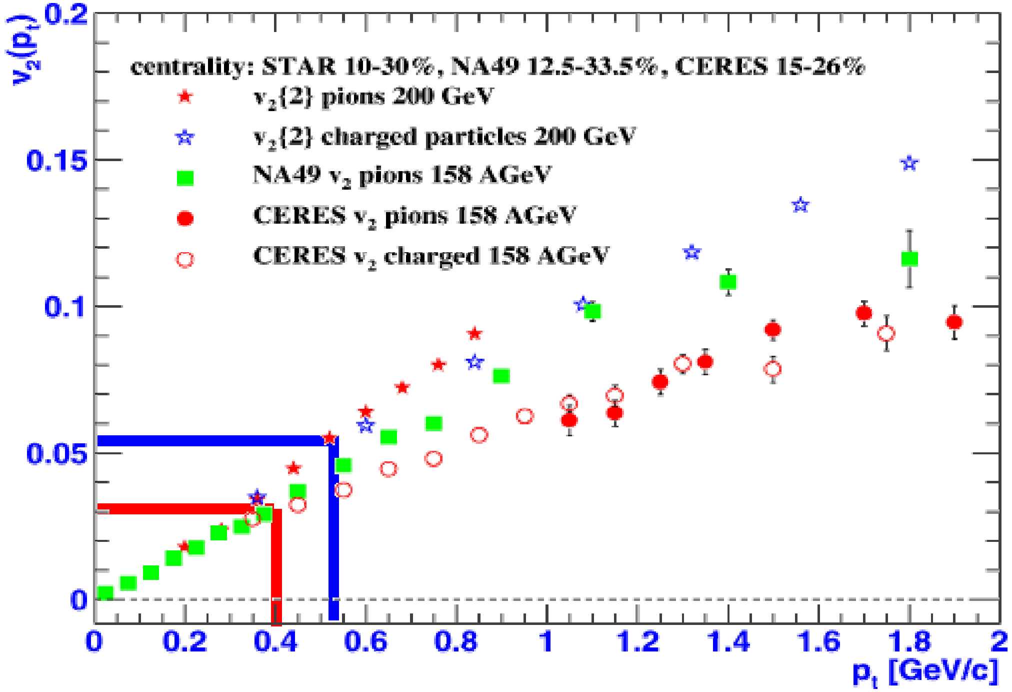




• Is part of the effect non-hydro ?

$$\langle p_T \rangle_{SPS} \sim 400\ MeV$$

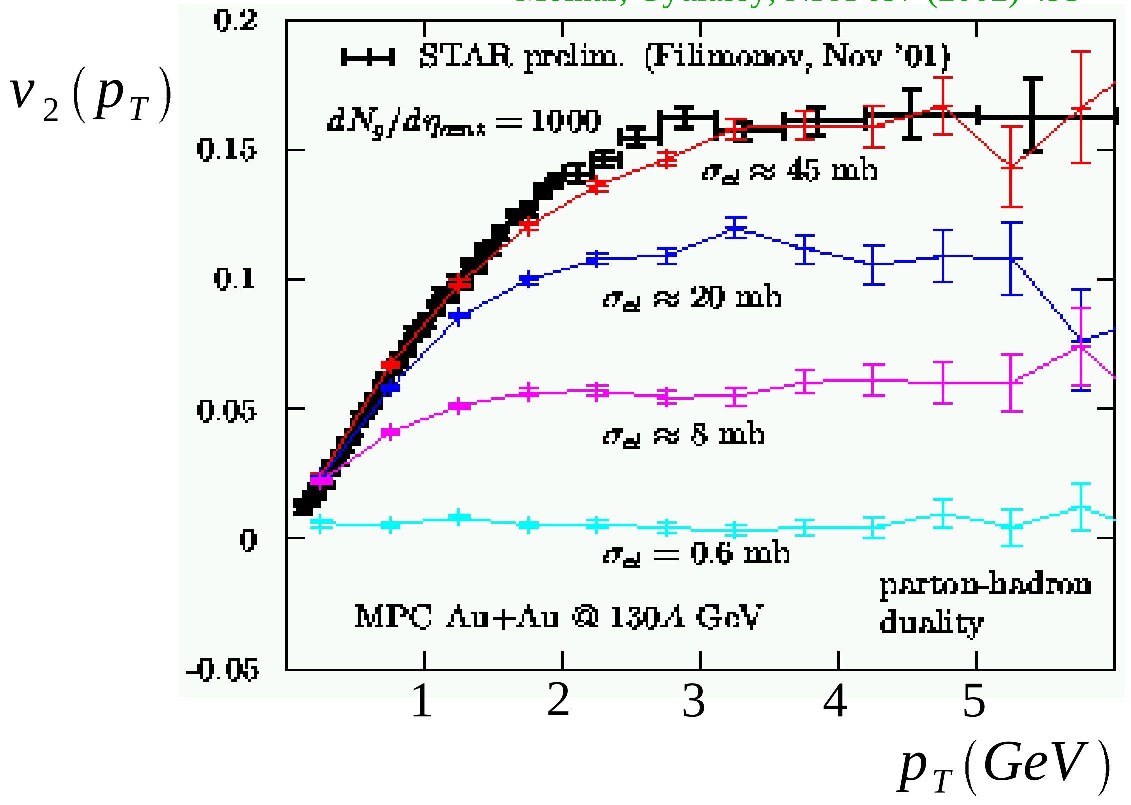
$$\langle p_T \rangle_{RHIC} \sim 500\ MeV$$



Caveat: Opacity Problem

- Microscopic parton cascades reproduce hydrodynamic behavior with unnatural large partonic cross sections only

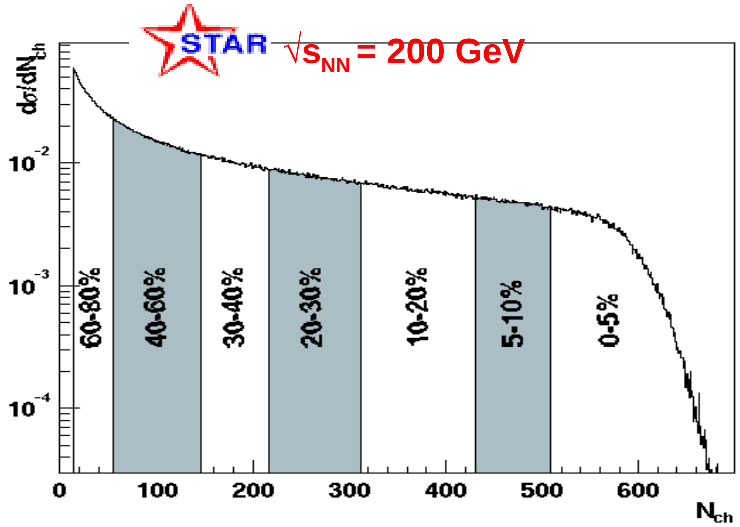
Molnar, Gyulassy, NPA 697 (2002) 495



→ Agreement with and deviations from hydrodynamics are a very active field of current research

Summary of Lecture 1

- Minimum Bias Multiplicity Distribution:
  - shape determined by nuclear geometry
  - a robust centrality measure



- Particle Production with respect to the reaction plane
  - large asymmetry at RHIC (80 % more particles in plane)
  - signal of collectivity
  - if hydrodynamical description applies then elliptic flow gives access to the QCD equation of state