Selected Topics in the Theory of Heavy Ion Collisions

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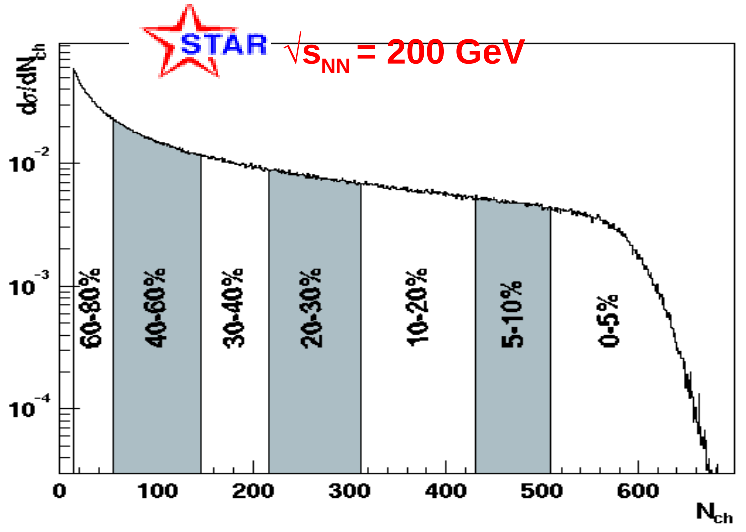
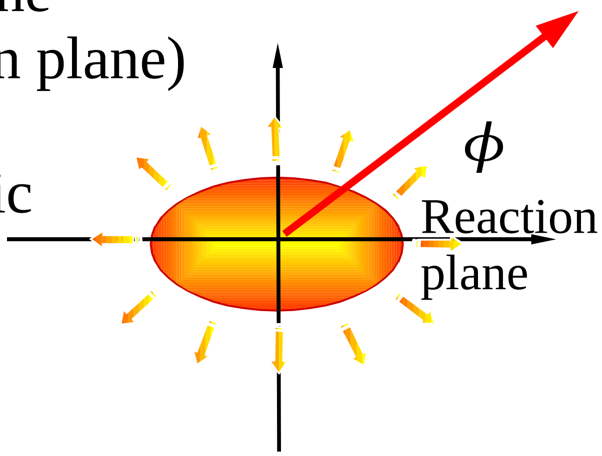
Summary of Lecture 1

- Minimum Bias Multiplicity Distribution:
  - shape determined by nuclear geometry
  - a robust centrality measure



- Particle Production with respect to the reaction plane
  - large asymmetry at RHIC (80 % more particles in plane)
  - signal of collectivity
  - if hydrodynamical description applies then elliptic flow gives access to the QCD equation of state



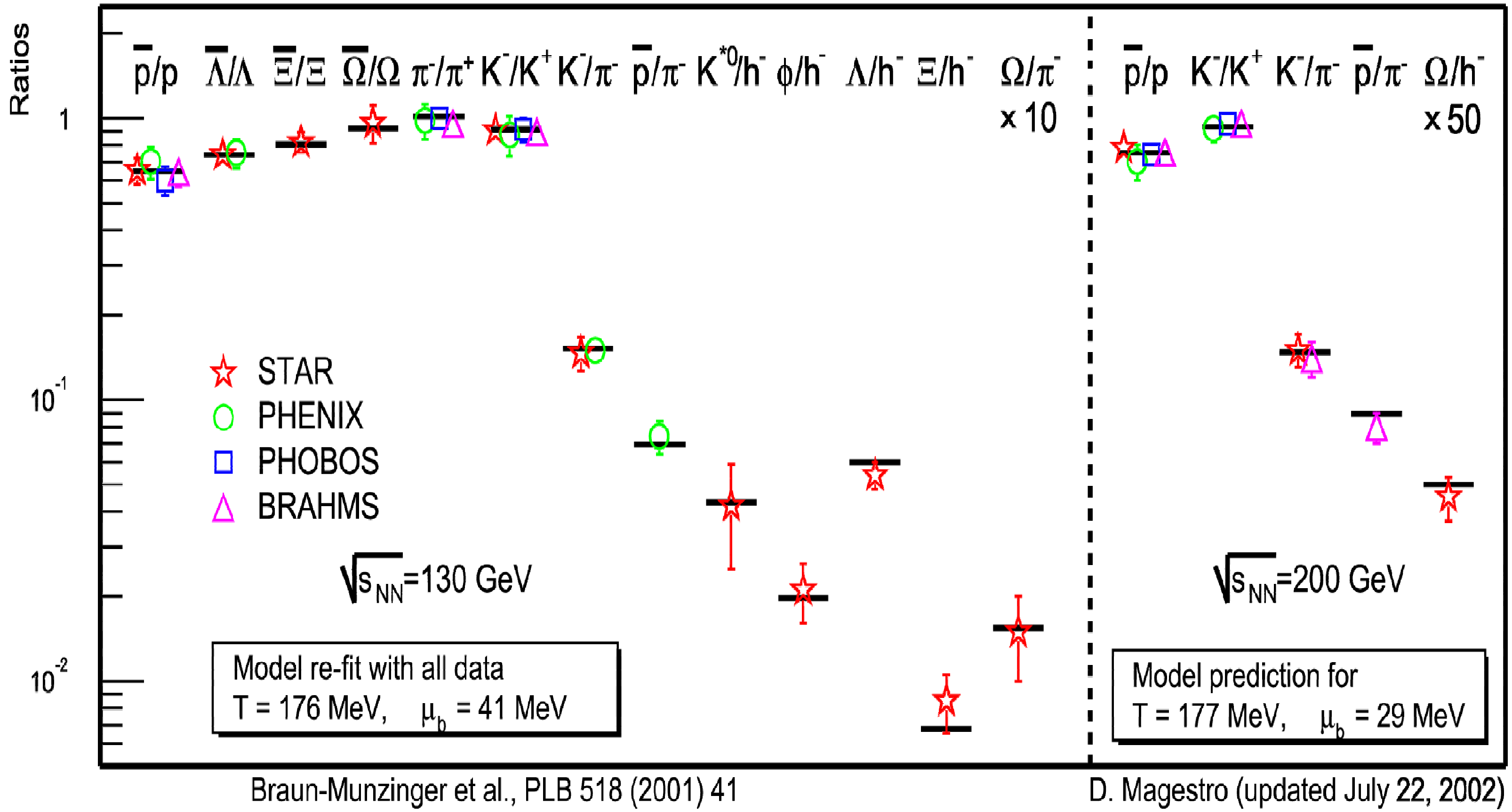
Lecture 2:
Hadron Thermodynamics
Concepts, Applications and Limitations

a. Hadronic Particle Ratios
- Hagedorn's limiting temperature and the QCD phase transition
- The model of statistical hadronization
- Thermal deviations from a grand canonical description

b. Low-pt Hadronic Spectra
- Models combining hydrodynamics and statistical hadronization
- Deviations at intermediate pt

The Run 1 Observable at RHIC and LHC

- Hadrochemistry of particle ratios



<u>Aim:</u> understand the framework in which these data are discussed currently

Hadron Thermodynamics

- Ideal gas of identical neutral scalar particles of mass m_0 in box V characterized by grand canonical partition function (Boltzmann statistics)

$$Z(T,V)=\sum_N \frac{1}{N!}\left[\frac{V}{(2\pi)^3}\int d^3p \exp(-\sqrt{p^2+m_0^2}/T)\right]^N \longrightarrow \ln Z(T,V)=\frac{VTm_0^2}{2\pi^2}K_2\left(\frac{m_0}{T}\right)$$

- For $T \gg m_0$, energy density and particle density are

$$\epsilon(T)=\frac{1}{V}\frac{\partial \ln Z(T,V)}{\partial(1/T)}\simeq\frac{3}{\pi^2}T^4 \qquad n(T)=\frac{\partial \ln Z(T,V)}{\partial V}\simeq\frac{1}{\pi^2}T^3 \qquad \frac{\epsilon(T)}{n(T)}\simeq 3T$$

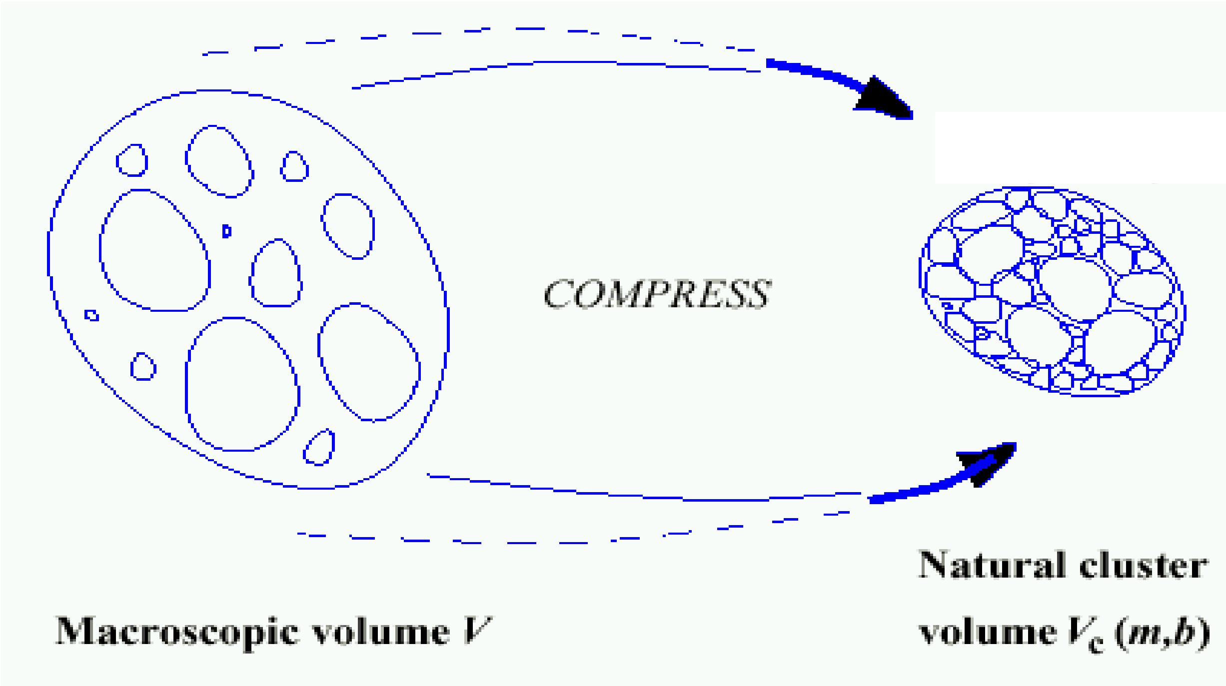
- Now include resonance formation: hadron resonance gas

$$\ln Z(T,V)=\sum_i \frac{VTm_i^2}{2\pi^2}\rho(m_i)K_2\left(\frac{m_i}{T}\right)$$

Hadron dynamics enters only via the density of resonance states $\rho(m_i)$

What is $\rho(m_i)$?

Hagedorn's Statistical Bootstrap Model



Bootstrap hypothesis: (self-similarity of a hadronic resonance gas)

if a Hadron gas is compressed to the volume of a hadron, V_c ,
then is is just another massive hadronic resonance in $\rho(m)$
(Hagedorn: fireballs consist of fireballs, which consist of fireballs ...)

$$\rho(m=\sqrt{p^2})=\delta(m-m_0)+\sum_N \frac{1}{N!}\frac{V_c^{N-1}}{(2\pi)^{3(N-1)}}\int\prod_{i=1}^{N}\left[dm_i\,\rho(m_i)\,d^3p_i\right]\delta^{(4)}\left(p-\sum_i p_i\right)$$

Nahm's Solution of the Statistical Bootstrap Model

- Statistical Bootstrap Model defined by:

$$\rho(m=\sqrt{p^2})=\delta(m-m_0)+\sum_N \frac{1}{N!}\frac{V_c^{N-1}}{(2\pi)^{3(N-1)}}\int\prod_{i=1}^{N}\left[dm_i\,\rho(m_i)\,d^3p_i\right]\delta^{(4)}\left(p-\sum_i p_i\right)$$

- Nahm's analytical solution:

$$\rho(m,V_c)=\frac{const.}{m^3}\exp\left[\frac{m}{T_H}\right]$$

$$\frac{V_c T_H^3}{2\pi^2}\left(m_0/T_H\right)^2 K_2\left(m_0/T_H\right)=2\ln(2)-1$$

- Hagedorn temperature T_H (determined by range of strong interactions):
  - Hagedorn's estimate: $m_0=m_\pi,\quad V_c=\frac{4\pi}{3}m_\pi^{-3}$

$$\longrightarrow\ T_H\approx 150\ MeV$$

  - Chiral limit: $m_0\to 0,\quad T_H=\left[\pi^2(2\ln 2-1)\right]^{1/3}V_c^{-1/3},\quad V_c=\frac{4\pi}{3}r_h^{3},\quad r_h\sim 1\,fm$

$$\longrightarrow\ T_H\approx 200\ MeV$$

The Exponential Hadron Mass Spectrum

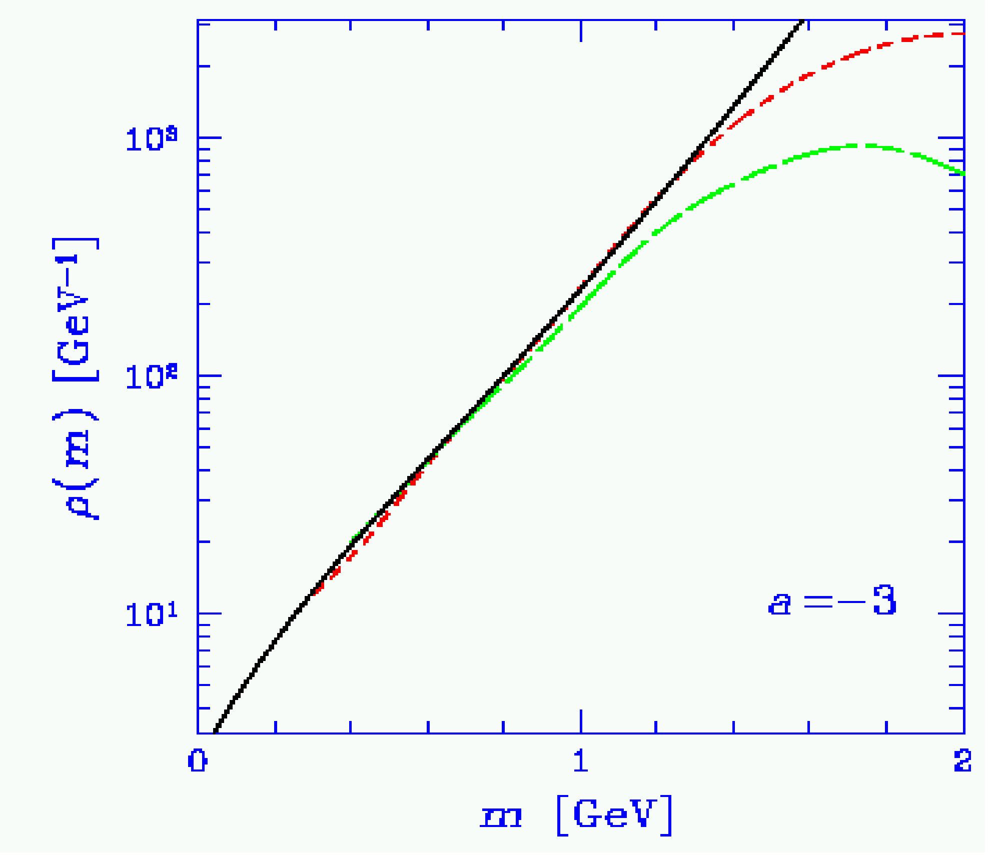
- Black line: fit to

$$\rho(m) = c\left(m_a^2 + m^2\right)^{-3/2} \exp\left[\frac{m}{T_H}\right]$$

$$m_a = 0.66\,GeV\,, \quad T_H = 158\,MeV$$

- Green line:
  1411 states of 1967

- Red line:
  4627 states of 1996



- Experimental lines include Gaussian smoothing, $\sigma_{\overline{m}} = \Gamma_{\overline{m}}/2 \sim 200\,MeV$

$$\rho(m) = \sum_{\overline{m} = m_\pi, m_\rho, \ldots} \frac{g_{\overline{m}}}{\sqrt{2\pi\sigma_{\overline{m}}}} \exp\left[\frac{-(m-\overline{m})^2}{2\sigma_{\overline{m}}^2}\right]$$

Interpretation of the Hagedorn Temperature

- Insert Nahm's solution into gc partition function for hadron gas

$$\ln Z(T,V) \simeq \frac{VT}{2\pi^2} \int dm\, m^2 \rho(m) K_2(m/T)$$

$$\sim V \frac{T^{3/2}}{(2\pi)^{3/2}} \int dm\, m^{-3/2} \exp\left[-m\left(\frac{1}{T} - \frac{1}{T_H}\right)\right]$$

Cabibbo and Parisi observe: $\ln Z(T,V)$ does not diverge at $T = T_H$
but its derivates do diverge.

This signals a phase transition.

T_H ultimate temperature of hadronic matter.

- For $T \rightarrow T_H$
  - momenta of the resonances in the gas do not increase
  - heavier and heavier resonances appear (infinitely many at T_H)

- For $T > T_H$
  - inconsistent with a gas of hadronic constituents
  - consistent with a gas of the partonic constituents of hadrons

T_H signals change of the relevant thermodynamic degrees of freedom

→ Quark Gluon Plasma

Opening Brackets:

The QCD Phase Diagram

The QCD Partition Function

- Evaluate partition function for QCD hamiltonian

$$Z = Tr\, \exp\left[-\frac{1}{T}\left(\widehat{H} - \mu_i \widehat{N}_i \right) \right]$$

Two approaches:

- perturbation theory: applies if $\alpha_s(T) \ll 1 \longrightarrow T \gg T_c$

- lattice at finite T: applies if $a^{lattice}_{spacing} \sim \frac{1}{T}$ sufficiently large $T \sim O(T_c)$

- Gas of free quarks and gluons

$$P_{free\ QCD} = \frac{\pi^2}{45} N_g T^4 + '\, quark\ degrees\ of\ freedom\, '$$

for an interacting quark gluon plasma

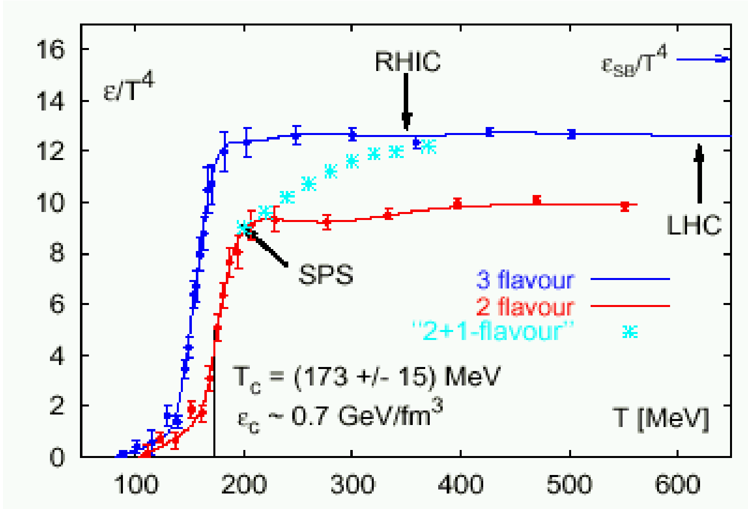
$$\frac{P^{glue}_{QCD}}{P^{glue}_{free\ QCD}} = 1 - \frac{15}{4}\left(\frac{\alpha_s}{\pi}\right) + 30\left(\frac{\alpha_s}{\pi}\right)^{3/2} + O(\alpha_s^2)$$

but perturbation theory has bad convergence properties

Results from Finite Temperature Lattice QCD

- Deconfinement
  phase transition

$$\epsilon_c^{lattice} < \epsilon_{Bjorken} = \frac{1}{\tau_0 \pi R^2} \frac{dE_T}{dy}$$

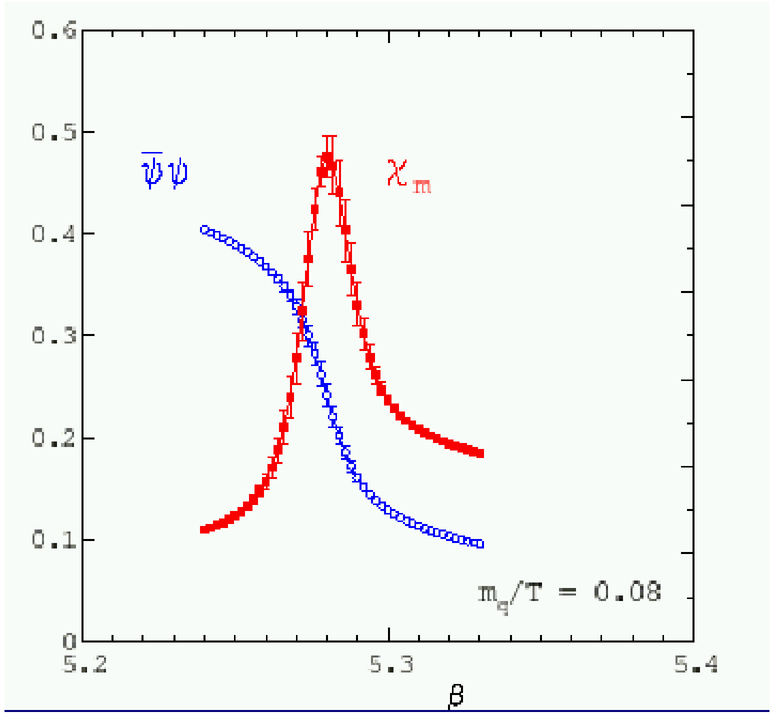


- Chiral symmetry
  - broken by the QCD vacuum

$$\langle \overline{\Psi}_L \psi_R + \overline{\Psi}_R \psi_L \rangle \simeq (250\ MeV)^3$$

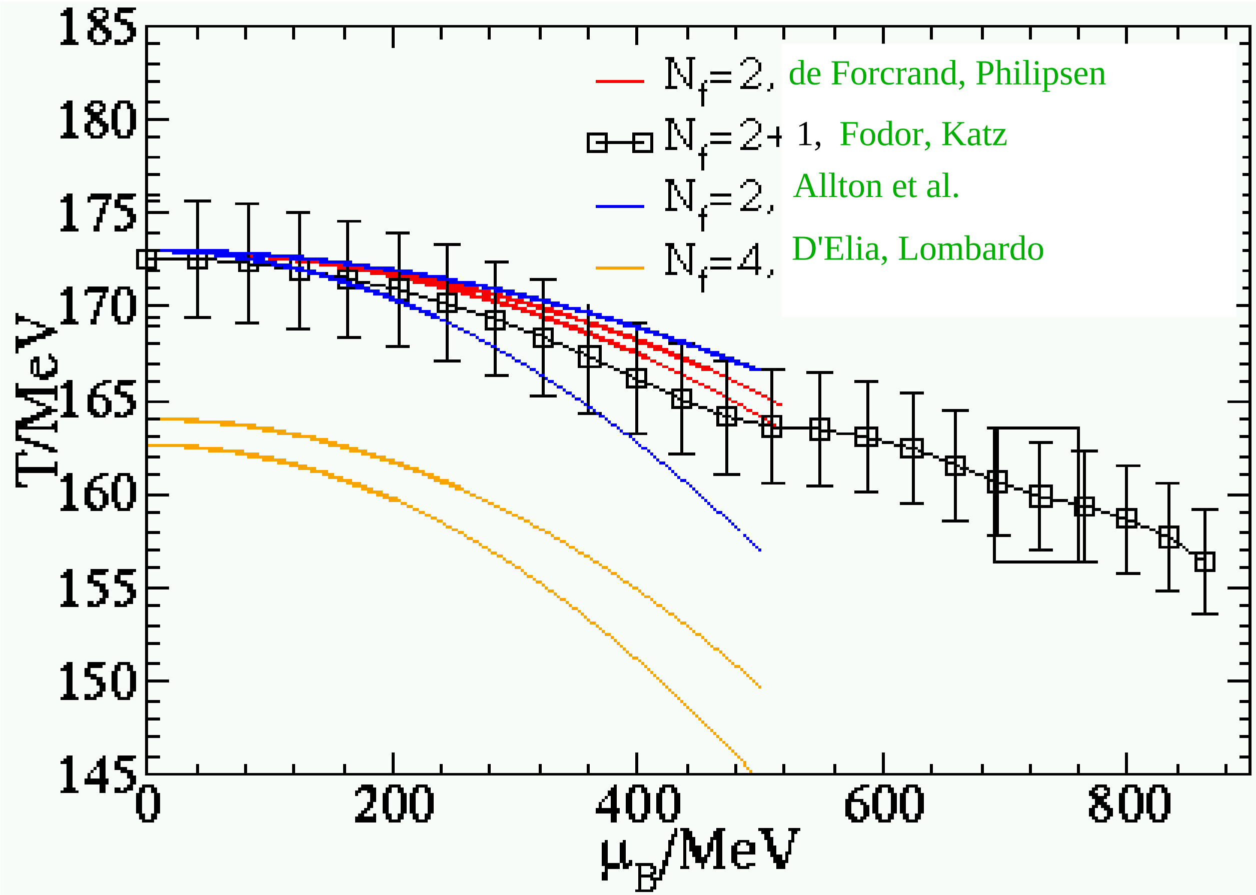
  - restored at high temperature

$$\langle \overline{\Psi}_L \psi_R + \overline{\Psi}_R \psi_L \rangle \to 0$$



The Lattice QCD Phase Diagram

- For finite baryochemical potential



Phase-space integrated Hadroproduction
in Central Heavy Ion Collisions

The Grand Canonical Description

Model of Statistical Hadronization

Basic assumption: a heavy ion collision creates a Hagedorn fireball which releases a thermal resonance mass spectrum determined by statistics

$$\ln Z(T,V,\vec{\mu}) = \sum_i \frac{V g_i}{2\pi^2} \int \pm p^2 \, dp \ln\left[1 \pm \lambda_i \exp\left(-\epsilon_i = \sqrt{p^2 + m_i^2}/T\right)\right]$$

- fugacities λ_i in terms of chemical potentials for baryon #, strangeness and electric charge

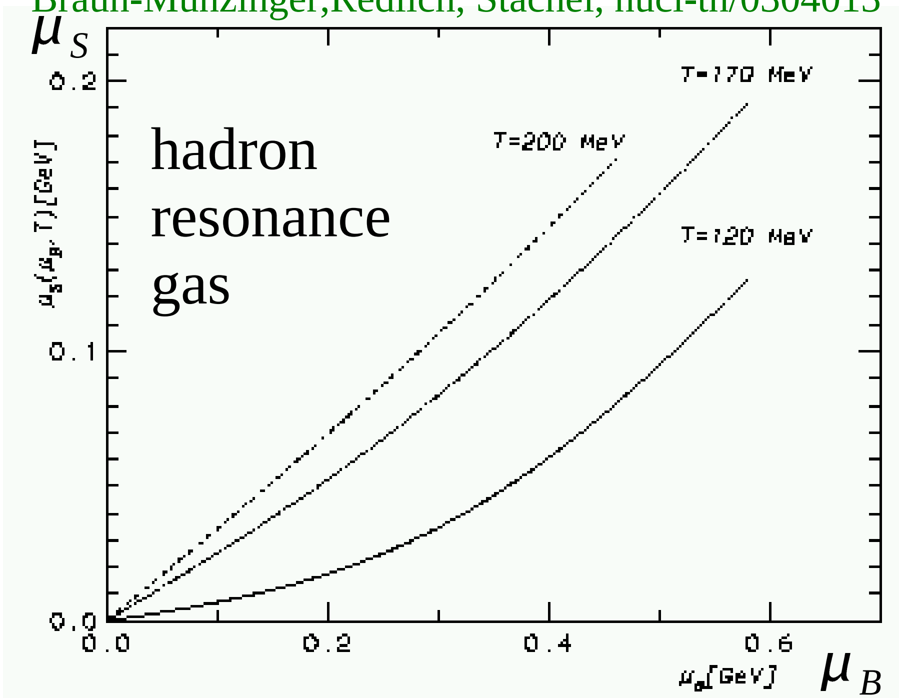
$$\lambda_i(T,\vec{\mu}) = \exp\left(\frac{B_i \mu_B + S_i \mu_S + Q_i \mu_Q}{T}\right)$$

- Conservation of strangeness eliminates μ_S

$$\langle N_s \rangle - \langle N_{\bar{s}} \rangle = \lambda_s \frac{\partial}{\partial \lambda_s} \ln(Z_s^{HG}) \equiv 0$$

$$\longrightarrow \quad \mu_S = \mu_S(T, \mu_B)$$

Braun-Munzinger,Redlich, Stachel, nucl-th/0304013



Model of Statistical Hadronization (cont'd)

- Number density of produced hadrons

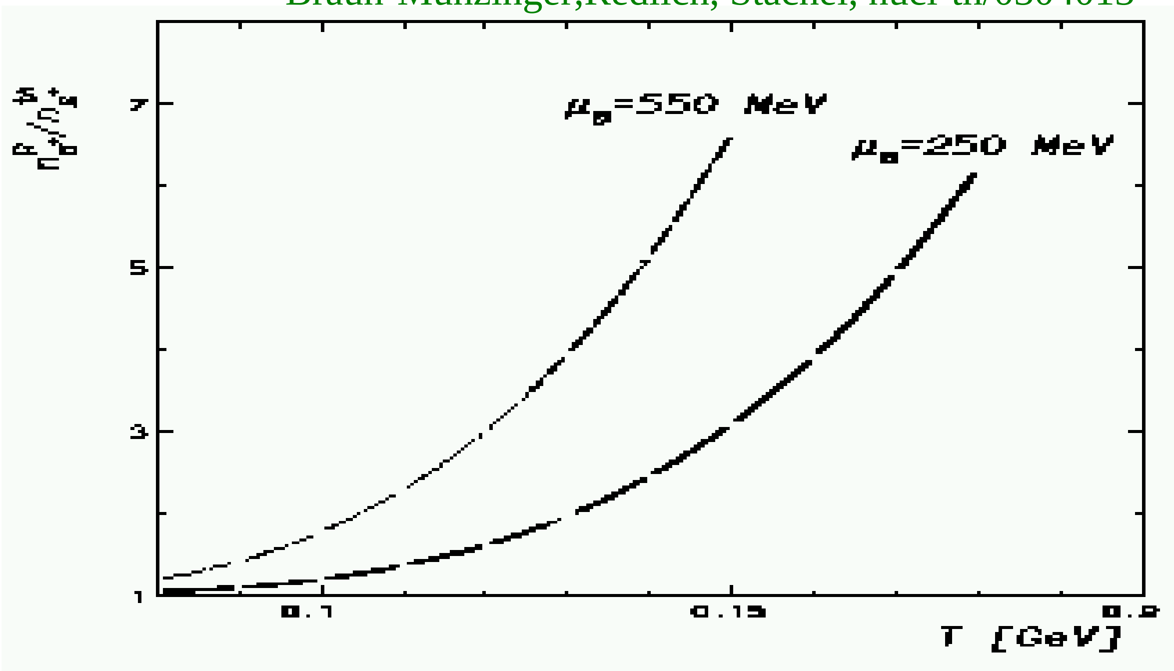
$$n_i(T,\mu)=\frac{\langle N_i\rangle^{therm}}{V}=\frac{T\,g_i}{2\pi^2}\sum_{k=1}^{\infty}\frac{(\pm 1)^{k+1}}{k}\lambda_i^k\, m_i^2\, K_2\left(\frac{k\,m_i}{T}\right)$$

Problem: derive this from partition function

$$\langle N_i\rangle(T,\vec{\mu})=\langle N_i\rangle^{th}(T,\vec{\mu})+\underbrace{\sum_j \Gamma_{j\rightarrow i}\langle N_j\rangle^{th.\,resonance}(T,\vec{\mu})}_{decay\ contributions}$$

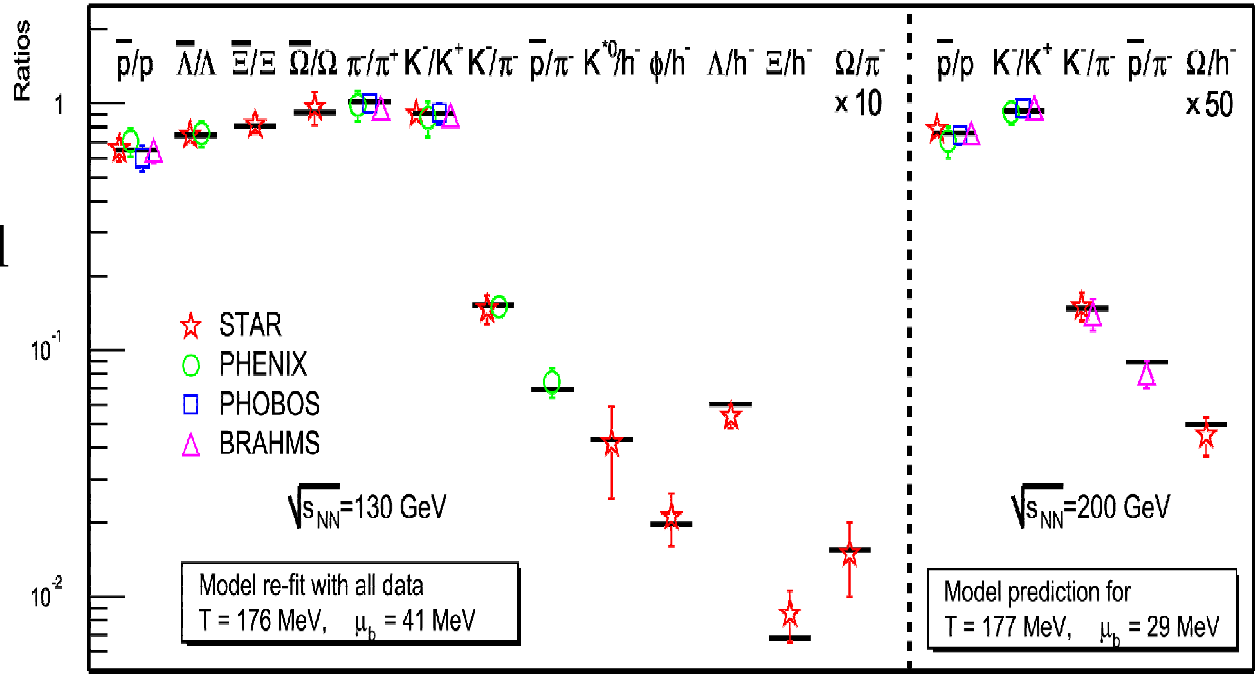
- Fraction of final state pions, coming from resonance decays

Braun-Munzinger,Redlich, Stachel, nucl-th/0304013



Comparison to Particle Ratios at RHIC

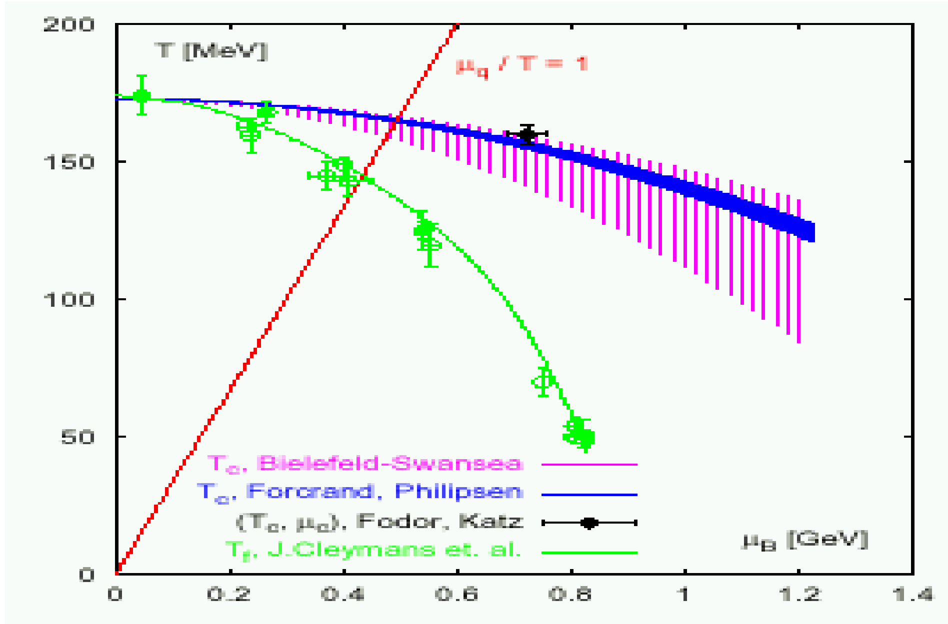
- Hadron yields and ratios fitted by statistical model in terms of (T, μ)



Braun-Munzinger et al., PLB 518 (2001) 41 D. Magestro (updated July 22, 2002)

- Parameters (T, μ) coincide approach phase boundary of lattice QCD at high energy.

(This is an empirical observation)



Deviations from the Grand Canonical Description

Deviations from Grand Canonical Ensemble

- Rate equation for particles with rare conserved charge $a+b \Leftrightarrow c+\bar{c}$

$$\frac{dP_{N_c}}{d\tau} = \underbrace{G\langle N_a\rangle\langle N_b\rangle P_{N_c-1}}_{Gain} + \underbrace{L(N_c+1)^2 P_{N_c+1}}_{Loss} - \underbrace{G\langle N_a\rangle\langle N_b\rangle P_{N_c}}_{Gain} - \underbrace{L N_c^2 P_{N_c}}_{Loss}$$

- Resulting time evolution of the average number $\langle N_c\rangle = \sum_{N_c=0}^{\infty} N_c P_{N_c}(\tau)$

$$\frac{d\langle N_c\rangle}{d\tau} = G\langle N_a\rangle\langle N_b\rangle - L\langle N_c^2\rangle$$

- Grand-canonical (gc) limit $\langle N_c\rangle \gg 1, \quad \Rightarrow \quad \langle N_c^2\rangle \approx \langle N_c\rangle^2$

$$\langle N_c(\tau)\rangle = \langle N_c\rangle_{equil}^{gc} \tanh(\tau/\tau_0) \qquad \tau_0 = 1/L\sqrt{\epsilon}$$

$$\langle N_c\rangle_{equil}^{gc} = \sqrt{\epsilon} \equiv \sqrt{G\langle N_a\rangle\langle N_b\rangle/L}$$

- Opposite limit $\langle N_c\rangle \ll 1, \quad \Rightarrow \quad \langle N_c^2\rangle \approx \langle N_c\rangle$

$$\langle N_c(\tau)\rangle^c = \langle N_c\rangle_{equil}^{c}\left(1-e^{-\tau/\tau_0}\right) \qquad \tau_0 = 1/L$$

$$\langle N_c\rangle_{equil}^{c} = \epsilon \ll \langle N_c\rangle_{equil}^{gc}$$

suppression with respect to grand canonical formalism

Equilibrium Average Number of Rare Particles

- Rewrite rate equation

introduce generating functional $g(x,\tau) = \sum_{N_c=0}^{\infty} x^{N_c} P_{N_c}(\tau)$

$$\Rightarrow \frac{dg(x,\tau)}{d\tau} = L(1-x)(x g'' + g' - \epsilon g)$$

$$\langle N_c \rangle_{equil}^{gc} = \sqrt{\epsilon} \equiv \sqrt{G \langle N_a \rangle \langle N_b \rangle / L}$$

- Equilibrium solution:

$$x g_{eq}'' + g_{eq}' - \epsilon g_{eq} = 0 \quad \longrightarrow \quad g_{eq}(x) = \frac{I_0(2\sqrt{\epsilon x})}{I_0(2\sqrt{\epsilon})}$$

- Average number of particles:

$$\langle N_c \rangle_{equil} = g_{eq}'(1) = \sqrt{\epsilon} \frac{I_1(2\sqrt{\epsilon})}{I_0(2\sqrt{\epsilon})}$$

$$\xrightarrow{\epsilon \to \infty} = \sqrt{\epsilon} \quad \text{grand canonical limit}$$

$$\xrightarrow{\epsilon \ll 1} = \epsilon$$

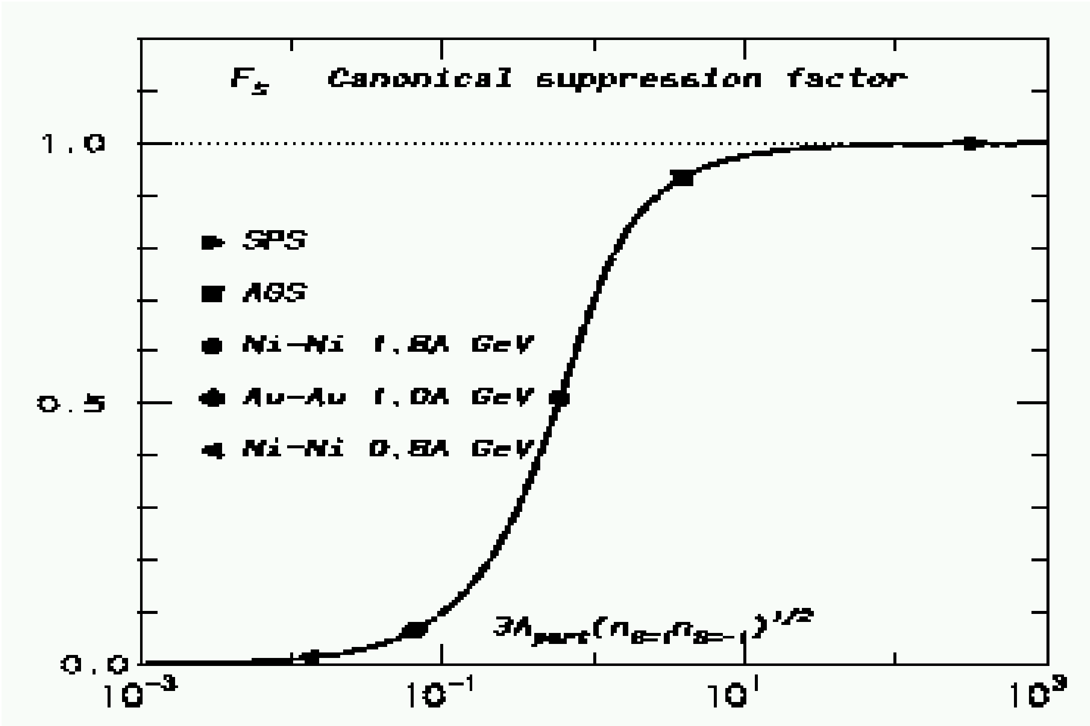
If conserved charge is rare, the production in pairs $c + \overline{c}$
is suppressed compared to the grand canonical limit

Canonical Strangeness Suppression at Low Energies

At low center of mass energies, $N_s + N_{\bar{s}}$ is rare.

$$\langle N_{s+\bar{s}} \rangle^{canonical} = \langle N_{s+\bar{s}} \rangle^{grand\ canonical} \underbrace{I_1(2\sqrt{\epsilon})/I_0(2\sqrt{\epsilon})}_{F_s}$$

$$\epsilon \propto \langle N_{s+\bar{s}} \rangle^{grand\ canonical}$$



The canonical strangeness suppression factor accounts for the observed reduction compared to the grand canonical limit.

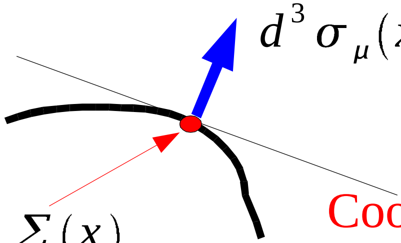
Combining Statistics and Dynamics

Combining Hydrodynamics + Stat. Hadronization

- Recall from first lecture the standard freeze-out condition of hydrodynamics:
  if $T(x) < T_{fo}$ then energy in space-time hypersurface element $\Sigma(x)$
  is converted to hadronic spectrum, assuming statistical hadronization

$$E\frac{dN_i}{d^3p} = \frac{g_i}{(2\pi)^3}\int_\Sigma p\cdot d^3\sigma(x)\, f_i(p\cdot u(x), x)$$

$$f_i(E,x) = \frac{1}{\exp\left[(E-\mu_i(x))/T(x)\right]\pm 1}$$



- Collective flow results in Doppler-shifted hadronic spectra

transverse flow

$$u^\mu(x) = \left(\cosh(\eta)\cosh(\rho), \vec{n}\sinh(\rho(x_T)), \sinh(\eta)\sinh(\rho)\right)$$

$$p^\mu = \left(M_T\cosh(y), \vec{p}_T, M_T\sinh(y)\right), \quad M_T = \sqrt{m^2 + \vec{p}_T^{\,2}}$$

$$\frac{p\cdot u(x)}{T} = \frac{M_T\cosh(\rho)}{T}\cosh(y-\eta) - \frac{\vec{p}_T\sinh(\rho)}{T}$$

emission from longitudinally comoving fluid element

slope of pt-spectra determined by combination of **radial flow** and temperature

$$T_{eff} \approx T\sqrt{\frac{1+\langle v_T\rangle}{1-\langle v_T\rangle}}$$

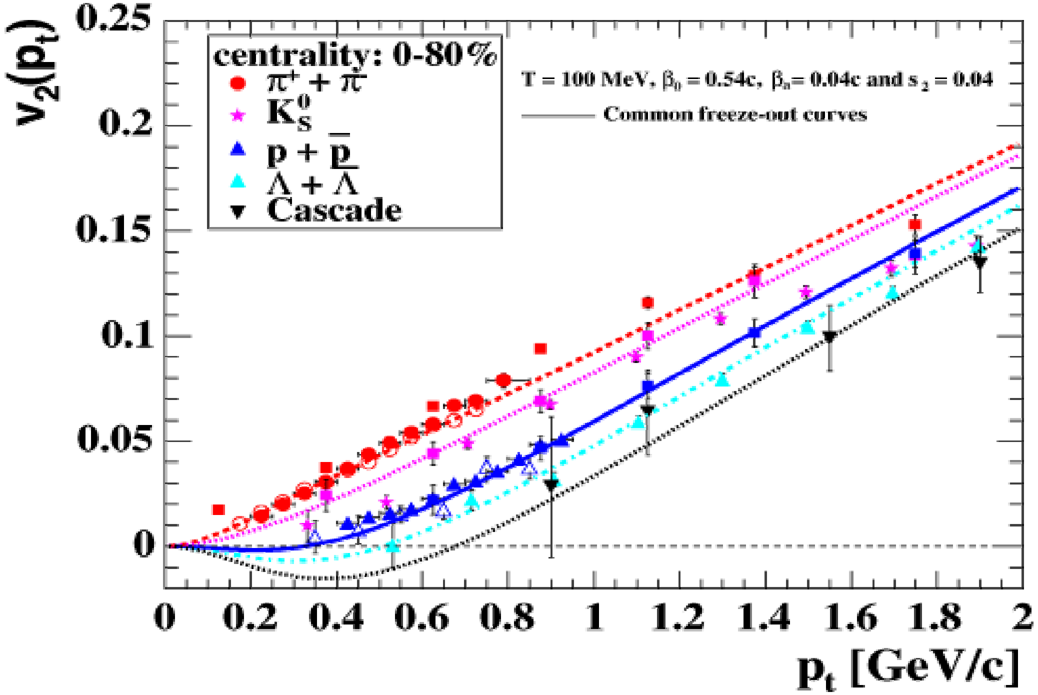
... compares well to low-pt Hadron Spectra at RHIC

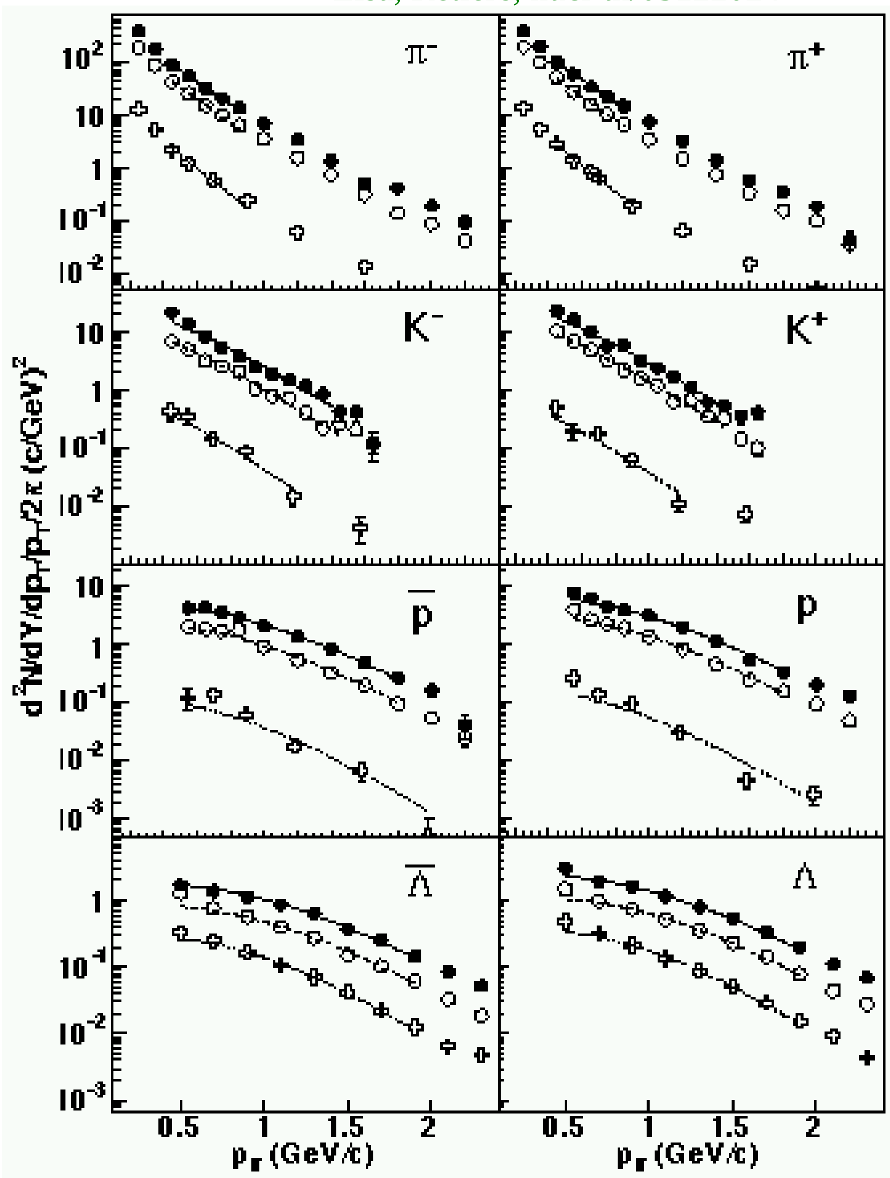
Lisa, Retiere, nucl-th/03122024

- Model fit assuming
  - local thermal equilibrium $T(\eta, b)$
  - collective flow u_ν

  $T_{fo} \sim 100\ MeV \quad v_{trans} > 0.5\ c$

- Mass-dependence of elliptic flow

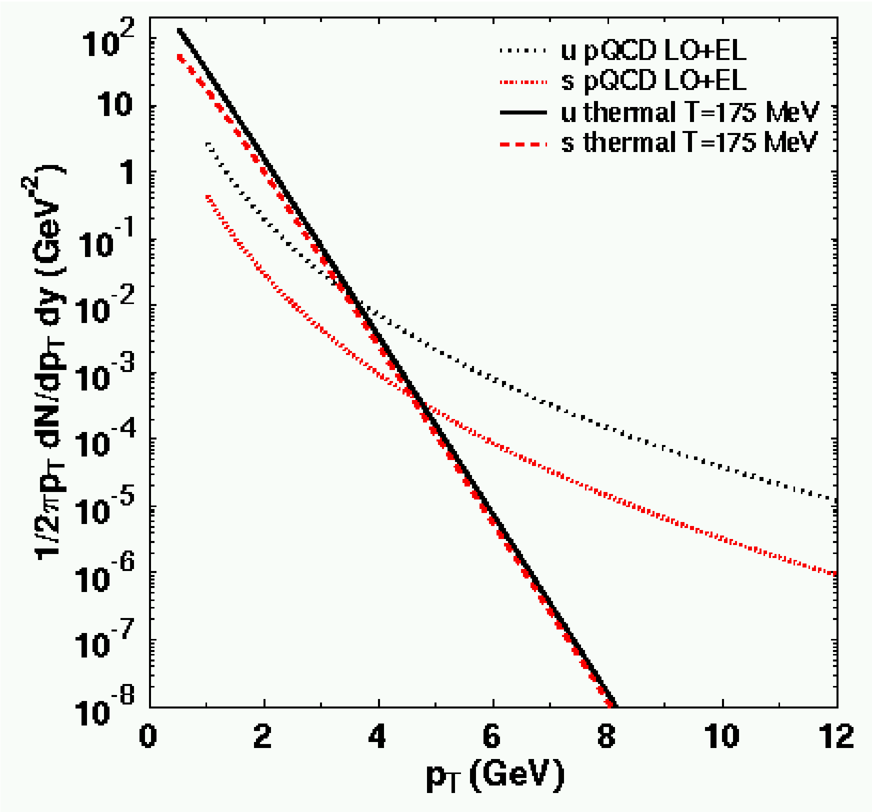




Model provides minimal parametrization of low-pt spectra
but does not address the microscopic dynamics determining these distributions

Limitations to Hydro + Statistical Hadronization

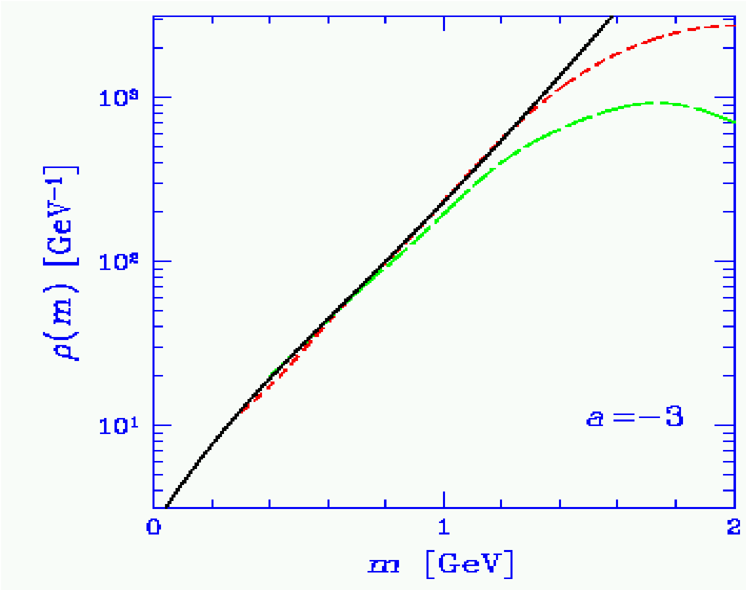
- at sufficiently high-pt, the spectrum is not thermal



- to study how these “out-of-equilibrium” processes evolve towards equilibrium tests the <u>dynamics</u> of thermalization processes

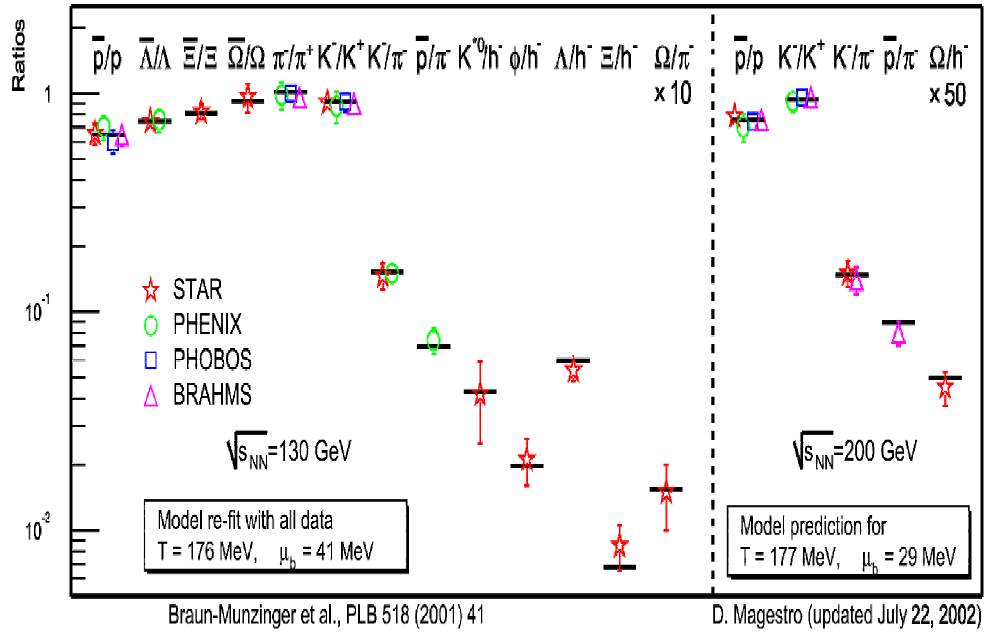
→ Lecture 3 + 4

Summary of Lecture 2: Hadron Thermodynamics



- Hadron resonance gas has a <u>limiting temperature</u> due to combinatorics of resonance formation
  - exponential increase in number of states
  - at T_H, Hagedorn's Statistical Bootstrap Model predicts phase transition
  - statistical QCD predicts properties of this phase transition to a plasma of deconfined partons

- <u>Model of statistical hadronization</u> assumes that hadroproduction determined by phase space
  - grand canonical description accounts for particle ratios
  - rare conserved quantum numbers lead to canonical suppression



- <u>Hydrodynamics + statistical hadronization</u>
  - minimal parametrization of low-pt hadronic spectra and elliptic flow
  - unclear whether this is indicative of thermalization processes