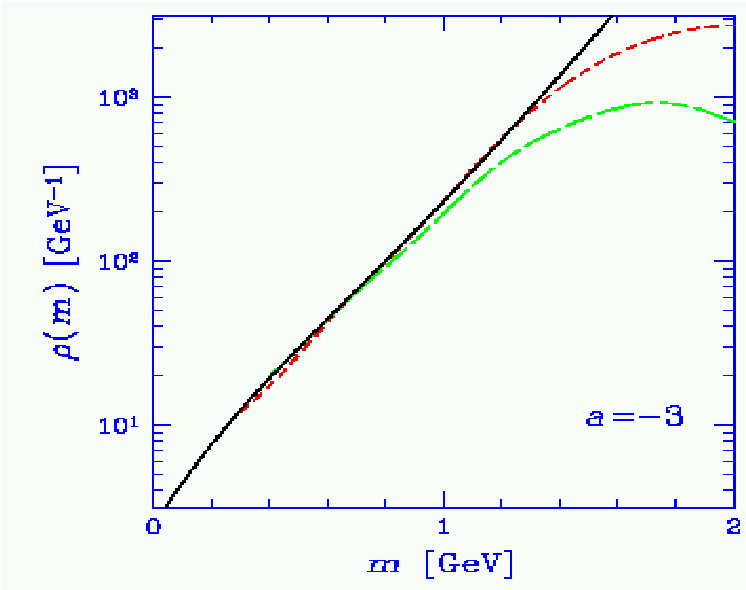
Selected Topics in the Theory of Heavy Ion Collisions

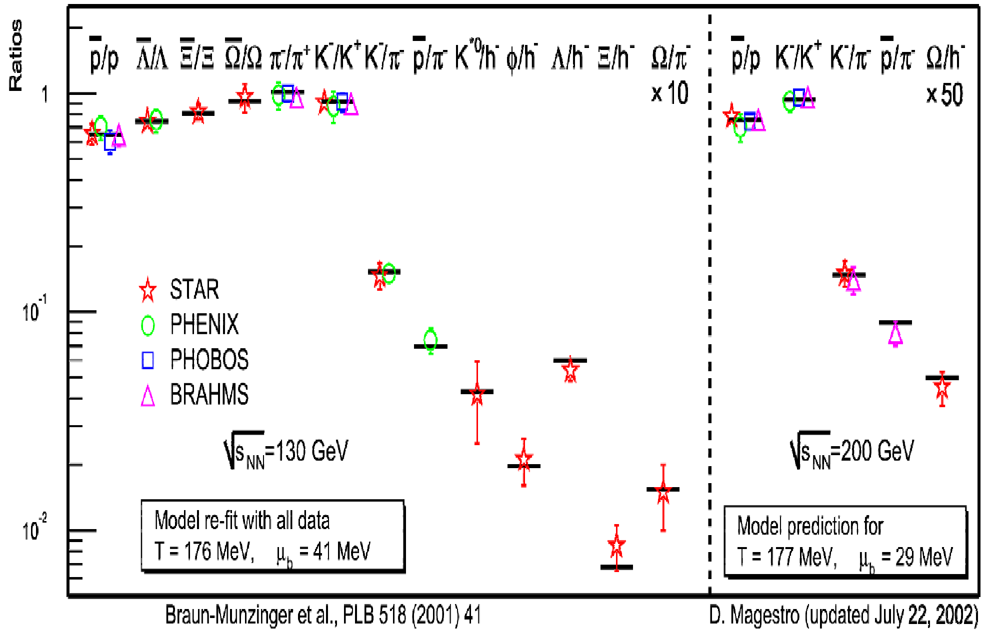
Urs Achim Wiedemann, Physics Department,
CERN, Theory Division

Summary of Lecture 2: Hadron Thermodynamics

- Hadron resonance gas has a limiting temperature due to combinatorics of resonance formation
  - exponential increase in number of states
  - at T_H, Hagedorn's Statistical Bootstrap Model predicts phase transition
  - statistical QCD predicts properties of this phase transition to a plasma of deconfined partons



- Model of statistical hadronization assumes that hadroproduction determined by phase space
  - grand canonical description accounts for particle ratios
  - rare conserved quantum numbers lead to canonical suppression



- Hydrodynamics + statistical hadronization
  - minimal parametrization of low-pt hadronic spectra and elliptic flow
  - unclear whether this is indicative of thermalization processes

Lecture 3:
a. The Space-time picture of the Bulk
b. Hard Processes Escaping the Bulk

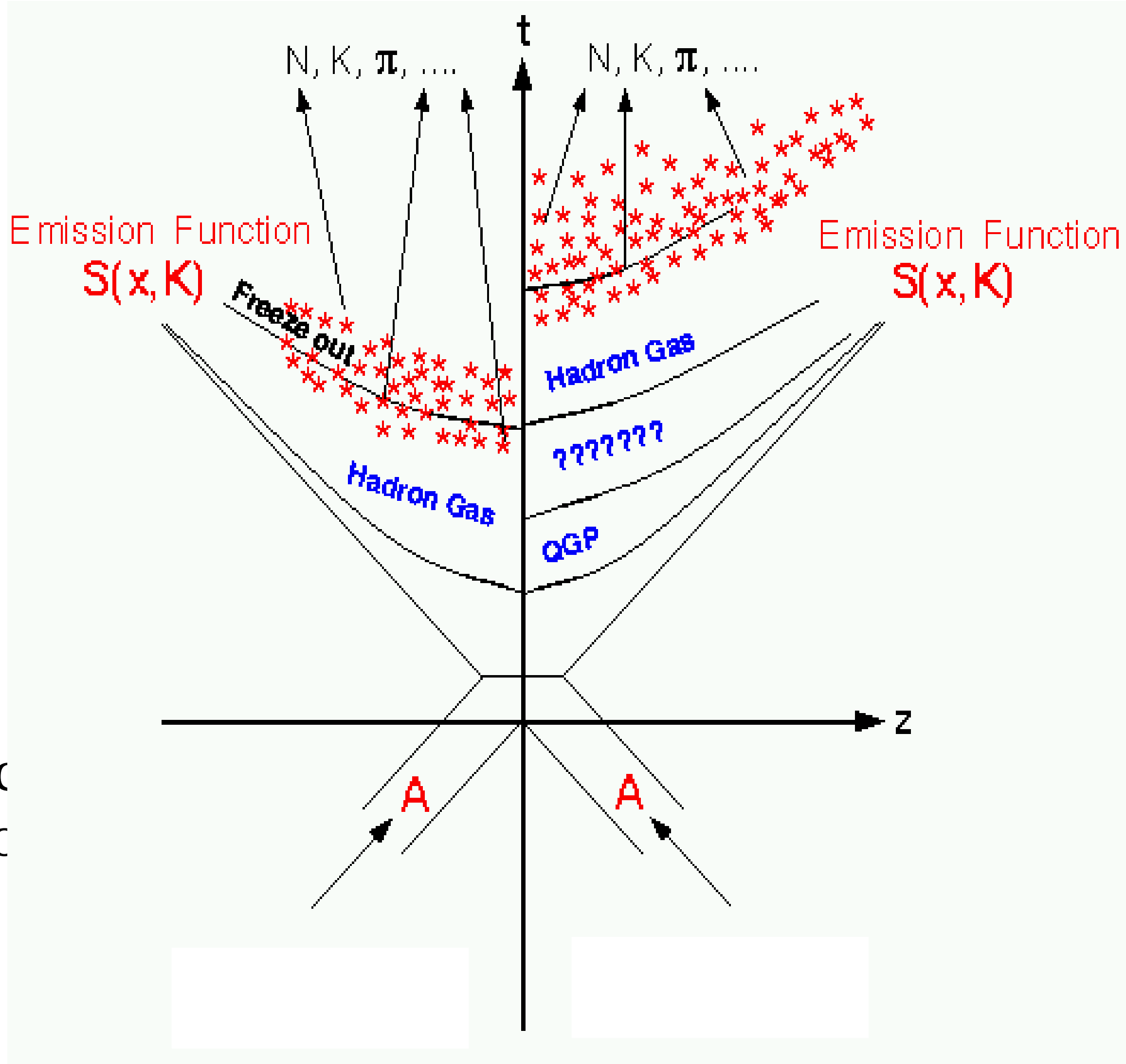
a. Identical Two Particle Correlation Functions
- Interferometry for Central Collisions
- The space-time picture of elliptic flow

b. Hard Probes in Heavy Ion Collisions
- Modifications of high-pt processes in nuclear matter
  – to be continued in lecture 4

The Emission Function S(x,K)

• How can we measure the space-time geometry of the particle emitting source ?

• **S(x,K)** is a quantum-mechanic Wigner phase space distributic



model 1 model 2

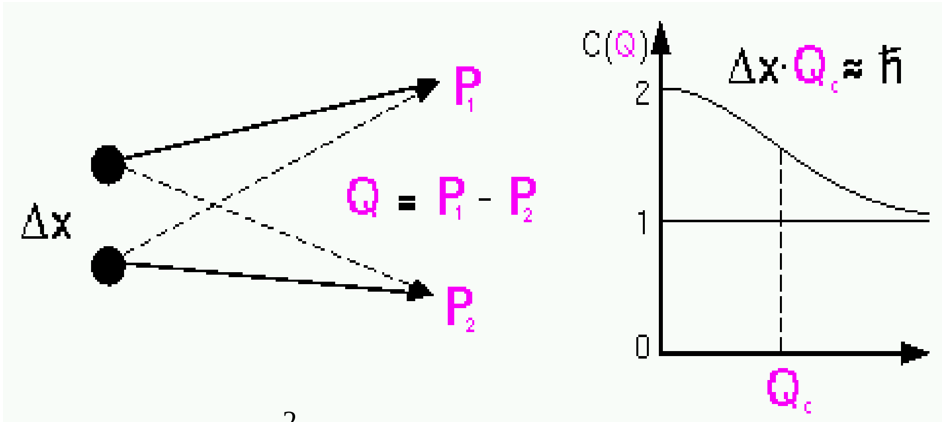
Why are two-particle correlations interesting ?

How to test phase space distribution **S(x,K)** of particle emitting source ?

- One-particle spectrum sensitive to momentum information only

$$E\frac{dN}{d^3p}=\int d^4x\,S(x,p)=\frac{1}{2\pi}\frac{dN}{p_t\,dp_t\,d\eta}\left[1+2v_2(p_t)\cos\left(2(\phi-\psi_r)\right)\right]$$

- Q-momentum dependence of two-particle spectrum sensitive to space-time information



Problem 1: derive this

$$C(K,Q)=1+\frac{\left|\int d^4x\,S(x,K)\exp(iK\cdot x)\right|^2}{\int d^4x\,S(x,P_1)\int d^4y\,S(y,P_2)}=1+\lambda\exp\left[-\sum_{ij}R_{ij}^2(K)\,q_i\,q_j\right]$$

The out-side-long System

$$C(K_t,Q) = 1 + \lambda\, e^{\left[-R_o^2(K_t)q_o^2 - R_2^2(K_t)q_2^2 - R_l^2(K_t)q_l^2 - 2R_{ol}^2(K_t)q_o q_l\right]}$$

- Different HBT radii measure different combinations of spatial and temporal information, encoded in the space-time variances which are the Gaussian widths of S(x,K)

$$\langle f\rangle(K_t) \equiv \frac{\int d^4x\, f(x) S(x,K)}{\int d^4x\, S(x,K)}$$

Problem 2: derive this

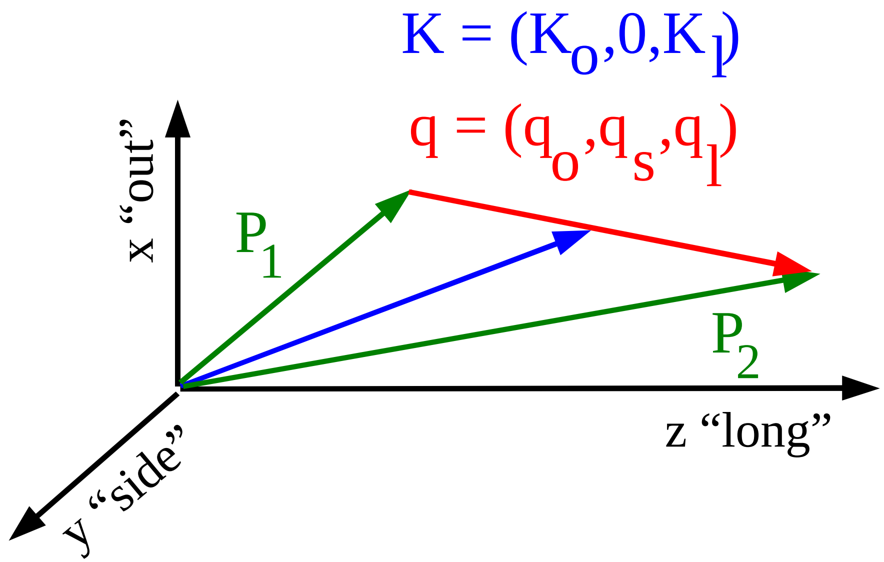
$$R_o^2(K_t) = \langle(\tilde{x} - \beta_t \tilde{t})^2\rangle$$

$$R_s^2(K_t) = \langle\tilde{y}^2\rangle$$

$$R_l^2(K_t) = \langle(\tilde{z} - \beta_l \tilde{t})^2\rangle$$

$$R_{ol}^2(K_t) = \langle(\tilde{x} - \beta_t \tilde{t})(\tilde{z} - \beta_l \tilde{t})\rangle$$

$$\beta \equiv \frac{\vec{K}}{K_0}$$

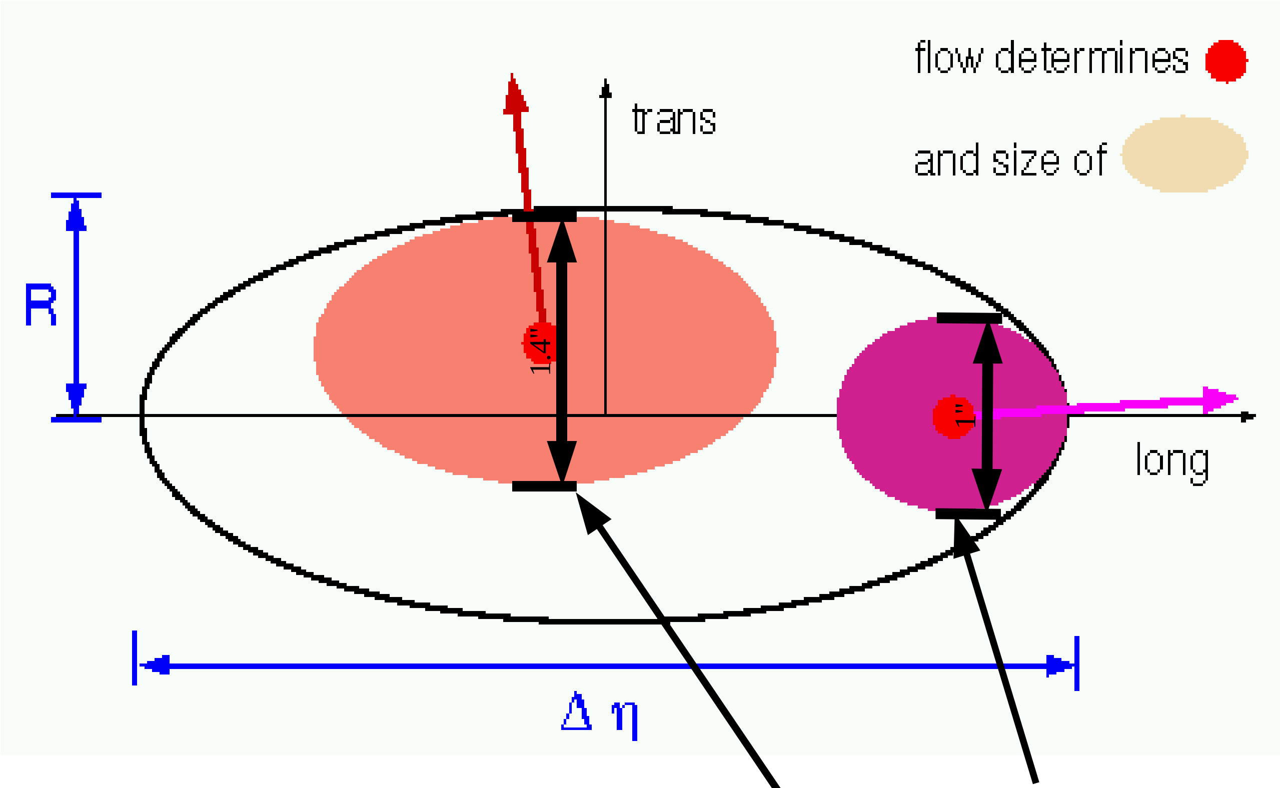


- Distances are always measured w.r.t. the source center, $\tilde{x} \equiv x - \langle x\rangle$
- Momentum dependence of HBT radii can be exploited to extract spatio-temporal information, (over)simplified example:

$$R_o^2(K_t) - R_s^2(K_t) \approx \beta_t^2 \langle\tilde{t}^2\rangle$$

HBT radii measure homogeneity regions

- They do NOT measure the entire source size



$$R_s^2(K_{t1}) \neq R_s^2(K_{t2})$$

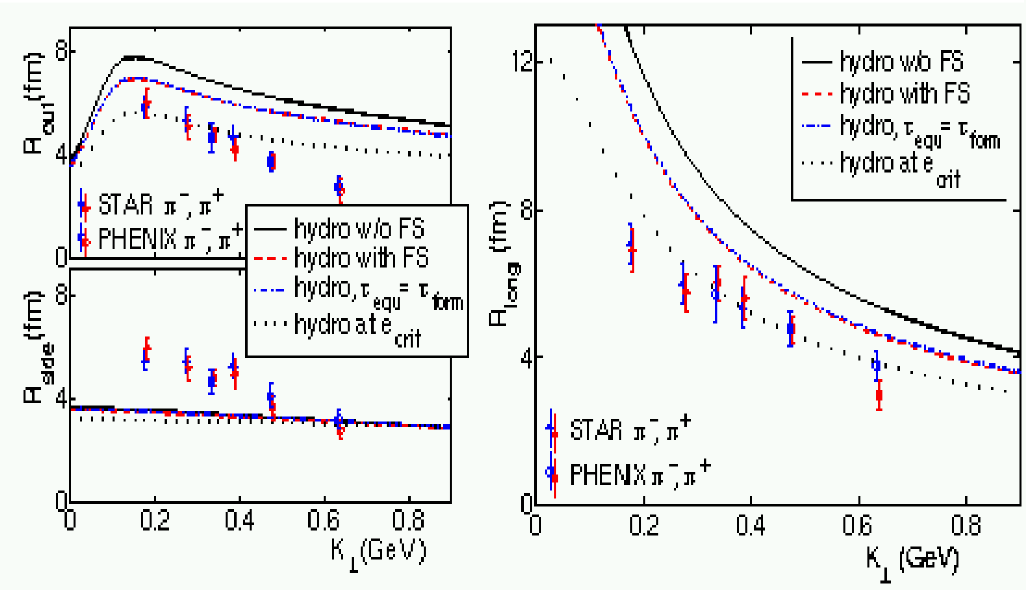
- Kt-dependence of HBT radii contains dynamical information

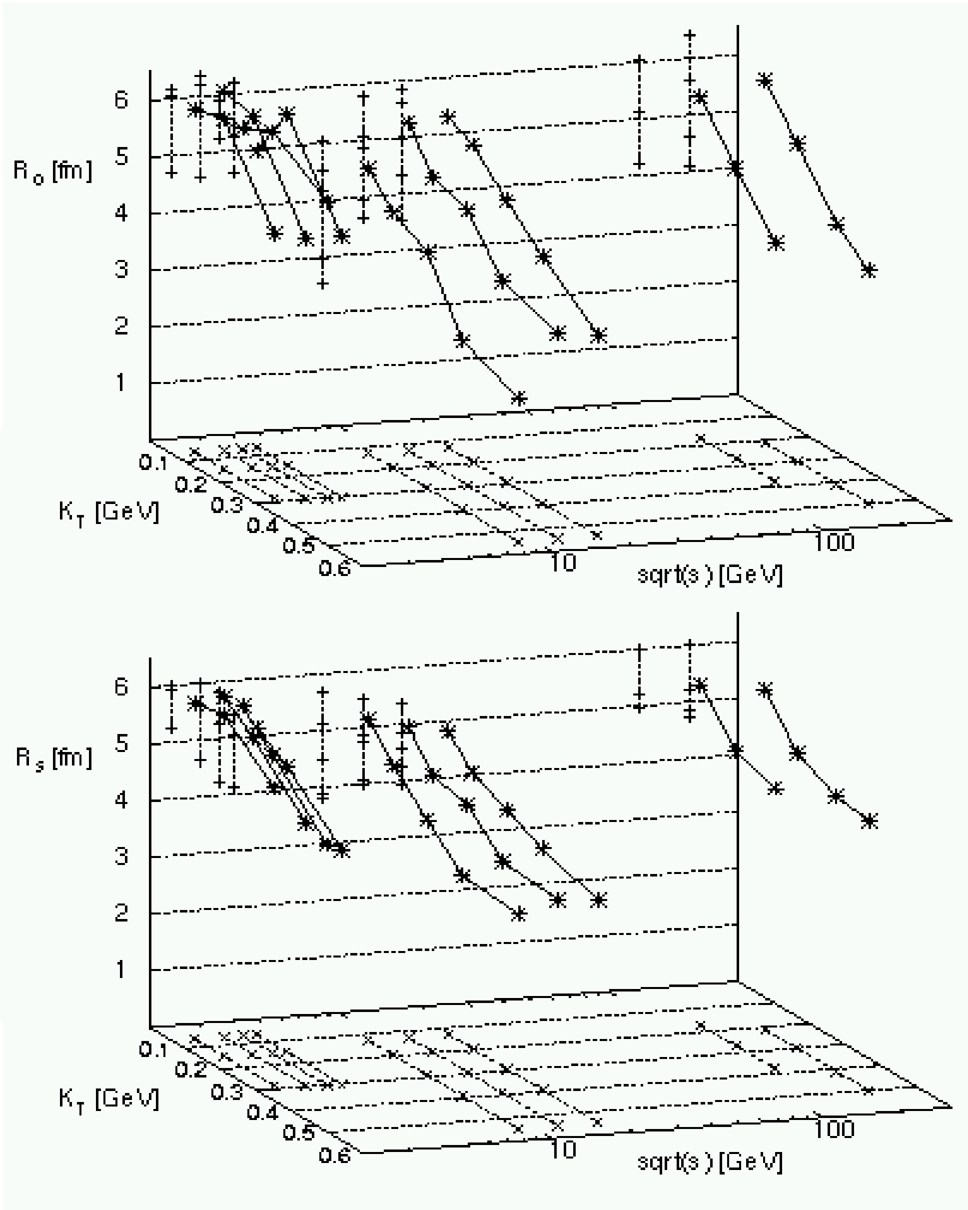
  (Kt defines orientation and wave-length filter of the observer's eye)

The HBT puzzle at RHIC

- Why do radii show such a weak dependence on cms energy ?

- Why do hydrodynamical models fail to reproduce size and slope of HBT radii ?





Unsolved problem

(may require an improved freeze-out description)

The HBT space-time picture of the reaction plane

HBT particle correlations

measures geometry + dynamics

$$S(x, K)$$

$$\lambda_{mfp} = R_{system}$$

determined at freeze-out,
not described by hydro

Elliptic Flow

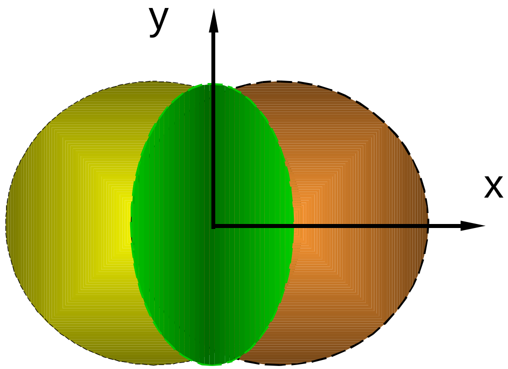
depends on geometry but
does not measure it

$$S(\cancel{x}, K)$$

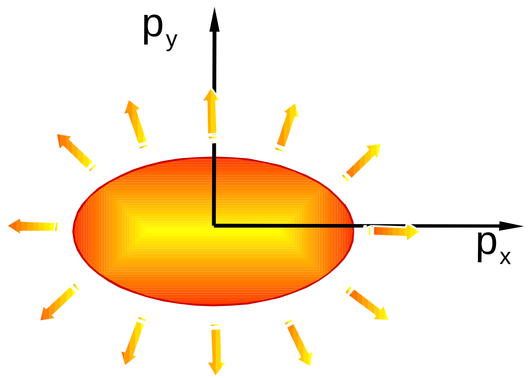
$$\lambda_{mfp} \ll R_{system}$$

not sensitive to freeze-out,
hallmark of hydrodynamic behavior

Coordinate space



Momentum space

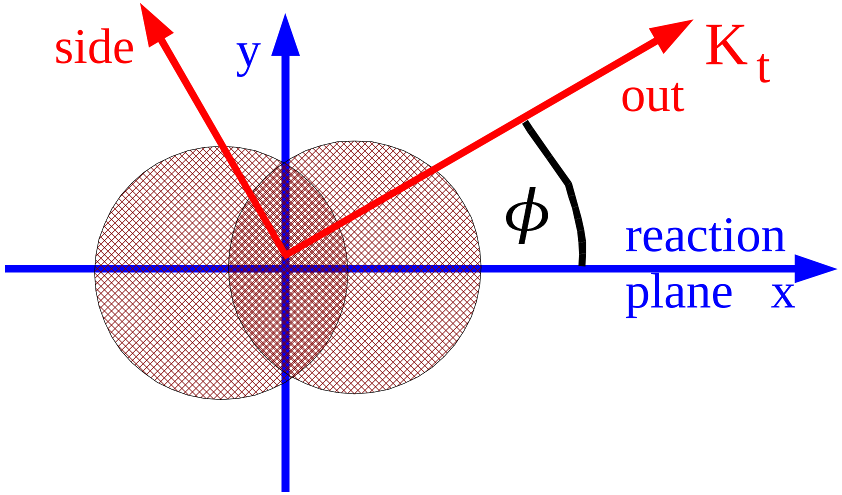


coordinate space anisotropy
implies pressure gradient
implies momentum space anisotropy

The out-side-long System for Anisotropic Sources

$$C(K_t, Q) = 1 + \lambda \exp\left[-\sum_{ij} R_{ij}^2(Y, K_t, \phi) q_i q_j \right]$$

Problem 3: derive this



$$R_s^2 = S_{11} \sin^2\phi + S_{22} \cos^2\phi - S_{12} \sin 2\phi$$

$$R_o^2 = S_{11} \cos^2\phi + S_{22} \sin^2\phi + S_{12} \sin 2\phi - 2\beta_t S_{01} \cos\phi - 2\beta_t S_{02} \sin\phi + \beta_t^2 S_{00}$$

$$R_{os}^2 = S_{12} \cos 2\phi + (S_{22}/2 - S_{11}/2) \sin 2\phi + \beta_t S_{01} \sin\phi - \beta_t S_{02} \cos\phi$$

$$R_l^2 = S_{33} - \beta_l S_{03} + \beta_l^2 S_{00}$$

$$R_{ol}^2 = (S_{13} - \beta_l S_{01}) \cos\phi + (S_{23} - \beta_l S_{02}) \sin\phi - \beta_t S_{03} + \beta_l \beta_t S_{00}$$

$$R_{sl}^2 = (S_{23} - \beta_l S_{02}) \cos\phi - (S_{13} - \beta_l S_{01}) \sin\phi$$

- Spatial correlation tensor

$$S_{\mu\nu}(\phi) = \langle \tilde{x}_\mu \tilde{x}_\nu \rangle \equiv \frac{\int d^4x\, \tilde{x}_\mu \tilde{x}_\nu S(x, K)}{\int d^4x\, S(x, K)}$$

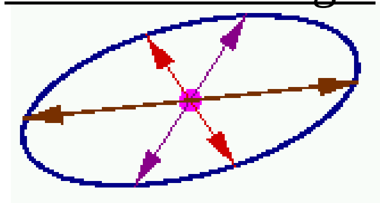
parametrizes **implicit** ϕ - dependence

ϕ - dependence on HBT radii contains **geometrical** and **dynamical** information

Azimuthal Dependence at Mid-Rapidity

- Symmetries: only 5 non-vanishing components

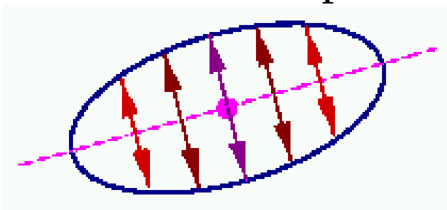
reflection at origin



$(\tilde{x},\tilde{y},\tilde{z}) \rightarrow (-\tilde{x},-\tilde{y},-\tilde{z})$

$\Rightarrow S_{01} = S_{02} = S_{03} = 0$

w.r.t. reaction plane



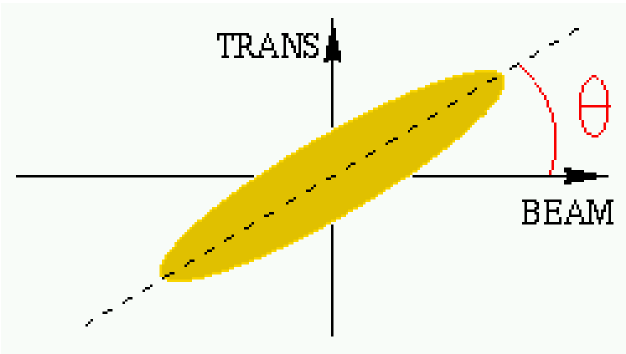
$\langle \widetilde{x_\mu \tilde{y}} \rangle(\phi) = -\langle \widetilde{x_\mu \tilde{y}} \rangle(-\phi)$

$\Rightarrow S_{12} = S_{23} = S_{02} = 0$

- Case: no (implicit) ϕ - dependent position-momentum correlations:

four main axis of emission ellipsoid and the **tilt angle**

$$\Theta = \frac{1}{2}\tan^{-1}\left(\frac{2S_{13}}{S_{33}-S_{11}}\right)$$



$$R_s^2 = 0.5(S_{11}+S_{22}) + 0.5(S_{22}-S_{11})\cos 2\phi$$

$$R_o^2 = 0.5(S_{11}+S_{22}) - 0.5(S_{22}-S_{11})\cos 2\phi + \beta_t^2 S_{00}$$

$$R_{os}^2 = 0.5(S_{22}-S_{11})\sin 2\phi$$

$$R_l^2 = S_{33} + \beta_l^2 S_{00}$$

$$R_{ol}^2 = S_{13}\cos\phi$$

$$R_{sl}^2 = -S_{13}\sin\phi$$

First harmonics at midrapidity !

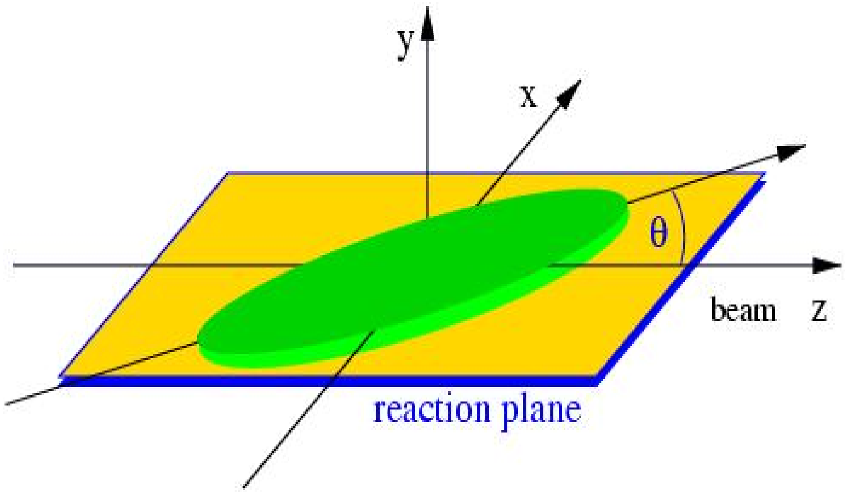
Anisotropic HBT at the AGS

- Consistency check:

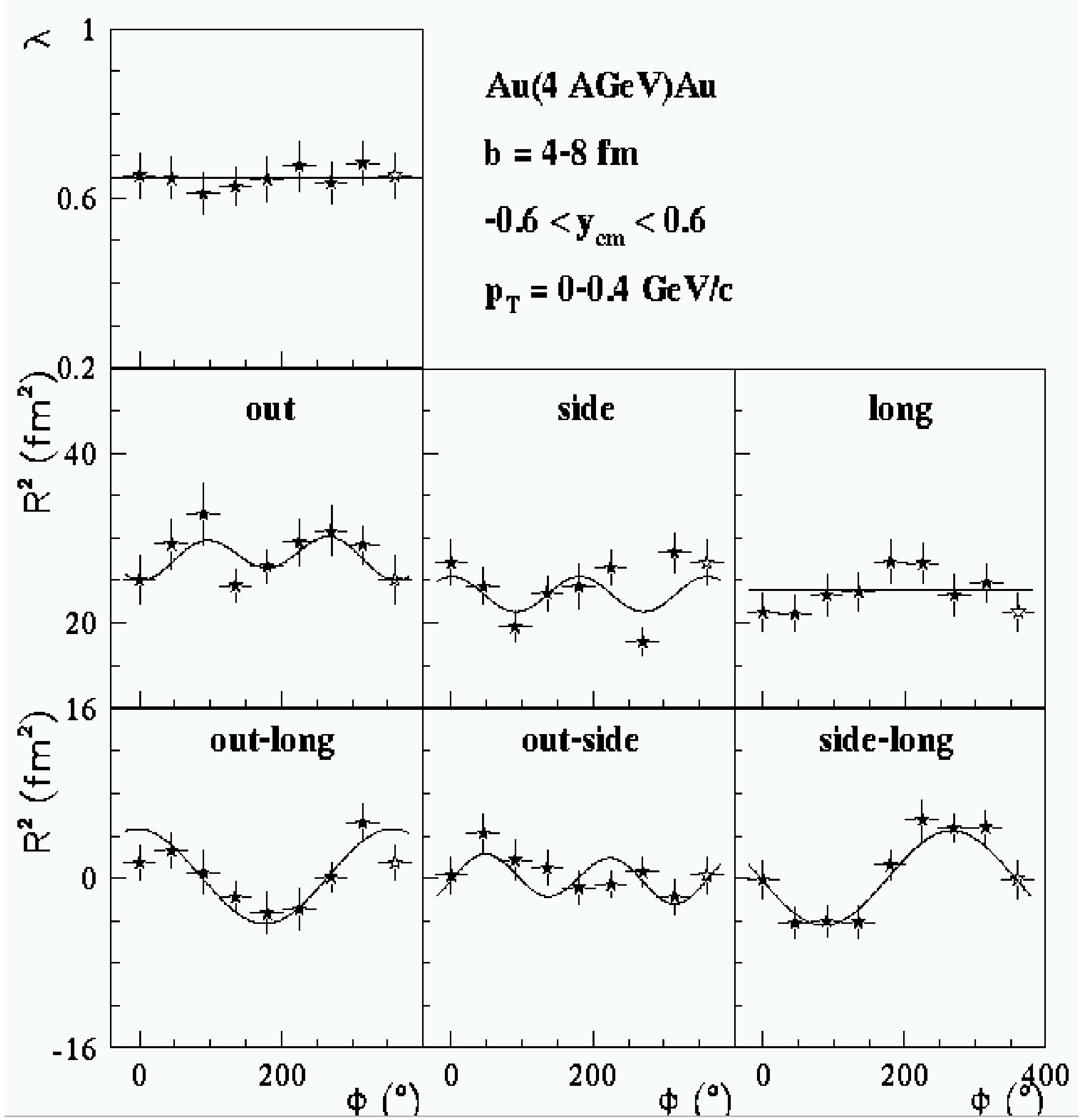
$$-R^{c}_{o,2}{}^{2} = R^{c}_{s,2}{}^{2} = R^{s}_{os,2}{}^{2}$$

- Very large tilt angle:

$$\Theta = \frac{1}{2}\tan^{-1}\left(\frac{2\,S_{13}}{S_{33}-S_{11}}\right) \simeq \frac{\pi}{4}$$



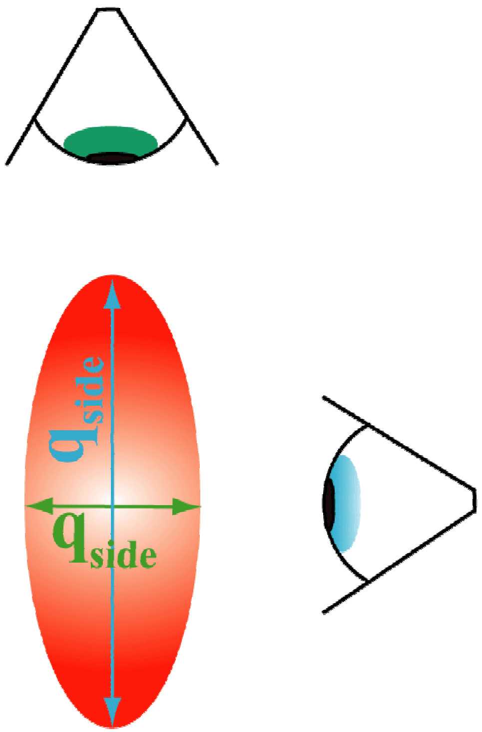
E895 Coll., M.A. Lisa et al. PLB 496 (2000) 1

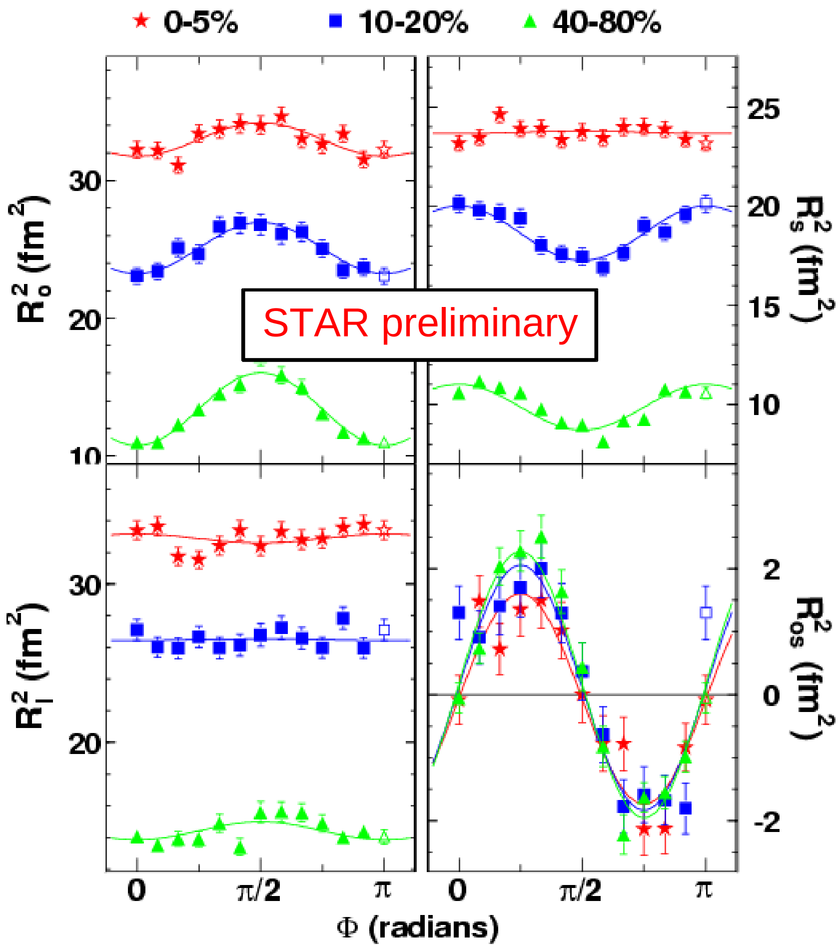


Azimuthal dependence of HBT "sees" reaction plane

- Use identical two-particle correlations

$$C(q, K) = 1 + \lambda e^{i q_i q_j R_{ij}^2(\phi)}$$



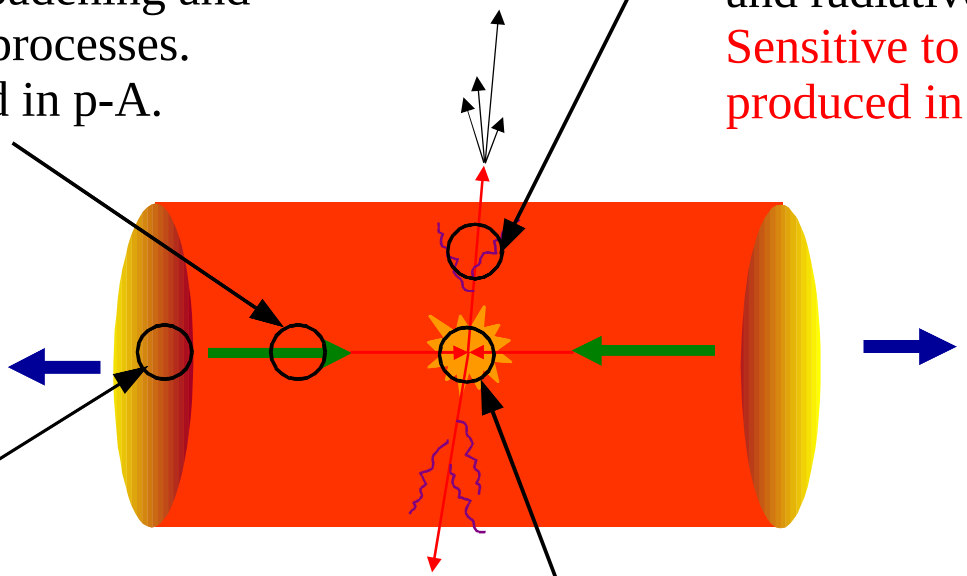


Hard Processes Escaping the Bulk

High-Q^2 QCD in a Dense Medium

- Initial state scattering effects, Cronin-type pt-broadening and possible inelastic processes. Can be determined in p-A.

- Final state pt-broadening, collisional and radiative energy loss. Sensitive to properties of the matter produced in A+A collisions.



- Parton distribution functions (pdfs) of incoming nuclei show modifications compared to pdfs for nucleons.

$$f_{i/A}(x, Q^2) \neq A\, f_{i/p}(x, Q^2)$$

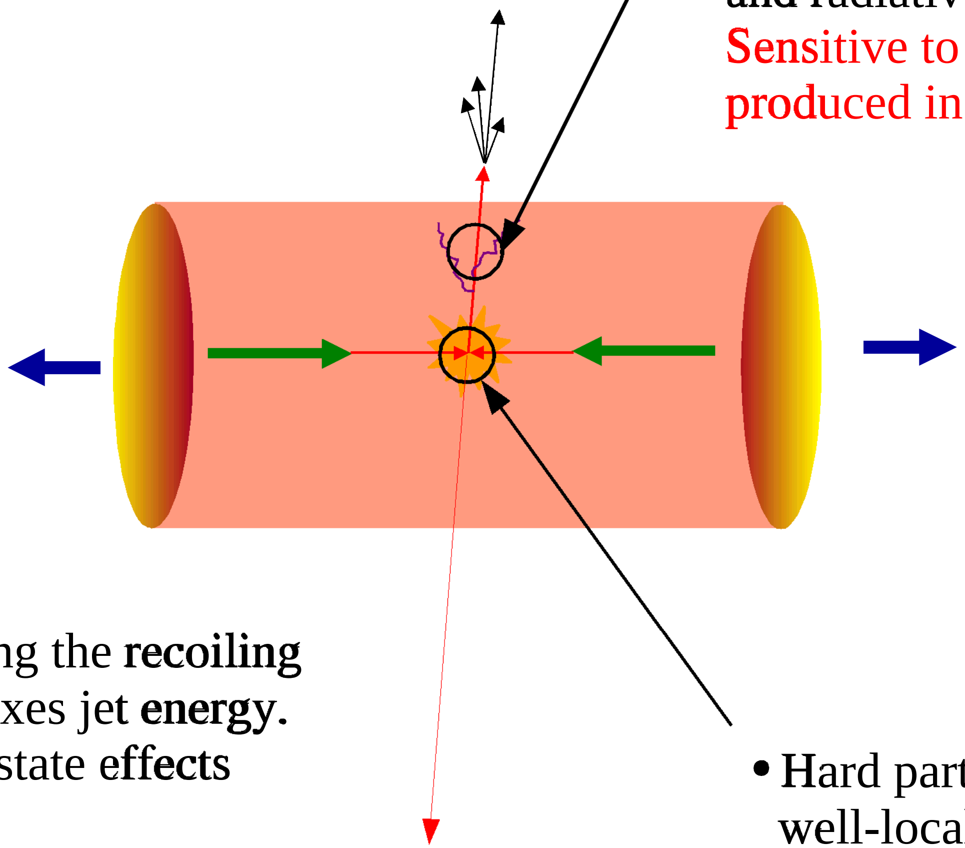
Typically a 20% effect.
Can be determined in e-A DIS.

- Hard partonic 2->2 process is well-localized on a scale

$$r \sim \tau \sim 1/Q \ll 1/T$$

→ unaffected by the medium

(Dis)Advantage of 'Self-Gauging' Hard Processes



- Final state pt-broadening, collisional and radiative energy loss. Sensitive to properties of the matter produced in A+A collisions.

- **Advantage:** measuring the **recoiling** Z-boson or photon fixes jet **energy.** Insensitive to initial state **effects**

- Hard partonic 2->2 process is well-localized on a scale

$$r \sim \tau \sim 1/Q \ll 1/T$$

→ unaffected by the medium

Z-boson
Photon

- Disadvantage: theses processes are rare and difficult to measure.

Bremsstrahlung (potential scattering) in QED/QCD

- Radiation in both QED and QCD

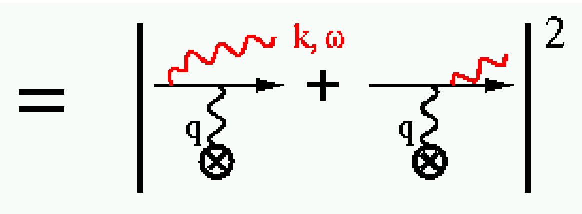
$$\frac{dI_A}{d\ln(x)\,dk_t} \propto \int \frac{d^2q_t\, d^2r_t}{(2\pi)^4} \frac{q_t^2}{k_t^2(k_t-q_t)^2} \sigma_A(r_t) \exp[i\, q_t \cdot r_t]$$

radiation term

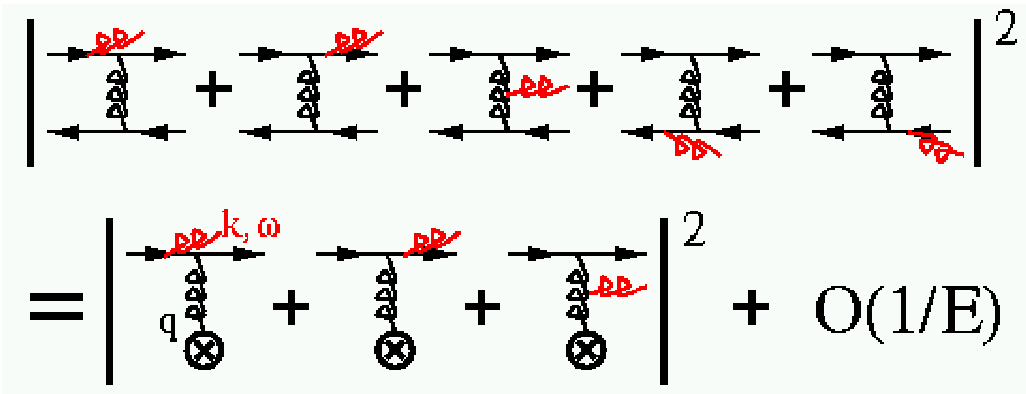
potential scattering determined by 'dipole cross section':

$$\sigma_A(r_t) = 2\int \frac{d^2q_t}{(2\pi)^2} |a_A(q_t)|^2 \left(1 - \exp[i\, q_t \cdot r_t]\right)$$

- QED Bethe-Heitler spectrum



- QCD Gunion-Bertsch spectrum



$$\sigma_{QED}(r_t) = \sigma_A(x\, r_t)$$

only difference

$$x = \omega / E$$

$$\sigma_{QCD}(r_t) = \sigma_A(r_t)$$

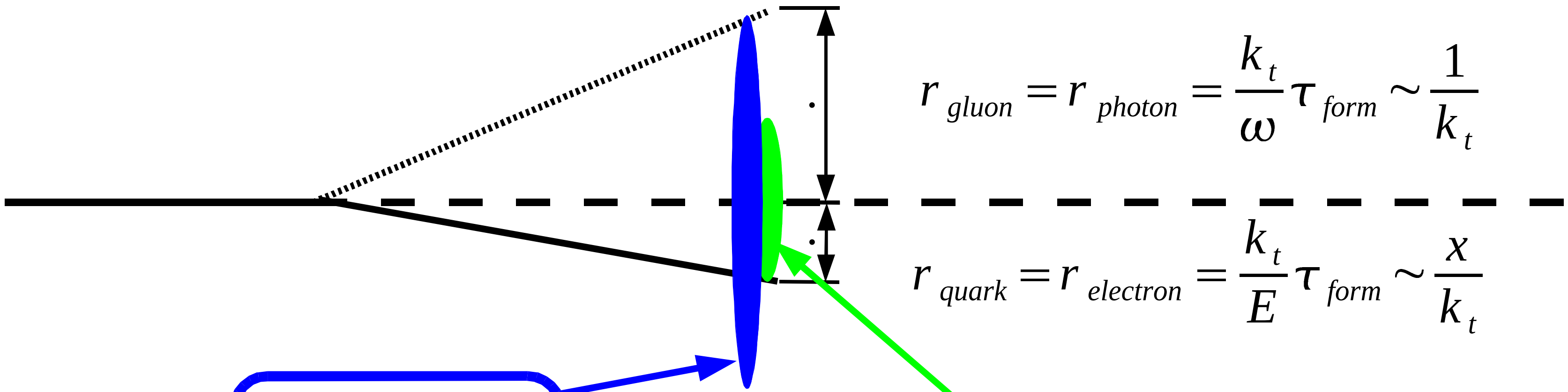
Configuration Space Picture of Bremsstrahlung

- Consider incoming projectile (electron or quark) with its higher Fock states

$$projectile = q + q\,g + \ldots$$

- Gluon in incoming wavefunction is 'freed' (decoheres) if it interacts with the scattering center with significantly different amplitude.
  This depends on transverse distance of different Fock components.

characteristic time scale for photon/gluon formation $\tau_{form} \sim \left(\frac{k_t^2}{2\,\omega}\right)^{-1} \sim \left(\frac{k_t^2}{2\,x\,E}\right)^{-1}$



$$r_{gluon} = r_{photon} = \frac{k_t}{\omega}\tau_{form} \sim \frac{1}{k_t}$$

$$r_{quark} = r_{electron} = \frac{k_t}{E}\tau_{form} \sim \frac{x}{k_t}$$

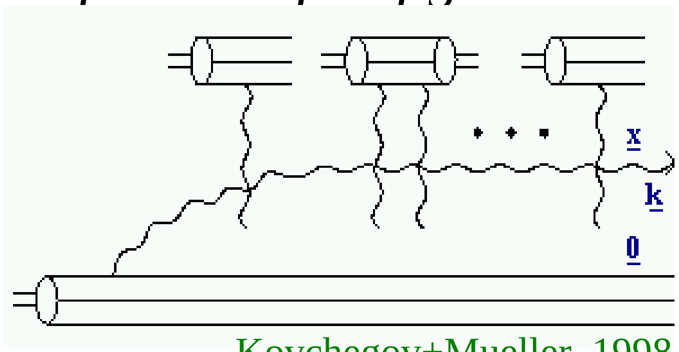
$$\sigma_{QCD}\left(r_{gluon}\right)$$

$$\sigma_{QED}\left(r_{electron} = x\,r_{photon}\right)$$

since the photon does not scatter

Gluon Production in p+A Collisions

- Incoming quark carries Weizsacker-Williams gluon cloud $quark = q + q\,g + \ldots$



Kovchegov+Mueller, 1998

- Gluon in quark interacts with many scattering centers. In the high-energy limit, these scattering centers act coherently as one single effective scattering center.

$$N_{gluons}(k_t) \propto \int \frac{d^2 q_t}{(2\pi)^2} \frac{q_t^2}{k_t^2 (k_t - q_t)^2} \exp\left[-\frac{q_t^2}{\hat{q}\, L} \right]$$

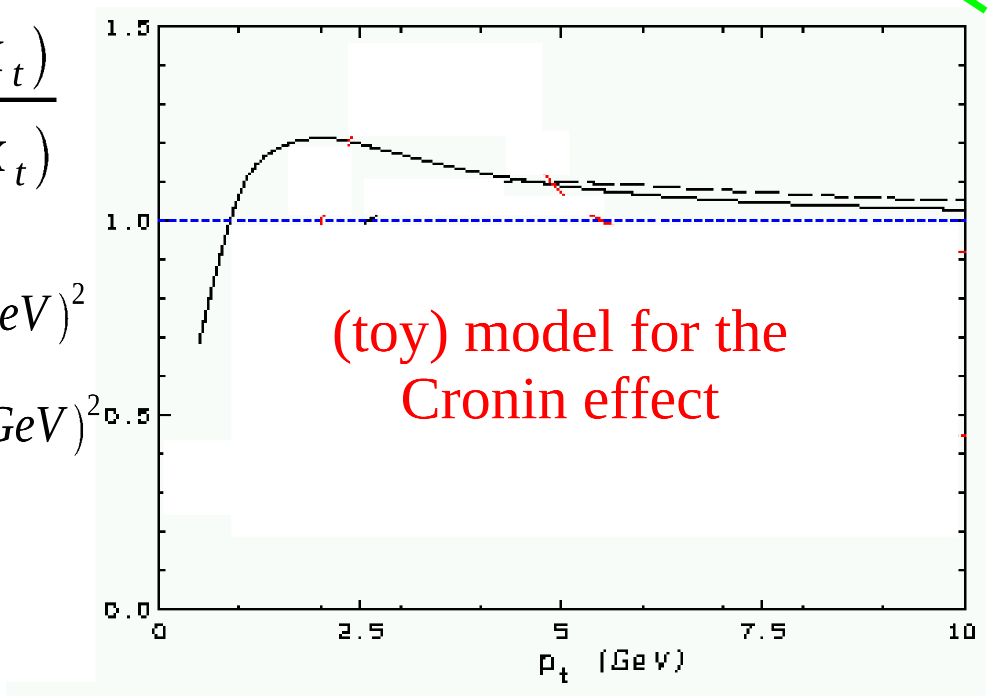
use $n\,\sigma(r_t) \approx \hat{q}\, r_t^2$

average momentum transfer from effective coh. scattering

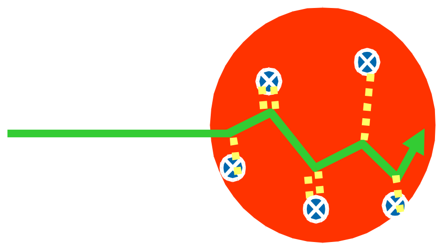
$$\frac{N_{gluons}^{\hat{q} L\ nucleus}(k_t)}{N_{gluons}^{\hat{q} L\ nucleon}(k_t)}$$

$(\hat{q}\, L)_{nucleus} \sim (2\, GeV)^2$

$(\hat{q}\, L)_{nucleon} \sim (0.2\, GeV)^2$

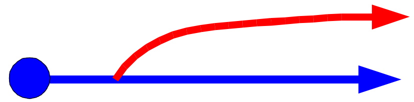


- Multiple scattering leads to redistribution of the gluons in transverse phase space



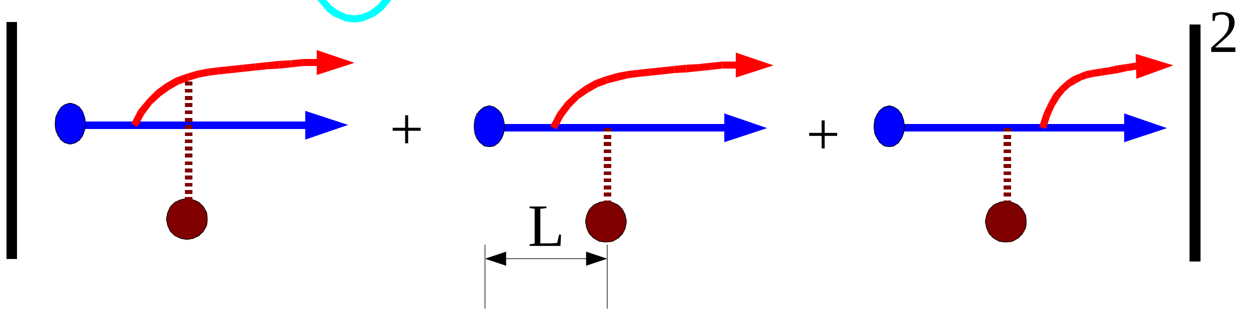
Gluon Radiation off a Produced Quark

- No scattering center $\omega \frac{dI^{N=0}}{d\omega\, dk_t} \propto \frac{1}{k_t^2}$



- One scattering center

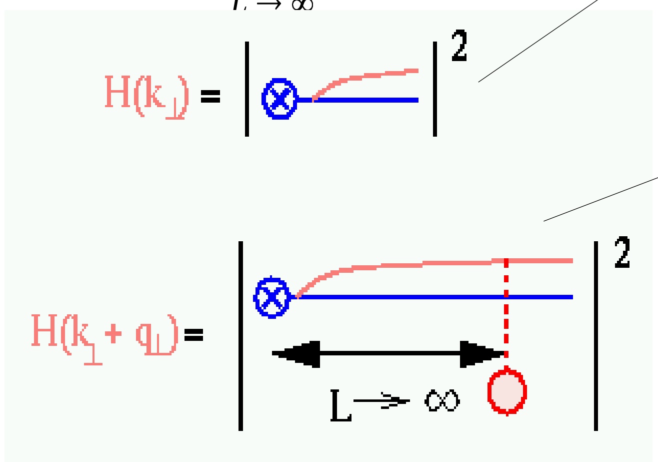
medium-modified parton splitting function

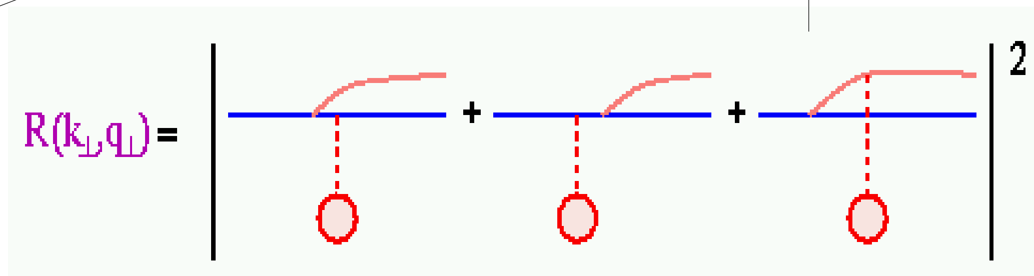


$$\omega \frac{dI^{N=1}}{d\omega\, dk_t} \propto \int \frac{dq_t}{(2\pi)^2} n_0 |a(q_t)|^2 \frac{-k_t \cdot q_t}{Q\, Q_1} \frac{LQ_1 - \sin(LQ_1)}{Q_1} \qquad Q_1 = \frac{(k_t+q_t)^2}{2\omega}$$

- Radiation interpolates between the totally coherent and totally incoherent limit

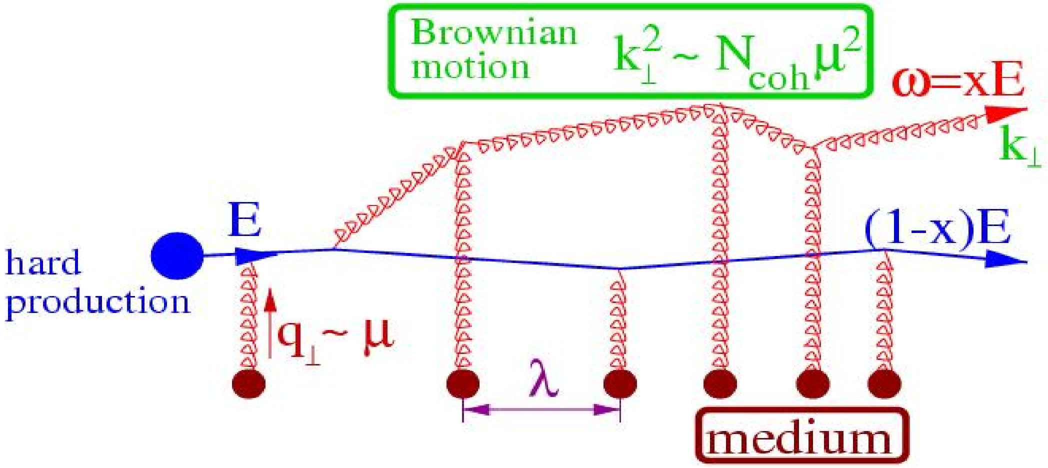
$$\omega \frac{dI^{N=0+1}}{d\omega\, dk_t} \underbrace{\rightarrow}_{L\rightarrow\infty} (1-Prob_1) H(k_t) + Prob_1 \int |a(q_t)|^2 \left(H(k_t+q_t) + R(k_t, q_t) \right)$$





The medium-modified Final State Parton Shower

Baier, Dokshitzer, Mueller, Peigne, Schiff (1996); Zakharov (1997); Wiedemann (2000); Gyulassy, Levai, Vitev (2000); Wang ...



Medium characterized by transport coefficient:

$$\hat{q} = \frac{\mu^2}{\lambda} \sim n_{density}$$

- How much energy is lost ?

Phase accumulated in medium: $\left\langle \frac{k_t^2}{2\,\omega} \Delta z \right\rangle \simeq \frac{\hat{q}\, L^2}{2\,\omega} \equiv \frac{\omega_c}{\omega}$

Characteristic gluon energy

Number of coherent scatterings: $N_{coh} \simeq \frac{t_{coh}}{\lambda}, \quad \langle k_t^2 \rangle = \hat{q}\, t_{coh} \longrightarrow t_{coh} \simeq \frac{\omega}{2k_t^2} \simeq \sqrt{\frac{\omega}{\hat{q}}}$

Gluon energy distribution: $\omega \frac{dI_{med}}{d\,\omega\, dz} \simeq \frac{1}{N_{coh}} \omega \frac{dI_1}{d\,\omega\, dz} \simeq \alpha_s \sqrt{\frac{\hat{q}}{\omega}}$

Average energy loss $\Delta E \simeq \int^{\omega_c} d\,\omega\, \omega \frac{dI_{med}}{d\,\omega} \sim \alpha_s\, \omega_c = \alpha_s \frac{1}{2} \hat{q}\, L^2$

Some Numbers:

- Transport coefficient: $q = \frac{(1\,GeV)^2}{fm}$, in-medium pathlength: $L = 5\ fm$

  Average momentum broadening: $\langle k_t^2 \rangle \simeq q\,L = (1\,GeV)^2$

  Characteristic gluon energy: $\omega_c = \frac{1}{2} q\,L^2 = 62.5\,GeV$ → $\Delta E_{loss} > 10\ GeV$

- Time scales: Hadronization time scale: $\tau_{hadr} > \frac{1}{Q_0}\frac{E}{Q_0}, Q_0 = 1\,GeV$

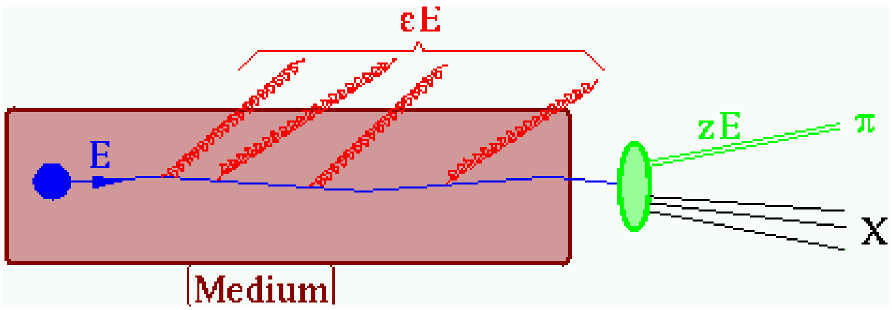
  Thermalization time scale: $\tau_{therm} = L_{max} = \sqrt{\frac{4\,E}{\alpha_s C_R\, q}}$

  $(\Delta E \sim E_q)$

$E_q = 10\ GeV$	(RHIC)	$\tau_{hadr} > 2\ fm$	$\tau_{therm} \sim 4.5\ fm$	Stuck in medium
$E_q = 100\ GeV$	(LHC)	$\tau_{hadr} > 20\ fm$	$\tau_{therm} \sim 13.5\ fm$	Medium-modified
$E_q = 10^{10}\ GeV$		$\tau_{hadr} > 2\ \mu m$	$\tau_{therm} \sim 10^5\ fm$	

The Nuclear Modification Factor

- Reduced parton energy implies reduced leading hadron pt

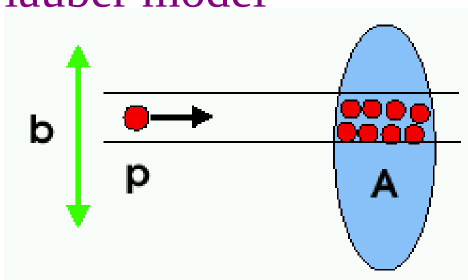


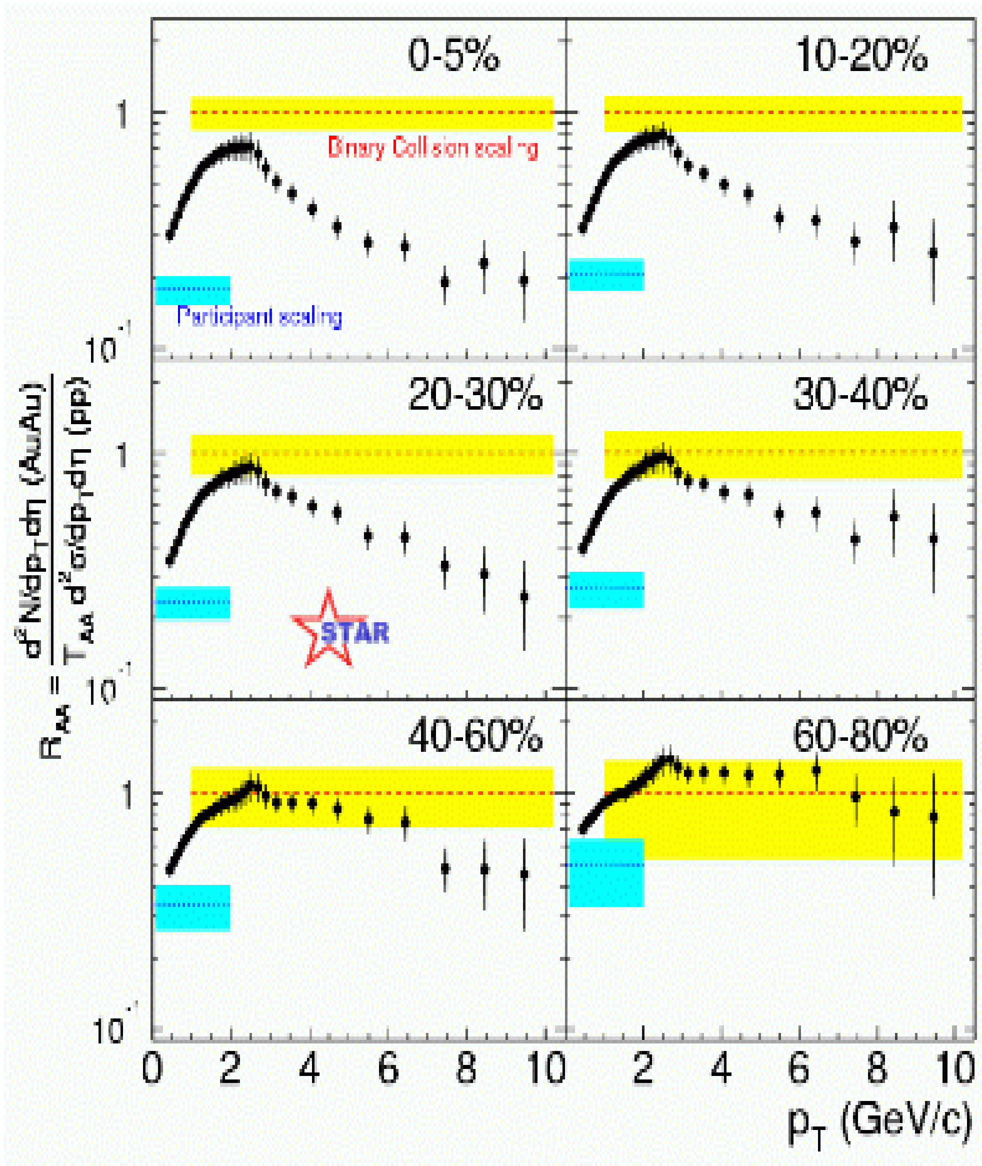
- The nuclear modification factor

$$R_{AA}(p_t,\eta)=\frac{dN^{AA}/dp_t\,d\eta}{n_{coll}\,dN^{NN}/dp_t\,d\eta}$$

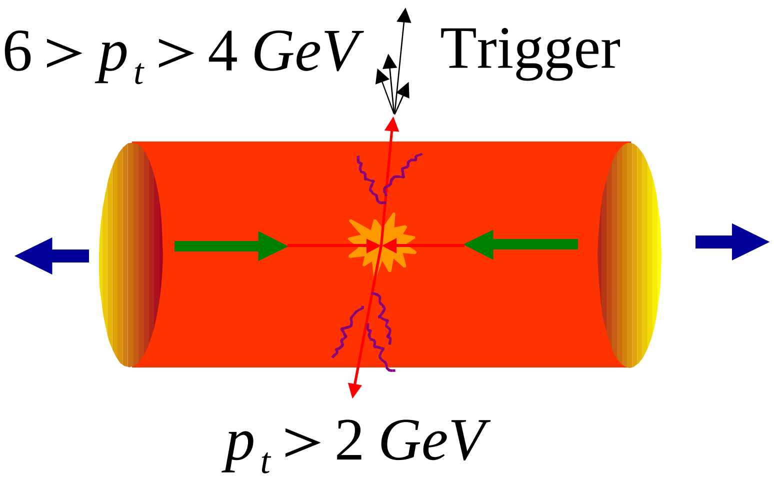
$$n_{coll} = \frac{T_{AA}(b)\,\sigma_{NN}}{\sigma_{AA}(b)}$$

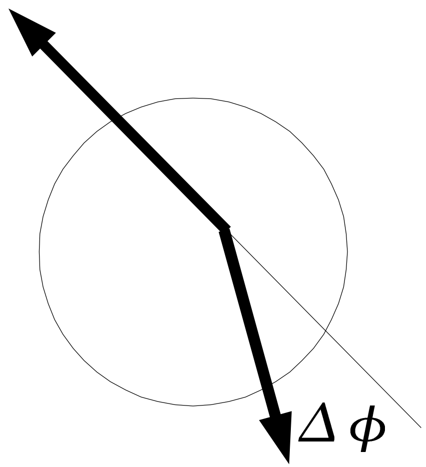
Glauber model

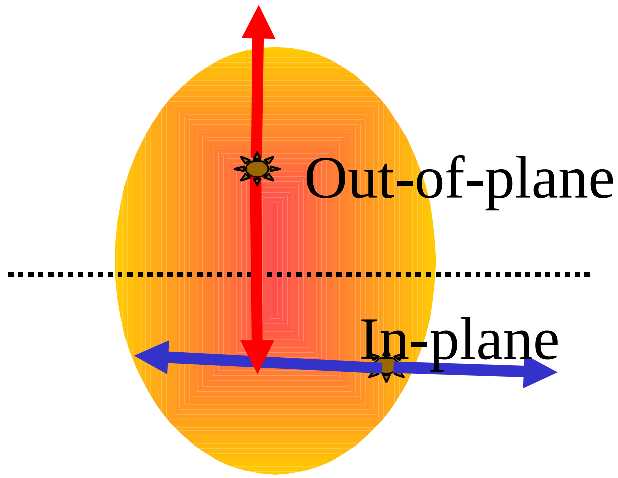


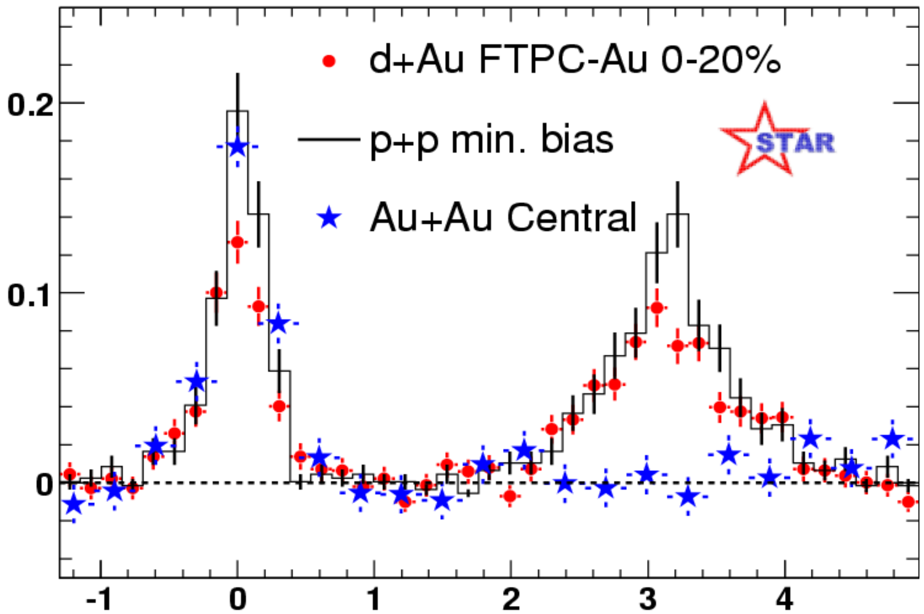


Back-to-Back Correlations: p+p vs. d+Au vs Au-Au

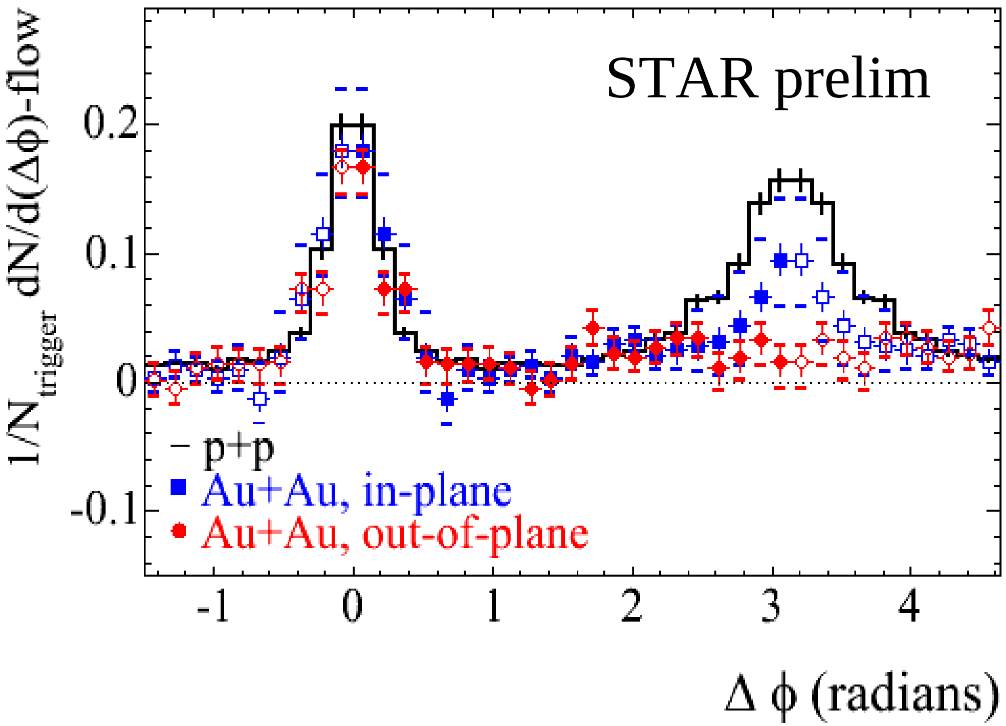






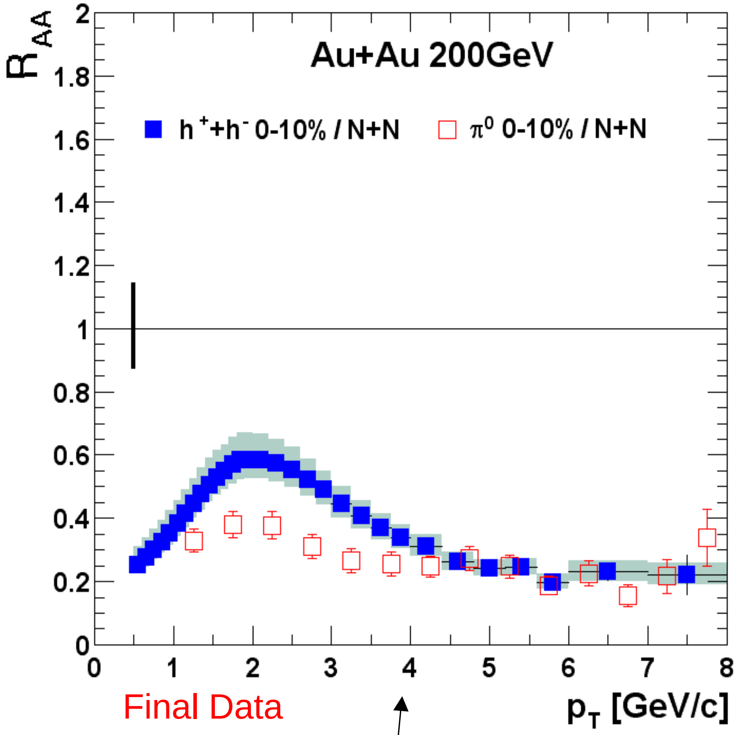


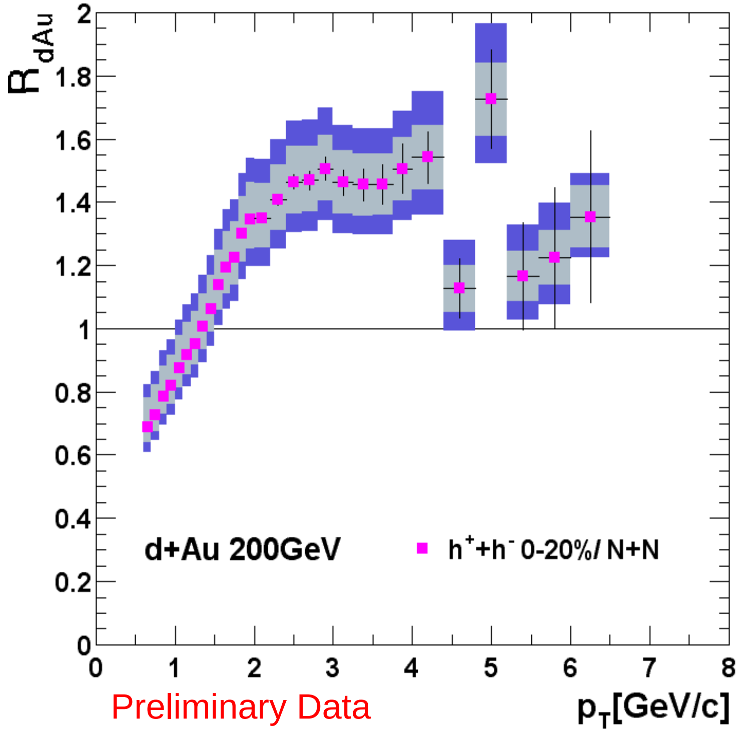
Adams et al. PRL 91 (2003)
Adler et al. PRL 90:082302 (2003)

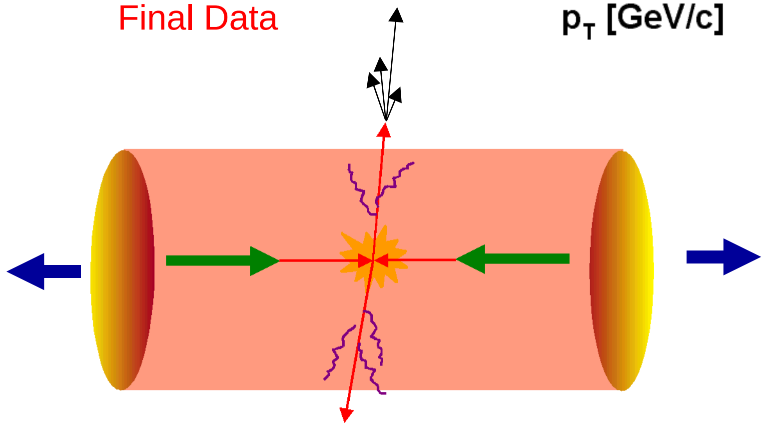


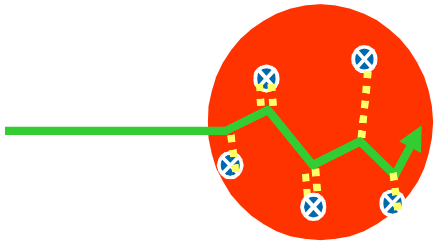
Centrality Dependence: Au+Au vs. d+Au

- Final state suppression
- Initial state enhancement



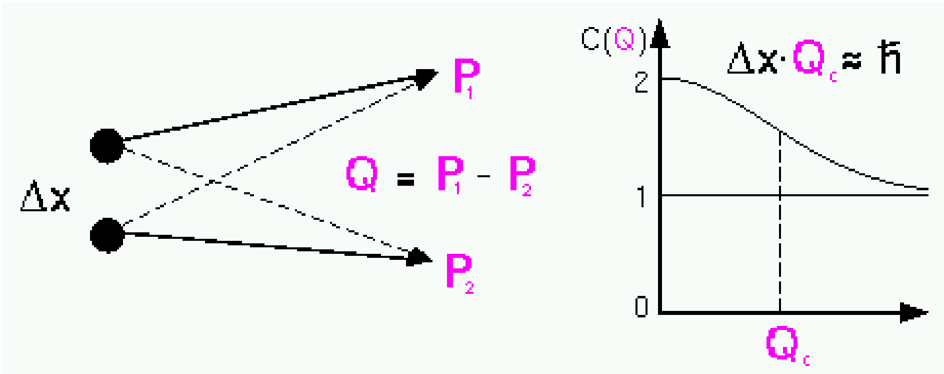




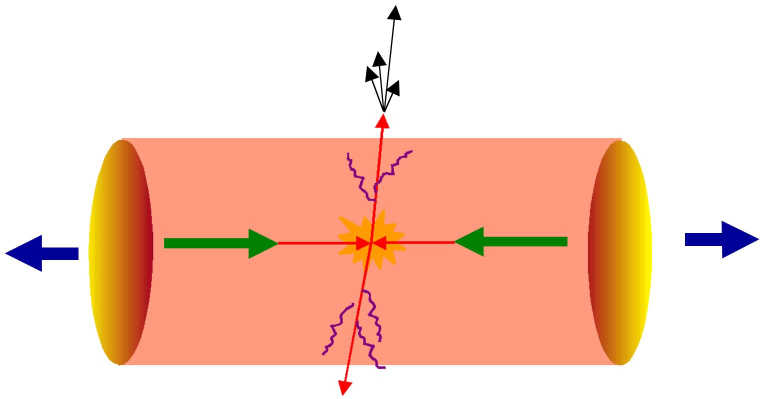


Summary of Lecture 3

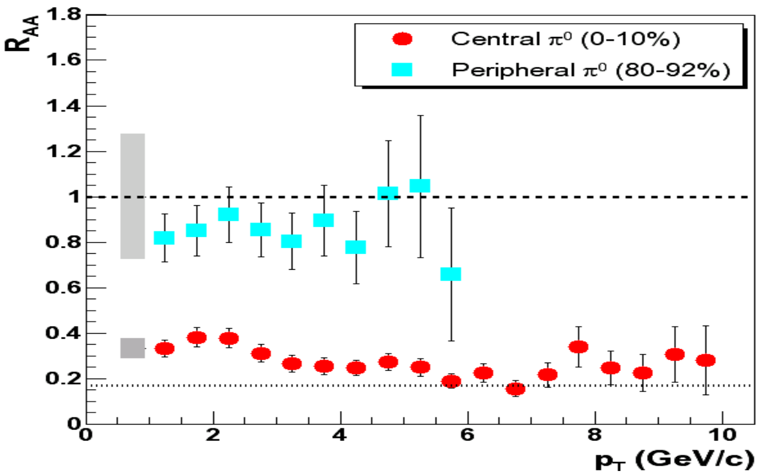
- Identical two-particle Correlations
  measure the space-time extension
  of collision region at freeze-out



- High-Q^2 processes in dense matter
  Parton propagation in matter results in
  - pt-broadening in initial and final state
  - energy loss of leading parent parton



- Observable Consequences of "jet quenching"
  - suppressed leading hadron spectra
  - exp. test that this suppression is a final state effect
  - dependence on in-medium pathlength/centrality



- More consequences of "jet quenching"

→ Lecture 4