Frammentazione del quark bottom nel decadimento del top

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1. **Decadimento del top a NLO**
2. **Funzioni di frammentazione perturbative**
3. **Risommazione collineare e soffice**
4. **Spettro di energia del quark b e di adroni B**
5. **Conclusioni**

G.C. and A.D. Mitov, NPB 623 (2002) 247

M. Cacciari, G.C. and A.D. Mitov, JHEP 0212 (2002) 015

Factorization:

$$\sigma = f \ \otimes \ \hat{\sigma} \ \otimes \ D \ + \ \mathcal{O}\left(\left(\frac{1}{Q}\right)^p\right)$$

σ: **hadron-level cross section**

f: **parton distribution functions**

$\hat{\sigma}$: **parton-level cross section (perturbative QCD)**

D: **fragmentation functions**

$$(f \otimes g)(x) = \int_x^1 \frac{dz}{z} f(z) g\left(\frac{x}{z}\right)$$

Top decay:

$$t(q) \to b(p_b) W(p_W) X(p_X) \quad ; \quad b(p_b) \to B(p_B)$$

$$x_E = \frac{2p_b \cdot q}{m_t^2} = \frac{2E_b}{m_t} \quad ; \quad x_b = \frac{x_E}{x_{E,\mathrm{max}}} = \frac{x_E}{1 + \frac{m_b^2}{m_t^2} - \frac{m_W^2}{m_t^2}}$$

$$x_B = \frac{1}{x_{E\mathrm{max}}} \frac{2p_B \cdot q}{m_t^2} = \frac{E_B}{E_b} x_b$$

$$\frac{1}{\Gamma_0} \frac{d\Gamma_{\mathrm{had}}}{dx_B}(t \to B) = \frac{1}{\Gamma_0} \int_{x_B}^1 \frac{dz}{z} \frac{d\Gamma_{\mathrm{part}}}{dz}(t \to b) D_{np}^B\left(\frac{x_B}{z}\right)$$

$\Gamma_0 = \Gamma(t \to bW)$

Computation of $d\Gamma_{\mathrm{part}}/dz$ in NLO QCD

Top decay at $\mathcal{O}(\alpha_S)$ (NLO)

$$t(q) \to b(p_b)W(p_W)\,(g(p_g)) \quad m_b \neq 0 \quad w = \frac{m_W^2}{m_t^2}$$

Virtual diagrams: ultraviolet singularities $(|p_g| \to \infty)$

Virtual and real diagrams: soft singularities $(|p_g| \to 0)$

Dimensional regularization: $d \to 4 - 2\epsilon \;\; g_s \to g_s \mu_r^\epsilon$

On-shell mass-renormalization scheme
(m_t**,** m_b **pole masses)**

In $d\Gamma/dx_b$ **soft singularities cancel after adding real and virtual diagrams**

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dx_b} = \delta(1-x_b) + \frac{\alpha_S(\mu)}{2\pi} A_1(x_b)$$

$$\begin{aligned}
A_1(x_b) &= C_F \left\{ \left[\frac{1+x_b^2}{(1-x_b)_+} + \frac{3}{2}\delta(1-x_b) \right] \ln\frac{m_t^2}{m_b^2} \right. \\
&+ 2\frac{1+x_b^2}{(1-x_b)_+} \ln[(1-w)x_b] - \frac{4x_b}{(1-x_b)_+} + \frac{4w(1-w)}{1+2w}\frac{x_b(1-x_b)}{1-(1-w)x_b} \\
&+ \delta(1-x_b) \left[4\mathrm{Li}_2(1-w) + 2\ln w \ln(1-w) - \frac{2\pi^2}{3} \right. \\
&- \left.\left. \frac{2(1-w)}{1+2w}\ln(1-w) - \frac{2w}{1-w}\ln w - 4 \right] \right\}
\end{aligned}$$

$$\alpha_S(\mu) = \frac{1}{b_0 \ln(\mu^2/\Lambda^2_{\overline{\mathrm{MS}}})} \left\{ 1 - \frac{b_1 \ln\left[\ln(\mu^2/\Lambda^2_{\overline{\mathrm{MS}}})\right]}{b_0^2 \ln(\mu^2/\Lambda^2_{\overline{\mathrm{MS}}})} \right\}$$

$$\int_{x_{b,\mathrm{min}}}^1 dx_b \frac{f(x_b)}{(1-x_b)_+} = \int_{x_{b,\mathrm{min}}}^1 dx_b \frac{f(x_b)-f(1)}{1-x_b}$$

Altarelli–Parisi splitting function:

$$P_{qq}(x_b) = C_F \left[\frac{1+x_b^2}{(1-x_b)_+} + \frac{3}{2}\delta(1-x_b) \right] = C_F \left(\frac{1+x_b^2}{1-x_b} \right)_+$$

$$\frac{m_t^2}{m_b^2} \simeq \mathcal{O}(10^3) \qquad\qquad \alpha_S(\mu)\ln\frac{m_t^2}{m_b^2} \simeq \mathcal{O}(1)$$

$x_b \to 1$**: soft-gluon radiation**

$$\begin{aligned}
\Gamma &= \Gamma_0 \left\{ 1 + \frac{C_F \alpha_S(\mu)}{2\pi} \left[\frac{2w(1-2w)(1+w)}{(1-w)^2(1+2w)} - \frac{5+4w}{1+2w}\ln(1-w) \right. \right. \\
&+ \left.\left. 2\mathrm{Li}_2(1-w) - 2\mathrm{Li}_2(w) - \frac{5+9w-6w^2}{2(1-w)(1+2w)} - \pi^2 \right] \right\}
\end{aligned}$$

Perturbative fragmentation functions

B. Mele and P. Nason, NPB 361 (1991) 626

Heavy quarks produced at $E \gg m$, afterwards they slow down and fragment into massive objects

$$\begin{aligned}\frac{1}{\Gamma_0}\frac{d\Gamma}{dx_b}(x_b, m_b \neq 0) &= \frac{1}{\Gamma_0}\sum_i \int_{x_b}^1 \frac{dz}{z}\frac{d\hat{\Gamma}_i}{dz}(z, m_i = 0, \mu_F) D_i\left(\frac{x_b}{z}, \mu_F, m_b\right) \\ &+ \mathcal{O}\left[\left(\frac{m_b}{m_t}\right)^p\right]\end{aligned}$$

$d\Gamma/dx_b$ **: production rate of a massive b, neglecting powers of** m_b/m_t

$d\hat{\Gamma}_i/dz$ **: production rate of a massless parton i**

$D_i(x_b, \mu_F, m_b)$ **: perturbative fragmentation function for the transition** $i \to b$

μ_F **:factorization scale**

$B(t \to bW) = 1 \Rightarrow i = b$

$\Gamma_b(m_b) = \Gamma_b(m = 0) \otimes D_p(m_b) \quad \Gamma_B = \Gamma_b(m = 0) \otimes D_p(m_b) \otimes D_{np}$

In dimensional regularization:

$$\frac{1}{\Gamma_0}\frac{d\hat{\Gamma}_b}{dz} = \delta(1-z) + \frac{\alpha_S(\mu)}{2\pi}\left[P_{qq}(z)\left(-\frac{1}{\epsilon}+\gamma_E - \ln 4\pi + \ln\frac{m_t^2}{\mu_F^2}\right) + \hat{A}_1(z)\right]$$

$\overline{\mathrm{MS}}$ coefficient function and PFF:

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dx_b}(m_b) = \left(\frac{1}{\Gamma_0}\frac{d\hat{\Gamma}_b}{dx_b}(m_b=0)\right)^{\overline{\mathrm{MS}}} \otimes D_b^{\overline{\mathrm{MS}}}(m_b)$$

Initial condition $D(x_b,\mu_{0F})$ is process independent (Mele–Nason, M. Cacciari and S. Catani, NPB 617 (2001) 167)

$$D_b(x_b,\mu_{0F},m_b) = \delta(1-x_b) + \frac{\alpha_S(\mu_0)C_F}{2\pi}\left[\frac{1+x_b^2}{1-x_b}\left(\ln\frac{\mu_{0F}^2}{m_b^2} - 2\ln(1-x_b) - 1\right)\right]_+$$

DGLAP equations for PFF's:

$$\frac{d}{d\ln\mu_F^2}D_i(x_b,\mu_F,m_b) = \sum_j \int_{x_b}^1 \frac{dz}{z}P_{ij}\left(\frac{x_b}{z},\alpha_S(\mu_F)\right)D_j(z,\mu_F,m_b)$$

Mellin-space transform:

$$D_N(\mu_F,m_b) = \int_0^1 dx\; x_b^{N-1}D(x_b,\mu_F,m_b)$$

$$\frac{dD_N(\mu_F,m_b)}{d\ln\mu_F^2} = \frac{\alpha_S(\mu_F)}{2\pi}\left[P_N^{(0)} + \frac{\alpha_S(\mu_F)}{2\pi}P_N^{(1)}\right]D_N(\mu_F,m_b)$$

$$\begin{aligned} D_N(\mu_F,m_b) &= D_N(\mu_{0F},m_b)\exp\left\{C_{1,0}\alpha_S(\mu_F) + C_{1,1}\alpha_S(\mu_F)\ln(\mu_F^2/\mu_{0F}^2)\ldots\right. \\ &+ \left. C_{n,n-1}\alpha_S^n(\mu_F)\ln^{n-1}(\mu_F^2/\mu_{0F}^2) + C_{n,n}\alpha_S^n(\mu_F)\ln^n(\mu_F^2/\mu_{0F}^2) + \ldots\right\} \end{aligned}$$

Resummation of leading logarithms $\alpha_S^n\ln^n(\mu_F^2/\mu_{0F}^2)$
and next-to-leading logarithms $\alpha_S^n\ln^{n-1}(\mu_F^2/\mu_{0F}^2)$
$\mu_{0F}\simeq m_b$ **and** $\mu_F\simeq m_t$
Resummation of NLL $\ln(m_t^2/m_b^2)$ **(Collinear resummation)**

Soft-gluon radiation

Region $x_b \to 1$ corresponds to soft-gluon radiation in top decay

Coefficient function and initial condition of perturbative fragmentation function contain terms behaving like $1/(1-x_b)_+$ or $[\ln(1-x_b)/(1-x_b)]_+$ for $x_b \to 1$

In Mellin space:

$$\frac{1}{(1-x_b)_+} \to \ln N \qquad \left[\frac{1}{1-x_b}\ln(1-x_b)\right]_+ \to \ln^2 N$$

$$\begin{aligned}
\hat{\Gamma}_N \ \simeq\ & 1 + \frac{\alpha_S(\mu) C_F}{2\pi}\left\{2\ln^2 N + \left[4\gamma + 2 - 4\ln(1-w) - 2\ln\frac{m_t^2}{\mu_F^2}\right]\ln N\right. \\
+ & \left(\frac{3}{2} - 2\gamma\right)\ln\frac{m_t^2}{\mu_F^2} + 2\gamma^2 + 2\gamma\left[1 - 2\ln(1-w)\right] \\
+ & 2\ln w \ln(1-w) - 2\frac{1-w}{1+2w}\ln(1-w) - \frac{2w}{1-w}\ln w \\
+ & \left. 4\mathrm{Li}_2(1-w) - 6 - \frac{\pi^2}{3} + \mathcal{O}\left(\frac{1}{N}\right)\right\}
\end{aligned}$$

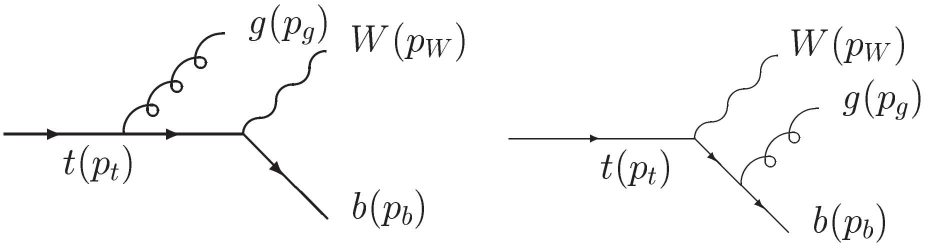
$$\begin{aligned}
D_N(\mu_0, \mu_{0F}) \ =\ & 1 + \frac{\alpha_S(\mu_0^2)}{\pi} C_F \left[-\ln^2 N + \left(\ln\frac{m^2}{\mu_{0F}^2} - 2\gamma_E + 1\right)\ln N\right. \\
+ & \left. 1 - \frac{\pi^2}{6} + \gamma_E - \gamma_E^2 + \left(\gamma_E - \frac{3}{4}\right)\ln\frac{m^2}{\mu_{0F}^2} + \mathcal{O}\left(\frac{1}{N}\right)\right]
\end{aligned}$$

Soft resummation in PFF initial condition is process independent (Cacciari–Catani to NLL)

Soft resummation in coefficient function is process dependent

Soft resummation of the coefficient function

Eikonal matrix element for $t(p_t) \to b(p_b) W(p_W) g(p_g)$



$$\mathcal{M}^{a,\mu} = \mathcal{M}_0^{\mu}\; g_s T^a \epsilon_\nu \left(\frac{p_t^\nu}{p_t \cdot p_g} - \frac{p_b^\nu}{p_b \cdot p_g} \right)$$

$$q^2 = (p_b + p_g)^2 (1-z) = 2E_g^2 (1 - \cos\theta_{bg}) \simeq E_g^2 \sin^2\theta_{bg}$$

$$\hat{\Gamma}_N = \frac{C_F}{\pi} \int_0^1 dz \frac{z^{N-1} - 1}{1-z} \int_0^{m_t^2(1-w)^2(1-z)^2} dq^2 \left[\frac{\alpha_S}{q^2} - \frac{\alpha_S}{m_t^2 (1-w)^2 (1-z)^2} \right]$$

$$\begin{aligned} \Delta_N \;=\; & \exp\Big\{ \int_0^1 dz \frac{z^{N-1}-1}{1-z} \Big[\int_{\mu_F^2}^{m_t^2(1-w)^2(1-z)^2} \frac{dq^2}{q^2} A\left[\alpha_S(q^2)\right] \\ & + \; S\left[\alpha_S\left(m_t^2(1-w)^2(1-z)^2\right)\right] \Big] \Big\} = \exp\left[\ln N g_1 + g_2\right] \end{aligned}$$

$$A(\alpha_S) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n A^{(n)} \quad ; \quad S(\alpha_S) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n S^{(n)}$$

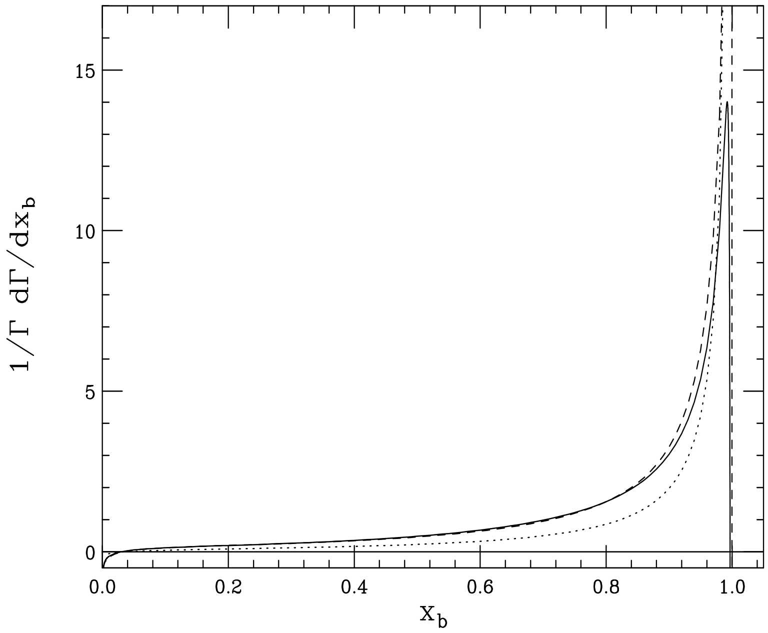
$g_1 \ln N$ **resums LL** $A^{(1)}$ **:** $\alpha_S \ln^2 N, \alpha_S^2 \ln^4 N \ldots \alpha_S^n \ln^{n+1} N;$

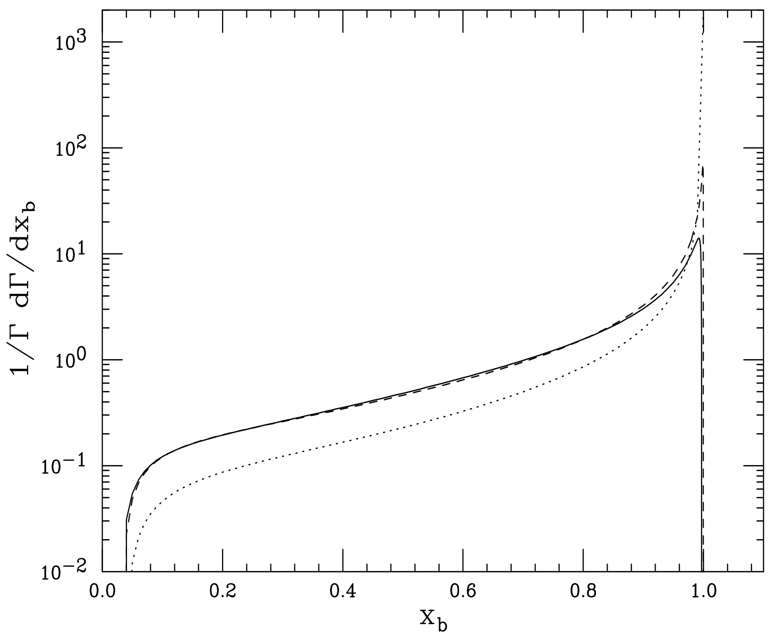
g_2 **resums NLL** $A^{(2)}, S^{(1)}$ **:** $\alpha_S \ln N, \alpha_S^2 \ln^2 N \ldots \alpha_S^n \ln^n N$

b-quark spectrum in top decay

$m_t = 175$ GeV $m_b = 5$ GeV $m_W = 80$ GeV $\Lambda_{\overline{\rm MS}} = 200$ MeV

$\mu = \mu_F = m_t$ $\mu_0 = \mu_{0F} = m_b$

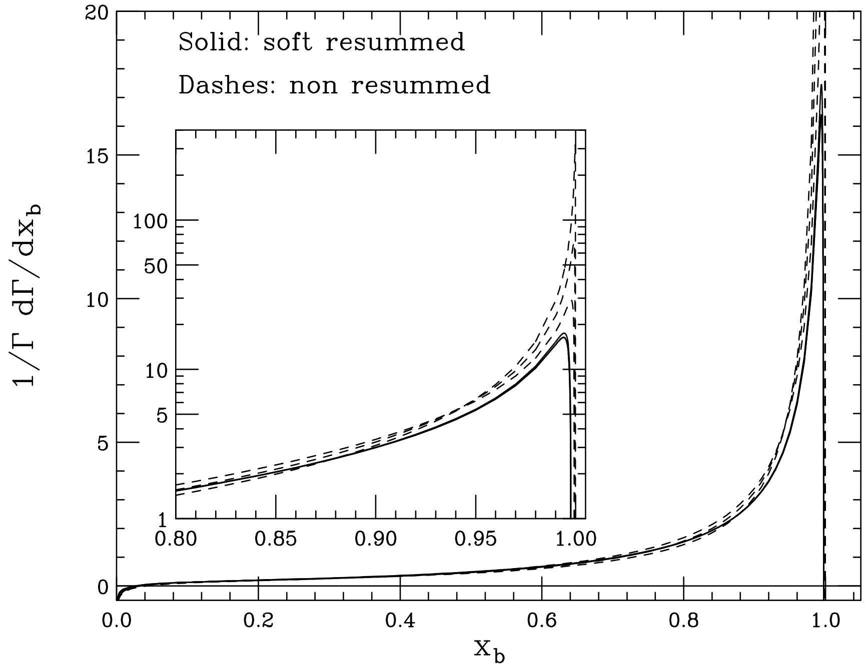




Solid: soft and collinear resummation Dashes: only collinear resummation

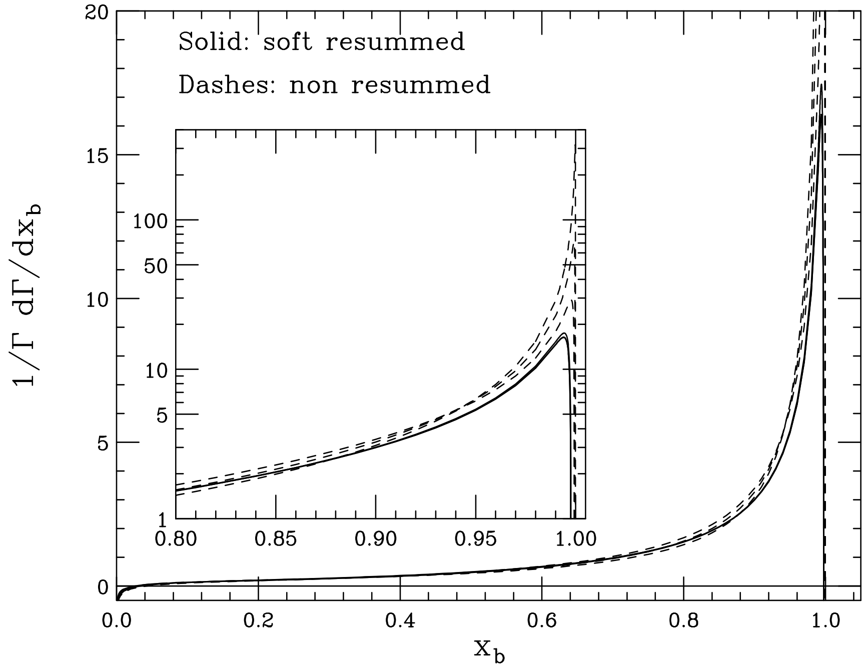
Dots: massive NLO without resummation

Dependence on the factorization scale μ_F



$m_t/2 < \mu_F < 2m_t$

Dependence on the factorization scale μ_{0F}



$m_b/2 < \mu_{0F} < 2m_b$

Hadron-level results

$$\frac{d\Gamma_{\rm had}}{dx_B}(t \to B) = \frac{d\Gamma_{\rm part}}{dx_b}(t \to b) \otimes D_{np}(b \to B)$$

$d\Gamma_{\rm part}(t \to b)/dx_b$ **according to PFF approach**

Non-perturbative fragmentation functions:

Power law with two tunable parameters:

$$D_{np}(x;\alpha,\beta) = \frac{1}{B(\beta+1,\alpha+1)}(1-x)^\alpha x^\beta$$

Model of Kartvelishvili et al.:

$$D_{np}(x;\delta) = (1+\delta)(2+\delta)(1-x)x^\delta$$

Model of Peterson et al.:

$$D_{np}(x;\epsilon) = \frac{A}{x[1-1/x-\epsilon/(1-x)]^2}$$

Parameters α, β, δ, ϵ **from fits to** $e^+e^- \to b\bar{b}$ **data**

$$\frac{d\sigma_{\rm had}}{dx_B}(e^+e^- \to B) = \frac{d\sigma_{\rm part}}{dx_b}(e^+e^- \to b\bar{b}) \otimes D_{np}(b \to B)$$

$d\sigma_{\rm part}/dx_b$ **must be computed within the same framework as** $d\Gamma_{\rm part}/dx_b$

Results of fits of fragmentation models to e^+e^- data

Preliminary: correlations between data points are neglected

ALEPH Collaboration, A. Heister et al., PLB 512 (2001) 30: only B mesons

SLD Collaboration, K. Abe et al., PRL 84 (2000) 4300: both b-flavoured mesons and baryons

$0.18 \lesssim x_B \lesssim 0.94$

With NLL soft resummation :

	ALEPH	SLD
α	0.51 ± 0.15	2.04 ± 0.38
β	13.35 ± 1.46	25.18 ± 3.27
$\chi^2(\alpha,\beta)$/dof	$2.56/14$	$11.50/16$
δ	17.76 ± 0.62	16.59 ± 0.49
$\chi^2(\delta)$/dof	$10.54/15$	$22.19/17$
ϵ	$(1.77 \pm 0.16) \times 10^{-3}$	$(1.61 \pm 0.14) \times 10^{-3}$
$\chi^2(\epsilon)$/dof	$29.83/15$	$158.15/17$

Without soft resummation :

	ALEPH	SLD
α	0.66 ± 0.13	2.05 ± 0.28
β	12.39 ± 1.04	22.10 ± 2.13
$\chi^2(\alpha,\beta)$/dof	$7.12/14$	$40.23/16$
δ	14.97 ± 0.44	14.57 ± 0.37
$\chi^2(\delta)$/dof	$13.30/15$	$58.63/17$
ϵ	$(2.87 \pm 0.21) \times 10^{-3}$	$(2.33 \pm 0.19) \times 10^{-3}$
$\chi^2(\epsilon)$/dof	$52.76/15$	$275.69/17$

Power law: excellent fits but large errors

Kartvelishvili model: good fits

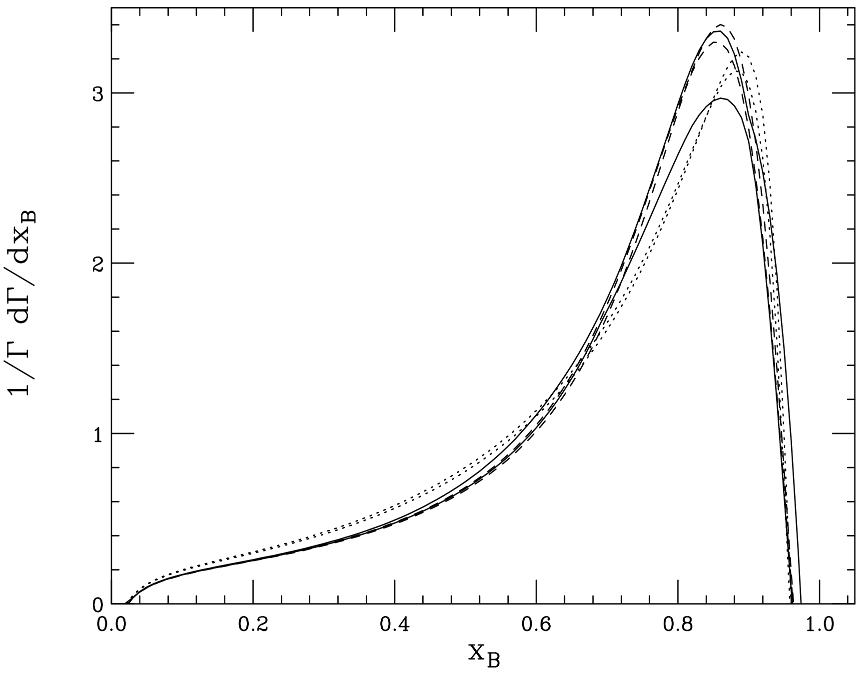
Peterson model marginally consistent only with ALEPH data

SLD and ALEPH α and β agree within 3σ, δ and ϵ within 2σ

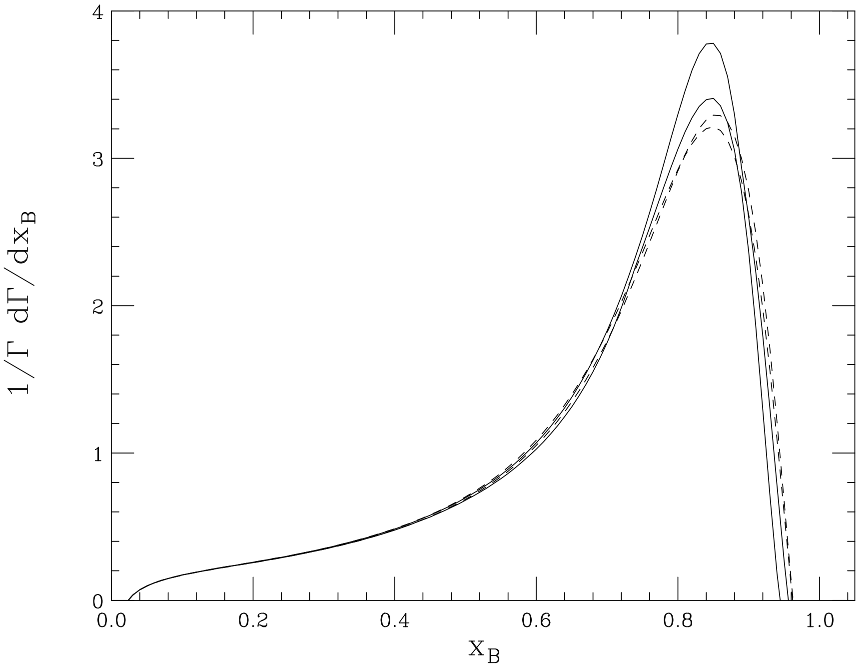
With no soft resummation, power laws still fit ALEPH data

b-flavoured hadron spectrum in top decay

ALEPH fit Solid: power law; Dashes: Kartvelishvili; Dots: Peterson

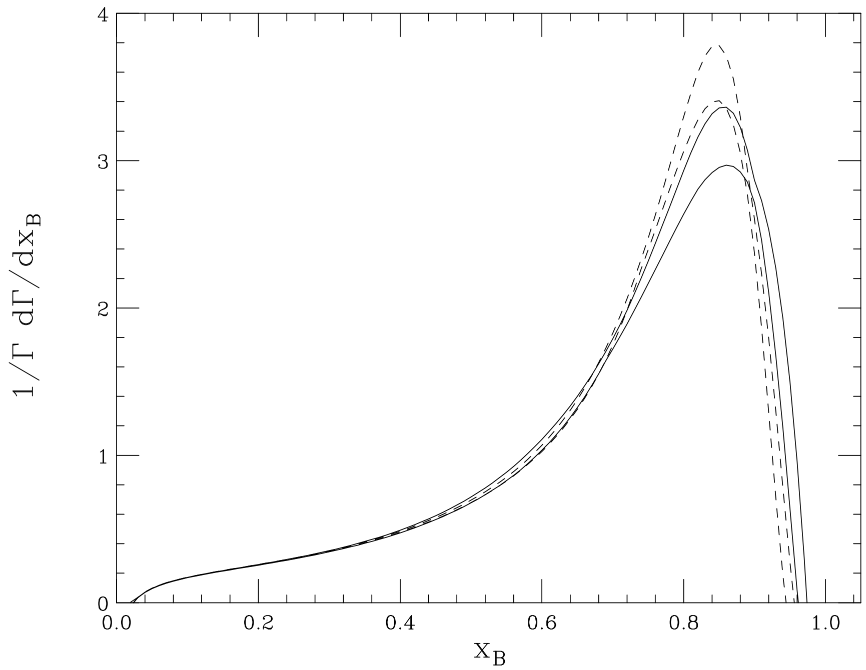


SLD Solid: power law; Dashes: Kartvelishvili et al.



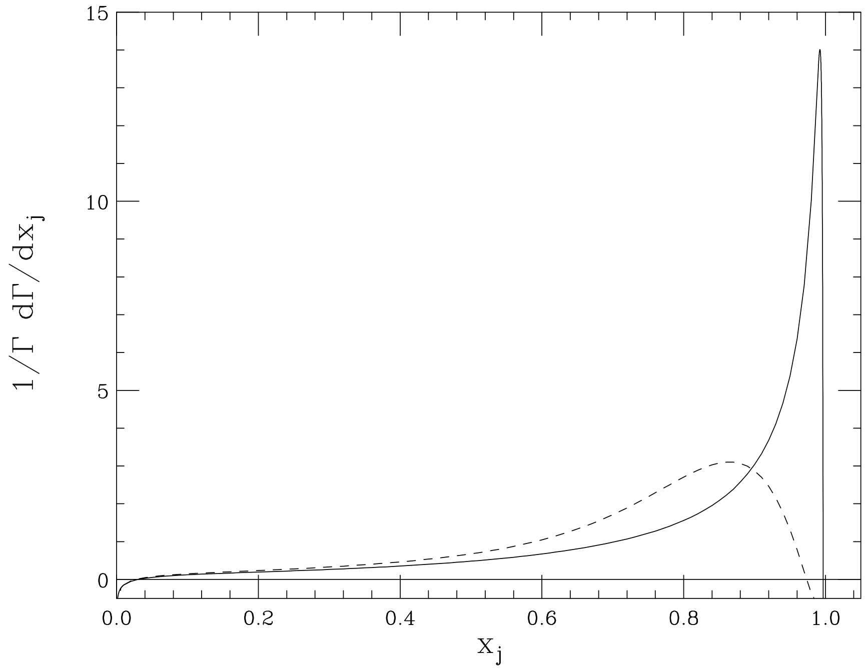
Comparison of ALEPH (solid) and SLD (dashes) fits

Power law; soft resummation included



Comparison of hadron- and parton-level results

Solid: parton level; dashes: hadron level, ALEPH fit



$j = b$: **solid** $j = B$: **dashes**

Fits in moment space

e^+e^- **annihilation** $\sigma_N^B = \sigma_N^b D_N^{np}$

σ_N^B **measured ;** σ_N^b **calculated ;** D_N^{np} **fitted**

top decay: $\Gamma_N^B = \Gamma_N^b D_N^{np} = \Gamma_N^b \sigma_N^B / \sigma_N^b$

Fits to DELPHI data (ICHEP 2002 Note, DELPHI 2002-069 CONF 603)

	$\langle x \rangle$	$\langle x^2 \rangle$	$\langle x^3 \rangle$	$\langle x^4 \rangle$
e^+e^- data σ_N^B	0.7153±0.0052	0.5401±0.0064	0.4236±0.0065	0.3406±0.0064
e^+e^- NLL σ_N^b [A]	0.7666	0.6239	0.5246	0.4502
e^+e^- NLL σ_N^b [B]	0.7801	0.6436	0.5479	0.4755
D_N^{np} [A]	0.9331	0.8657	0.8075	0.7566
D_N^{np} [B]	0.9169	0.8392	0.7731	0.7163
t-decay NLL Γ_N^b [A]	0.7750	0.6417	0.5498	0.4807
t-decay NLL Γ_N^b [B]	0.7884	0.6617	0.5737	0.5072
t-decay Γ_N^B [A]	0.7231	0.5555	0.4440	0.3637
t-decay Γ_N^B [B]	0.7228	0.5553	0.4435	0.3633

$[A]$: $\Lambda_{\overline{\rm MS}} = 0.226$ **GeV**, $m_b = 4.75$ **GeV**

$[B]$: $\Lambda_{\overline{\rm MS}} = 0.2$ **GeV**, $m_b = 5$ **GeV**

Conclusions

Hadron spectrum as factorization of perturbative and non-perturbative contributions

b-quark spectrum in top decay presents large mass and soft logarithms

PFF's and DGLAP evolution to resum NLL $\log(m_t^2/m_b^2)$

NLL soft-gluon resummation in the coefficient function and in the PFF initial condition

Big impact of collinear and soft resummation on b-energy spectrum

Very small dependence on factorization and renormalization scales

Fits of hadronization models to ALEPH and SLD data in x_b space and to DELPHI in N space

b-flavoured hadron spectrum in x_b and moment space

In progress:

Application to top physics at Tevatron and LHC

Comparison of PFF approach with Monte Carlo event generators (HERWIG):

MC generators are LO+LL parton showers (only some NLL) but exclusive final states

PFF formalism is NLO+NLL but quite inclusive (x_B), though it may be extended to study other observables

Fits of Monte Carlo hadronization parameters to the data