# Determinazione accurata delle masse dei bosoni di Higgs supersimmetrici

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# The Minimal Supersymmetric Standard Model

- Superfield Content: $\left\{ \begin{array}{ll} G^a\,,\ W^a\,,\ B & \text{(vector)} \\ L\,,\ Q\,,\ E^c\,,\ U^c\,,\ D^c\,,\ H_1\,,\ H_2 & \text{(chiral)} \end{array} \right.$

- MSSM superpotential [ SU(3)×SU(2)×U(1) + R-parity ]

$$W = \mu H_1 H_2 + h^E H_1 L E^c + h^D H_1 Q D^c + h^U H_2 Q U^c$$

- Supersymmetry must be broken without introducing quadratic divergences $\longrightarrow$ "soft" SUSY-breaking terms:

$$\begin{aligned} -\mathcal{L}_{\rm soft} &= \frac{1}{2} \sum_A M_A \overline{\lambda}_A \lambda_A + \sum_i m_i^2 \, |\phi_i|^2 \\ &+ \; B H_1 H_2 + h^E A^E H_1 L E^c + h^D A^D H_1 Q D^c + h^U A^U H_2 Q U^c \end{aligned}$$

- $m_i^2\,,\ A^E\,,\ A^U\,,\ A^D$ are matrices in generation space

  $\longrightarrow$ the MSSM contains 105 new parameters !!!

- The MSSM phenomenology becomes extremely complex unless we adopt some simplifying assumptions.

# The Higgs sector of the MSSM at tree-level

- Two $SU(2) \times U(1)$ doublets: $H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix}$ , $H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}$

$$H_i^0 = \frac{v_i + S_i + i\,P_i}{\sqrt{2}} \qquad\qquad \tan\beta = \frac{v_2}{v_1}$$

- The soft SUSY-breaking mass terms for $H_1^0$ and $H_2^0$ are responsible for electroweak symmetry breaking (EWSB):

$$\begin{aligned} V_{\rm tree} &= (m_{H_1}^2 + \mu^2)\,|H_1^0|^2 + (m_{H_2}^2 + \mu^2)\,|H_2^0|^2 \\ &+ B\,\left(H_1^0 H_2^2 + \text{h.c.}\right) + \frac{1}{8}\,(g^2 + g'^2)\,\left(|H_1^0|^2 - |H_2^0|^2\right)^2 \end{aligned}$$

- Five physical states: $h\,,\ H\,,\ A^0\,,\ H^+\,,\ H^-$

- Tree–level mass matrix for the CP–even sector:

$$\left(\mathcal{M}_S^2\right)^{\rm tree} = \begin{pmatrix} m_Z^2\,c_\beta^2 + m_A^2\,s_\beta^2 & -\left(m_Z^2 + m_A^2\right) s_\beta\,c_\beta \\ -\left(m_Z^2 + m_A^2\right) s_\beta\,c_\beta & m_Z^2\,s_\beta^2 + m_A^2\,c_\beta^2 \end{pmatrix}$$

$\rightarrow$ $m_h$ and $m_H$ are predicted in terms of $m_Z$, $m_A$ and $\tan\beta$

- Tree–level mass relation: $m_h^2 \leq \cos^2 2\beta\, m_Z^2$ !!!

- Radiative corrections can push $m_h$ well above the tree–level bound (e.g. $m_h \leq 135$ GeV for typical parameter choices) and introduce a dependence on many MSSM parameters.

## High-energy boundary conditions and RG evolution

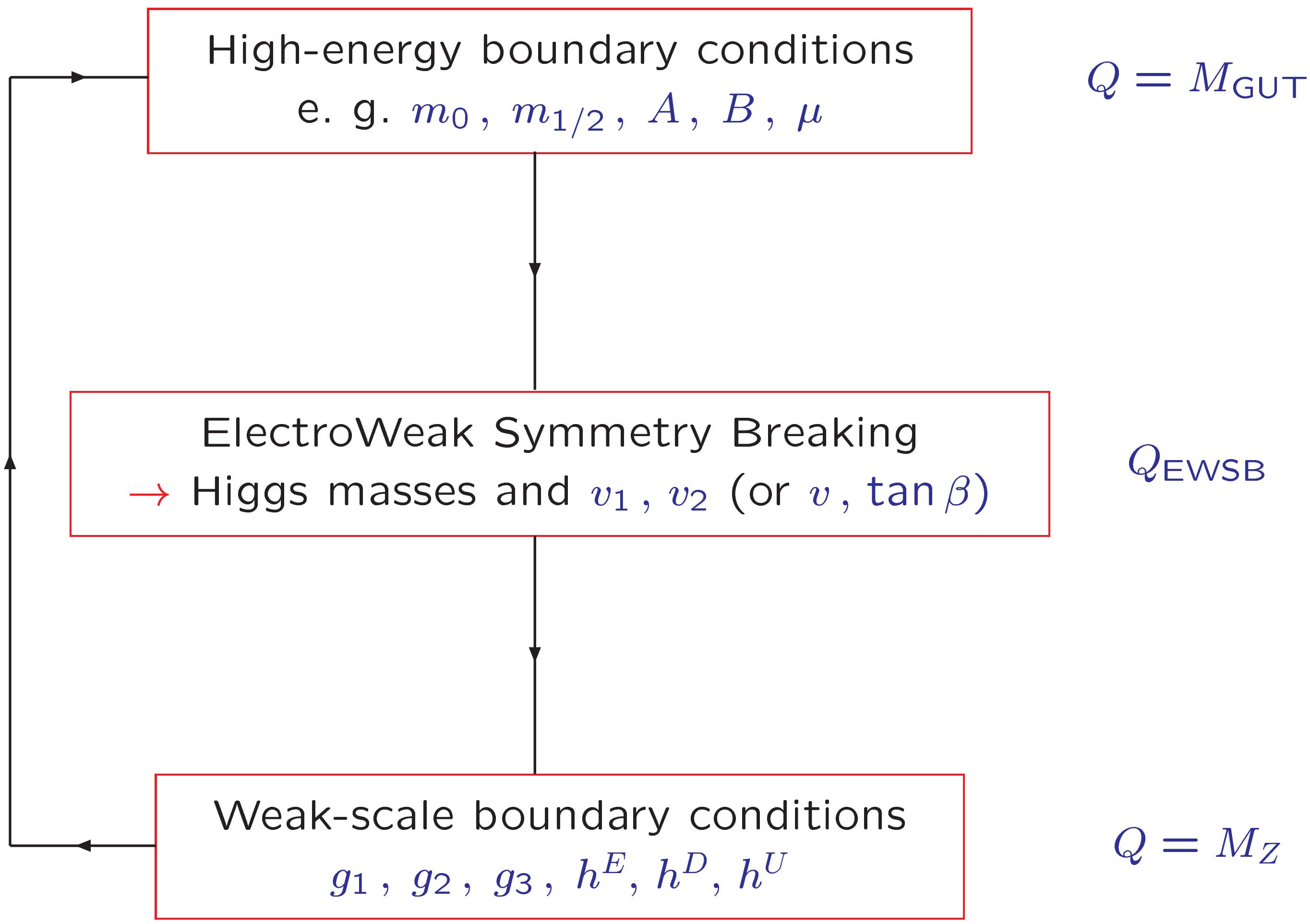
- Determining the MSSM mass spectrum becomes feasible if at some high-energy scale $Q = M_{\rm GUT}$ the structure of the soft SUSY-breaking terms is dictated by an underlying theory, e. g. :
  - mSUGRA: $m_0\,,\; m_{1/2}\,,\; A\,,\; B\,,\; \mu$
  - GMSB: $\Lambda\,,\; M_{\rm mess}\,,\; N_{\rm mess}\,,\; B\,,\; \mu$
  - AMSB: $m_{3/2}\,,\; m_0\,,\; B\,,\; \mu$

- The gauge couplings $g_i$ and the Yukawa couplings $h^E$, $h^D$, $h^U$ are given as input at $Q = M_Z$. They are computed from the known values of the fermion masses and of the SM parameters $G_F\,,\; M_Z\,,\; \alpha_{\rm em}\,,\; \alpha_s$.

- The $\overline{\rm DR}$-renormalized parameters of the MSSM at the weak scale are obtained by Renormalization Group (RG) evolution.

- The EWSB conditions are imposed at the weak scale and the masses and VEVs of the Higgs fields are computed (as well as the other MSSM particle masses).

## RG evolution between different energy scales

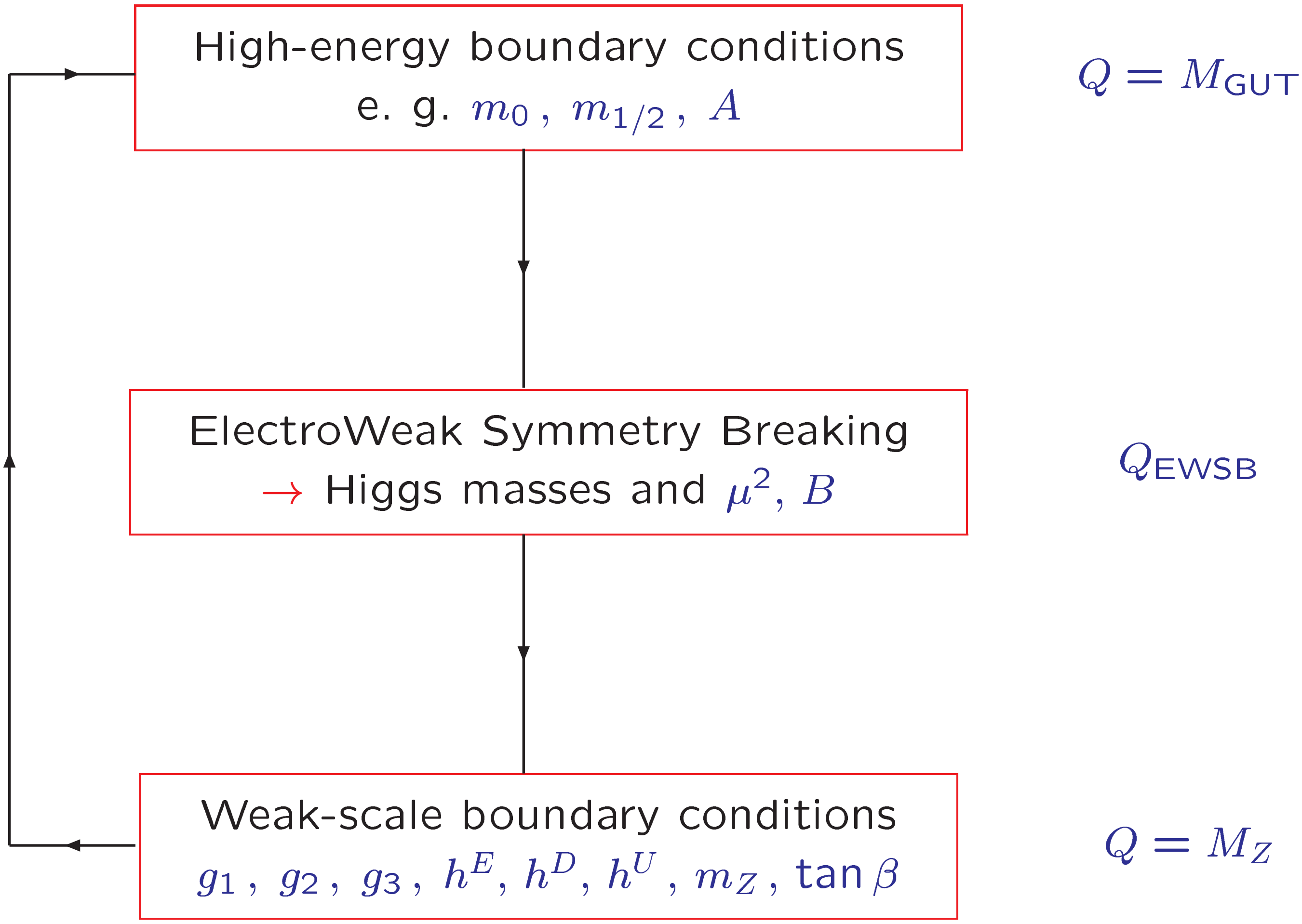



- The physical Higgs masses should not depend on $Q_{\mathrm{EWSB}}$, which can be anywhere between $M_Z$ and the TeV scale.

- A choice of high-energy boundary conditions is satisfactory if the resulting VEVs $v_1$ , $v_2$ reproduce the correct value of the running Z-boson mass:

$$m_Z^2 \;=\; M_Z^2 + \mathrm{Re}\,\Pi^T_{ZZ}(M_Z^2) \;=\; \frac{1}{4}\,(g^2 + g'^2)\,(v_1^2 + v_2^2)$$

# A phenomenological approach




- Assume successful EWSB and take $v_1$ and $v_2$ (or $m_Z$ and $\tan\beta$) as input parameters at $Q = M_Z$.

- The EWSB conditions now allow to determine $\mu^2$ and $B$.

- $\mathrm{sign}(\mu)$ is an extra (scale-independent) input parameter.

# EWSB conditions and Higgs boson masses

- The EWSB conditions allow to determine $\mu^2$ and $B$:

$$\mu^2 = -\frac{m_Z^2}{2} - \frac{1}{2}\tan 2\beta \left[ \left( m_{H_1}^2 - \frac{t_1}{v_1} \right) \cot\beta - \left( m_{H_2}^2 - \frac{t_2}{v_2} \right) \tan\beta \right]$$

$$B = -\frac{m_Z^2}{2}\sin 2\beta - \frac{1}{2}\tan 2\beta \left( m_{H_1}^2 - \frac{t_1}{v_1} - m_{H_2}^2 + \frac{t_2}{v_2} \right)$$

- The CP–odd and charged Higgs boson masses are:

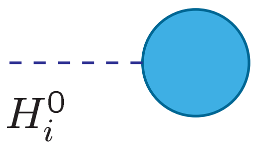
$$M_A^2 = 2\,B/\sin 2\beta - \mathrm{Re}\,\Pi_{AA}(M_A^2) + s_\beta^2 \frac{t_1}{v_1} + c_\beta^2 \frac{t_2}{v_2}$$

$$M_{H^\pm}^2 = m_A^2 + M_W^2 + \mathrm{Re}\left[ \Pi_{AA}(M_A^2) - \Pi_{H^+H^-}(M_{H^\pm}^2) + \Pi_{WW}^T(M_W^2) \right]$$

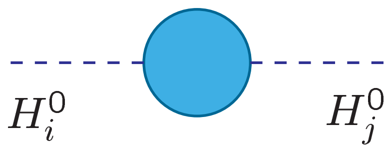
- The CP–even mass matrix is:

$$\mathcal{M}^2 = \begin{pmatrix} m_Z^2\, c_\beta^2 + m_A^2\, s_\beta^2 - \Pi_{11} + t_1/v_1 & -(m_Z^2 + m_A^2)\, s_\beta c_\beta - \Pi_{12} \\ -(m_Z^2 + m_A^2)\, s_\beta c_\beta - \Pi_{12} & m_Z^2\, s_\beta^2 + m_A^2\, c_\beta^2 - \Pi_{22} + t_2/v_2 \end{pmatrix}$$

$t_i \equiv$ tadpole :

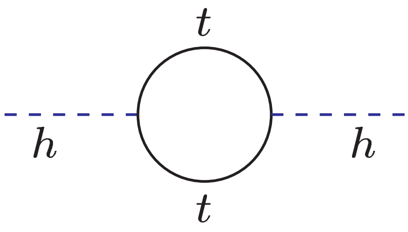



$\Pi_{ij} \equiv$ self – energy :

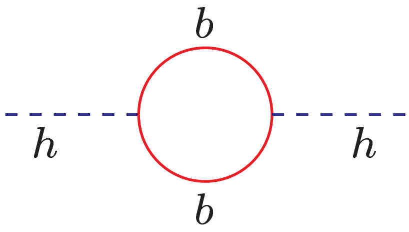


## Summary of the leading Higgs mass corrections

- Leading one-loop:

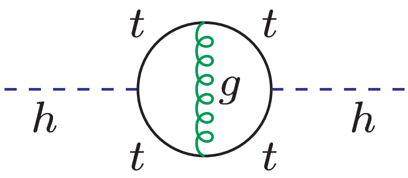



$\mathcal{O}(\alpha_t)$

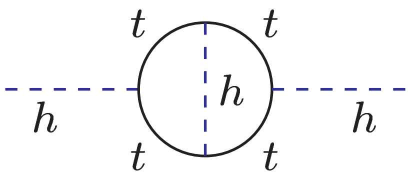



$\mathcal{O}(\alpha_b)$

- Leading two-loop (top):

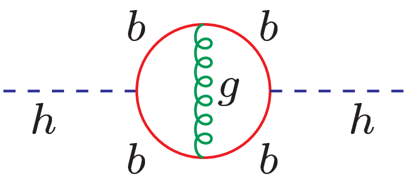



$\mathcal{O}(\alpha_t \alpha_s)$

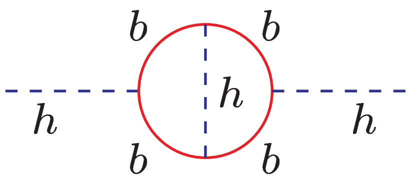



$\mathcal{O}(\alpha_t^2)$

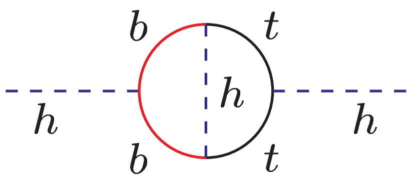
- Leading two-loop (bottom):




$\mathcal{O}(\alpha_b \alpha_s)$




$\mathcal{O}(\alpha_b^2)$




$\mathcal{O}(\alpha_t \alpha_b)$

# Programs computing the MSSM mass spectrum

- We present the new versions of three public programs for the computation of the MSSM mass spectrum:
  - *SuSpect* 2.3 (A.Djouadi, J.L.Kneur and G.Moultaka)
  - *SoftSusy* 1.8 (B.Allanach)
  - *SPheno* 2.2 (W.Porod)

- In the latest versions, all the codes include a two–loop computation of the Higgs masses and EWSB conditions performed in the $\overline{\mathrm{DR}}$ renormalization scheme.

- The full one–loop corrections are taken from Pierce-Bagger-Matchev-Zhang (PBMZ) 1996.

- The leading two–loop corrections in the limit of zero external momentum in the self–energies are taken from Brignole-Dedes-Degrassi-Slavich-Zwirner (BDDSZ) 2001-2003.

# Benchmark scenarios

- *Snowmass* points: six (out of ten) representative choices for the input parameters (see hep-ph/0202233)

| mSUGRA: | $m_0$ (GeV) | $m_{1/2}$ (GeV) | $A$ (GeV) | $\tan\beta$ | sign($\mu$) |
|---|---|---|---|---|---|
| SPS1a | 100 | 250 | -100 | 10 | $+$ |
| SPS2 | 1450 | 300 | 0 | 10 | $+$ |
| SPS4 | 400 | 300 | 0 | 50 | $+$ |
| SPS5 | 150 | 300 | -1000 | 5 | $+$ |
| **GMSB:** | $\Lambda$ (TeV) | $M_{\text{mess}}$ (TeV) | $N_{\text{mess}}$ | $\tan\beta$ | sign($\mu$) |
| SPS8 | 100 | 200 | 1 | 15 | $+$ |
| **AMSB:** | $m_{3/2}$ (TeV) | $m_0$ (GeV) | | $\tan\beta$ | sign($\mu$) |
| SPS9 | 60 | 450 | | 10 | $+$ |

# Results for the Higgs masses

- Light CP-even Higgs boson mass $m_h$:

| | SPS1a | SPS2 | SPS4 | SPS5 | SPS8 | SPS9 |
|---|---|---|---|---|---|---|
| *SoftSusy* | 111.0 | 115.3 | 112.9 | 114.2 | 113.9 | 116.2 |
| *SPheno* | 111.1 | 115.6 | 113.1 | 114.7 | 114.3 | 116.5 |
| *SuSpect* | 111.0 | 115.4 | 112.9 | 114.1 | 113.9 | 116.2 |

- Heavy CP-even Higgs boson mass $m_H$:

| | SPS1a | SPS2 | SPS4 | SPS5 | SPS8 | SPS9 |
|---|---|---|---|---|---|---|
| *SoftSusy* | 401.7 | 1523.1 | 335.5 | 692.4 | 542.8 | 1064.7 |
| *SPheno* | 401.2 | 1524.6 | 340.3 | 691.3 | 541.1 | 1059.4 |
| *SuSpect* | 401.3 | 1521.4 | 334.4 | 692.1 | 539.1 | 1064.1 |

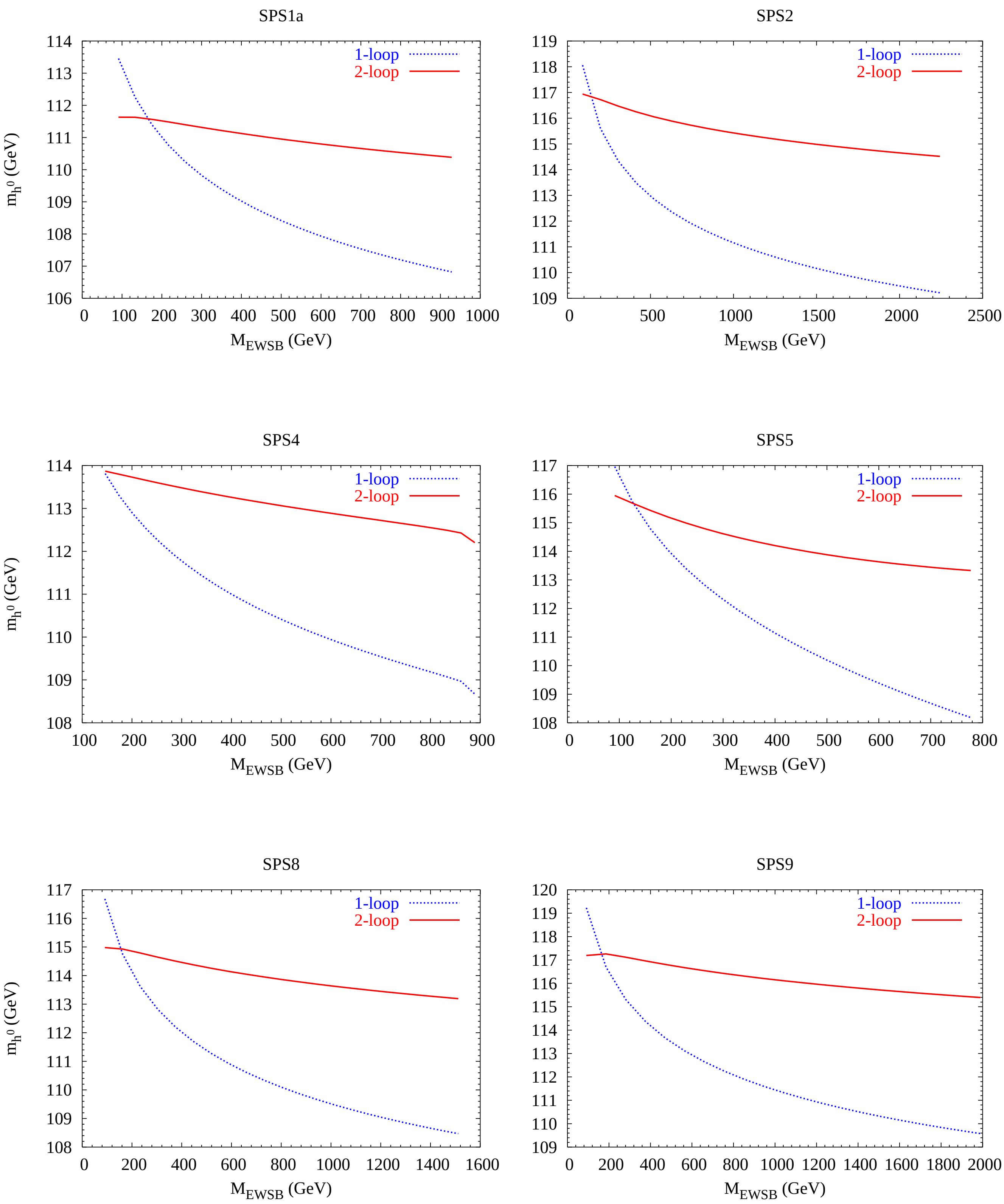
- CP–odd Higgs boson mass $m_A$:

| | SPS1a | SPS2 | SPS4 | SPS5 | SPS8 | SPS9 |
|---|---|---|---|---|---|---|
| *SoftSusy* | 401.4 | 1523.0 | 335.5 | 692.3 | 542.5 | 1064.5 |
| *SPheno* | 400.8 | 1524.5 | 340.3 | 691.5 | 540.8 | 1059.2 |
| *SuSpect* | 401.0 | 1521.3 | 334.4 | 692.1 | 538.8 | 1063.9 |

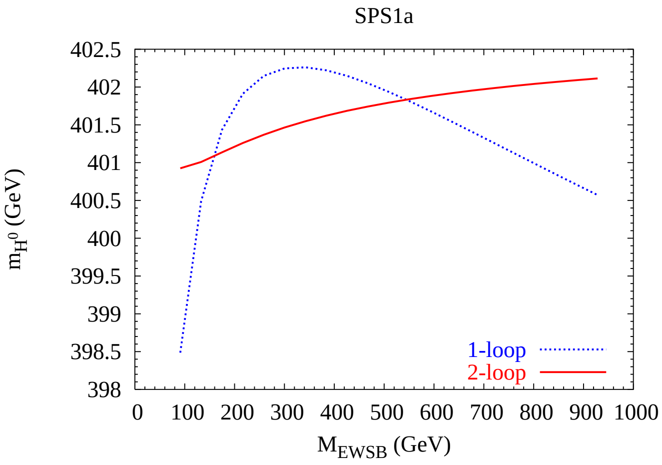
- Superpotential Higgs mass parameter $\mu$:

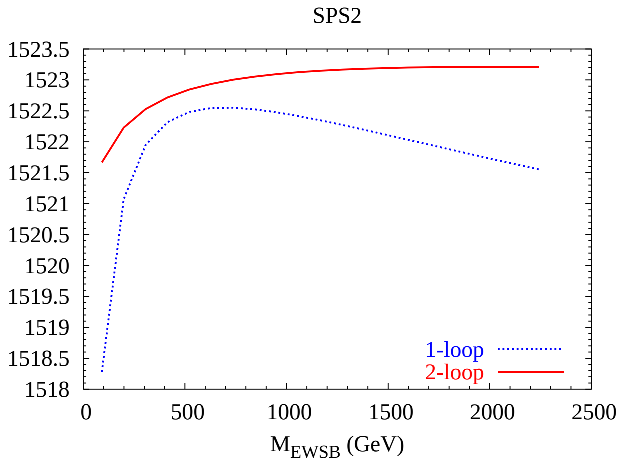
| | SPS1a | SPS2 | SPS4 | SPS5 | SPS8 | SPS9 |
|---|---|---|---|---|---|---|
| *SoftSusy* | 359.6 | 503.9 | 399.5 | 637.0 | 430.6 | 1020.0 |
| *SPheno* | 359.0 | 505.3 | 400.2 | 636.1 | 428.5 | 1014.6 |
| *SuSpect* | 359.7 | 500.2 | 399.1 | 636.7 | 428.0 | 1019.8 |

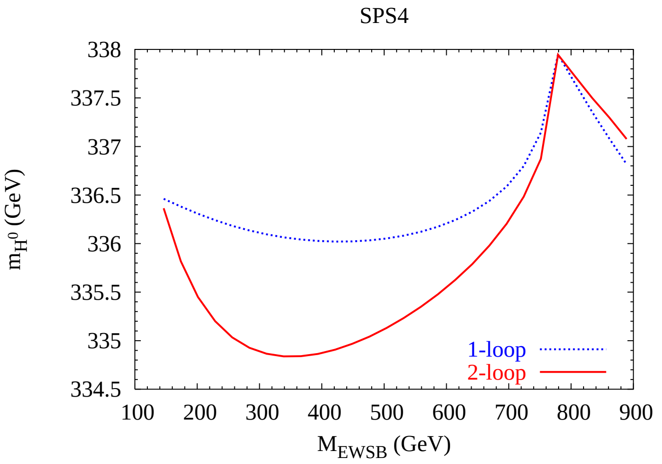
# Renormalization scale dependence of $m_h$

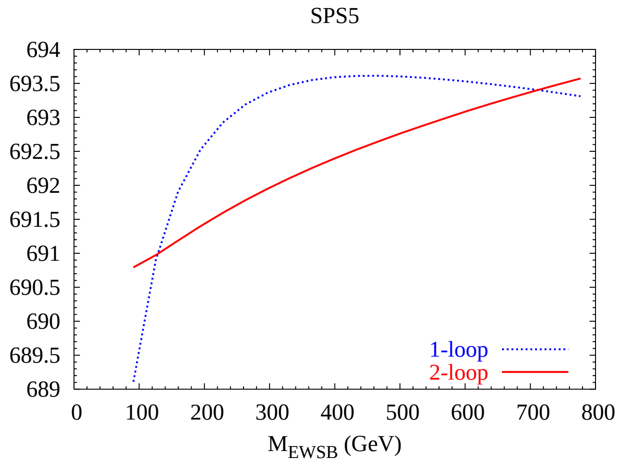


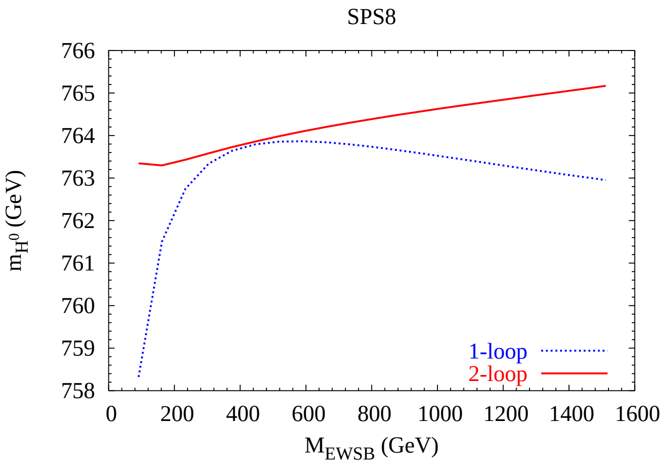
# Renormalization scale dependence of $m_H$

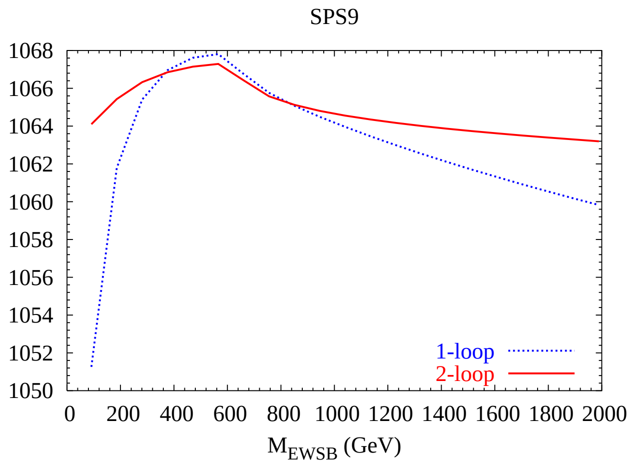


# Comparing the $\overline{\text{DR}}$ and OS calculations

- In the two–loop results implemented in *SoftSusy, SPheno* and *SuSpect* the MSSM parameters are expressed in the $\overline{\text{DR}}$ scheme (as they come naturally from the RG evolution).

- In alternative, we might express the MSSM input parameters in terms of physical (On–Shell) masses and mixing angles.

- The code *FeynHiggs* (S.Heinemeyer *et al.*) includes all the leading two–loop corrections in the OS renormalization scheme.

- The differences between the $\overline{\text{DR}}$ and OS calculations measure the uncertainty coming from higher–order corrections.

- Comparing the light CP-even Higgs boson mass $m_h$:

| | SPS1a | SPS2 | SPS4 | SPS5 | SPS8 | SPS9 |
|---|---|---|---|---|---|---|
| *SuSpect* | 111.0 | 115.4 | 112.9 | 114.1 | 113.9 | 116.2 |
| *FeynHiggs* * | 112.6 | 116.7 | 114.7 | 116.4 | 115.6 | 116.8 |

- Comparing the heavy CP-even Higgs boson mass $m_H$:

| | SPS1a | SPS2 | SPS4 | SPS5 | SPS8 | SPS9 |
|---|---|---|---|---|---|---|
| *SuSpect* | 401.3 | 1521.4 | 334.4 | 692.1 | 539.1 | 1064.1 |
| *FeynHiggs* * | 401.4 | 1521.4 | 333.9 | 691.9 | 539.1 | 1064.1 |

$*$ The MSSM input parameters for *FeynHiggs*, including $m_A$, are taken from the output of *SuSpect*.

# Summary

- *SuSpect* 2.3, *SoftSusy* 1.8 and *SPheno* 2.2 now include a fully consistent two-loop $\overline{\rm DR}$ computation of the Higgs tadpoles and masses, and agree well for several choices of the MSSM boundary conditions at the GUT scale.
The small residual differences are due to higher–order effects and they are understood.

- The inclusion of the two-loop corrections clearly improves the renormalization scale dependence of the Higgs masses. The residual $\sim$ 2–3 GeV variation in $m_h$ is a measure of the unknown higher–order effects.

- The $\sim 2$ GeV difference in $m_h$ w.r.t. the two-loop On-Shell computation of *FeynHiggs* 1.5.1 is another measure of the higher–order effects (compare with $\Delta m_h^{\rm theory} \simeq 3$ GeV).

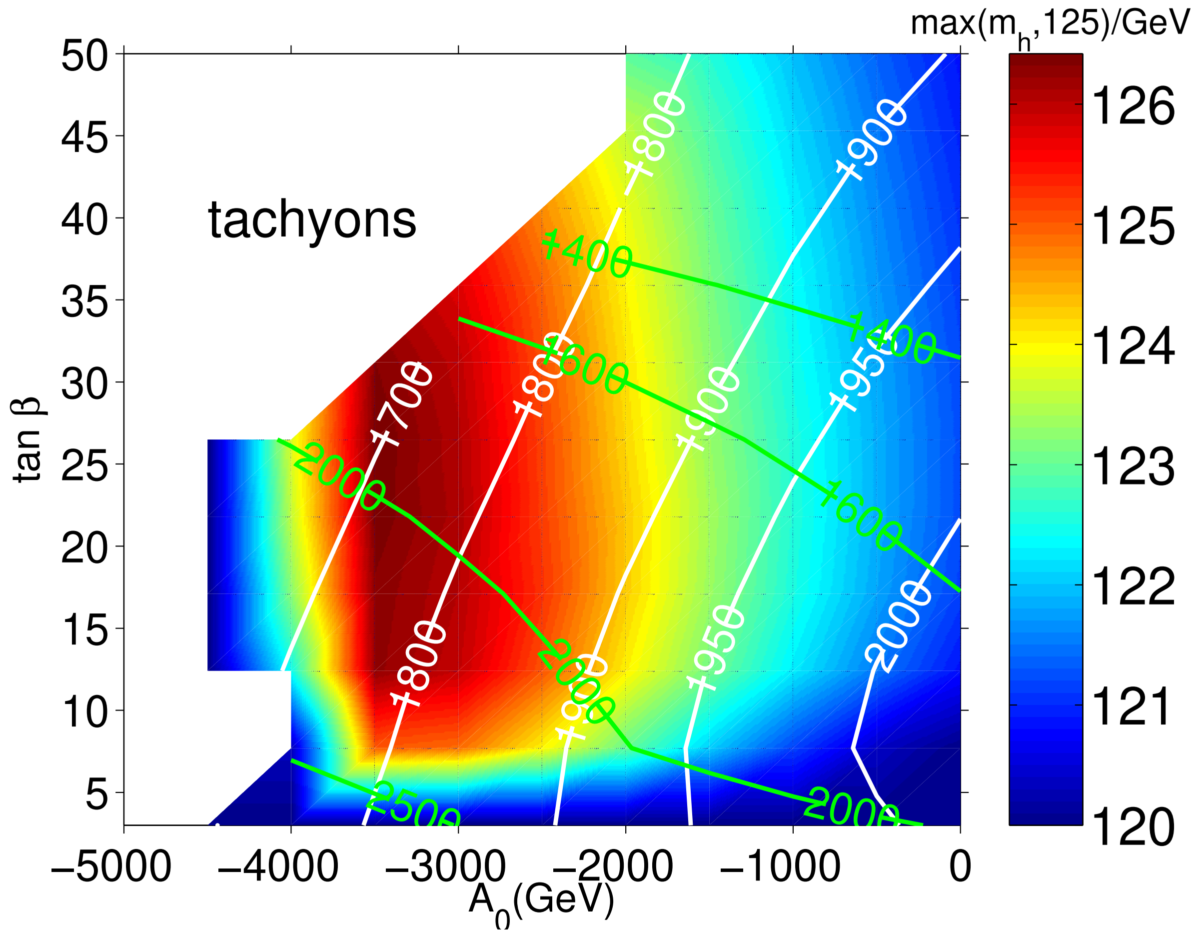
- Still in progress:
  - Bounds on $m_h$, $\tan\beta$ etc. in the general MSSM;
  - implications for various SUSY-breaking mechanisms (mSUGRA, GMSB, AMSB);
  - Estimate of the remaining uncertainties (both theoretical and parametric).

- Stay tuned...

# Maximal value of $m_h$

- Look for the mSUGRA parameter choice that maximizes $m_h$:

$$m_0 = m_{1/2} = 1\,\text{TeV}, \quad \mu > 0, \quad M_t = 175\,\text{GeV}$$




- The maximal $m_h$ depends on the chosen range for the squark masses *(is 3 TeV less natural than 2 TeV?)*

- The maximal $m_h$ depends critically on the measured $M_t$.