Determinazione della massa del quark b con regole di somma relativistiche

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G.C. and A.H. Hoang, Phys. Lett. B554 (2003) 133

Data from B-factories on B decays require precise knowledge of the b mass m_b

m_b is important to extract CKM matrix elements

Example: Width of semileptonic B decays:

$$\Gamma(B \to X_i \ell \nu) \propto |V_{ib}|^2 \frac{m_b^5}{m_W^4}$$

$$\frac{\delta V_{cb}}{V_{cb}} \simeq 2\% \ \Rightarrow \ \frac{\delta m_b}{m_b} \simeq 1\% \ \Rightarrow \ \delta m_b < 50 \text{ MeV}$$

Bottom quark mass definitions

Pole mass

$\Sigma^r(p^2 = m_b^2) = 0$

Processes where bottom is close to the mass-shell

Problems with renormalons: $\Sigma(m_b) \sim m_b \, \Sigma_n \, \alpha_S^n \, C_n \, n!$

$\overline{\mathrm{MS}}$ **mass**

Dimensional regularization $D = 4 - 2\epsilon$

Counterterm to subtract $(1/\epsilon + \gamma_E - \ln 4\pi)$

A renormalization scale μ **is introduced** $\bar{m}_b(\mu)$

Processes where the b **quarks are off-shell**

Threshold masses

Counterterms to subtract pole $1/\epsilon$ **and constants which do not contain** $n!$ **terms**

Processes whose characteristic energy is $Q \sim m_b$

Method

Moments of the $e^+e^- \to b\bar{b}$ cross section

$$P_n = \int \frac{ds}{s^{n+1}} R_{b\bar{b}}(s)$$

R ratio:

$$R_{b\bar{b}}(s) = \frac{\sigma(e^+e^- \to b\bar{b}X)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

Low-n moments ($n \leq 3$): relativistic moments

Advantage: Theoretical moments are calculable in fixed-order perturbative QCD

Disadvantage: Big contribution of continuum region to moments

$\overline{\mathrm{MS}}$ **mass is the appropriate mass for relativistic $b\bar{b}$ production**

$$R_{b\bar{b}}(s) = R^{(0)}_{b\bar{b}} + \left(\frac{\alpha_S}{\pi}\right) R^{(1)}_{b\bar{b}} + \left(\frac{\alpha_S}{\pi}\right)^2 R^{(2)}_{b\bar{b}}$$

K.G. Chetyrkin, A.H. Hoang, J.H. Kühn, M. Steinhauser and T. Teubner, EPJ C2 (1998) 137

J.H. Kühn and M. Steinhauser, NPB 619 (2001) 588; NPB 604 (2002) 415

KS: $\bar{m}_b(\bar{m}_b) = (4.21 \pm 0.05)$ GeV

All quarks are massless except the b quark

Theoretical moments to $\mathcal{O}(\alpha_S^2)$ $(r = m_c/m_b)$:

$$P_n = \frac{1}{(4\bar{m}_b(\mu))^n}\left\{f_n^0 + \frac{\alpha_S(\mu)}{\pi}\left[f_n^{10} + f_n^{11}\ln\left(\frac{\bar{m}_b{}^2(\mu)}{\mu^2}\right)\right]\right.$$
$$\left. + \left(\frac{\alpha_S(\mu)}{\pi}\right)^2\left[f_n^{20}(r) + f_n^{21}\ln\left(\frac{\bar{m}_b{}^2(\mu)}{\mu^2}\right) + f_n^{22}\ln^2\left(\frac{\bar{m}_b{}^2(\mu)}{\mu^2}\right)\right]\right\}$$

At $\mu = \bar{m}_b$:

$$P_n = \frac{1}{(4\bar{m}_b(\bar{m}_b))^n}\left\{f_n^0 + \frac{\alpha_S(\mu)}{\pi}f_n^{10} + \left(\frac{\alpha_S(\mu)}{\pi}\right)^2\left[f_n^{20}(r) - \frac{1}{4}\beta_0 f_n^{10}\ln\left(\frac{\bar{m}_b{}^2(\bar{m}_b)}{\mu^2}\right)\right]\right\}$$

Results for $m_c = 0$

n	1	2	3	4
f_n^0	0.2667	0.1143	0.0677	0.0462
f_n^{10}	0.6387	0.2774	0.1298	0.0508
f_n^{11}	0.5333	0.4571	0.4063	0.3694
$f_n^{20}(0)$	0.9446	0.8113	0.5172	0.3052
$[f_n^{20}(0)]_{\mathrm{KS}}$	0.7898	0.8080	0.5169	0.3051
f_n^{21}	0.8606	1.2700	1.1450	0.8682
f_n^{22}	0.0222	0.4762	0.8296	1.1240

Deviation in f_n^{20} with respect to KS due to inclusion of secondary $b\bar{b}$ production effects

Charm-mass effects $(r = m_c/m_b)$

r	0.1	0.2	0.3	0.4	0.5
$f_1^{20}(r) - f_1^{20}(0)$	−0.0021	−0.0078	−0.0164	−0.0266	−0.0382
$f_2^{20}(r) - f_2^{20}(0)$	−0.0028	−0.0091	−0.0187	−0.0302	−0.0430
$f_3^{20}(r) - f_3^{20}(0)$	−0.0024	−0.0101	−0.0204	−0.0330	−0.0466
$f_4^{20}(r) - f_4^{20}(0)$	−0.0030	−0.0109	−0.0219	−0.0348	−0.0491

Experimental moments

We shall consider the following contributions:

- **resonances** $\Upsilon(1S) - \Upsilon(6S)$
- **region 11.1-12 GeV (continuum 1)**
- **region 12 GeV -** m_Z **(continuum 2)**
- **region** $m_Z - \infty$ **(continuum 3)**

Narrow width approximation for a resonance k **:**

$$(P_n)_k = \frac{9\pi \ \Gamma_k^{e^+e^-}}{\alpha(10 \text{ GeV}) m_k^{2n+1}}$$

For $\Upsilon(1S)$**,** $\Upsilon(2S)$**,** $\Upsilon(3S)$ **(ARGUS) and** $\Upsilon(6S)$ **(CUSB and CLEO) we use the PDG results for** m_k **and** $\Gamma_k^{e^+e^-}$

In the $4S - 5S$ **region (**$\simeq 10.7$ **GeV) both CLEO and CUSB have observed an additional enhancement** B^*

$m(B^*) = (10.684 \pm 0.013)\,\text{GeV} \ \ \Gamma(B^* \to e^+e^-) = (0.20 \pm 0.11)\,\text{keV} \ \ \text{(CLEO)}$

PDG ignored such an enhancement and the PDG-based moments are systematically lower

Above $\Upsilon(6S) \Rightarrow$ **perturbative QCD**

Agreement of pQCD with LEP1 ($\sqrt{s} = m_Z$**) to** 1% **and with LEP2 to** 10% **(LEPEWWG, hep-ph/0212036)**

Continuum regions 2 and 3: our error is a fraction of pQCD full prediction (conservative error treatment)

KS: error just due to variations of μ**,** $\alpha_S(m_Z)$ **and** m_b

In the continuum region we use:

$$\alpha_S(m_Z) = 0.118 \pm 0.003 \quad ; \quad \bar{m}_b(\bar{m}_b) = (4.2 \pm 0.2)\ \text{GeV} \quad \text{(PDG)}$$

$$2.5\ \text{GeV} < \mu < 10\ \text{GeV}$$

Results on experimental moments

Contribution	P_1 $\times 10^3$ GeV^2	P_2 $\times 10^5$ GeV^4	P_3 $\times 10^7$ GeV^6	P_4 $\times 10^9$ GeV^8
$\Upsilon(1S)$	0.766(29)	0.856(32)	0.956(36)	1.068(40)
$\Upsilon(2S)$	0.254(16)	0.252(16)	0.251(15)	0.250(15)
$\Upsilon(3S)$	0.211(29)	0.196(27)	0.183(26)	0.171(24)
$[\Upsilon(4S) - \Upsilon(5S)]_{\text{PDG}}$	0.222(40)	0.192(34)	0.167(29)	0.145(25)
$[\Upsilon(4S) - \Upsilon(5S)]_{\text{CUSB}}$	0.257(42)	0.223(36)	0.194(31)	0.169(27)
$[\Upsilon(4S) - \Upsilon(5S)]_{\text{CLEO}}$	0.244(95)	0.213(82)	0.186(72)	0.162(62)
$[\Upsilon(4S) - \Upsilon(5S)]_{\text{our}}$	0.251(95)	0.218(82)	0.190(72)	0.165(62)
$\Upsilon(6S)$	0.048(11)	0.039(9)	0.032(7)	0.027(6)
11.1 GeV – 12.0 GeV	0.418(57)	0.314(44)	0.236(34)	0.178(27)
12.0 GeV – m_Z	2.467(26)	0.886(21)	0.414(13)	0.217(8)
$m_Z - \infty$	0.047(1)	0.000(0)	0.000(0)	0.000(0)

Solving $(P_n)_{th} = (P_n)_{exp}$ for $\bar{m}_b(\mu)$ or $\bar{m}_b(\bar{m}_b)$ one determines the $\overline{\rm MS}$ bottom quark mass

Four different methods:

1) Single moment fits $(n = 1, 2, 3)$ yield $\bar{m}_b(\bar{m}_b)$

2) Single moment fits $(n = 1, 2, 3)$ yield $\bar{m}_b(\mu)$, with $2.5 \text{ GeV} < \mu < 10 \text{ GeV}$, afterwards $\bar{m}_b(\bar{m}_b)$ is obtained using renormalization group equations

3) Fits of the ratio P_n/P_{n+1} $(n = 1, 2)$ yield $\bar{m}_b(\bar{m}_b)$

4) Fits of the ratio P_n/P_{n+1} $(n = 1, 2)$ yield $\bar{m}_b(\mu)$; afterwards $\bar{m}_b(\bar{m}_b)$ is obtained using renormalization group equations

Treatment of the errors

Resonances: half of the error correlated (linear sum); half uncorrelated (quadratic sum)

Continuum:
Theoretical error: variation of μ (2.5, 5 and 10 GeV), $\alpha_S(M_Z)$, $m_c = (1.3 \pm 0.2)$ GeV
Experimental error: a fraction of pQCD prediction

Total continuum error: the different errors are added linearly, since the separation into three regions is arbitrary and no data is available in the continuum

Results

10% uncertainty in the continuum region

	Method 1 (2)			Method 3 (4)	
n	1	2	3	1	2
central	4210 (4214)	4200 (4205)	4197 (4200)	4191 (4195)	4191 (4191)
$\Upsilon(1S)$	14 (13)	12 (12)	11 (11)	11 (11)	9 (9)
$\Upsilon(2S)$	7 (7)	6 (6)	5 (5)	4 (4)	3 (3)
$\Upsilon(3S)$	14 (14)	10 (10)	8 (8)	7 (7)	3 (3)
$4S-5S$	45 (44)	32 (32)	22 (22)	18 (18)	4 (4)
$\Upsilon(6S)$	5 (5)	3 (3)	2 (2)	2 (2)	0 (0)
combined	67 (67)	50 (50)	38 (38)	33 (33)	15 (15)
$[\text{region 1}]_{\text{th}}$	27_{th} (26_{th})	17_{th} (17_{th})	11_{th} (11_{th})	7_{th} (7_{th})	2_{th} (2_{th})
$[\text{region 2}]_{\text{th}}$	12_{th} (12_{th})	8_{th} (8_{th})	4_{th} (4_{th})	4_{th} (4_{th})	4_{th} (4_{th})
$[\text{region 2}]_{10\%}$	115 (114)	33 (33)	13 (13)	49 (49)	29 (29)
$[\text{region 3}]_{\text{th}}$	1_{th} (1_{th})	0_{th} (0_{th})	0_{th} (0_{th})	1_{th} (1_{th})	0_{th} (0_{th})
$[\text{region 3}]_{10\%}$	2 (2)	0 (0)	0 (0)	2 (2)	0 (0)
δm_c	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)
$\delta\alpha_s(M_Z)$	17 (18)	10 (11)	6 (6)	3 (3)	2 (2)
$\delta\mu$	23 (5)	16 (14)	11 (27)	15 (27)	3 (50)
combined	184 (166)	77 (75)	41 (57)	76 (88)	37 (85)
total	251 (233)	127 (125)	79 (95)	110 (121)	51 (99)

From P_3 with methods 1 and 2 one gets:

$$\bar{m}_b(\bar{m}_b) = (4.20 \pm 0.09)\ \text{GeV}$$

Larger uncertainties for smaller n

Result from KS: $\bar{m}_b(\bar{m}_b) = (4.21 \pm 0.05)\ \text{GeV}$

Agreement but larger error

Future CLEO data (I.Shipsey et al., hep-ex/0203033): measurement of $\Gamma(\Upsilon \to e^+e^-)$ at the level of 2% for $\Upsilon(1S) - \Upsilon(3S)$
Reduction of 10 MeV of the error on $\bar{m}_b(\bar{m}_b)$ and possibly 20 MeV after a similar improvement for the widths of $\Upsilon(4S) - \Upsilon(6S)$

Conclusions

Analysis of $\bar{m}_b(\bar{m}_b)$ and uncertainties

$R_{b\bar{b}}$ **ratio to $\mathcal{O}(\alpha_S^2)$**

Theoretical and experimental low-n moments

Reconsidered the region between resonances $\Upsilon(4S)$ and $\Upsilon(5S)$ and included the intermediate B^* state

Inclusion of charm mass and secondary b-quark production: small effect

Conservative treatment of the experimental continuum region (error is a fraction of pQCD prediction)

Determination of bottom mass from fits of the third moment P_3 and assuming a 10% error in the experimental continuum

Larger uncertainties on $\bar{m}_b(m_b)$ for fits of lower n moments

Final result reads: $\bar{m}_b(\bar{m}_b) = (4.20 \pm 0.09)$ **GeV**