Complete 1-loop calculations in the chargino/neutralino sector of the MSSM and SPA conventions

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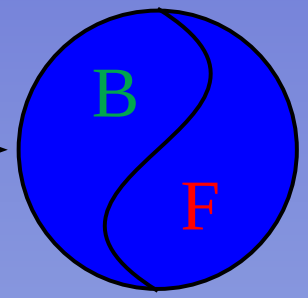
Outline

- Charginos & Neutralinos in the MSSM
- Motivation
- Renormalization
  - calculation of counterterms
  - renormalization schemes
- Transition $\overline{\mathrm{DR}} \rightarrow \mathrm{OS}$, SPA
- Separation of "QED-like" parts
- Exemplary results: $e^+e^- \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^0 (\gamma)$; $e^+e^- \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^- (\gamma)$
- Summary & outlook

MSSM



Ingredients: SM + SUSY

Contents of fields:

$\gamma, Z^0, W^\pm$	g		$\tilde{\gamma}, \tilde{Z}^0, \tilde{W}^\pm$	$\tilde{g}$		
$\begin{pmatrix}\nu_e\\ e\end{pmatrix}$	$\begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}$	$\begin{pmatrix}\nu_\tau\\ \tau\end{pmatrix}$	$\tilde{\nu}_e, \tilde{e}$	$\tilde{\nu}_\mu, \tilde{\mu}$	$\tilde{\nu}_\tau, \tilde{\tau}$	
$\begin{pmatrix}u\\ d\end{pmatrix}$	$\begin{pmatrix}c\\ s\end{pmatrix}$	$\begin{pmatrix}t\\ b\end{pmatrix}$	$\tilde{u}, \tilde{d}$	$\tilde{c}, \tilde{s}$	$\tilde{t}, \tilde{b}$	
SU(2) doublet H_1			ψ_{H_1}	H_2	ψ_{H_2}	

SUSY breaking: "soft"

- explicit mass terms for SUSY-partners, Higgs-bosons
- trilinear Higgs-sfermion-sfermion couplings

Charginos & Neutralinos

Charginos: mass eigenstates of charged higgsinos/gauginos

- mass matrix: $X = \begin{pmatrix} M_2 & \sqrt{2}\, M_W\, \sin\beta \\ \sqrt{2}\, M_W\, \cos\beta & \mu \end{pmatrix}$
- diagonalization: $U^*\, X\, V^\dagger = M^{\rm diag}_{\tilde\chi^+}$
- dirac spinors: $\tilde\chi^+_i \quad (i = 1, 2)$

Neutralinos: mass eigenstates of uncharged higgsinos/gauginos

- mass matrix:

$$Y = \begin{pmatrix} M_1 & 0 & -M_Z\, s_W\, c_\beta & M_Z\, s_W\, s_\beta \\ 0 & M_2 & M_Z\, c_W\, c_\beta & -M_Z\, c_W\, s_\beta \\ -M_Z\, s_W\, c_\beta & M_Z\, c_W\, c_\beta & 0 & -\mu \\ M_Z\, s_W\, s_\beta & -M_Z\, c_W\, s_\beta & -\mu & 0 \end{pmatrix}$$

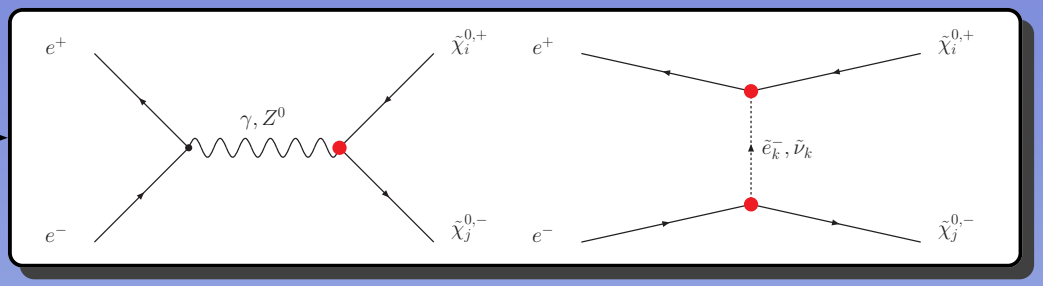
- diagonalization: $N^*\, Y\, N^\dagger = M^{\rm diag}_{\tilde\chi^0}$
- majorana spinors: $\tilde\chi^0_j \quad (j = 1, \ldots, 4)$

Importance

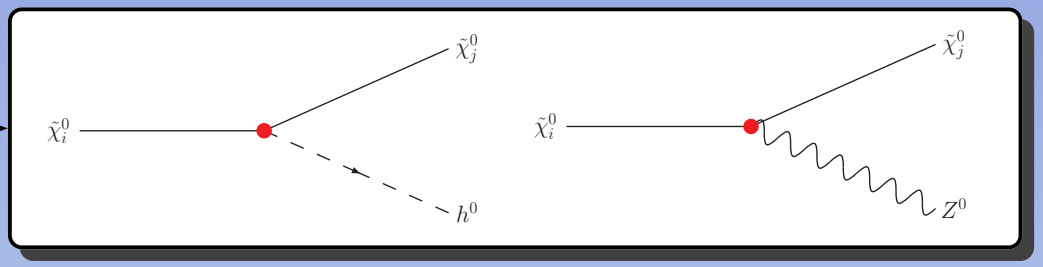
Parameters

$M_1,\ M_2,\ \mu$

$(\tan\beta)$



$m_{\tilde{\chi}_1^+}, m_{\tilde{\chi}_2^+}\ ;\ m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}, m_{\tilde{\chi}_3^0}, m_{\tilde{\chi}_4^0}$



Prospects:

- discovery of SUSY particles
- testing MSSM-predicted relations in the Chargino/Neutralino sector
- gaining insight into SUSY breaking mechanism

Quantum corrections

Radiative corrections important:

- large mass shifts
  - → D. Pierce and A. Papadopoulos, Phys. Rev. D **50** (1994) 565
  - → H. Eberl, M. Kincel, W. Majerotto and Y. Yamada, Phys. Rev. D **64** (2001) 115013
  - → T. Fritzsche and W. Hollik, Eur. Phys. J. C **24** (2002) 619
- large corrections for cross sections
  - → M. A. Diaz, S. F. King and D. A. Ross, Nucl. Phys. B **529** (1998) 23
  - → T. Blank, W. Hollik [arXiv:hep-ph/0011092]
  - → W. Oeller, H. Eberl, W. Majerotto [arXiv:hep-ph/0402134]
- high experimental accuracy
  - → H. U. Martyn and G. A. Blair, arXiv:hep-ph/9910416.
  - → K. Desch, J. Kalinowski, G. Moortgat-Pick, M. M. Nojiri and G. Polesello, JHEP **0402** (2004) 035
  - → J. A. Aguilar-Saavedra *et al.* [ECFA/DESY LC Physics Working Group Collaboration], arXiv:hep-ph/0106315.

Renormalization

reason: calculation of loop diagrams $\longrightarrow$ divergent results

- regularization: $\infty + \text{finite} \longrightarrow f(\epsilon) + \text{finite}$ $\left(\lim_{\epsilon \to 0} f(\epsilon) = \infty\right)$

  SUSY: dimensional reduction (DR)

- renormalization: rescaling of all parameters and fields

$$\left.\begin{array}{rcl} X & \longrightarrow & X + \delta X \\ \Phi & \longrightarrow & \left(1 + \frac{\delta Z}{2}\right) \Phi \end{array}\right\}$$

Automation

- counterterms:

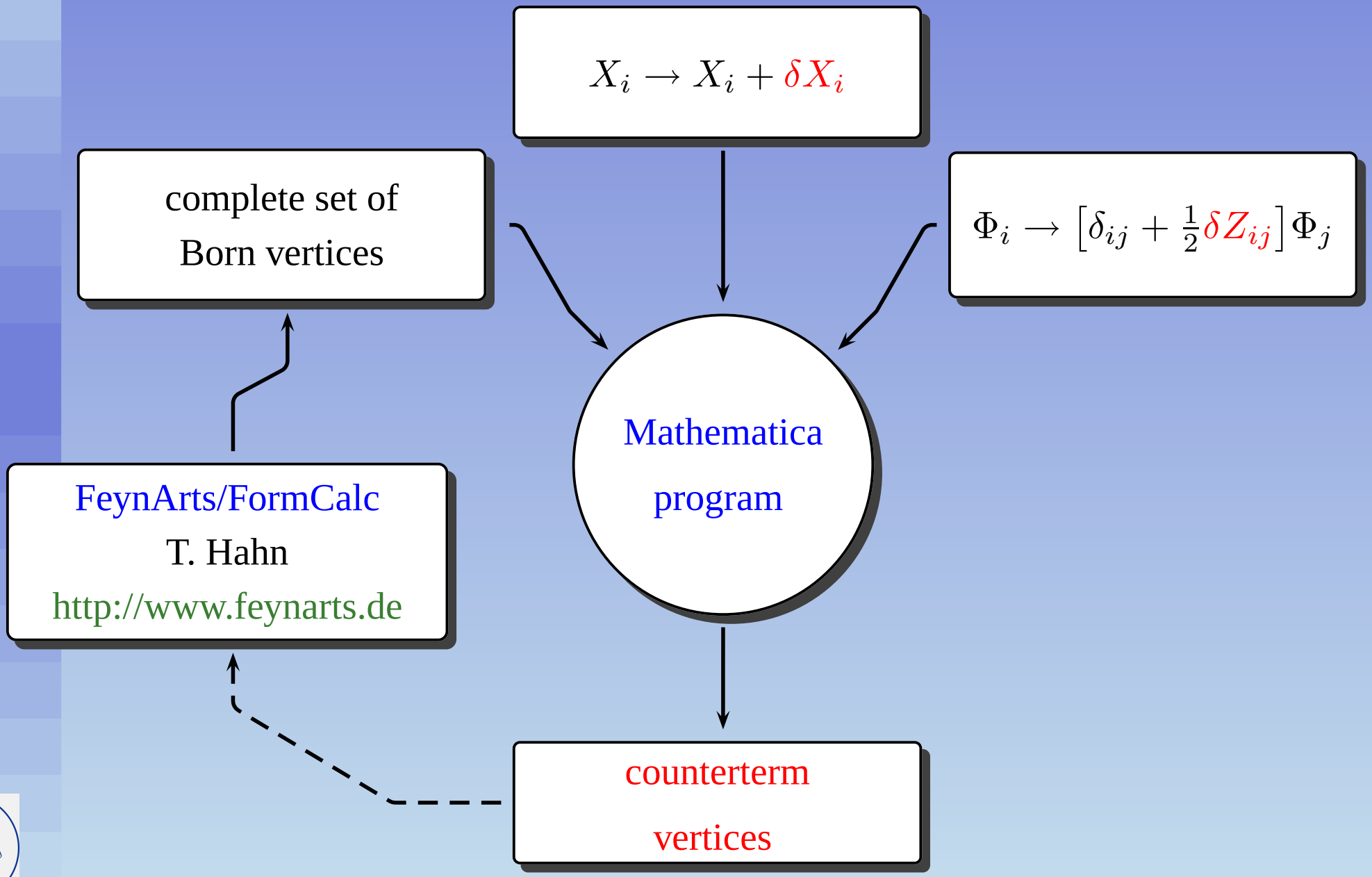
$$\mathcal{L}(\Phi, X) \quad \longrightarrow \quad \mathcal{L}_{\text{Born}}(\Phi, X) + \mathcal{L}_{\text{CT}}(\Phi, X, \delta Z, \delta X)$$

- renormalization conditions: fixing the values of δX,δZ

  S-matrix elements und Green's functions finite

Conditio sine qua non: symmetries must remain preserved

Automatic generation of CTs



[FFV] 2 Charginos – Gauge Boson

$$\underset{30}{C}(-\tilde{\chi}^-_{c2}, \tilde{\chi}^-_{c1}, \gamma) = \left[\frac{\mathrm{i}\,e \left\{ 2 \left\{ \begin{array}{l} c_W\, s_W \left[\delta Z^R_{\tilde{\chi}^+}\right]_{c1,c2}{}^* + c_W\, s_W \left[\delta Z^R_{\tilde{\chi}^+}\right]_{c2,c1} + \left(-[\delta Z_{ZA}]_{12}\, s_W{}^2 + (c_W\, s_W)\, ([\delta Z_{ZA}]_{22} + 2\,\delta Z_e)\right) \delta_{c1,c2} + \\ [\delta Z_{ZA}]_{12}\, U^*_{c1,1}\, U_{c2,1} \end{array} \right\} + [\delta Z_{ZA}]_{12}\, U^*_{c1,2}\, U_{c2,2} \right\}}{4\, c_W\, s_W} \right.$$

$$\left. \frac{\mathrm{i}\,e \left\{ 2 \left\{ \begin{array}{l} c_W\, s_W \left[\delta Z^L_{\tilde{\chi}^+}\right]_{c2,c1}{}^* + c_W\, s_W \left[\delta Z^L_{\tilde{\chi}^+}\right]_{c1,c2} + \left(-[\delta Z_{ZA}]_{12}\, s_W{}^2 + (c_W\, s_W)\, ([\delta Z_{ZA}]_{22} + 2\,\delta Z_e)\right) \delta_{c1,c2} + \\ [\delta Z_{ZA}]_{12}\, V^*_{c2,1}\, V_{c1,1} \end{array} \right\} + [\delta Z_{ZA}]_{12}\, V^*_{c2,2}\, V_{c1,2} \right\}}{4\, c_W\, s_W} \right]$$

$$\underset{31}{C}(-\tilde{\chi}^-_{c2}, \tilde{\chi}^-_{c1}, Z) = \left[\frac{\left\{ \begin{array}{l} \left(2\,\delta s_W\, s_W{}^2 + c_W{}^2\, (2\,\delta s_W + c_W\, [\delta Z_{ZA}]_{21} - s_W\, ([\delta Z_{ZA}]_{11} + 2\,\delta Z_e))\right) \left(2\,\mathrm{i}\,e\, s_W{}^2\, \delta_{c1,c2}\right) + \\ 4\, c_W{}^3\, s_W{}^2 \sum_{n=1}^{2} -\frac{\mathrm{i}\,e \left[\delta Z^R_{\tilde{\chi}^+}\right]_{n,c1}}{4\, c_W\, s_W} \left(2\, s_W{}^2\, \delta_{n,c2} - 2\, U^*_{n,1}\, U_{c2,1} - U^*_{n,2}\, U_{c2,2}\right) + \\ 4\, c_W{}^3\, s_W{}^2 \sum_{n=1}^{2} -\frac{\mathrm{i}\,e \left[\delta Z^R_{\tilde{\chi}^+}\right]_{n,c2}{}^*}{4\, c_W\, s_W} \left(2\, s_W{}^2\, \delta_{c1,n} - 2\, U^*_{c1,1}\, U_{n,1} - U^*_{c1,2}\, U_{n,2}\right) + \\ (\mathrm{i}\,e) \left(\left(2\, c_W{}^2\, \delta s_W + 2\, c_W{}^2\, \delta Z_e\, s_W + c_W{}^2\, [\delta Z_{ZA}]_{11}\, s_W - 2\,\delta s_W\, s_W{}^2\right) \left(2\, U^*_{c1,1}\, U_{c2,1} + U^*_{c1,2}\, U_{c2,2}\right)\right) \end{array} \right\}}{4\, c_W{}^3\, s_W{}^2} \right.$$

$$\left. \frac{\left\{ \begin{array}{l} \left(2\,\delta s_W\, s_W{}^2 + c_W{}^2\, (2\,\delta s_W + c_W\, [\delta Z_{ZA}]_{21} - s_W\, ([\delta Z_{ZA}]_{11} + 2\,\delta Z_e))\right) \left(2\,\mathrm{i}\,e\, s_W{}^2\, \delta_{c1,c2}\right) + \\ 4\, c_W{}^3\, s_W{}^2 \sum_{n=1}^{2} -\frac{\mathrm{i}\,e \left[\delta Z^L_{\tilde{\chi}^+}\right]_{n,c2}}{4\, c_W\, s_W} \left(2\, s_W{}^2\, \delta_{c1,n} - 2\, V^*_{n,1}\, V_{c1,1} - V^*_{n,2}\, V_{c1,2}\right) + \\ 4\, c_W{}^3\, s_W{}^2 \sum_{n=1}^{2} -\frac{\mathrm{i}\,e \left[\delta Z^L_{\tilde{\chi}^+}\right]_{n,c1}{}^*}{4\, c_W\, s_W} \left(2\, s_W{}^2\, \delta_{n,c2} - 2\, V^*_{c2,1}\, V_{n,1} - V^*_{c2,2}\, V_{n,2}\right) + \\ (\mathrm{i}\,e) \left(\left(2\, c_W{}^2\, \delta s_W + 2\, c_W{}^2\, \delta Z_e\, s_W + c_W{}^2\, [\delta Z_{ZA}]_{11}\, s_W - 2\,\delta s_W\, s_W{}^2\right) \left(2\, V^*_{c2,1}\, V_{c1,1} + V^*_{c2,2}\, V_{c1,2}\right)\right) \end{array} \right\}}{4\, c_W{}^3\, s_W{}^2} \right]$$

[FFV] 2 Leptons – Gauge Boson

$$\underset{9}{C}(-e_{j2}, e_{j1}, \gamma) = \left[\begin{array}{c} \left(-[\delta Z_{ZA}]_{12} - 2\,[\delta Z_{ZA}]_{12}\, s_W{}^2 + (2\, c_W\, s_W)\, ([\delta Z_{ZA}]_{22} + 2\,\delta Z_e) + 4\, c_W\, s_W\, \mathrm{Re}\left[\left[\delta Z^L_e\right]_{j2,j2}\right]\right) \frac{\mathrm{i}\,e\,\delta_{j1,j2}}{4\, c_W\, s_W} \\ \left(-[\delta Z_{ZA}]_{12}\, s_W + c_W\, ([\delta Z_{ZA}]_{22} + 2\,\delta Z_e) + 2\, c_W\, \mathrm{Re}\left[\left[\delta Z^R_e\right]_{j2,j2}\right]\right) \frac{\mathrm{i}\,e\,\delta_{j1,j2}}{2\, c_W} \end{array} \right]$$

Renormalization schemes

$\overline{\mathrm{DR}}$ scheme:

- Loop integrals: $\frac{2}{\epsilon} - \gamma + \log 4\pi + \log \mu^2 \to \log \mu^2_{\overline{\mathrm{DR}}}$

+ easy to implement

– observables are scale dependent in finite order perturbation theory

+ natural choice for GUT-inspired parameter sets (mSUGRA)

OS scheme:

- renormalization constants fixed by physical conditions

– renormalization constants complicated

+ observables are scale independent

+ well suited for calculations of cross sections and decay rates
(e.g. pole masses $\to$ correct kinematical thresholds)

Remarks: on-shell scheme

Two different technical realisations

- H. Eberl, M. Kincel, W. Majerotto and Y. Yamada,
  Phys. Rev. D **64** (2001) 115013 [arXiv:hep-ph/0104109]
- T. Fritzsche and W. Hollik,
  Eur. Phys. J. C **24** (2002) 619 [arXiv:hep-ph/0203159]

Physical equivalence (e.g. mass spectrum)

- formal parameters μ, M_1, M_2 different
- relations between observables in agreement

$\longrightarrow$ relations between pole masses $m_{\tilde{\chi}^+_{1,2}}$, $m_{\tilde{\chi}^0_{1,\ldots 4}}$ ok

$\overline{\mathrm{DR}} \to \mathrm{OS}$

SPA conventions

GUT $\xrightarrow{\text{RGE}}$ low energy $\overline{\mathrm{DR}}$ parameters $\longrightarrow$ OS parameters $\longrightarrow$ Γ, σ, m_i

$\overline{\mathrm{DR}}$ parameters M_1, M_2, μ $\xrightarrow{\text{Loop}}$ phys. 1-loop masses of $\tilde{\chi}_1^+, \tilde{\chi}_2^+, \tilde{\chi}_1^0$ $\xrightarrow{\text{Tree}}$ OS parameters M_1, M_2, μ

SPS1a

```
M1  =  99.1
M2  = 192.7
MUE = 352.4
```

```
MCha(1) = 176.013 +  8.889
MCha(2) = 378.527 + 10.312
MNeu(1) =  96.154 +  4.004
```

SPS1a-OS

```
M1  = 103.02
M2  = 201.56
MUE = 363.06
```

Separation of QED-like parts

Full calculation inevitable

- naive omission of diagrams with virtual photons not feasible
- soft photonic contributions necessary to get IR-finite results ($\rightarrow E^{\max}_{\gamma,\mathrm{soft}}$)
- including hard bremsstrahlung

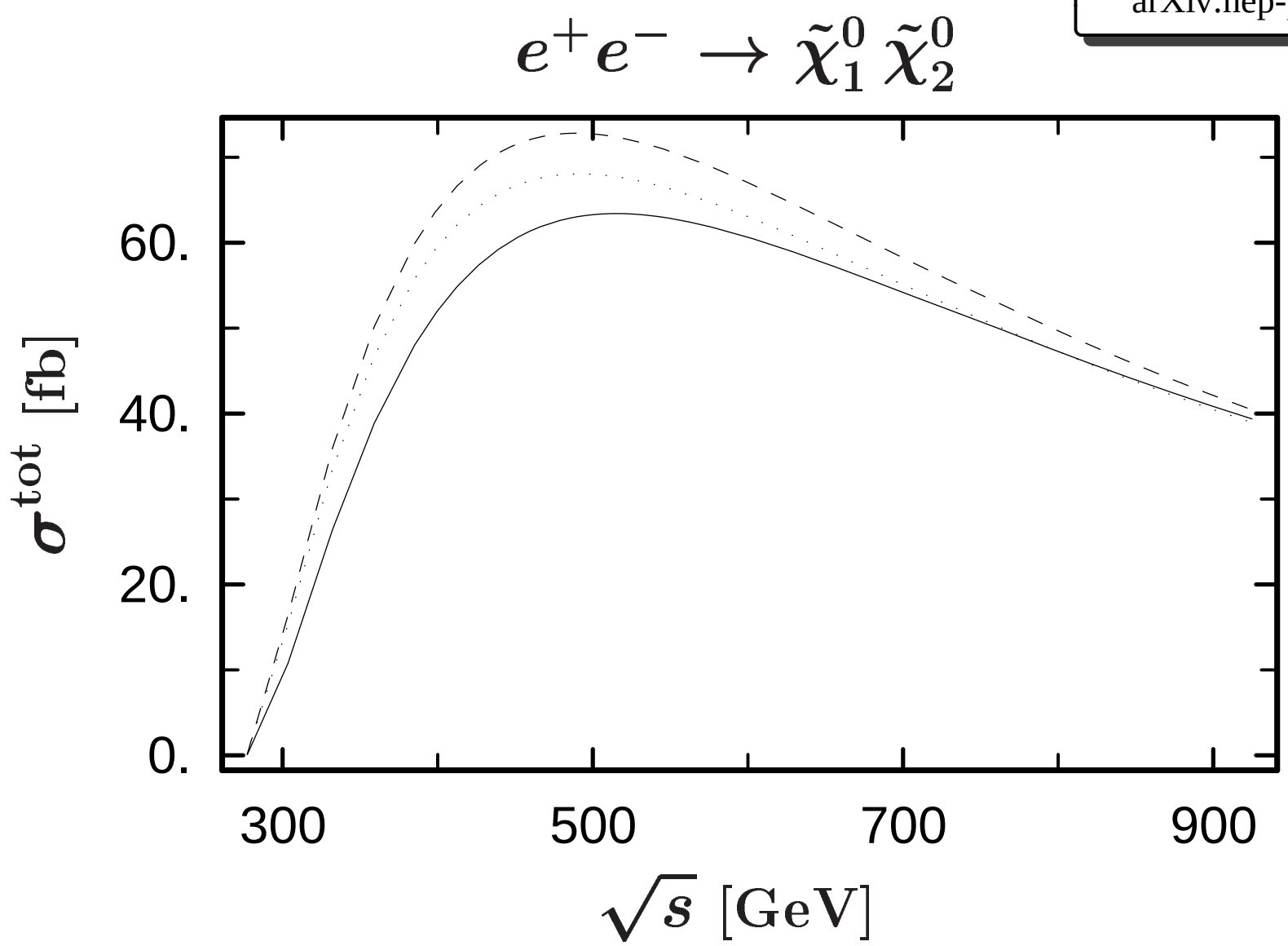
Separation (W. Oeller *et al.* arXiv:hep-ph/0402134)

$$
\begin{aligned}
\sigma &= \sigma_{\mathrm{QED}} + \sigma_{\mathrm{remainder}} \\
\sigma_{\mathrm{QED}} &:= \sigma^{\mathrm{hard}} + \frac{\alpha}{\pi}\left\{\log\left(\frac{4[E^{\max}_{\gamma,\mathrm{soft}}]^2}{s}\right)\left[\log\left(\frac{s}{m_e^2}\right)-1\right]+\frac{3}{2}\log\left(\frac{s}{m_e^2}\right)\right\}\sigma_0 \\
\sigma_{\mathrm{remainder}} &:= \sigma^{\mathrm{virt+soft}} - \frac{\alpha}{\pi}\left\{\log\left(\frac{4[E^{\max}_{\gamma,\mathrm{soft}}]^2}{s}\right)\left[\log\left(\frac{s}{m_e^2}\right)-1\right]+\frac{3}{2}\log\left(\frac{s}{m_e^2}\right)\right\}\sigma_0
\end{aligned}
$$

- gauge invariant
- $\sigma_{\mathrm{remainder}}$ free of large soft and universal collinear photon contributions

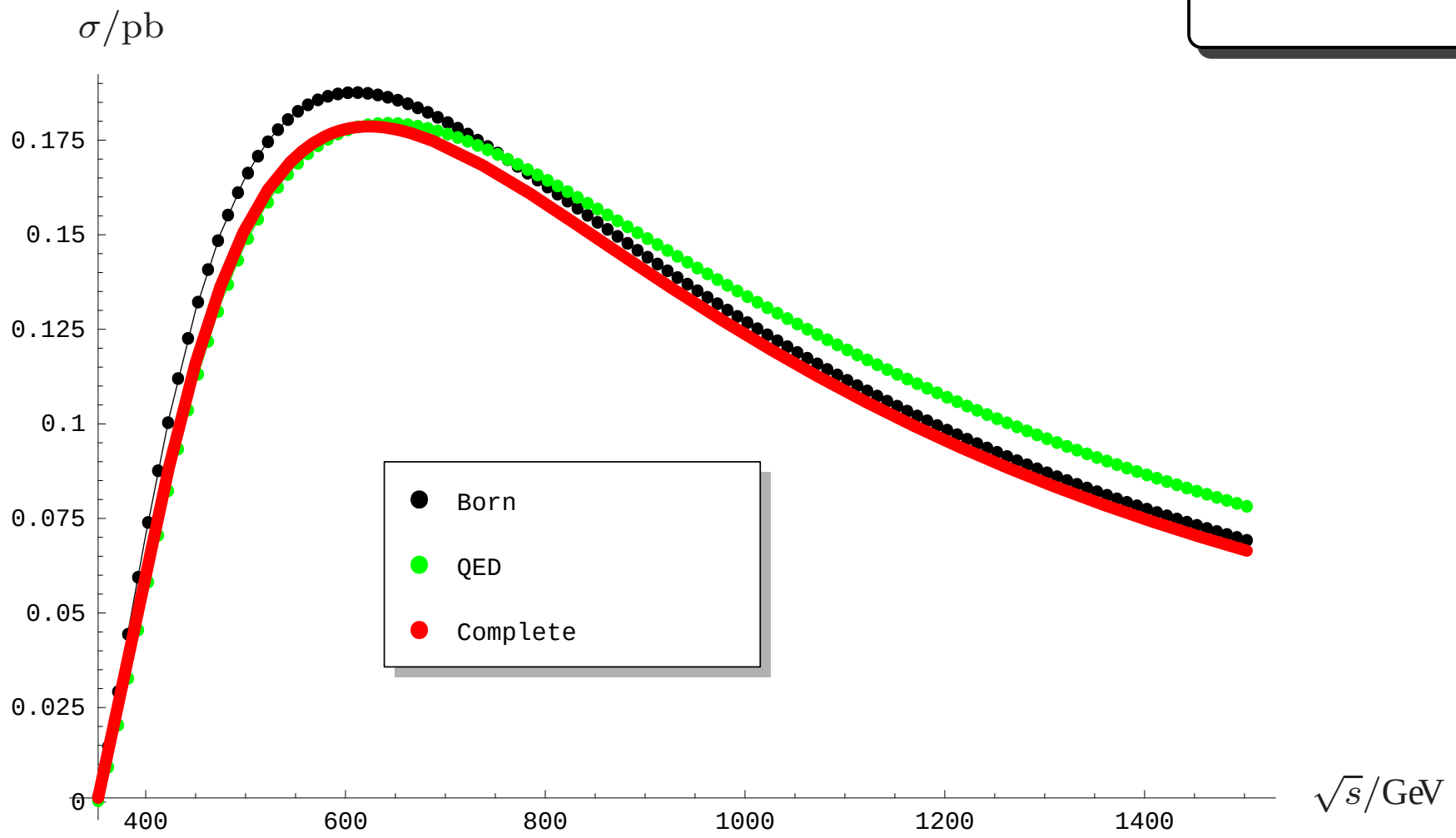
SPS1a: $e^+ e^- \to \tilde{\chi}_1^0 \tilde{\chi}_2^0 (\gamma)$

W. Oeller *et al.*
arXiv:hep-ph/0402134



SPS1a: $e^+ e^- \to \tilde{\chi}_1^+ \tilde{\chi}_1^- (\gamma)$

splitting
hard/soft photon:
$10^{-3} \cdot \sqrt{s}$



Summary & outlook

- Charginos & Neutralinos: comprehensive sector of the MSSM
  - discovery of SUSY particles
  - testing relations predicted by the MSSM
  - gaining insight into SUSY breaking mechanism
- Quantum corrections are important
- SPA conventions to channel efforts
- Tools for 1-loop calculations are ready and tested
  - FeynArts modelfile with all CT-couplings
  - decays with 2 or 3 particles in the final state
  - production processes with 2 or 3 particle final states
- Ongoing work
  - multiparticle final states