The SPA project and conventions – a proposal for a high precision Supersymmetry Parameter Analysis

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for the SPA group:

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www-flc.desy.de/spa/

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Motivation & Aim

- At a future e^+e^- LC measurements with high precision possible
  $\longrightarrow$ requires equally accurate theoretical calculations including radiative corrections

- Allows high precision determination of SUSY parameters

- For such a Supersymmetry Parameter Analysis (SPA) a clearly defined framework necessary

  - for comparison of different theoretical calculations
  - for extracting the parameters from data
  - for extrapolating the parameters to high scale

SPA Conventions

- Masses of SUSY particles and Higgs bosons are given as pole masses
- All SUSY parameters in $\mathcal{L}$, including $\tan\beta$, are given in the $\overline{\mathrm{DR}}$ scheme (dimensional reduction) at the scale $\hat{M} = 1$ TeV
- All elements in mass matrices, rotation matrices, and corresponding mixing angles at tree-level are given in $\overline{\mathrm{DR}}$ scheme at $\hat{M} = 1$ TeV, except for the Higgs sector where the mixing angle is defined on-shell
- SM input parameters: G_F, α, m_Z, $\alpha_s(m_Z)$ and the fermion masses
- Branching ratios and cross sections are expressed in terms of pole masses and $\overline{\mathrm{DR}}$ SUSY parameters

Why these conventions ?

- Quite generally, SUSY parameters depend on the renormalization scheme (on-shell, $\overline{\mathrm{DR}}$, $\overline{\mathrm{MS}}$)

- It would be desirable to adopt a pure on-shell scheme (no scale dependence!) as in the SM. Would need for all parameters special counter term fixing conditions, which should be gauge invariant and SUSY conserving; no consensus yet
  $\rightarrow$ We have therefore chosen the $\overline{\mathrm{DR}}$ scheme, which is technically most convenient and simple (shifts from $\overline{\mathrm{MS}}$ to $\overline{\mathrm{DR}}$ are known), $\hat{M} = 1$ TeV is a reasonable scale for our analysis

- All parameters in mass and rotation matrices (at tree-level) are $\overline{\mathrm{DR}}$ parameters at 1 TeV for consistency

- Note that $m_W(\mathrm{pole})$ is calculated from the SM input parameters G_F, α, m_Z, $\alpha_s(m_Z)$

Standard Model Parameters

Calculate m_W(pole): (Degrassi - Fanchiotti - Sirlin, Pierce et al.)

$$m_W^2 = m_Z^2 \hat{\rho} \left(\frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\alpha^{\overline{\mathrm{DR}}}(m_Z)\pi}{\sqrt{2} G_F m_Z^2 \hat{\rho} (1 - \Delta\hat{r})}} \right)$$

$$\Delta\hat{r} = \hat{\rho} \frac{\Pi^T_{WW}(0)}{m_W^2} - \frac{\Pi^T_{ZZ}(m_Z^2)}{m_Z^2} + \delta_{VB}$$

(have included 2–loop + SUSY contributions)

In the calculations we use $\overline{\mathrm{DR}}$ weak mixing angle:

$$(\cos\theta_W)^{\overline{\mathrm{DR}}} \equiv \hat{c}, \quad \hat{c}^2 \hat{s}^2 = \frac{\pi \alpha^{\overline{\mathrm{DR}}}(m_Z)}{\sqrt{2} m_Z^2 G_\mu (1 - \Delta\hat{r})}$$

Standard Model Parameters, cont.

Important: m_b running:

$m_b^{\overline{\rm MS},SM}(m_Z)$ from $m_b^{\overline{\rm MS}}(m_b)$ via 4-loop RGE's, and

$$m_b^{\overline{\rm DR},SM}(m_Z) = m_b^{\overline{\rm MS},SM}(m_Z)\left(1 - \frac{\alpha_s^{\overline{\rm DR}}}{3\pi} - \frac{23(\alpha_s^{\overline{\rm DR}})^2}{72\pi^2} + \frac{4g_2^2}{128\pi^2} - \frac{13g'^2}{1152\pi^2}\right)$$

Including SUSY (Carena et al.)

$$m_b^{\overline{\rm DR}}(m_Z) = \frac{m_b^{\overline{\rm DR},SM}(m_Z)}{1 - \frac{Re(\hat{\Sigma}_b(m_Z))}{m_b^{\overline{\rm DR}}(m_Z)}}$$

similar for m_t: starts from pole mass $\to m_t^{\overline{\rm DR}}(m_Z)$
(with 2-loop gluonic shift + 1-loop SUSY)

$$\text{Yukawa coupling } Y_f^{\overline{\rm DR}}(m_Z) = \frac{\sqrt{2}m_f^{\overline{\rm DR}}(m_Z)}{v_i(m_Z)}, \quad \text{with } v^2(m_Z) = 4\frac{m_Z^2 + Re(\Pi_{ZZ}^T(m_Z))}{g'^2(m_Z) + g_2^2(m_Z)}$$

From scale m_Z to scale $\hat{M} = 1$ TeV, use 2-loop RGE's (Martin-Vaughn, Yamada, Jack-Jones)

SUSY masses, mixing matrices

- All SUSY parameters including $\tan\beta$ are given at $\hat{M} = 1$ TeV

- All elements in mass matrices and rotation matrices are taken at $\hat{M} = 1$ TeV
  (in $\overline{\rm DR}$ scheme; use rotation matrices such that all eigenvalues are positive to allow an easy inclusion of CP violating phases)

- Pole masses of SUSY particles are calculated at complete one-loop level

$$m^{\rm (pole)}_{\rm SUSY} = m^{\overline{\rm DR}}_{\rm SUSY}(\hat{M}) - \delta m^{\rm one-loop}_{\rm SUSY}(\hat{M})$$

- Take pole masses and running mixing matrices for calculations of branching ratios and cross sections (here as a first step: tree-level formulas used, no cuts and no ISR included)

Reference point for probing the proposed SPA

- Take mSUGRA point SPS1a:
  $M_{1/2} =$ 250 GeV, $M_0 =$ 100 GeV, $A_0 =$ -100 GeV, $\tan\beta =$ 10, sign(μ)$= +$
  However, SPA is not bound to this scenario!
  Parameters, masses, branching ratios, cross sections obtained by using SPheno 2.2.0
  hep-ph/0301101; http://www-theorie.physik.unizh.ch/∼porod/SPheno.html

m_e	$5.109989\ 10^{-4}$	$m_u(Q)$	$3\ 10^{-3}$	$m_d(Q)$	$7\ 10^{-3}$
m_μ	0.105658357	$m_c(Q)$	1.2	$m_s(Q)$	0.12
m_τ	1.777	m_t^{pole}	$175.$	$m_b(m_b)$	4.2
m_Z	91.1876	G_F	$1.16639\ 10^{-5}$ GeV^{-2}	$1/\alpha$	137.0359998
$\Delta\alpha_{had}$	0.027690	$\alpha_s^{\overline{\rm MS}}(m_Z)$	0.119		

Numerical values for the SM input. Masses are given in GeV. Scale $Q = 2$ GeV.

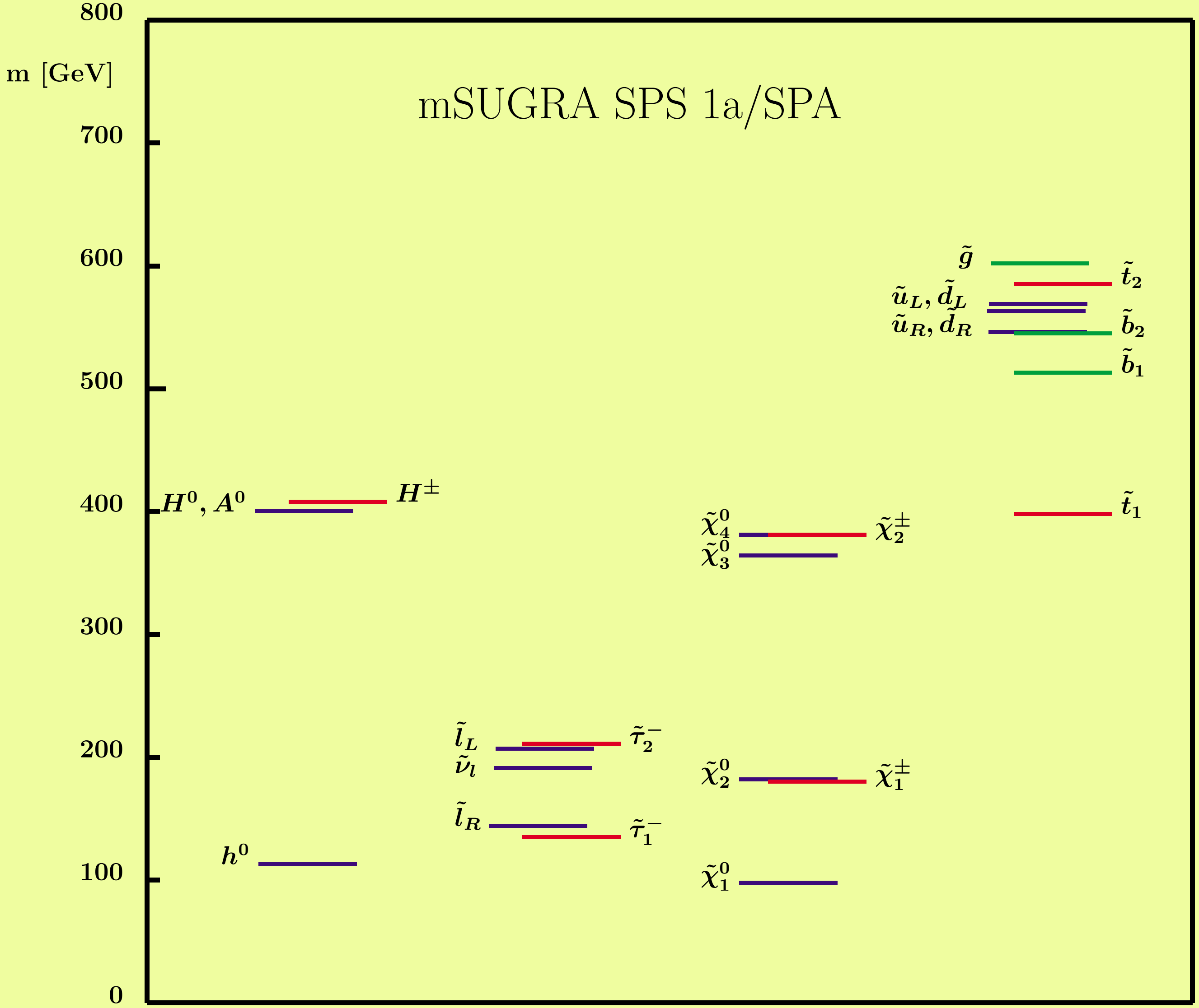
$$\rightarrow m_W = 80.3971 \text{ GeV}$$

SUSY input parameters at $\hat{M}$= 1 TeV

g'	0.36354	M_1	103.01
g	0.64804	M_2	192.84
g_s	1.08412	M_3	571.44
Y_τ	0.09958	A_τ	-249.8
Y_t	0.88176	A_t	-487.7
Y_b	0.13143	A_b	-766.9
μ	362.35	$\tan\beta$	10.0
$M^2_{L_1}$	$3.7821 \cdot 10^4$	$M^2_{L_3}$	$3.7513 \cdot 10^4$
$M^2_{E_1}$	$1.8399 \cdot 10^4$	$M^2_{E_3}$	$1.7773 \cdot 10^4$
$M^2_{Q_1}$	$28.177 \cdot 10^4$	$M^2_{Q_3}$	$23.416 \cdot 10^4$
$M^2_{U_1}$	$26.198 \cdot 10^4$	$M^2_{U_3}$	$16.734 \cdot 10^4$
$M^2_{D_1}$	$25.972 \cdot 10^4$	$M^2_{D_3}$	$25.682 \cdot 10^4$
$M^2_{H_1}$	$3.2864 \cdot 10^4$	$M^2_{H_2}$	$-11.804 \cdot 10^4$

Extracted SUSY parameters and Yukawa couplings at $\hat{M} = 1$ TeV [mass units in GeV]

SPS1a/SPA1a mass spectrum



Sleptons

$\tilde{l}$	m [GeV]	Γ [GeV]	decay	$\mathcal{B}$
$\tilde{e}_R$	143.96	0.21	$\tilde{\chi}_1^0 e^-$	1.000
$\tilde{e}_L$	207.4	0.27	$\tilde{\chi}_1^0 e^-$	0.476
			$\tilde{\chi}_2^0 e^-$	0.182
			$\tilde{\chi}_1^- \nu_e$	0.342
$\tilde{\nu}_e$	191.5	0.19	$\tilde{\chi}_1^0 \nu_e$	0.849
			$\tilde{\chi}_2^0 \nu_e$	0.036
			$\tilde{\chi}_1^+ e^-$	0.115
$\tilde{\mu}_R$	143.9	0.21	$\tilde{\chi}_1^0 \mu^-$	1.000
$\tilde{\mu}_L$	207.4	0.27	$\tilde{\chi}_1^0 \mu^-$	0.476
			$\tilde{\chi}_2^0 \mu^-$	0.182
			$\tilde{\chi}_1^- \nu_\mu$	0.342

$\tilde{l}$	m [GeV]	Γ [GeV]	decay	$\mathcal{B}$
$\tilde{\nu}_\mu$	191.5	0.19	$\tilde{\chi}_1^0 \nu_\mu$	0.849
			$\tilde{\chi}_2^0 \nu_\mu$	0.036
			$\tilde{\chi}_1^+ \mu^-$	0.115
$\tilde{\tau}_1$	134.8	0.15	$\tilde{\chi}_1^0 \tau^-$	1.000
$\tilde{\tau}_2$	211.0	0.32	$\tilde{\chi}_1^0 \tau^-$	0.525
			$\tilde{\chi}_2^0 \tau^-$	0.167
			$\tilde{\chi}_1^- \nu_\tau$	0.308
$\tilde{\nu}_\tau$	190.6	0.18	$\tilde{\chi}_1^0 \nu_\tau$	0.869
			$\tilde{\chi}_2^0 \nu_\tau$	0.031
			$\tilde{\chi}_1^+ \tau^-$	0.100

Slepton pole masses, widths and branching ratios $\mathcal{B} > 1\%$ in SPA 1a from SPheno 2.2.0

Neutralinos

$\tilde{\chi}$	m [GeV]	Γ [GeV]	decay	$\mathcal{B}$
$\tilde{\chi}_1^0$	97.12		stable	
$\tilde{\chi}_2^0$	181.2	0.022	$\tilde{e}_R^\pm e^\mp$	0.062
			$\tilde{\mu}_R^\pm \mu^\mp$	0.064
			$\tilde{\tau}_1^\pm \tau^\mp$	0.869
$\tilde{\chi}_3^0$	367.7	2.0	$\tilde{\chi}_1^\pm W^\mp$	0.591
			$\tilde{\chi}_1^0 Z^0$	0.115
			$\tilde{\chi}_2^0 Z^0$	0.209
			$\tilde{\chi}_1^0 h^0$	0.022
			$\tilde{\chi}_2^0 h^0$	0.013
			$\tilde{\tau}_1^\pm \tau^\mp$	0.010
			$\tilde{\tau}_2^\pm \tau^\mp$	0.013

$\tilde{\chi}$	m [GeV]	Γ [GeV]	decay	$\mathcal{B}$
$\tilde{\chi}_4^0$	384.7	2.6	$\tilde{\chi}_1^\pm W^\mp$	0.512
			$\tilde{\chi}_1^0 Z^0$	0.021
			$\tilde{\chi}_2^0 Z^0$	0.019
			$\tilde{\chi}_1^0 h^0$	0.072
			$\tilde{\chi}_2^0 h^0$	0.148
			$\tilde{e}_L^\pm e^\mp$	0.017
			$\tilde{\mu}_L^\pm \mu^\mp$	0.017
			$\tilde{\tau}_2^\pm \tau^\mp$	0.030
			$\tilde{\nu}_e \nu_e$	0.047
			$\tilde{\nu}_\mu \nu_\mu$	0.047
			$\tilde{\nu}_\tau \nu_\tau$	0.047

Neutralino pole masses, widths and branching ratios $\mathcal{B} > 1\%$ in SPA 1a from SPheno 2.2.0

Charginos & Gluinos

$\tilde{\chi}$	m [GeV]	Γ [GeV]	decay	$\mathcal{B}$
$\tilde{\chi}_1^+$	179.6	0.016	$\tilde{\tau}_1^+ \nu_\tau$	0.959
			$\tilde{\chi}_1^0 W^+$	0.033
$\tilde{\chi}_2^+$	384.4	2.5	$\tilde{e}_L^+ \nu_e$	0.049
			$\tilde{\mu}_L^+ \nu_\mu$	0.049
			$\tilde{\tau}_2^+ \nu_\tau$	0.054
			$\tilde{\nu}_e e^+$	0.018
			$\tilde{\nu}_\mu \mu^+$	0.018
			$\tilde{\nu}_\tau \tau^+$	0.025
			$\tilde{\chi}_1^0 W^+$	0.068
			$\tilde{\chi}_2^0 W^+$	0.287
			$\tilde{\chi}_1^+ Z^0$	0.240
			$\tilde{\chi}_1^+ h^0$	0.190

$\tilde{g}$	m [GeV]	Γ [GeV]	decay	$\mathcal{B}$
$\tilde{g}$	607.6	4.6	$\tilde{u}_R \bar{u}$	0.099
			$\tilde{u}_L \bar{u}$	0.047
			$\tilde{d}_R \bar{d}$	0.100
			$\tilde{d}_L \bar{d}$	0.035
			$\tilde{t}_1 \bar{t}$	0.104
			$\tilde{b}_1 \bar{b}$	0.224
			$\tilde{b}_2 \bar{b}$	0.105

Chargino and gluino pole masses, widths and branching ratios $\mathcal{B} > 1\%$ in SPA 1a from SPheno 2.2.0

3rd gen. Squarks

$\tilde{q}_3$	m [GeV]	Γ [GeV]	decay	$\mathcal{B}$
$\tilde{t}_1$	401.7	2.0	$\tilde{\chi}_1^0 t$	0.196
			$\tilde{\chi}_2^0 t$	0.119
			$\tilde{\chi}_1^+ b$	0.666
			$\tilde{\chi}_2^+ b$	0.011
$\tilde{t}_2$	590.2	7.5	$\tilde{\chi}_1^0 t$	0.030
			$\tilde{\chi}_2^0 t$	0.088
			$\tilde{\chi}_3^0 t$	0.041
			$\tilde{\chi}_4^0 t$	0.194
			$\tilde{\chi}_1^+ b$	0.224
			$\tilde{\chi}_2^+ b$	0.195
			$\tilde{t}_1 Z^0$	0.194
			$\tilde{t}_1 h^0$	0.033

$\tilde{q}_3$	m [GeV]	Γ [GeV]	decay	$\mathcal{B}$
$\tilde{b}_1$	518.7	3.8	$\tilde{\chi}_1^0 b$	0.048
			$\tilde{\chi}_2^0 b$	0.343
			$\tilde{\chi}_1^- t$	0.445
			$\tilde{t}_1 W^-$	0.150
$\tilde{b}_2$	550.9	1.0	$\tilde{\chi}_1^0 b$	0.234
			$\tilde{\chi}_2^0 b$	0.154
			$\tilde{\chi}_3^0 b$	0.043
			$\tilde{\chi}_4^0 b$	0.064
			$\tilde{\chi}_1^- t$	0.206
			$\tilde{t}_1 W^-$	0.298

Third generation squark pole masses, widths and branching ratios $\mathcal{B} > 1\%$ in SPA 1a from SPheno 2.2.0

1st gen. Squarks

$\tilde{q}_1$	m [GeV]	Γ [GeV]	decay	$\mathcal{B}$
$\tilde{u}_R$	551.3	1.2	$\tilde{\chi}_1^0 u$	0.986
$\tilde{u}_L$	569.2	5.6	$\tilde{\chi}_2^0 u$	0.319
			$\tilde{\chi}_4^0 u$	0.010
			$\tilde{\chi}_1^+ \bar{d}$	0.652
			$\tilde{\chi}_2^+ \bar{d}$	0.012

$\tilde{q}_1$	m [GeV]	Γ [GeV]	decay	$\mathcal{B}$
$\tilde{d}_R$	551.0	0.30	$\tilde{\chi}_1^0 d$	0.986
$\tilde{d}_L$	574.6	5.4	$\tilde{\chi}_1^0 d$	0.025
			$\tilde{\chi}_2^0 d$	0.308
			$\tilde{\chi}_4^0 d$	0.014
			$\tilde{\chi}_1^- \bar{u}$	0.610
			$\tilde{\chi}_2^- \bar{u}$	0.041

First generation squark pole masses, widths and branching ratios $\mathcal{B} > 1\%$ in SPA 1a from SPheno 2.2.0

Higgs particles

H	m [GeV]	Γ [GeV]	decay	$\mathcal{B}$
h^0	110.6	$2.7 \cdot 10^{-3}$	$\tau^-\tau^+$	0.112
			$b\bar{b}$	0.766
			$c\bar{c}$	0.037
			$W^\pm W^{\mp *}$	0.040
			gg	0.042
H^0	403.4	0.77	$\tau^-\tau^+$	0.104
			$b\bar{b}$	0.657
			$t\bar{t}$	0.047
			$\tilde{\chi}^0_1\tilde{\chi}^0_1$	0.021
			$\tilde{\chi}^0_1\tilde{\chi}^0_2$	0.061
			$\tilde{\chi}^0_2\tilde{\chi}^0_2$	0.018
			$\tilde{\chi}^+_1\tilde{\chi}^-_1$	0.052
			h^0h^0	0.013

H	m [GeV]	Γ [GeV]	decay	$\mathcal{B}$
A^0	403.0	1.2	$\tau^-\tau^+$	0.066
			$b\bar{b}$	0.418
			$t\bar{t}$	0.123
			$\tilde{\chi}^0_1\tilde{\chi}^0_1$	0.021
			$\tilde{\chi}^0_1\tilde{\chi}^0_2$	0.087
			$\tilde{\chi}^0_2\tilde{\chi}^0_2$	0.074
			$\tilde{\chi}^+_1\tilde{\chi}^-_1$	0.199
H^+	411.4	0.67	$\nu_\tau\tau^+$	0.121
			$t\bar{b}$	0.639
			$\tilde{\chi}^+_1\tilde{\chi}^0_1$	0.208
			$\tilde{\nu}_\tau\tilde{\tau}^+_1$	0.021

Higgs pole masses and branching ratios $\mathcal{B} > 1\%$ in SPA 1a from SPheno 2.2.0

Summary & Outlook

- Have presented a consistent framework for a SUSY Parameter Analysis ("SPA project") and defined appropriate conventions
- We have applied it (for testing) to the SPS1a scenario
- Next steps:
  - Compare different theoretical calculations (including full one-loop corrections) within these conventions (see also talks by T. Fritzsche, A. Freitas, and K. Kovařík at this conference)
  - Apply the scheme (including the one-loop calculations) to methods for extracting the SUSY parameters from measurements in a global fit (e.g.: SFITTER, FITTINO) (see talk by P. Wienemann (P. Bechtle, K. Desch) at this conference)
  - This project is an evolving process, improvements will be necessary in the future.