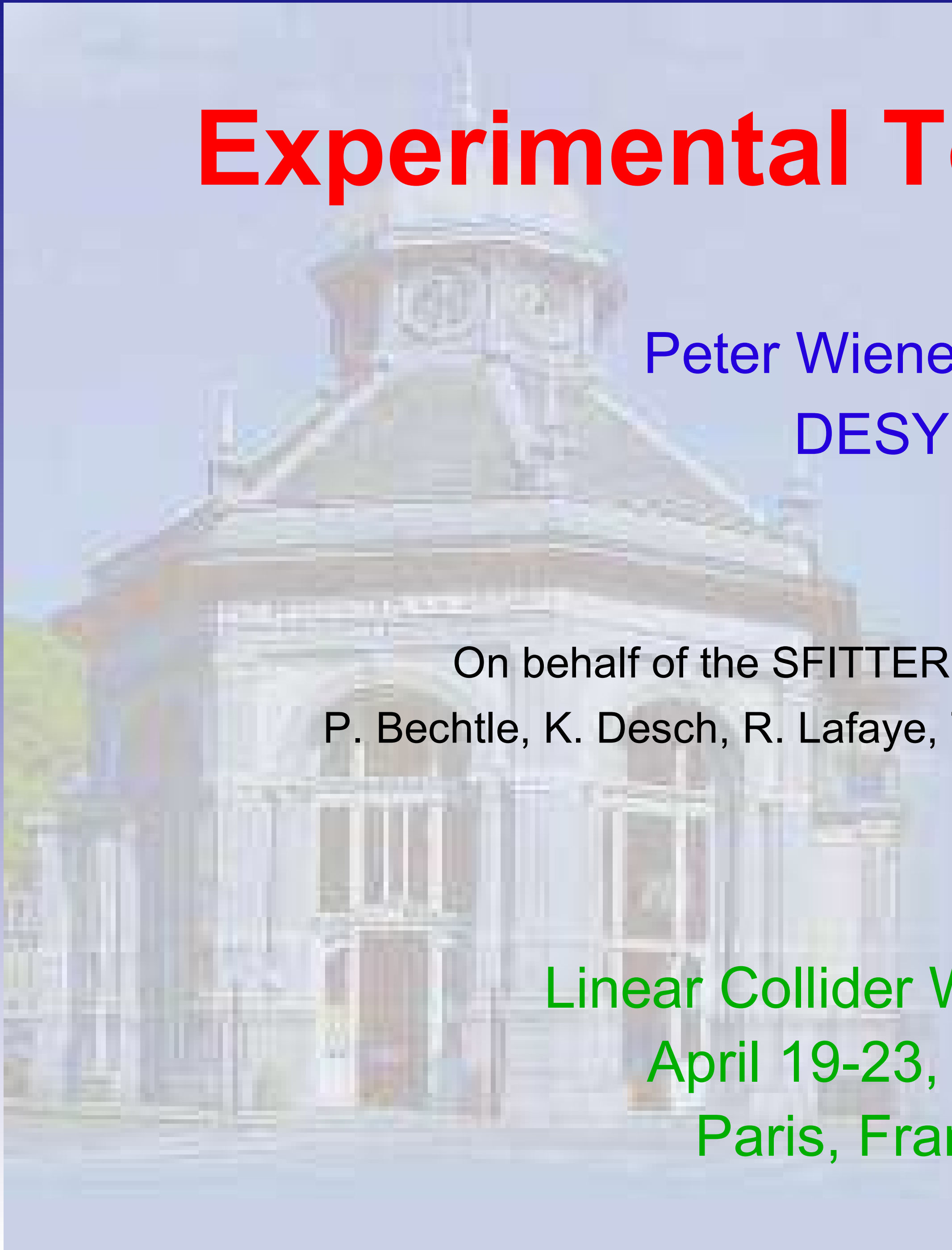

Experimental Tools for SPA

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DESY

On behalf of the SFITTER and Fittino authors:
P. Bechtle, K. Desch, R. Lafaye, T. Plehn, P. W., D. Zerwas

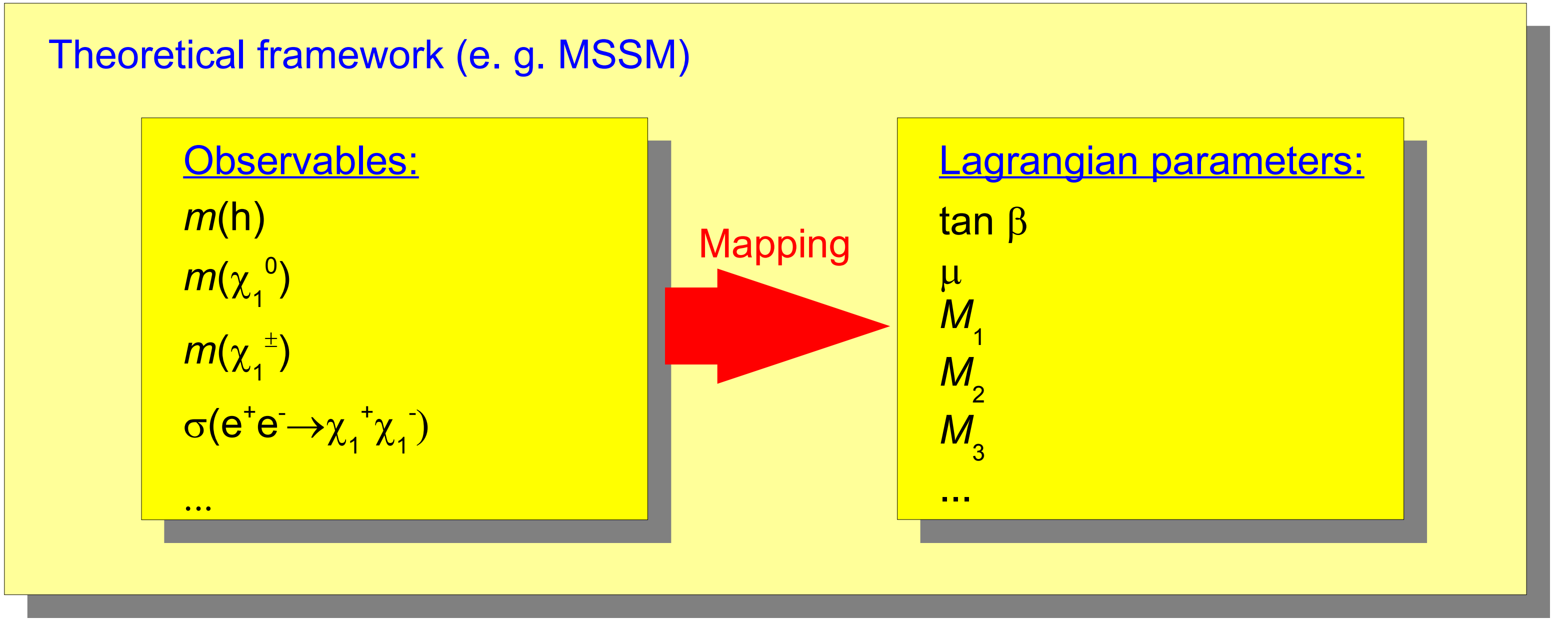
Linear Collider Workshop
April 19-23, 2004
Paris, France

The Challenge

Once SUSY has been established in experiments, Lagrangian parameters need to be extracted from measurements.

Stumbling block: Lagrangian parameters ≠ observables

Instead experiments provide: σ, BR, asymmetries, ...



Theoretical framework (e. g. MSSM)

Observables:

$m(h)$

$m(\chi_1^0)$

$m(\chi_1^\pm)$

$\sigma(e^+e^- \rightarrow \chi_1^+ \chi_1^-)$

...

Mapping

Lagrangian parameters:

$\tan\beta$

μ

M_1

M_2

M_3

...

Need a procedure to connect observables to Lagrangian parameters within a certain theoretical framework

The Challenge (2)

At tree level, some sectors (e. g. chargino, chargino+neutralino) can be treated separately.

At loop level, in principle every observable depends on every parameter.

Complicated mutual dependence of the various parameters.

Approximate picture (not quite correct since non-linear mapping):

$$\begin{bmatrix} P_1 \\ P_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} \boxed{} & & & 0 \\ & \boxed{} & & \\ & & \boxed{} & \\ 0 & & & \ddots \end{bmatrix} \begin{bmatrix} O_1 \\ O_2 \\ \vdots \end{bmatrix}$$

Tree level

$$\begin{bmatrix} P_1 \\ P_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} \boxed{} & & & \neq 0 \\ & \boxed{} & & \\ & & \boxed{} & \\ \neq 0 & & & \ddots \end{bmatrix} \begin{bmatrix} O_1 \\ O_2 \\ \vdots \end{bmatrix}$$

Loop level

The SPA Project

SPA = Supersymmetry Parameter Analysis

Common effort of theorists and experimentalists working on physics at the LHC and a future linear collider

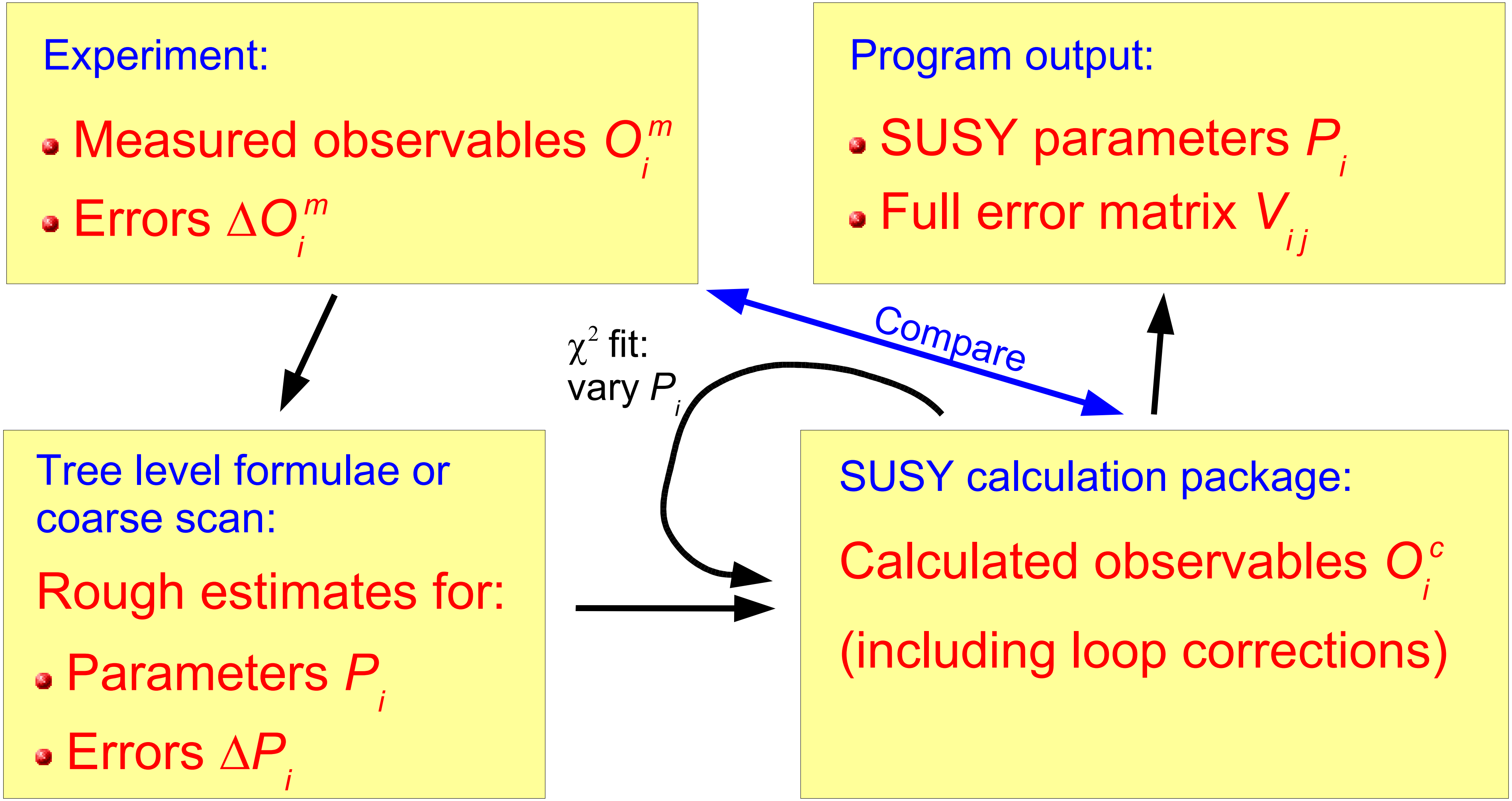
Goals:

- Determination of the SUSY Lagrangian parameters at the electroweak scale
- Extrapolation to a high scale to reconstruct the fundamental parameters and the SUSY breaking mechanism

Information about the project:

`http://spa.desy.de/spa`

Iterative Approach



Experiment:

- Measured observables O_i^m
- Errors ΔO_i^m

Program output:

- SUSY parameters P_i
- Full error matrix V_{ij}

χ^2 fit:
vary P_i

Compare

Tree level formulae or coarse scan:

Rough estimates for:

- Parameters P_i
- Errors ΔP_i

SUSY calculation package:

Calculated observables O_i^c

(including loop corrections)

The Tools

Approach implemented in two new programs presented at the EuroGDR Meeting in December 2003:

- **SFITTER** by R. Lafaye, T. Plehn and D. Zerwas
- **Fittino** by P. Bechtle, K. Desch and P. W.

Both determine SUSY parameters from observables in a global fit.

Program Components

SFITTER:

- SUSPECT (masses)
- MSMlib (BR, σ_{e+e-})
- Prospino 2.0 (NLO σ_{pp})
- MINUIT

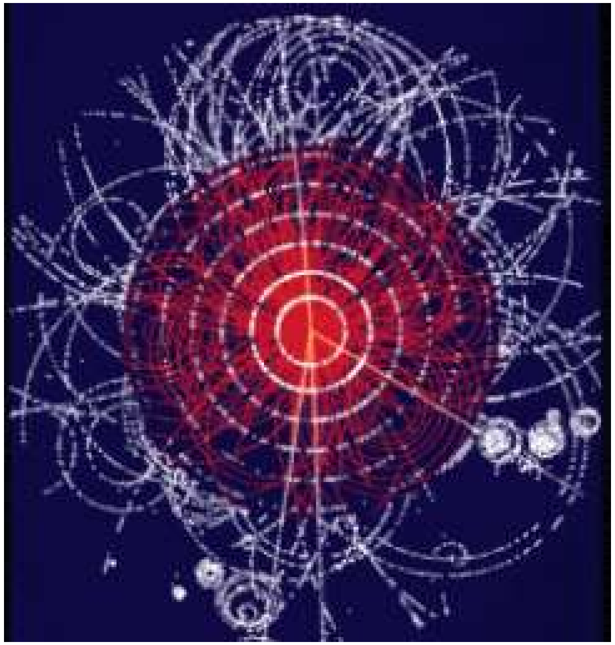
Fittino:

- SPheno 2.2.0 (masses, BR, σ_{e+e-})
- MINUIT

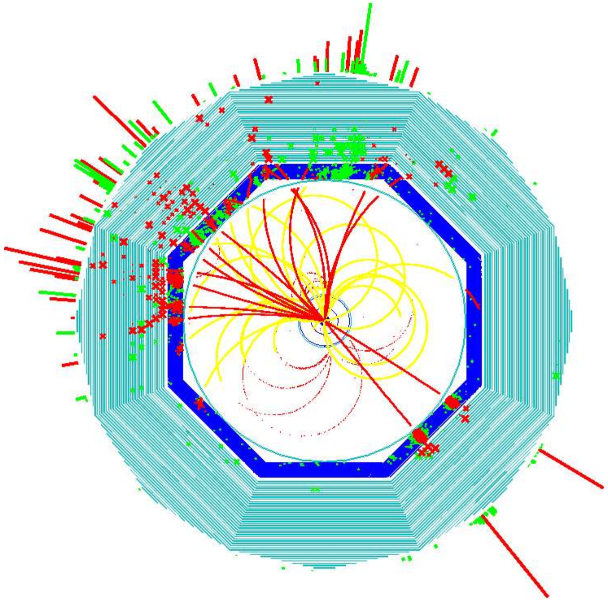
Both programs use the Les Houches Accord:

- Easy interfacing between the components
- Components can be easily exchanged or added

The Protagonists



- LHC:
  - quite comprehensive sparticle spectrum
  - typical accuracy 1-10 %

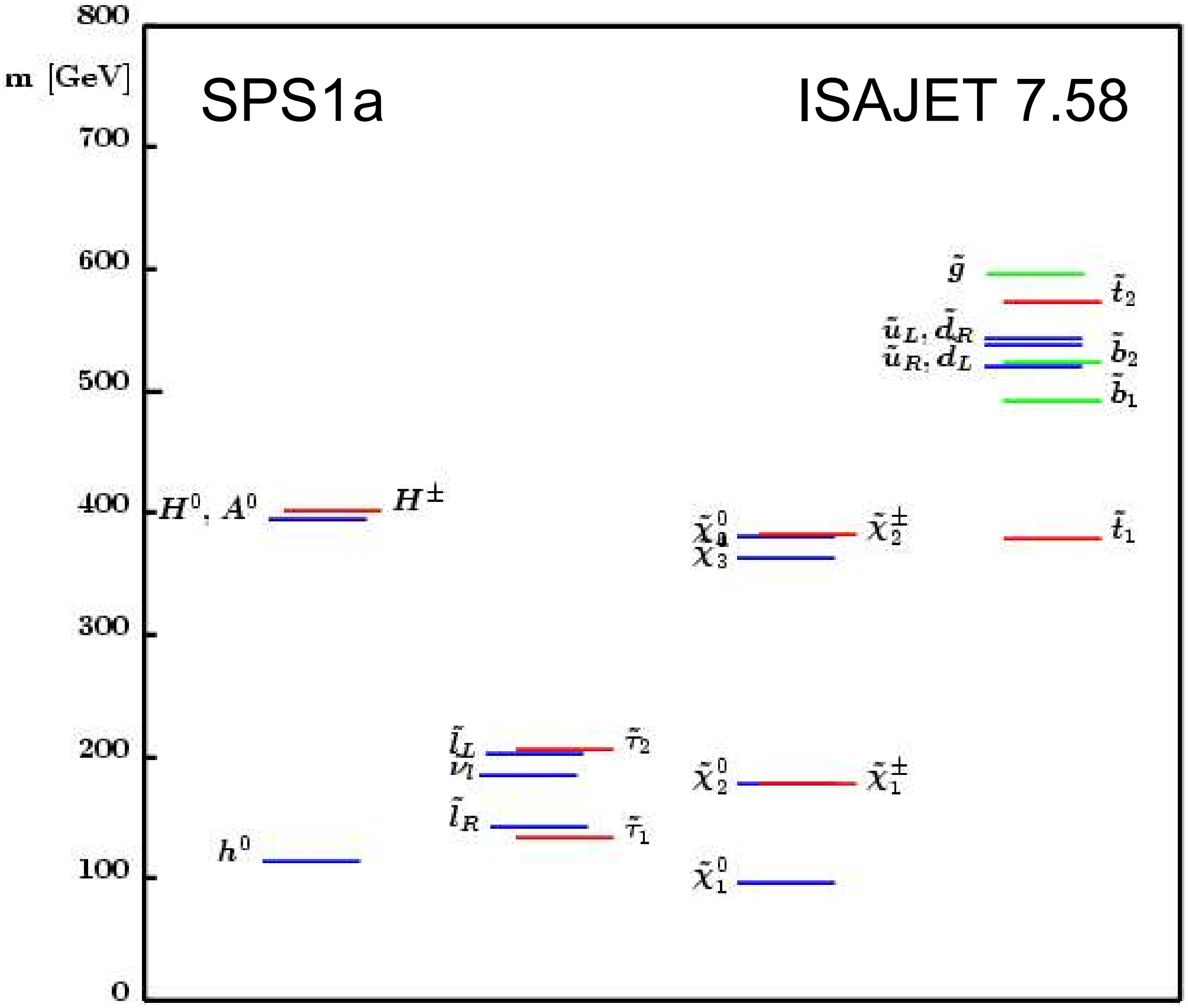


- 0.5 – 1 TeV Linear Collider:
  - only light sparticles are accessible
  - typical precision 0.1-1 %

The Fit Scenarios

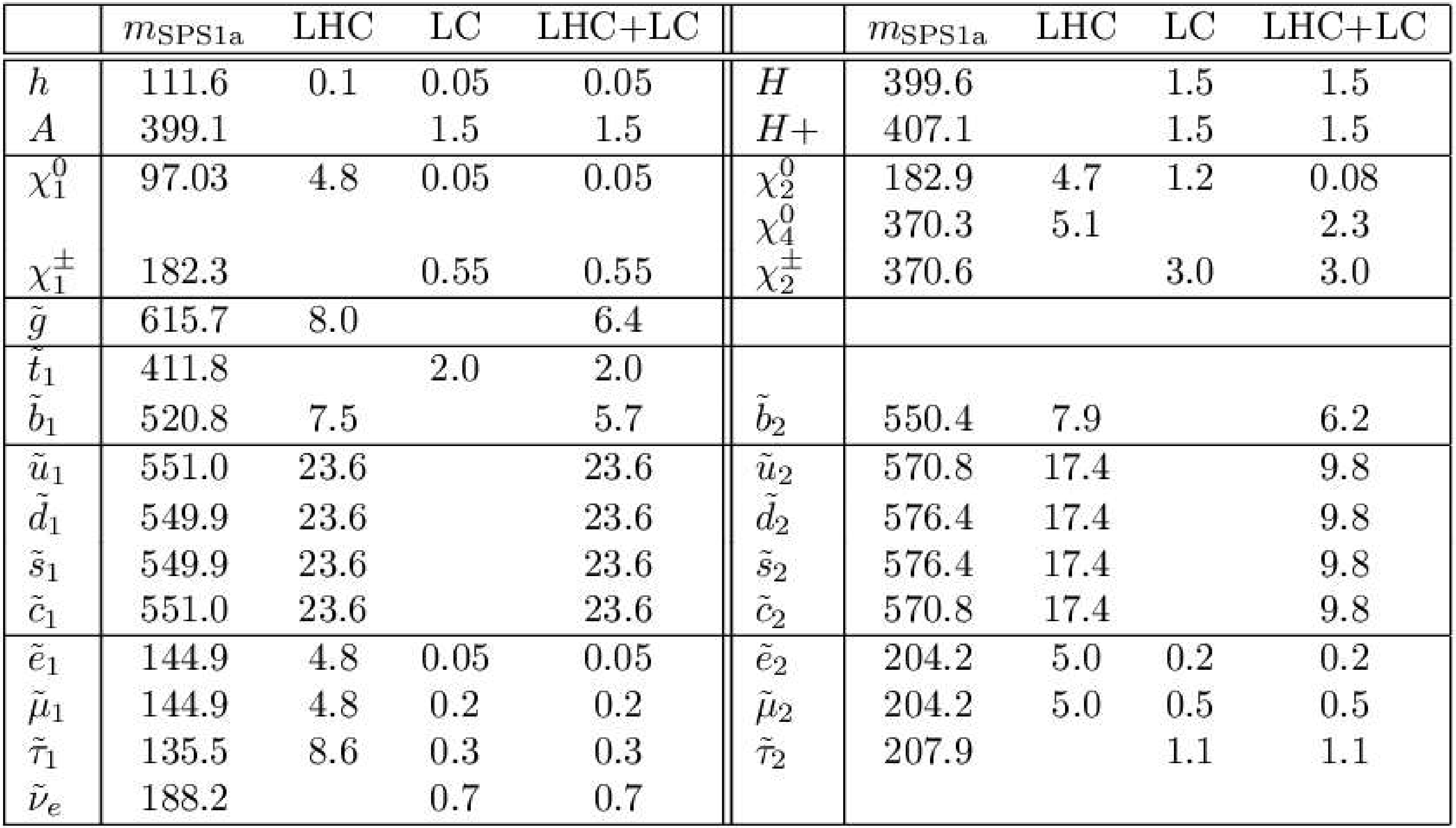
SPS1a example fits for three different scenarios:

- LHC only
- LC only
- LHC+LC



The SFITTER Inputs

Measurement precisions assumed for the SFITTER fits:



	m_{SPS1a}	LHC	LC	LHC+LC		m_{SPS1a}	LHC	LC	LHC+LC
h	111.6	0.1	0.05	0.05	H	399.6		1.5	1.5
A	399.1		1.5	1.5	$H+$	407.1		1.5	1.5
χ_1^0	97.03	4.8	0.05	0.05	χ_2^0	182.9	4.7	1.2	0.08
					χ_4^0	370.3	5.1		2.3
$\chi_1^\pm$	182.3		0.55	0.55	$\chi_2^\pm$	370.6		3.0	3.0
$\tilde{g}$	615.7	8.0		6.4					
$\tilde{t}_1$	411.8		2.0	2.0					
$\tilde{b}_1$	520.8	7.5		5.7	$\tilde{b}_2$	550.4	7.9		6.2
$\tilde{u}_1$	551.0	23.6		23.6	$\tilde{u}_2$	570.8	17.4		9.8
$\tilde{d}_1$	549.9	23.6		23.6	$\tilde{d}_2$	576.4	17.4		9.8
$\tilde{s}_1$	549.9	23.6		23.6	$\tilde{s}_2$	576.4	17.4		9.8
$\tilde{c}_1$	551.0	23.6		23.6	$\tilde{c}_2$	570.8	17.4		9.8
$\tilde{e}_1$	144.9	4.8	0.05	0.05	$\tilde{e}_2$	204.2	5.0	0.2	0.2
$\tilde{\mu}_1$	144.9	4.8	0.2	0.2	$\tilde{\mu}_2$	204.2	5.0	0.5	0.5
$\tilde{\tau}_1$	135.5	8.6	0.3	0.3	$\tilde{\tau}_2$	207.9		1.1	1.1
$\tilde{\nu}_e$	188.2		0.7	0.7					

The Fittino Inputs

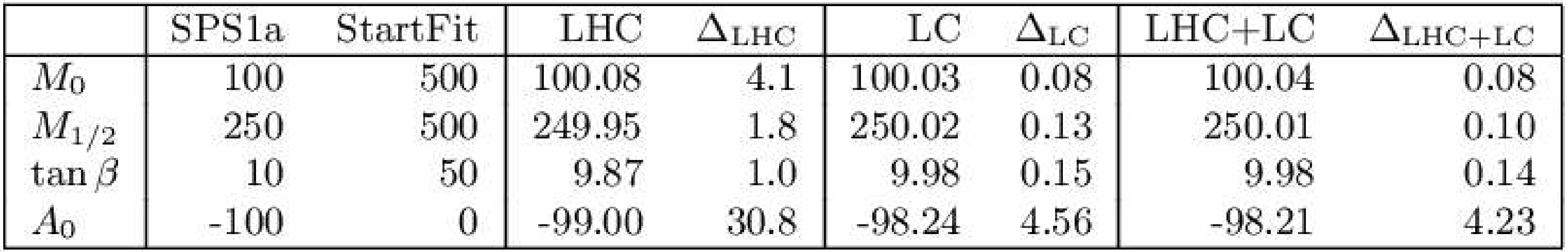
For the Fittino fits, the following measurements have been used:

Measurement	Value	Uncertainty
m_Z	91.1187 GeV	0.0021 GeV
m_W	80.3382 GeV	0.039 GeV
m_c	1.2 GeV	0.2 GeV
m_b	4.2 GeV	0.5 GeV
m_t	174.3 GeV	0.3 GeV
m_τ	1.77699 GeV	0.00029 GeV
α_s	0.1172	0.0002
G_F	$1.16639 \cdot 10^{-5}$ GeV^{-2}	$1 \cdot 10^{-11}$ GeV^{-2}
$1/\alpha$	127.934	0.027
$\sin^2\theta_W$	0.23113	0.00015
m_{h^0}	110.2 GeV	0.5 GeV
m_{H^0}	400.8 GeV	1.3 GeV
m_{A^0}	399.8 GeV	1.3 GeV
$m_{H^\pm}$	407.7 GeV	1.1 GeV
$m_{\tilde{u}_L}$	583.5 GeV	9.8 GeV
$m_{\tilde{u}_R}$	566.5 GeV	23.6 GeV
$m_{\tilde{d}_L}$	586.7 GeV	9.8 GeV
$m_{\tilde{d}_R}$	566.3 GeV	23.6 GeV
$m_{\tilde{c}_L}$	583.6 GeV	9.8 GeV
$m_{\tilde{c}_R}$	566.5 GeV	23.6 GeV
$m_{\tilde{s}_L}$	586.7 GeV	9.8 GeV
$m_{\tilde{s}_R}$	566.3 GeV	23.6 GeV
$m_{\tilde{t}_R}$	417.5 GeV	2.0 GeV
$m_{\tilde{b}_R}$	532.1 GeV	5.7 GeV
$m_{\tilde{b}_L}$	565.6 GeV	6.2 GeV
$m_{\tilde{\nu}_{eL}}$	192.3 GeV	0.7 GeV
$m_{\tilde{e}_L}$	208.0 GeV	0.2 GeV
$m_{\tilde{e}_R}$	143.91 GeV	0.05 GeV
$m_{\tilde{\mu}_L}$	208.0 GeV	0.5 GeV
$m_{\tilde{\mu}_R}$	143.9 GeV	0.2 GeV
$m_{\tilde{\tau}_R}$	134.3 GeV	0.3 GeV
$m_{\tilde{\tau}_L}$	211.8 GeV	1.1 GeV
$m_{\tilde{g}}$	630.4 GeV	6.4 GeV
$m_{\tilde{\chi}_1^0}$	95.74 GeV	0.05 GeV
$m_{\tilde{\chi}_2^0}$	182.40 GeV	0.08 GeV
$m_{\tilde{\chi}_1^\pm}$	180.46 GeV	0.55 GeV
$m_{\tilde{\chi}_2^\pm}$	380.0 GeV	3.0 GeV

Measurement	Value	Uncertainty
$\sigma\ (e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0,\ \sqrt{s} = 500\text{ GeV},\ P_{e^-} = 0.8,\ P_{e^+} = 0.6\ )$	22.7 fb	2.0 fb
$\sigma\ (e^+e^- \to \tilde{\chi}_2^0\tilde{\chi}_2^0,\ \sqrt{s} = 500\text{ GeV},\ P_{e^-} = 0.8,\ P_{e^+} = 0.6\ )$	19.5 fb	2.0 fb
$\sigma\ (e^+e^- \to \tilde{e}_L\tilde{e}_L,\ \sqrt{s} = 500\text{ GeV},\ P_{e^-} = 0.8,\ P_{e^+} = 0.6)$	205.0 fb	4.0 fb
$\sigma\ (e^+e^- \to \tilde{\mu}_L\tilde{\mu}_L,\ \sqrt{s} = 500\text{ GeV},\ P_{e^-} = 0.8,\ P_{e^+} = 0.6)$	36.8 fb	4.0 fb
$\sigma\ (e^+e^- \to \tilde{\tau}_1\tilde{\tau}_1,\ \sqrt{s} = 500\text{ GeV},\ P_{e^-} = 0.8,\ P_{e^+} = 0.6)$	39.1 fb	4.0 fb
$\sigma\ (e^+e^- \to \tilde{\chi}_1^\pm\tilde{\chi}_1^\mp,\ \sqrt{s} = 500\text{ GeV},\ P_{e^-} = 0.8,\ P_{e^+} = 0.6)$	46.7 fb	1.0 fb
$\sigma\ (e^+e^- \to Z\ h^0,\ \sqrt{s} = 500\text{ GeV},\ P_{e^-} = 0.8,\ P_{e^+} = 0.6\ )$	11.13 fb	0.21 fb
$\sigma\ (e^+e^- \to \tilde{\chi}_1^\pm\tilde{\chi}_1^\mp,\ \sqrt{s} = 500\text{ GeV},\ P_{e^-} = -0.8,\ P_{e^+} = -0.6\ )$	104.8 fb	3.5 fb
$\sigma\ (e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0,\ \sqrt{s} = 500\text{ GeV},\ P_{e^-} = -0.8,\ P_{e^+} = -0.6)$	43.9 fb	2.0 fb
$\sigma\ (e^+e^- \to \tilde{\chi}_2^0\tilde{\chi}_2^0,\ \sqrt{s} = 500\text{ GeV},\ P_{e^-} = -0.8,\ P_{e^+} = -0.6)$	43.8 fb	2.0 fb
$\sigma\ (e^+e^- \to \tilde{e}_L\tilde{e}_L,\ \sqrt{s} = 500\text{ GeV},\ P_{e^-} = -0.8,\ P_{e^+} = -0.6)$	97.4 fb	4.0 fb
$\sigma\ (e^+e^- \to \tilde{e}_L\tilde{e}_R,\ \sqrt{s} = 500\text{ GeV},\ P_{e^-} = -0.8,\ P_{e^+} = -0.6)$	223.7 fb	4.0 fb
$\sigma\ (e^+e^- \to \tilde{e}_R\tilde{e}_R,\ \sqrt{s} = 500\text{ GeV},\ P_{e^-} = -0.8,\ P_{e^+} = -0.6)$	29.0 fb	2.0 fb
$\sigma\ (e^+e^- \to \tilde{\mu}_L\tilde{\mu}_L,\ \sqrt{s} = 500\text{ GeV},\ P_{e^-} = -0.8,\ P_{e^+} = -0.6)$	22.7 fb	2.0 fb
$\sigma\ (e^+e^- \to \tilde{\tau}_1\tilde{\tau}_1,\ \sqrt{s} = 500\text{ GeV},\ P_{e^-} = -0.8,\ P_{e^+} = -0.6)$	25.7 fb	2.0 fb
BR ($h^0 \to b\bar{b}$)	0.82	0.01
BR ($h^0 \to c\bar{c}$)	0.04	0.01
BR ($h^0 \to \tau^+\tau^-$)	0.14	0.01

mSUGRA Fit Results

- Fit with SFITTER
- Input observables not smeared within their errors, no systematic and theory errors included
- Fit start values: mean of upper and lower bound (not necessarily close to true value)



	SPS1a	StartFit	LHC	Δ_{LHC}	LC	Δ_{LC}	LHC+LC	$\Delta_{\mathrm{LHC+LC}}$
M_0	100	500	100.08	4.1	100.03	0.08	100.04	0.08
$M_{1/2}$	250	500	249.95	1.8	250.02	0.13	250.01	0.10
$\tan\beta$	10	50	9.87	1.0	9.98	0.15	9.98	0.14
A_0	-100	0	-99.00	30.8	-98.24	4.56	-98.21	4.23

- True SPA1a values well reconstructed for all parameters
- Missing strongly interacting particles at a LC do not significantly worsen parameter determination due to mSUGRA unification
- Scalar and gaugino mass very precise from LC

General MSSM

Without assuming a certain SUSY breaking scenario, the MSSM contains 105 SUSY parameters (masses, phases, mixing angles)

→ too many parameters for a fit

Simplifying assumptions:

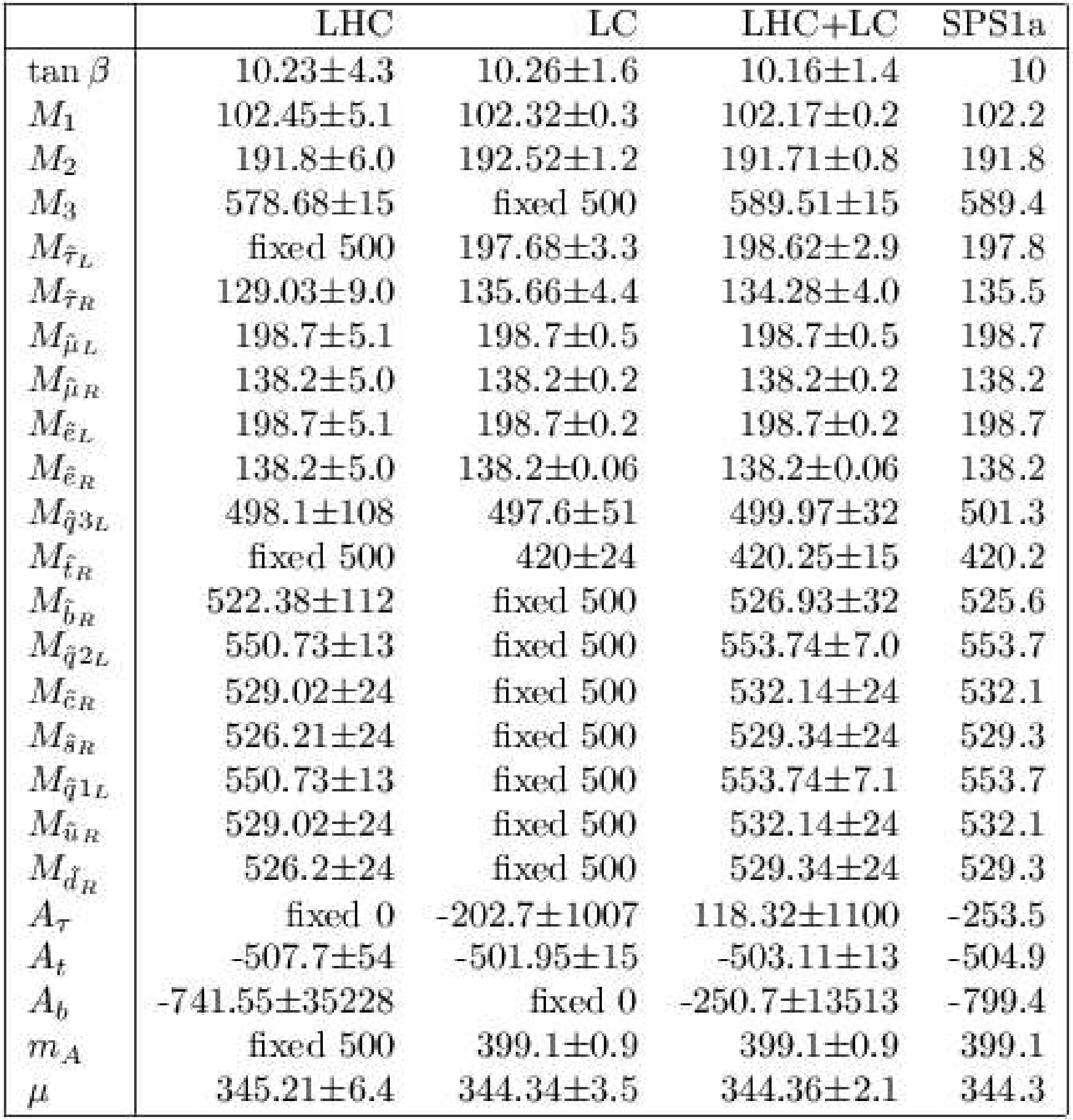
- all phases = 0
- no mixing between generations
- no mixing within first two generations

→ 24 parameters remain

General MSSM Fit Results

- Fit with SFITTER
- Input observables not smeared within their errors, no systematic and theory errors included
- Fit start values: M_1, M_2, μ and tan β from coarse scan, other parameters fixed to true values

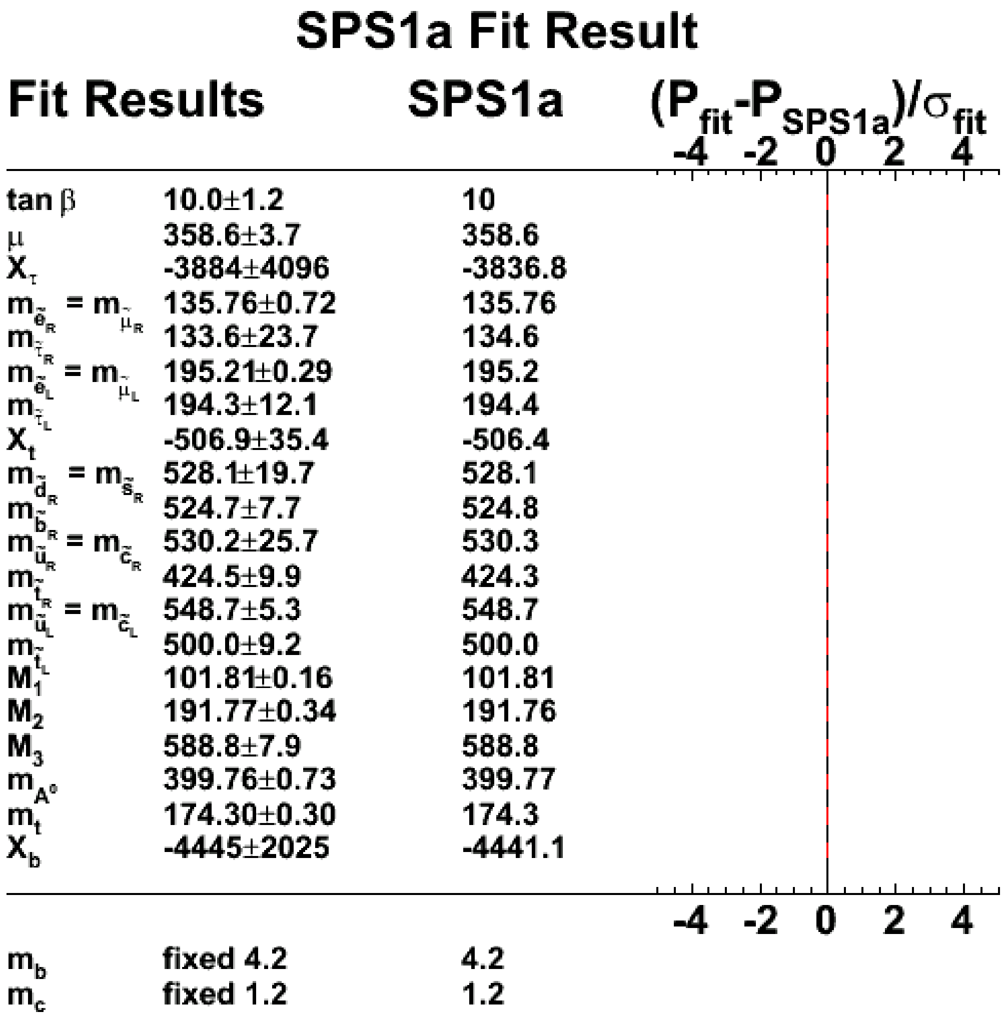
- Parameters well reconstructed
- Only combined LHC and LC input allows a complete fit without fixing parameters



	LHC	LC	LHC+LC	SPS1a
$\tan\beta$	10.23±4.3	10.26±1.6	10.16±1.4	10
M_1	102.45±5.1	102.32±0.3	102.17±0.2	102.2
M_2	191.8±6.0	192.52±1.2	191.71±0.8	191.8
M_3	578.68±15	fixed 500	589.51±15	589.4
$M_{\tilde{\tau}_L}$	fixed 500	197.68±3.3	198.62±2.9	197.8
$M_{\tilde{\tau}_R}$	129.03±9.0	135.66±4.4	134.28±4.0	135.5
$M_{\tilde{\mu}_L}$	198.7±5.1	198.7±0.5	198.7±0.5	198.7
$M_{\tilde{\mu}_R}$	138.2±5.0	138.2±0.2	138.2±0.2	138.2
$M_{\tilde{e}_L}$	198.7±5.1	198.7±0.2	198.7±0.2	198.7
$M_{\tilde{e}_R}$	138.2±5.0	138.2±0.06	138.2±0.06	138.2
$M_{\tilde{q}3_L}$	498.1±108	497.6±51	499.97±32	501.3
$M_{\tilde{t}_R}$	fixed 500	420±24	420.25±15	420.2
$M_{\tilde{b}_R}$	522.38±112	fixed 500	526.93±32	525.6
$M_{\tilde{q}2_L}$	550.73±13	fixed 500	553.74±7.0	553.7
$M_{\tilde{c}_R}$	529.02±24	fixed 500	532.14±24	532.1
$M_{\tilde{s}_R}$	526.21±24	fixed 500	529.34±24	529.3
$M_{\tilde{q}1_L}$	550.73±13	fixed 500	553.74±7.1	553.7
$M_{\tilde{u}_R}$	529.02±24	fixed 500	532.14±24	532.1
$M_{\tilde{d}_R}$	526.2±24	fixed 500	529.34±24	529.3
A_τ	fixed 0	-202.7±1007	118.32±1100	-253.5
A_t	-507.7±54	-501.95±15	-503.11±13	-504.9
A_b	-741.55±35228	fixed 0	-250.7±13513	-799.4
m_A	fixed 500	399.1±0.9	399.1±0.9	399.1
μ	345.21±6.4	344.34±3.5	344.36±2.1	344.3

General MSSM Fit Results

- Fit with Fittino
- Input observables not smeared within their errors, no syst.+theory errors (except for m_h)
- Start values: From tree level formulae
- Assumed universality in 1st and 2nd generation
- All parameters well reconstructed



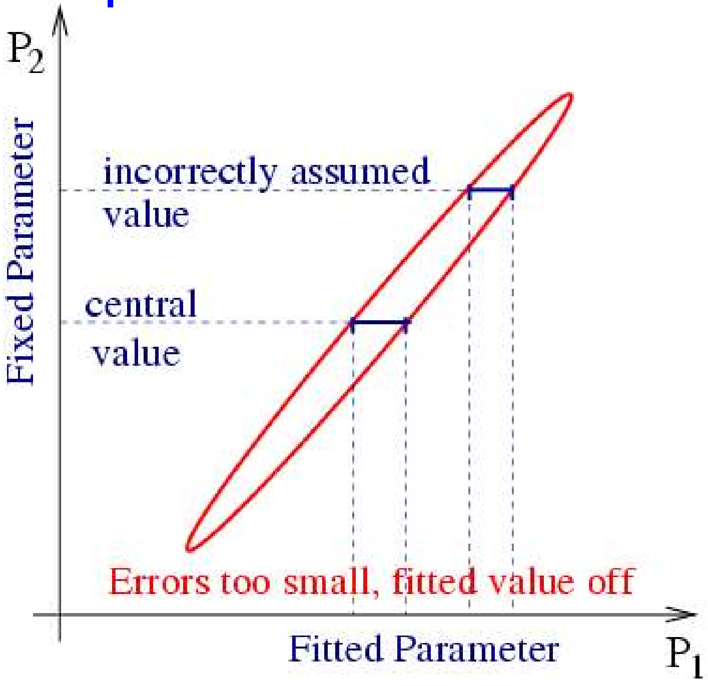
SPS1a Fit Result

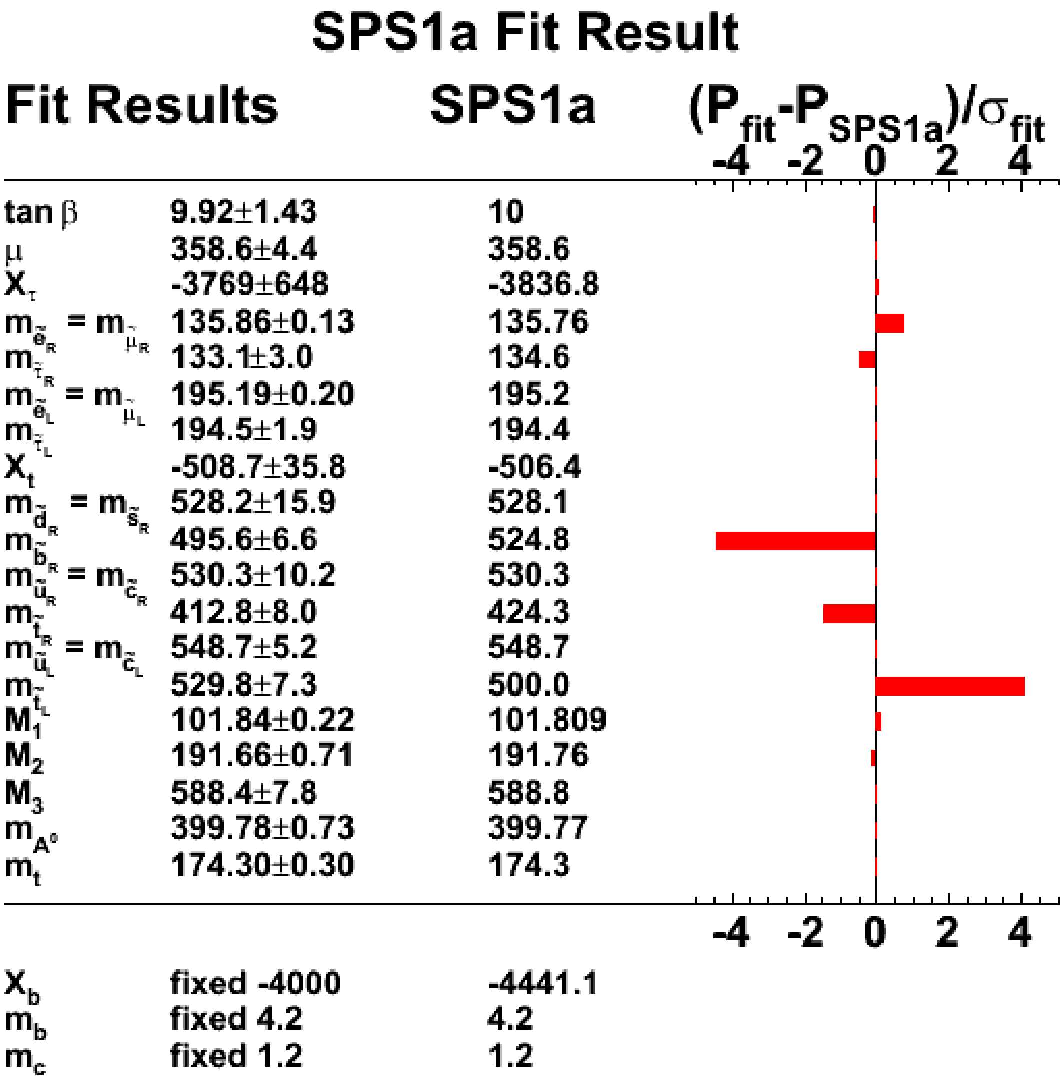
	Fit Results	SPS1a	$(P_{fit}-P_{SPS1a})/\sigma_{fit}$
$\tan\beta$	10.0 ± 1.2	10	
μ	358.6 ± 3.7	358.6	
X_τ	-3884 ± 4096	-3836.8	
$m_{\tilde{e}_R} = m_{\tilde{\mu}_R}$	135.76 ± 0.72	135.76	
$m_{\tilde{\tau}_R}$	133.6 ± 23.7	134.6	
$m_{\tilde{e}_L} = m_{\tilde{\mu}_L}$	195.21 ± 0.29	195.2	
$m_{\tilde{\tau}_L}$	194.3 ± 12.1	194.4	
X_t	-506.9 ± 35.4	-506.4	
$m_{\tilde{d}_R} = m_{\tilde{s}_R}$	528.1 ± 19.7	528.1	
$m_{\tilde{b}_R}$	524.7 ± 7.7	524.8	
$m_{\tilde{u}_R} = m_{\tilde{c}_R}$	530.2 ± 25.7	530.3	
$m_{\tilde{t}_R}$	424.5 ± 9.9	424.3	
$m_{\tilde{u}_L} = m_{\tilde{c}_L}$	548.7 ± 5.3	548.7	
$m_{\tilde{t}_L}$	500.0 ± 9.2	500.0	
M_1	101.81 ± 0.16	101.81	
M_2	191.77 ± 0.34	191.76	
M_3	588.8 ± 7.9	588.8	
m_{A^0}	399.76 ± 0.73	399.77	
m_t	174.30 ± 0.30	174.3	
X_b	-4445 ± 2025	-4441.1	
m_b	fixed 4.2	4.2	
m_c	fixed 1.2	1.2	

General MSSM Fit Results

- Same fit but fixed X_b (weak sensitivity)
- Fixing X_b to wrong value distorts 3rd generation mass spectrum
- Observables from all sectors are essential to avoid fixing parameters

Explanation:





SPS1a Fit Result

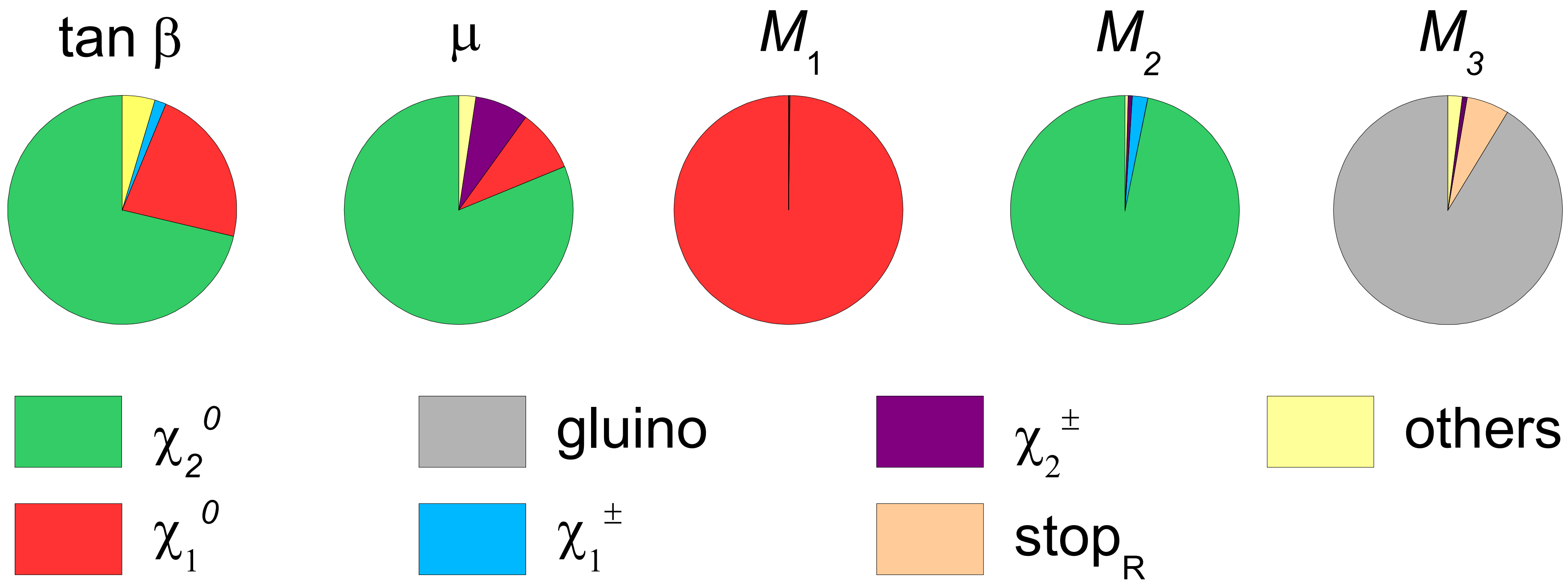
	Fit Results	SPS1a
$\tan\beta$	9.92±1.43	10
μ	358.6±4.4	358.6
X_τ	-3769±648	-3836.8
$m_{\tilde{e}_R} = m_{\tilde{\mu}_R}$	135.86±0.13	135.76
$m_{\tilde{\tau}_R}$	133.1±3.0	134.6
$m_{\tilde{e}_L} = m_{\tilde{\mu}_L}$	195.19±0.20	195.2
$m_{\tilde{\tau}_L}$	194.5±1.9	194.4
X_t	-508.7±35.8	-506.4
$m_{\tilde{d}_R} = m_{\tilde{s}_R}$	528.2±15.9	528.1
$m_{\tilde{b}_R}$	495.6±6.6	524.8
$m_{\tilde{u}_R} = m_{\tilde{c}_R}$	530.3±10.2	530.3
$m_{\tilde{t}_R}$	412.8±8.0	424.3
$m_{\tilde{u}_L} = m_{\tilde{c}_L}$	548.7±5.2	548.7
$m_{\tilde{t}_L}$	529.8±7.3	500.0
M_1	101.84±0.22	101.809
M_2	191.66±0.71	191.76
M_3	588.4±7.8	588.8
m_{A^0}	399.78±0.73	399.77
m_t	174.30±0.30	174.3
X_b	fixed -4000	-4441.1
m_b	fixed 4.2	4.2
m_c	fixed 1.2	1.2

$\Delta\chi^2$ Contributions

Sensitivity of MSSM parameters on the various observables:

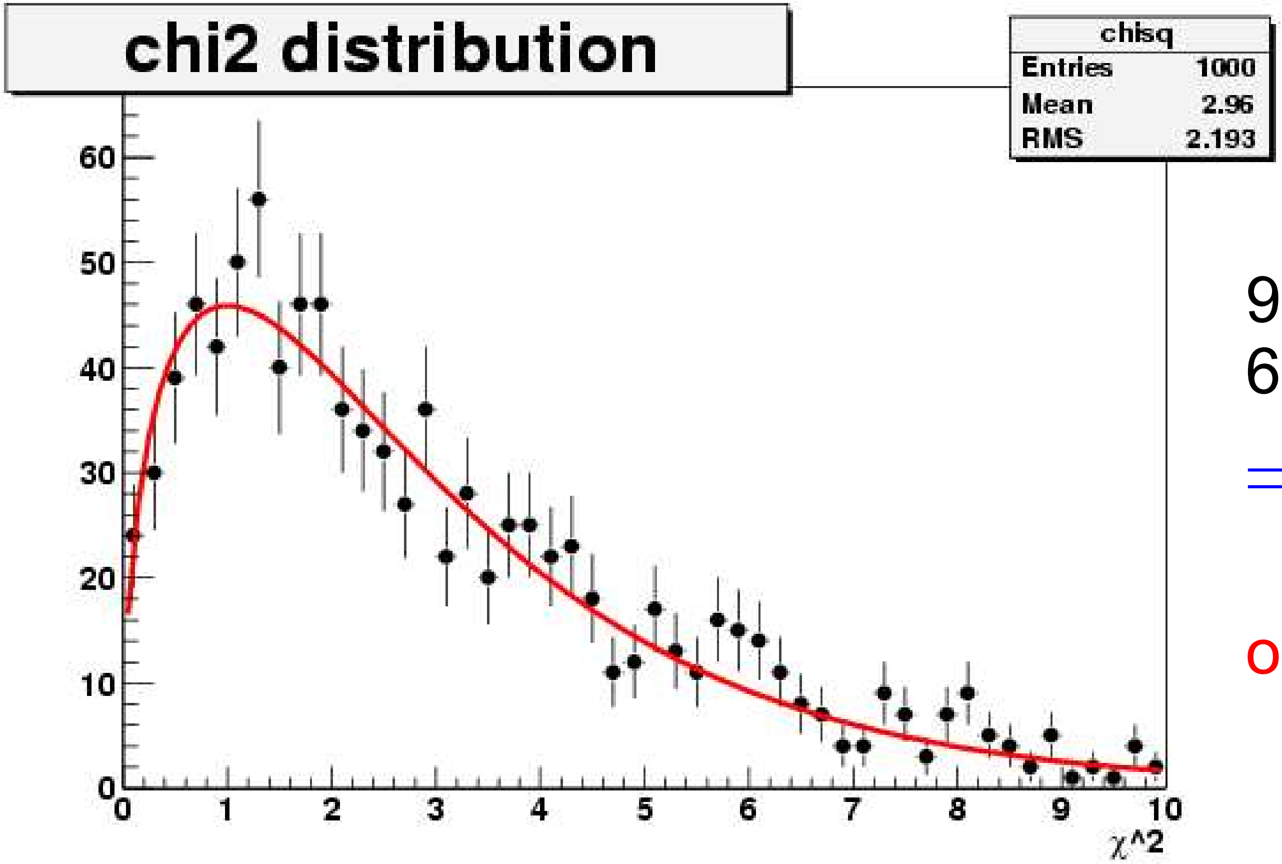
Individual $\Delta\chi^2$ contributions: Vary parameter by $\pm 1\sigma$ and determine $\Delta\chi^2$ of the various observables

Example: General MSSM Fit



Fit with Smeared Measurements

- Reduced observable and parameter set
  9 observables, 6 fit parameters
- Simulate 1000 different measurements of the observables
- Smear observables within their errors and carry out fit



9 observables,
6 fit parameters:

$\Rightarrow$ expected:
$\langle\chi^2\rangle = 3$

observed: $\langle\chi^2\rangle = 2.96$

Summary

- Two powerful tools, SFITTER and Fittino, are available for SUSY parameter analysis.
- Both have been successfully tested in example fits for the SPS1a scenario.
- Precision of mSUGRA parameters driven by LC measurements.
- Observables from all sectors are essential in general MSSM fits. Here, LHC and LC perfectly complement each other.

Outlook

- Find observables to get a better handle on A_τ and A_b.
- Check fitting procedure for other Snowmass points.
- Take correlations between input observables into account.
- Other SUSY packages welcome to cross-check and extend the fit results.