Summary of SUSY studies
Outline

• Supersymmetry Parameter Analysis
  – Goals
  – Conventions
  – Tools
  – Radiative corrections
  – Measuring SUSY parameters: neutralinos, sleptons, stops
  – LHC/LC analyses
  – Reconstructing supersymmetric theories

• CP violation
• SUSY for Cosmology
• Neutrino masses and SUSY: (s)Lepton flavour violation
Preview

• High-luminosity e+e- LC is a machine for precision measurements
• What level of precision can be achieved?
  – Many experimental studies on measurements of different SUSY parameters
  – Theoretical predictions at loop-level

• Discovery of Higgs and SUSY would be fantastic, the real question then is: What can we learn from precision measurements?
  – Reconstructing fundamental supersymmetric theory at the GUT scale, understanding SUSY breaking mechanism…
  – Learning more about cosmology
• Should keep an eye on other experiments that can give us a clue on where supersymmetry might be (or cannot be)
  – Hadronic colliders, rare decays: bsgamma.., cosmology: relic density direct searches for darkmatter…
SPA project

Supersymmetry has been discovered at LHC and some supersymmetric particles (sleptons, charginos, neutralinos) are kinematically accessible at linear collider.

- Goals
  - High-precision determination of SUSY parameters at EW scale
  - Extrapolation to high scale to reconstruct the fundamental theory: elucidating the supersymmetry breaking mechanism

Theoretical knowledge at one-loop or more

Use combined experimental simulations at LHC+LC

Group was started within ECFA SUSY group, should include also groups from Asia and America.

Please join the effort
SPA-Goals

• How precisely can one determine masses, cross-sections, branching ratios, couplings..

• What precision can be achieved on parameters of MSSM Lagrangian
  – Lagrangian parameters are not measured directly
  – Many relations between sparticle masses already at tree-level, even worse at loop-level
  – Some parameters are not directly related to one observable: $\mu$, $\tan \beta$ ..
  – Choice of renormalization condition : no obvious best choice
  – In practice, fitting procedure: comparison data/Monte-Carlo

• Reconstructing the fundamental theory: going back to the high scale
  – Test of unification of couplings, masses etc..
  – Which supersymmetry breaking mechanism?
### Parameters

<table>
<thead>
<tr>
<th>Physical parameters</th>
<th>MSSM parameters</th>
<th>GUT scale parameters</th>
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<tbody>
<tr>
<td>Masses, branching ratios, Cross-sections</td>
<td>$M_1$, $M_2$, $M_3$</td>
<td>mSUGRA: $M_0$, $m_{1/2}$, $A$, $\tan\beta$, $\text{sgn}(\mu)$</td>
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<tr>
<td>Neutralino/chargino</td>
<td>$\mu$, $\tan\beta$</td>
<td>String inspired models</td>
</tr>
<tr>
<td>Sleptons</td>
<td>$M^2_{L_1}$, $M^2_{E_1}$, $\mu$, $\tan\beta$, $A_f$</td>
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<td>Squarks</td>
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<td>Higgs (h,H,A)</td>
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Parameters

Physical parameters
- Masses, branching ratios, Cross-sections

MSSM parameters
- Neutralino/chargino: $M_1, M_2, M_3, \mu, \tan \beta$
- Sleptons: $M^2_{L_1}, M^2_{E_1}$
- Squarks: $M^2_{Q_1}, M^2_{U_1}, M^2_{D_1}$
- Higgs (h, H, A)

GUT scale parameters
- mSUGRA: $M_0, m_{1/2}, A, \tan \beta, \text{sgn}(\mu)$
- String inspired models
- GMSB
- AMSB
- Top-down

Bottom up
- Fit
- RGE

LCWS-Paris, 23/04/2004
SPA-Goals

• How precisely can one determine masses, cross-sections, branching ratios, couplings..

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• Reconstructing the fundamental theory: going back to the high scale
  – Test of unification of couplings, masses etc..
  – Which supersymmetry breaking mechanism?
SPA conventions

Clearly defined framework needed for:
- comparison of theoretical calculations
- extracting parameters from data
- extrapolation to high scale

- As a starting point/testing point: SPS1a in mSUGRA
  - favourable point for both LC and LHC and many analyses already performed
  - Agree with existing constraints at $2\sigma$, although WMAP prefers lighter sleptons

SPA is more general than this scenario

Framework for communication between tools:
- Susy Les Houches Accord
SPA conventions

Agreement on parameter definitions:

- The masses of SUSY particles and Higgs bosons are given as pole masses.
- All SUSY Lagrange parameters including $\tan \beta$ are given in the $\overline{D}\bar{R}$ scheme defined at the scale $\tilde{M} = 1$ TeV.
- Mass matrices, rotation matrices and corresponding mixing angles at tree level are given in the $\overline{D}\bar{R}$ scheme at $\tilde{M} = 1$ TeV, except for the Higgs sector where the mixing angle is defined in the on-shell scheme.
- The Standard Model input parameters are $G_F$, $\alpha$, $m_Z$, $\alpha_s$ and the fermion masses.
- Branching ratios are calculated using pole masses and running mixing matrices.
- Cross sections are calculated using pole masses and running mixing matrices.
SPS1a/SPA1a

\[ m_0 \quad 100 \text{ GeV} \]
\[ m_{1/2} \quad 250 \text{ GeV} \]
\[ A_0 \quad -100 \text{ GeV} \]
\[ \tan \beta \quad 10 \]
\[ \text{sign } \mu \quad + \]

Spheno 2.2.0
www-theorie.physik.unizh.ch/~porod/Spheno.html
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<p>| $\tilde{\chi}^+_1$ | 97.12 | stable |
| $\tilde{\chi}^0_2$ | 181.2 | 0.022 | $\tilde{e}_R e^+$ | 0.062 |
| &amp; | | | $\tilde{\mu}_R \mu^+$ | 0.064 |
| &amp; | | | $\tilde{\tau}_1 \tau^+$ | 0.869 |
| $\tilde{\chi}^0_3$ | 367.7 | 2.0 | $\tilde{\chi}^0 W^+$ | 0.591 |
| &amp; | | | $\tilde{\chi}^0 Z^0$ | 0.115 |
| &amp; | | | $\tilde{\chi}^-_1 Z^0$ | 0.209 |</p>
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+ cross-sections (Spheno2.2)
SPA document

- Draft of a document defining conventions as started circulating within ECFA SUSY group.

- Will soon be circulating within much wider community
  - YOUR INPUT IS NEEDED

- Once agreement is reached, much easier for authors of various codes to provide the appropriate translation between their own conventions and SPA.
Radiative corrections

• Complete one-loop corrections to neutralino-chargino sector
  – Blank, Hollik, hep-ph/0011092 (chargino pairs)
  – Fritzsch, Hollik, hep-ph/0203159 (masses)
  – Oller, Eberl, Majerotto, hep-ph/0402134 (neutralino pairs)

• Complete one-loop corrections to sfermion pair production including third generation
  – Arhrib, Hollik, hep-ph/0311149

• Full one-loop for squark-> q+neutralino,q+chargino
  – Guasch, Hollik, Sola, hep-ph/0207364

• Full one-loop for pseudoscalar Higgs/sfermion/sfermion

• Tool for calculating Sparticles decays, including some QCD corrections: SDECAY
  – M. Muehleitner, Djouadi, Mambrini

Weak Corrections are important, sometimes comparable to SUSY-QCD and definitely relevant for high precision measurements at LC
Neutralinos: SPS1a

\[ e^+e^- \rightarrow \tilde{\chi}^0_1 \tilde{\chi}^0_2 \]

Oller et al, hep-ph/0402134

\[ \sigma_{\text{tot}} \] vs. \[ \sqrt{s} \text{ [GeV]} \]

\[ \Delta \sigma \text{ [%]} \] vs. \[ \sqrt{s} \text{ [GeV]} \]

Full, no ISR

Weak

Non-universal QED

G. Bélanger

LCWS-Paris, 23/04/2004
Weak corrections are important at high energies
Analysis of the Slepton sector

- Determination of slepton parameters: masses, mixings, couplings...
- One loop calculations completed

Simulations/decays: Martyn, Nauenberg '03

old: $\mu_R \rightarrow \mu \tilde{\chi}^0_1$
new: $\tilde{\nu}_e \rightarrow e^- \tilde{\chi}^+_1$ and $\tilde{\tau}_1 \rightarrow \tau \rightarrow 3\pi$. 

G. Bélanger

LCWS-Paris, 23/04/2004
Measuring physical parameters: Sneutrinos

Sneutrino spectrum: One $\tilde{\nu}$ decaying invisibly

$$e^+ e^- \rightarrow \tilde{\nu}_e \tilde{\nu}_e \rightarrow \nu_e \chi_1^0 e^+ \tilde{\chi}_1^\mp \rightarrow \nu_e \chi_1^0 e^{\pm} \tau^\mp \nu_\tau \tilde{\chi}_1^0 \rightarrow e^{\pm} \mu^{\mp} + E$$

Nauenberg et al. '02
Threshold scans

Simulations/production:

\[ e^+e^- \rightarrow \mu^+\mu^- \rightarrow \mu^+\mu^- + E \]

\[ \alpha \beta^3 \]

Freitas, v. Manteuffel, Martyn, Nauenberg, Zerwas

\[ e^+e^- \rightarrow \bar{\nu}_e^*\bar{\nu}_e \]

\[ \rightarrow e^+\chi_1^0 \nu_e\bar{\chi}_1^- \]

\[ \rightarrow e^+\tau^- + E \]

\[ \alpha \beta \]

\[ \alpha \beta^3 \]

incl. beamstrahlung, ISR, etc.

G. Bélanger

LCWS-Paris, 23/04/2004
3rd generation

Determination of $\tilde{\tau}$ masses as before
$m_{\tilde{\tau}_2}$ at SPS1a not yet clear

Mixing angle $\theta_{\tilde{\tau}}$ from $\sigma(\tilde{\tau}_1 \tilde{\tau}_1)$ with polarized $e^\pm$ beams
$\rightarrow \tilde{\tau}_1, \tilde{\tau}_2$ couple differently to $Z$

$\Rightarrow \cos 2\theta_{\tilde{\tau}} = -0.84 \pm 0.04$

Martyn '03

Ultimate goal:
Extract $A_{\tau}$ using
$$A_{\tau} = \frac{m_{\tilde{\tau}_2}^2 - m_{\tilde{\tau}_1}^2}{m_{\tau}} \sin 2\theta_{\tilde{\tau}} + \mu \tan \beta$$
from $\chi$ sector
$$\text{intern: } \tau \text{ polarization}$$
$$\text{extern: } \chi \text{ or Higgs sector}$$
Boos et al. '03

difficult due to large cancellations
<table>
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<td>$-$</td>
<td>1.1(^{(d)})</td>
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\(^{(a)}\) from $\tilde{e}_R$ spectrum using selectron mass determined at threshold  
\(^{(b)}\) from $\tilde{\mu}_R$ spectrum using $\tilde{\chi}_1^0$ mass as input  
\(^{(c,d)}\) estimate for threshold scan [P. Grannis]
The kinematics of cascade decays

Let's look at $e^+e^- \rightarrow \tilde{\chi}_2^0\tilde{\chi}_2^0$, with $\tilde{\chi}_2^0 \rightarrow u\bar{u} \rightarrow n\tilde{\chi}_1^0$ (or eg. $e^+e^- \rightarrow \tilde{\chi}_3^0\tilde{\chi}_3^0$, with $\tilde{\chi}_3^0 \rightarrow Z\tilde{\chi}_2^0 \rightarrow ZZ\tilde{\chi}_1^0$)

Assume $M_{\tilde{\chi}_i^0}$ are known to some extent. Count unknowns and equations:

- Two $\tilde{\chi}_1^0$ four-momenta $= 8$ unknowns
- $E$ & $\vec{p}$ conservation + four mass-relations $= 8$ constraints.

Hence, it should be possible to fully reconstruct the four-momenta, and since the SM particles are ($Z$, $\ell$) are observed and measured in the detector the Four-momentum of the intermediate SUSY particle is measurable in each event.

There was no need to assume any value of the mass of this particle. The reconstruction is hence a direct measurement of the sparticle mass.

Note that this is no rare process: As soon as pair-production threshold of the NNLSNP is passed, the NLSP can be reconstructed.

In eg, SPS1a, there are more $\tau$s produced in $\tilde{\chi}_2^0\tilde{\chi}_2^0$ decays, than in direct $\tau$-pair production!
Slepton masses in cascade decays

One might suspect that a number of things might cause problems:

- Beam-strahlung NO
- ISR NO
- Input assumptions on $M_{\tilde{\chi}_2^0}$ NO
- Background NO

The main background is $\bar{e}_L\bar{e}_R$, with $\bar{e}_L \rightarrow \tilde{\chi}_2^0 e \rightarrow \tilde{\ell} \ell$.

$\sigma = 83 \text{ MeV}/c^2$

90 events in the peak (11% efficiency).

$\delta(M_\ell) = \frac{\sigma}{\sqrt{N}} = 8.7 \text{ MeV}/c^2$.

Fitted mass = 174.74 GeV/c^2 (input was 174.73 GeV/c^2).

M. Berggren, LCWS
Cascade decays.....

- Possible to fully reconstruct the intermediate state in cascade decays
Cascade decays…..

- Possible to fully reconstruct the intermediate state in cascade decays

You used to think that a sparticle is just an end-point or an edge?

Not!
It’s a peak!!
Measuring SUSY parameters

• Neutralino/chargino sector
  – determination of $\mu$, $M_1$, $M_2$, $\tan\beta$
  – New study: Neutralino production using tau identification
    Sobloher, Desch

• Stop sector
  – Different methods to get precise mass determination few permil
    Finch, Nowak, Sopczak

• Stop quark
Reconstructing supersymmetric theories

- Using measurements of masses, cross-sections, branching ratios at LHC and LC, can extract the SUSY parameters
  - Gaugino parameters, scalar masses, Higgs/Higgsino, trilinear couplings

- Coherent LHC/LC analyses can improve the precision of each machine independently

- Reconstruction of fundamental SUSY theory
  - Top-down approach: fits of high-scale parameters in a given model to experimental data
  - Bottom-up approach: use experimental data to reconstruct the susy theory at high scale, need complete information from LHC and LC
Coherent LHC/LC analyses

- LHC can measure precisely mass differences, LC can measure precisely LSP mass \(\rightarrow\) improve LHC precision on squark masses
- End-point measurements and determination of masses:
  In heavy neutralino decays, \(\tilde{\chi}_2^0 \rightarrow ll \rightarrow ll\tilde{\chi}_1^0\)
  knowing the slepton mass improves the precision on the heavy neutralino from edge in invariant mass measurement

<table>
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<tr>
<th></th>
<th>(M_1)</th>
<th>(M_2)</th>
<th>(\mu)</th>
<th>(\tan \beta)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theo</td>
<td>99.1</td>
<td>192.7</td>
<td>352.4</td>
<td>10</td>
</tr>
<tr>
<td>(L_{C_{500}})</td>
<td>99.1 ± 0.2</td>
<td>192.7 ± 0.6</td>
<td>352.8 ± 8.9</td>
<td>10.3 ± 1.5</td>
</tr>
<tr>
<td>(LHC + L_{C_{500}})</td>
<td>99.1 ± 0.1</td>
<td>192.7 ± 0.3</td>
<td>352.4 ± 2.1</td>
<td>10.2 ± 0.6</td>
</tr>
</tbody>
</table>

Moortgat-Pick, LCWS04
# Expected accuracies

<table>
<thead>
<tr>
<th></th>
<th>Mass, ideal</th>
<th>“LHC”</th>
<th>“LC”</th>
<th>“LHC+LC”</th>
<th>$\Delta_{th}$ (GeV)</th>
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<tbody>
<tr>
<td>$\tilde{\chi}^\pm_1$</td>
<td>179.7</td>
<td>0.55</td>
<td>0.55</td>
<td>1.2</td>
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<tr>
<td>$\tilde{\chi}^0_1$</td>
<td>97.2</td>
<td>4.8</td>
<td>0.05</td>
<td>0.05</td>
<td>0.34</td>
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<tr>
<td>$\tilde{\chi}^0_2$</td>
<td>180.7</td>
<td>4.7</td>
<td>1.2</td>
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<td>$\tilde{\chi}^0_4$</td>
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<td>3-5</td>
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<td>$\tilde{\epsilon}_R$</td>
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<td>0.24</td>
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<tr>
<td>$\tilde{\tau}_1$</td>
<td>134.8</td>
<td>5-8</td>
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<td>0.59</td>
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<td>-</td>
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<tr>
<td>$\tilde{q}_R$</td>
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<td>7-12</td>
<td>-</td>
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<tr>
<td>$\tilde{q}_L$</td>
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<td>-</td>
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<td>$\tilde{t}_1$</td>
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<td>2.0</td>
<td>4.4</td>
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<tr>
<td>$\tilde{b}_1$</td>
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<td>5.7</td>
<td>7.4</td>
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<tr>
<td>$\tilde{b}_2$</td>
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<td>-</td>
<td>6.2</td>
<td>8.2</td>
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<td>$\tilde{g}$</td>
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<td>$H^0$</td>
<td>399.8</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
<td>0.7</td>
</tr>
</tbody>
</table>
Tools: SFITTER/Fittino

- New tools for determining MSSM parameters in global fit to LHC+LC data:
  - SFITTER (Lafaye, Plehn, D. Zerwas)
    - Suspect, MSMLib, Prospino, Minuit
    - Input values: masses
    - Include experimental errors
    - Fit in MSSM and mSUGRA
  - FITTINO (Bechtle, Desch, Wienemann)
    - Spheno, MINUIT
    - Input values: masses and e+e- cross-sections
    - Include experimental errors + theoretical error on mh
- SUSY Les Houches Accord
  - Easy interfacing
SFITTER/Fittino: Iterative approach

Extracting the parameters of the SUSY Lagrangian

Experiment:
- Measured observables $O_i^m$
- Errors $\Delta O_i^m$

Program output:
- SUSY parameters $P_i$
- Full error matrix $V_{ij}$

Tree level formulae or coarse scan:
- Rough estimates for:
  - Parameters $P_i$
  - Errors $\Delta P_i$

$\chi^2$ fit: vary $P_i$

Compare

SUSY calculation package:
- Calculated observables $O_i^c$
  (including loop corrections)

Peter Wienemann
LCWS Paris, April 19-23, 2004

G. Bélanger
LCWS-Paris, 23/04/2004
Global fit results: SPS1A

- Fit with SFITTER
- Input observables not smeared within their errors, no systematic and theory errors included
- Fit start values: $M_1$, $M_2$, $\mu$
  and $\tan\beta$ from coarse scan, other parameters fixed to true values
- Parameters well reconstructed
- Only combined LHC and LC input allows a complete fit without fixing parameters

<table>
<thead>
<tr>
<th></th>
<th>LHC</th>
<th>LC</th>
<th>LHC+LC</th>
<th>SPS1a</th>
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<tr>
<td>$\tan\beta$</td>
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<td>10.26±1.6</td>
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<td>$M_2$</td>
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<td>197.68±3.3</td>
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<td>135.66±4.4</td>
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<td>$m_{\psi}$</td>
<td>345.21±6.4</td>
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<td>344.36±2.1</td>
<td>344.3</td>
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</tbody>
</table>
Reconstructing supersymmetric theories

- Using measurements of masses, cross-sections, branching ratios at LHC and LC, can extract the SUSY parameters
- Coherent LHC/LC analyses can improve the precision of each machine independently
- Reconstruction of fundamental SUSY theory
  - Top-down approach: fits of high-scale parameters in a given model to experimental data
    - Theoretical uncertainties in evaluation of SUSY spectra are of the order of experimental errors at LHC.
    - Errors could also be larger in specific models (e.g., focus point in mSUGRA)
    - Need improved theoretical calculations certainly with precision expected at LC
  - Bottom-up approach: use experimental data to reconstruct the susy theory at high scale, *need complete information from LHC and LC*
mSUGRA : Top-Down

\[ \tan \beta = 10, \ M_0 = 200 \ \text{GeV}, \ M_{1/2} = 250 \ \text{GeV}, \ A_0 = -100, \]
\[ \text{sign}(\mu) = 1 \ (1 \sigma \ \text{errors}) \]

![Graphs showing M0 vs. M_{1/2} and tan\beta vs. A0 with different legend entries.](image)

Allanach, et al., hep-ph/0403133

G. Bélanger

LCWS-Paris, 23/04/2004
$\tan \beta = 10$, $M_0 = 200$ GeV, $M_{1/2} = 250$ GeV, $A_0 = -100$, $\text{sign}(\mu) = 1$

1 σ error bands
“LHC+LC” analyses

W. Porod, LCWS2004
Slepton Yukawa

**SUSY (softly broken):**

Gauge coupling $g = \text{Yukawa coupling } \tilde{g}$

---

**Selectron channel:**

$g'$ U(1) coupl.

$g$ SU(2) coupl.

---

**Results:**

$\frac{\delta g'}{g'} \approx 0.2\%$  \hspace{1cm} $\frac{\delta \tilde{g}}{\tilde{g}} \approx 0.7\%$

Freitas, v. Manteuffel, Zerwas '03

---

$\sqrt{s} = 500$ GeV

$\int L = 500$ fb$^{-1}$

---

Freitas et al.

---

G. Bélanger

LCWS-Paris, 23/04/2004
What if ....

SPS1a just a (optimistic) scenario
SUSY might be quite different than SPS1a…. or mSUGRA…. or MSSM

Other possibilities:

   Nearly degenerate slepton/neutralino (WMAP)
   Large tanβ, focus point
   Complex parameters (baryogenesis)
   Lepton flavour violation (neutrino masses)
   NMSSM

   …..
CP violation in SUSY

- CP violation might explain baryon asymmetry in universe
- In general MSSM introduce complex parameters in Higgs potential and in soft SUSY breaking terms,
  - $\mu$, M1, At

- Phases will affect both CP conserving and CP violating observables

- Strong constraints from EDM of $e, n, Hg, \ldots$, $a_\mu$ on CP phases in sfermion ($1^{\text{st}}$ 2$^{\text{nd}}$ generation) and chargino/neutralino sector
  - Cancellation between diagrams: some phases can be large

Choi et al. hep-ph/0403054
CP conserving observables

- **Determination of phases in neutralino/chargino sector**
  - Choi Djouadi Zerwas Kalinowski Guchait Kneur Moultaka ....

- **Production cross-sections for selectron, chargino, neutralino sensitive to phases : Correlation of observables**
  - Choi, Drees, Grassmaier, hep-ph/0403054

- **Impact of phases in Branching ratios of third generation sfermions**
  - Bartl et al, hep-ph/0207186
  - Bartl et al, hep-ph/0311338

possible determination of Af and phase
\( \tilde{t} \) and \( \tilde{b} \) sectors


\( \tilde{t}_1 \) partial decay widths and branching ratios in scenario:

\[
\begin{align*}
& m_{\tilde{t}_L} > m_{\tilde{t}_R}, \quad m_{\tilde{t}_1} = 379 \text{ GeV}, \quad m_{\tilde{t}_2} = 575 \text{ GeV}, \quad m_{\tilde{b}_1} = 492 \text{ GeV}, \\
& |A_t| = 466 \text{ GeV}, \quad |A_b| = 759 \text{ GeV}, \quad \varphi_{A_b} = 0, \quad |\mu| = 352 \text{ GeV}, \quad \varphi_\mu = 0, \\
& M_2 = 193 \text{ GeV}, \quad |M_1| = M_2 5/3 \tan^2 \theta_W, \quad \varphi_{M_1} = 0, \quad \tan \beta = 10
\end{align*}
\]

(SPS 1a inspired)

\[
\Gamma(\tilde{t}_1)/\text{GeV} \quad B(\tilde{t}_1)
\]

\[
\begin{align*}
& \tilde{t}_1 \rightarrow \tilde{\chi}_1^0 b \\
& \tilde{t}_1 \rightarrow \tilde{\chi}_2^0 t
\end{align*}
\]

\[
\varphi_{A_t}/\pi
\]

\[ \text{pronounced phase dependence of } \Gamma(\tilde{t}_1 \rightarrow \tilde{\chi}_1^0 b): \text{ effect of } \varphi_\tilde{t} \sim \varphi_{A_t} \]

\[
\Rightarrow \left[ \delta(\text{Im}(A_t))/|A_t| = 2 - 3 \%, \quad \delta(\text{Re}(A_t))/|A_t| = 2 - 3 \% \right]
\]

G. Bélanger

LCWS-Paris, 23/04/2004
CP –odd observables

- CP asymmetry in neutralino production $\rightarrow$ leptonic three-body decay

$$e^+ e^- \rightarrow \tilde{\chi}_1^0 + \tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 + \tilde{\chi}_1^0 \ell^+ \ell^-$$

Can reach 15% in SPS1a-like scenario, more sensitive to phase of M1

$S.Hesselbach, LCWS$

Also triple products in two-body decays

$A. Bartl et al$

\[ A_T = \frac{\int \text{sign}\{\vec{p}_{e^-} \cdot (\vec{p}_{\ell^+} \times \vec{p}_{\ell^-})\} |T|^2 dlips}{\int |T|^2 dlips} \]
Light sbottom quark

• World average: \( \sin(2\beta) = 0.734 \pm 0.055 \), using \( B \to J/\psi K_S \)
• 2003 using \( B \to \phi K_S \),
  
  \[
  \text{Belle: } \sin(2\beta) = -0.96 \pm 0.50 \pm 0.09 \\
  \text{BaBar: } \sin(2\beta) = 0.45 \pm 0.43 \pm 0.07
  \]

both data are in 2.1\( \sigma \) disagreement

Average: \( S_{\phi K_S} = -0.15 \pm 0.33 \) still 2.7\( \sigma \) from world average

• Call for new physics in \( b \to s \) CPV effect

Large mixing possible in \( (M^2_{RR}) \) for generation 2/3

Light sbottom quark \( \sim 200 \text{GeV} \): can be searched for at Tevatron, LHC, LC

K. Cheung, LCWS
Neutrino masses and lepton flavour violation

Observation of neutrino masses and mixings in neutrino oscillation can be described in MSSM with 3 RH neutrinos: See-saw mechanism

\[ W_\nu = -\frac{1}{2} \nu R^T M \nu R + \nu R^T Y_\nu L \cdot H_2. \]

Massive neutrinos affect RGE of slepton masses and trilinear couplings

- Flavour non-diagonal matrix elements (depend on neutrino mass/mixings)

\[
\frac{M_R}{v^2 \sin^2 \beta} \left( \sqrt{\Delta m_{12}^2} U_{a2} U_{b2}^* + \sqrt{\Delta m_{23}^2} U_{a3} U_{b3}^* \right) \ln \frac{M_X}{M_R}.
\]

- Lepton-flavour violation in rare decays AND in slepton pair production

\[
\sigma_{\text{pair}} \propto \frac{|(\delta m L)_{ij}|^2}{m^2 l^2} \sigma(e^+ e^- \rightarrow l_i^+ \tilde{l}_j^-) Br(l_j^+ \rightarrow l_i^+ \tilde{\chi}_0) Br(l_i^- \rightarrow l_j^- \tilde{\chi}_0).
\]
LFV-Correlation rare decays/slepton pair production

SUSY scenarios $C', G', B ', SPS1, l', \sqrt{s} = 800 \text{ GeV}$

800 GeV

SPS1a: $\sigma(\mu + 2\tilde{\chi}^0) = 0.1 \text{ fb} \equiv Br(\mu \rightarrow e\gamma) = 4 \times 10^{-12}$

SPS1a: $\sigma(\tau + 2\tilde{\chi}^0) = 1 \text{ fb} \equiv Br(\tau \rightarrow \mu\gamma) = 5 \times 10^{-9}$

$\text{If } Br(\mu \rightarrow e\gamma) = 10^{-13}$, LFV not observable at colliders

Ruckl, LCWS

G. Bélanger

LCWS-Paris, 23/04/2004
Neutrino masses and R-parity

- Observation of neutrino masses and mixings in neutrino oscillation can be explained in SUSY models with R-parity violation
- Add bilinear/ trilinear LFV
  - Decay properties of the lightest supersymmetric particle are correlated with neutrino mixing angles → testable at linear collider
Precision measurements for cosmology

• How precisely do the SUSY parameters need to be measured at LHC+LC colliders to have prediction for $\Omega h^2$ competitive with PLANCK (2%)
  $\Rightarrow$ Consistency check on cosmological model

Note: In mSUGRA scenario, large uncertainties in prediction of relic density largely due to uncertainties in sparticle spectrum, with measurements of MSSM parameters can have more precise predictions

• The most relevant parameter depends on the scenario:
  – Coannihilation: mass difference NLSP-LSP
  – Focus point: Higgsino/gaugino nature of LSP ($\mu$)
  – Higgs resonance annihilation: MA
Nearly degenerate slepton-neutralino

- Scenarios with nearly degenerate slepton/neutralinos, are cosmologically favoured (relic density of dark matter)

- The relic density depends sensitively on the NLSP-LSP mass difference:
  
  In mass range relevant for LC, typical $\Delta M(stau \chi) = 5-15 \text{ GeV}$

- Can this mass difference be measured precisely at LC?

- What is the impact of crossing-angle on this measurement?
Coannihilation scenario: precision

- 2 analyses optimal center-of-mass energy and energy spectra: Stau mass measurement at small $\Delta M$ is challenging
- Crossing angle is possible but lose 25% efficiency
- Relic density prediction from LC can compete with PLANCK in certain scenarios

Z. Zhang, LCWS04
Conclusions

- Recent progresses in estimating precision on determination of SUSY parameter at LC

- In one scenario (SPS1a) start to test the underlying supersymmetric models

- Might be able to confront cosmological models

- Need to look into other scenarios, e.g. Higgsino LSP/focus point

- Indications of physics beyond the standard model: new ways to be tested at LC
Strategy One: Optimal Center of Mass Energy for Stau Mass Measurement

Assuming negligible background and given
the integrated luminosity: \( L \)
the efficiency: \( \varepsilon \)
Signal cross section: \( \sigma = A \beta^3 \) (neglect ISR correction)
with \( A \sim 100 \), \( \beta = (1 - 4m^2/s)^{1/2} \)

\[ \text{Observed events: } N = LA\beta^3\varepsilon \]

One can easily derive
the relative stau mass precision:
\[ \frac{dm}{m} = s/12m^2 \left[ \beta/LA\varepsilon \right]^{1/2} \]
the optimal center of mass energy:
\[ E_{cm} = s^{1/2} = 2m / \left[ 1 - (N/LA\varepsilon)^{2/3} \right]^{1/2} \]

Note: This differs from a threshold scan measurement,
\( \Rightarrow \) Little sensitivity to the \( \sigma \) shape & corrections @ threshold

Z. Zhang, LCWS

G. Bélanger

LCWS-Paris, 23/04/2004
Scalar top

- Detailed study for SPS5
  - mstop=220.7 GeV, MLSP=120. GeV, decay to $\tilde{t}_1^0 \rightarrow \chi_1^- C$

Comparison of different methods:
Threshold scan, polarization, end-point ... precision .57-1.5 GeV

Finch et al, LCWS04