New-Physics Search in $\gamma \gamma \rightarrow t\bar{t}$

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1. Introduction

Discovery of Top-quark

We have now all the SM-fermions,
but · · · · · · ·

• Is the 3rd generation a copy of the 1st & 2nd.?

• Isn’t there any New-Physics in top-quark couplings?

*Based on collaboration with B. Grządkowski (Warsaw U), K.Ohkuma (Fukui U. Technology) and J. Wudka (UC Riverside).
So far, we have studied
\[ e^+e^- \rightarrow t\bar{t} \]
to explore possible non-SM top-quark couplings.
Here we perform a similar analysis in
\[ \gamma\gamma \rightarrow t\bar{t} \]

2. Basic Framework

What we aim to do is “A Model-Independent Analysis”.
For this purpose, what terms must be taken into account?

- In the case of \( e\bar{e} \rightarrow t\bar{t} \):

We are able to write down

“the most general covariant \( t\bar{t}\gamma/Z \) amplitude.”

\[
\Gamma^\mu_v = \frac{g}{2} \bar{u}(p_t) \left[ \gamma^\mu (A_v - B_v \gamma_5) + \frac{(p_t - \bar{p}_t)^\mu}{2m_t} (C_v - D_v \gamma_5) \right] v(p_t). \tag{1}
\]

\((v = \gamma \text{ or } Z)\)

However in \( \gamma\gamma \rightarrow t\bar{t} \), \( t \) or \( \bar{t} \) in \( t\bar{t}\gamma \) coupling is virtual.

\[ \Downarrow \]

Those which were dropped in \( e\bar{e} \rightarrow t\bar{t} \) thanks to the on-shell condition can contribute
Take \( \bar{\psi} \psi \) as an example. If \( \psi \) and \( \bar{\psi} \) are both on-shell,

\[
\bar{\psi} F(\Box) \psi \quad \implies \quad \bar{\psi} F(m^2) \psi
\]

So they are all equivalent to \( \bar{\psi} \psi \). However if \( \psi \) is virtual, we have infinite numbers of

\[
\bar{u} F(q^2) S_F(q) \cdots
\]

in an amplitude since \( F \) can be arbitrary.

We decided to perform an analysis in the framework of

**Effective Operator Approach à la Buchmüller & Wyler**

**Basic assumption**

- New Physics with Energy scale \( \Lambda \)
- Below \( \Lambda \), we have only SM particles

The leading non-SM interactions are dim.-6 operators:

\[
\begin{align*}
\mathcal{O}_{uB} &= i \bar{u} \gamma_\mu D_\nu u B^\mu{}^\nu \\
\mathcal{O}_{qB} &= i \bar{q} \gamma_\mu D_\nu q B^\mu{}^\nu \\
\mathcal{O}_{qW} &= i \bar{q} \gamma_\mu \tau^i D_\nu q W_{i}^{\mu\nu} \\
\mathcal{O}_{uW} &= (\bar{q} \sigma^{\mu\nu} \bar{u}) \bar{W}_{i}^{\mu\nu} \\
\mathcal{O}_{uW} &= (\bar{q} \sigma^{\mu\nu} \bar{u}) \bar{W}_{i}^{\mu\nu} \\
\mathcal{O}_{qB} &= (\bar{q} \sigma^{\mu\nu} \bar{u}) \bar{W}_{i}^{\mu\nu} \\
\mathcal{O}_{W^B} &= (\bar{q} \sigma^{\mu\nu} \bar{u}) \bar{W}_{i}^{\mu\nu} B^\mu{}^\nu \\
\mathcal{O}_{WB} &= (\bar{q} \sigma^{\mu\nu} \bar{u}) \bar{W}_{i}^{\mu\nu} B^\mu{}^\nu
\end{align*}
\]

\[
\mathcal{L} = \mathcal{L}_{\text{SM}} + \left[ \frac{1}{\Lambda^2} \sum_i \alpha_i \mathcal{O}_i + (\text{h.c.}) \right]
\]
One new discovery:

\[ \mathcal{O}_{uB}, \quad \mathcal{O}_{qB}, \quad \mathcal{O}_{qW} \]

are not independent of the others

\[ \mathcal{O}_{uB} = -ig_u\mathcal{O}_{uB} + [ -ig_u\mathcal{O}_{uB} ]^\dagger + \cdots \]
\[ \mathcal{O}_{qB} = ig_u\mathcal{O}_{uB} + [ ig_u\mathcal{O}_{uB} ]^\dagger + \cdots \]
\[ \mathcal{O}_{qW} = ig_u\mathcal{O}_{uB} + [ ig_u\mathcal{O}_{uB} ]^\dagger + \cdots \]

via some equations of motion

Independent operators lead to the following **Feynman rules**.

1. **CP-conserving \( t\bar{t}\gamma \) vertex**

   \[ \frac{\sqrt{2}}{\Lambda^2} \nu_a \gamma_1 k^\gamma \mu, \quad (3) \]

2. **CP-violating \( t\bar{t}\gamma \) vertex**

   \[ i\frac{\sqrt{2}}{\Lambda^2} \nu_a \gamma_2 k^\gamma \mu \gamma_5, \quad (4) \]

3. **CP-conserving \( \gamma\gamma H \) vertex**

   \[ -\frac{4}{\Lambda^2} \nu_a \gamma_1 \{ (k_1 k_2) g_{\mu\nu} - k_1^\mu k_2^\nu \}, \quad (5) \]

4. **CP-violating \( \gamma\gamma H \) vertex**

   \[ \frac{8}{\Lambda^2} \nu_a \gamma_2 k_1^\rho k_2^\sigma \epsilon_{\rho\sigma\mu\nu}, \quad (6) \]
Here, $k$ & $k_{1,2}$ are incoming photon momenta, $\alpha_{\gamma_1,\gamma_2,h_1,h_2}$ are defined as

$$\alpha_{\gamma_1} \equiv \sin \theta_W \text{Re}(\alpha_{uW}) + \cos \theta_W \text{Re}(\alpha_{uB}'),$$  \quad (7)$$

$$\alpha_{\gamma_2} \equiv \sin \theta_W \text{Im}(\alpha_{uW}) + \cos \theta_W \text{Im}(\alpha_{uB}'),$$  \quad (8)$$

$$\alpha_{h_1} \equiv \sin^2 \theta_W \text{Re}(\alpha_{\varphi W}) + \cos^2 \theta_W \text{Re}(\alpha_{\varphi B})$$
$$- 2 \sin \theta_W \cos \theta_W \text{Re}(\alpha_{WB}),$$  \quad (9)$$

$$\alpha_{h_1} \equiv \sin^2 \theta_W \text{Re}(\alpha_{\varphi W}) + \cos^2 \theta_W \text{Re}(\alpha_{\varphi B})$$
$$- \sin \theta_W \cos \theta_W \text{Re}(\alpha_{\tilde{W}B}).$$  \quad (10)$$

On the other hand, the general amplitude for $t \to bW$ can be written as

$$\Gamma_{Wtb}^\mu = -\frac{g}{\sqrt{2}} \bar{u}(p_b) \left[ \gamma^\mu P_L - \frac{i \sigma^{\mu\nu} k_\nu}{M_W} f^R_{2} P_R \right] u(p_t),$$  \quad (11)$$

where $P_{L,R} \equiv (1 \pm \gamma_5)/2$ and $f^R_2$ is

$$f^R_2 = \frac{1}{\Lambda^2} \left[ -\frac{4 M_W v}{g} \alpha_{uW} - \frac{M_W v}{2} \alpha_{Du} \right].$$  \quad (12)$$

for $m_b = 0$ and on-shell $W$ approximation.

Using them, we calculated the cross section of $\gamma\gamma \to \ell^\pm X$ via FORM.

HOWEVER, the result is too long to show here. Sorry!
3. Optimal-Observable Analysis

How can we determine several unknown parameters simultaneously?

⇒ Optimal-observable method

Brief summary of this method: Suppose we have a distribution

\[ \frac{d\sigma}{d\phi} (\equiv \Sigma(\phi)) = \sum_i c_i f_i(\phi) \]

where \( f_i(\phi) \) are calculable functions, and \( c_i \) are the parameters we try to determine.

Determining \( c_i \)

⇒ We make weighting functions \( w_i(\phi) \) which satisfies:

\[ \int w_i(\phi) \Sigma(\phi) d\phi = c_i \]

The one which minimizes the statistical uncertainty of \( c_i \) is

\[ w_i(\phi) = \sum_j X_{ij} f_j(\phi) / \Sigma(\phi) , \]

where \( X \) is the inverse matrix of

\[ M_{ij} \equiv \int \frac{f_i(\phi)f_j(\phi)}{\Sigma(\phi)} d\phi \]

This \( X \) gives

\[ \Delta c_i = \sqrt{X_{ii} \sigma_T / N} , \]
where $\sigma_T \equiv \int (d\sigma/d\phi)d\phi$, $N = L_{\text{eff}}\sigma_T$ is the total number of the events, $L_{\text{eff}}$ is the product of the integrated luminosity and detection efficiency.

We applied this procedure to the angular & energy distribution of $\gamma\gamma \rightarrow t\bar{t} \rightarrow \ell^+X$:

$$\frac{d\sigma}{dE_\ell d\cos \theta_\ell} = f_{\text{SM}}(E_\ell, \cos \theta_\ell) + \alpha_{\gamma_1} f_{\gamma_1}(E_\ell, \cos \theta_\ell) + \alpha_{\gamma_2} f_{\gamma_2}(E_\ell, \cos \theta_\ell)$$

$$+ \alpha_{h_1} f_{h_1}(E_\ell, \cos \theta_\ell) + \alpha_{h_2} f_{h_2}(E_\ell, \cos \theta_\ell) + \alpha_d f_d(E_\ell, \cos \theta_\ell)$$

(13)

in $e\bar{e}$-CM frame, where

- $f_{\text{SM}}$ is the SM contribution,
- $f_{\gamma_1, \gamma_2}$ are $CP$-conserving & $CP$-violating-$t\bar{t}\gamma$-vertices contribution,
- $f_{h_1, h_2}$ are $CP$-conserving & $CP$-violating-$\gamma\gamma H$-vertices contribution, and
- $f_d$ is from the anomalous $tbW$-vertex

$$\alpha_d = \text{Re}(f_2^R).$$
Parameters

Higgs mass: $m_H = 100, 300, 500 \text{ GeV}$

Polarizations of $e$ & $\bar{e}$: $P_e = P_{\bar{e}} = 1$

Polarization of the Laser:

(1) Linear Polarization

$$P_e = P_{\bar{e}} = 1, \quad P_t = P_{\bar{t}} = P_\gamma = P_{\bar{\gamma}} = 1/\sqrt{2} \text{ and } \chi(\equiv \varphi_1 - \varphi_2) = \pi/4$$

(2) Circular Polarization

$$P_e = P_{\bar{e}} = P_\gamma = P_{\bar{\gamma}} = 1.$$

Problems

It turned out that our results for $X_{ij}$ are very unstable:

even a tiny fluctuation of $M_{ij}$ changes $X_{ij}$ significantly.

\[\downarrow\]

Some of $f_i$ have similar shapes?

The only option in such a case is to refrain from determining all the

couplings at once through this process alone.

\[\downarrow\]

We assumed some parameters can be determined in other processes:

Result:

We found several sets of solution in two-parameter analysis
1) Linear polarization

- Independent of \( m_H \)

\[
\Delta \alpha_{\gamma 2} = \frac{73}{\sqrt{N_\ell}}, \quad \Delta \alpha_d = \frac{1.9}{\sqrt{N_\ell}},
\]

(14)

- \( m_H = 100 \text{ GeV} \)

\[
\Delta \alpha_{h2} = \frac{107}{\sqrt{N_\ell}}, \quad \Delta \alpha_d = \frac{1.6}{\sqrt{N_\ell}},
\]

(15)

- \( m_H = 300 \text{ GeV} \)

\[
\Delta \alpha_{h1} = \frac{3.4}{\sqrt{N_\ell}}, \quad \Delta \alpha_d = \frac{3.2}{\sqrt{N_\ell}},
\]

(16)

Here \( \sqrt{N_\ell} \simeq 63 \) for \( L_{\text{eff}}^{ee} = 500 \text{ fb}^{-1} \).

2) Circular polarization

- \( m_H = 100 \text{ GeV} \)

\[
\Delta \alpha_{h1} = \frac{9.0}{\sqrt{N_\ell}}, \quad \Delta \alpha_d = \frac{3.0}{\sqrt{N_\ell}},
\]

(17)

- \( m_H = 300 \text{ GeV} \)

\[
\Delta \alpha_{h1} = \frac{3.5}{\sqrt{N_\ell}}, \quad \Delta \alpha_d = \frac{3.0}{\sqrt{N_\ell}},
\]

(18)

\[
\Delta \alpha_{h2} = \frac{35}{\sqrt{N_\ell}}, \quad \Delta \alpha_d = \frac{3.1}{\sqrt{N_\ell}},
\]

(19)

- \( m_H = 500 \text{ GeV} \)

\[
\Delta \alpha_{h1} = \frac{7.7}{\sqrt{N_\ell}}, \quad \Delta \alpha_d = \frac{2.8}{\sqrt{N_\ell}},
\]

(20)
\[ \Delta \alpha_{h2} = 10 / \sqrt{N_{\ell}}, \quad \Delta \alpha_d = 2.8 / \sqrt{N_{\ell}}, \quad (21) \]

Here \( \sqrt{N_{\ell}} \simeq 48 \) for \( L_{ee}^{\text{eff}} = 500 \, \text{fb}^{-1} \).

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We also performed a similar analysis using

\[ \gamma \gamma \rightarrow t \bar{t} \rightarrow bX \]

Isn’t it harder to study \( b \)-quark distribution?

↓

\( b \)-quark tagging must be done to distinguish \( t \bar{t} \) events from possible background (WW production).

1) Linear polarization

• Independent of \( m_H \)

\[ \Delta \alpha_{\gamma 2} = 29 / \sqrt{N_b}, \quad \Delta \alpha_d = 2.6 / \sqrt{N_b}, \quad (22) \]

• \( m_H = 100 \, \text{GeV} \)

\[ \Delta \alpha_{h2} = 38 / \sqrt{N_b}, \quad \Delta \alpha_d = 2.4 / \sqrt{N_b}, \quad (23) \]
\[m_H = 300 \text{ GeV}\]
\[
\Delta \alpha_{\gamma_2} = 24/\sqrt{N_b}, \quad \Delta \alpha_{h_1} = 2.4/\sqrt{N_b}, \quad (24)
\]
\[
\Delta \alpha_{h_1} = 5.4/\sqrt{N_b}, \quad \Delta \alpha_d = 4.9/\sqrt{N_b}, \quad (25)
\]

\[m_H = 500 \text{ GeV}\]
\[
\Delta \alpha_{\gamma_2} = 23/\sqrt{N_b}, \quad \Delta \alpha_{h_1} = 5.0/\sqrt{N_b}, \quad (26)
\]
\[
\Delta \alpha_{h_1} = 18/\sqrt{N_b}, \quad \Delta \alpha_{h_2} = 22/\sqrt{N_b}, \quad (27)
\]
\[
\Delta \alpha_{h_1} = 8.0/\sqrt{N_b}, \quad \Delta \alpha_d = 3.3/\sqrt{N_b}, \quad (28)
\]

where \(\sqrt{N_b} \approx 140\) for \(L_{ee}^{\text{eff}} = 500 \text{ fb}^{-1}\).

2) Circular polarization

\[m_H = 100 \text{ GeV}\]
\[
\Delta \alpha_{h_1} = 14/\sqrt{N_b}, \quad \Delta \alpha_d = 5.2/\sqrt{N_b}, \quad (29)
\]

\[m_H = 500 \text{ GeV}\]
\[
\Delta \alpha_{h_1} = 10/\sqrt{N_b}, \quad \Delta \alpha_d = 4.2/\sqrt{N_b}, \quad (30)
\]

where \(\sqrt{N_b} \approx 100\) for \(L_{ee}^{\text{eff}} = 500 \text{ fb}^{-1}\).\(^{21}\)

The above results are for \(\Lambda = 1 \text{ TeV}\). When one takes the new-physics

\(^{21}\) We used the tree-level SM formula for computing \(N_b\), so that we have the same \(N_b\) for different \(m_H\).
scale to be $\Lambda' = \lambda \Lambda$, then all the above results ($\Delta \alpha_i$) are replaced with $\Delta \alpha_i / \lambda^2$, which means that the right-hand sides of eqs. (14)–(30) are multiplied by $\lambda^2$.

**Comparing two results**

The following parameter sets are measurable in

(1) Lepton analysis

$(\alpha_{\gamma 2}, \alpha_d), (\alpha_{h 1}, \alpha_d), (\alpha_{h 2}, \alpha_d)$

(2) $b$-quark analysis

$(\alpha_{\gamma 2}, \alpha_{h 1}), (\alpha_{\gamma 2}, \alpha_d), (\alpha_{h 1}, \alpha_{h 2}), (\alpha_{h 1}, \alpha_d), (\alpha_{h 2}, \alpha_d)$

### 4. Summary

- In order to explore possible anomalous top-quark couplings, we studied $t\bar{t}$ production/decay in $\gamma\gamma$ collisions.

- We assumed a New-Physics with an energy-scale $\Lambda$, and we have only the SM particles below $\Lambda$.

- All leading non-SM interactions are given in terms of dimension-6 effective operators à la Buchmüller & Wyler.

- We found some new “Equation-of-motion relations” among several operators, which reduced the number of operators necessary in our
analysis.

• We found it impossible to determine all the parameters in this process alone, but also found some stable solutions in two-parameter analysis.

• If we encounter phenomena which cannot be described in our framework, it will be an indication of some New-Physics beyond B& W scenario.

References
