**Herwig++ for $e^+e^-$ collisions**

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work with A Ribon, MH Seymour, P Stephens, BR Webber (Cambridge, CERN)

- Introduction/Motivation
- Some features of Herwig++
- Selection of LEP results for $e^+e^-$ Annihilation
- Jet Multiplicity at the Linear Collider
- Outlook


\[ e^+ e^- \] Event Generator

- hard scattering
- (QED) initial/final state radiation
- partonic decays, e.g. \( t \to bW \)
- parton shower evolution
- nonperturbative gluon splitting
- colour singlets
- colourless clusters
- cluster fission
- cluster \( \to \) hadrons
- hadronic decays
The new generator Herwig++

Complete rewrite of HERWIG in C++

- aiming at full multi-purpose generator for LHC and future colliders.
- Preserve main features of HERWIG such as
  - angular ordered parton shower
  - Cluster Hadronization
- New features and improvements
  - improved parton shower evolution for heavy quarks
  - consistent radiation from unstable particles

HERWIG's growth...
**Hard interactions**

- Basic ME's included in **ThePEG**, such as:
  \[ e^+ e^- \rightarrow q\bar{q}, \text{ partonic } 2 \rightarrow 2, \]
  we use them.
- Soft and hard **matrix element corrections** implemented for \( e^+ e^- \rightarrow q\bar{q}g \).
- **AMEGIC++** will provide arbitrary ME's for multiparton final states via **AMEGICInterface**.
- **CKKW ME+PS** foreseen.
- Other authors can easily include their own matrix elements (→ **safety** of OO code)
Quasi–Collinear Limit (Heavy Quarks)

Sudakov-basis $p, n$ with $p^2 = M^2$ (‘forward’), $n^2 = 0$ (‘backward’),

\[ p_q = zp + \beta_q n - q_\perp \]
\[ p_g = (1 - z)p + \beta_g n + q_\perp \]

Collinear limit for radiation off heavy quark,

\[ P_{gg}(z, q^2, m^2) = C_F \left[ \frac{1 + z^2}{1 - z} - \frac{2z(1 - z)m^2}{q^2 + (1 - z)^2m^2} \right] \]
\[ = \frac{C_F}{1 - z} \left[ 1 + z^2 - \frac{2m^2}{z\tilde{q}^2} \right] \]

\[
\rightarrow \quad \tilde{q}^2 \sim q^2 \text{ may be used as evolution variable.}
\]
Consider \((x, \bar{x})\) phase space for \(e^+e^- \rightarrow q\bar{q}g\)

- \(\times\) Larger dead region with new variables.
- \(\checkmark\) Smooth coverage of soft gluon region.
- \(\checkmark\) No overlapping regions in phase space.
Points \((x, \bar{x})\) in dead region chosen acc to LO \(e^+e^- \rightarrow q\bar{q}g\) matrix element and accepted acc to ME weight.

About 3\% of all events are actually hard \(q\bar{q}g\) events.

Red points have weight \(> 1\), practically no error by setting weight to one.

Event oriented according to given \(q\bar{q}\) geometry. Quark direction is kept with weight \(x^2/(x^2 + \bar{x}^2)\).
Soft Matrix Element Corrections

- Ratio $\text{ME/PS}$ compares emission with result from true ME if slightly away from soft/collinear region.
- **Veto** on ‘hardest emission so far’ in $p_\perp$.
- **Massive splitting function** very important!

Example with heavy quark, $m^2/Q^2 = 0.1$: 

\begin{figure}
\centering
\includegraphics[width=\textwidth]{chart.png}
\caption{Comparison with massless splitting function}
\end{figure}
## Hadron Multiplicities

<table>
<thead>
<tr>
<th>Particle</th>
<th>Experiment</th>
<th>Measured</th>
<th>Old Model</th>
<th>Herwig++</th>
<th>Fortran</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Charged</td>
<td>M,A,D,L,O</td>
<td>20.924 ± 0.117</td>
<td>20.22*</td>
<td>20.814</td>
<td>20.532*</td>
</tr>
<tr>
<td>γ</td>
<td>A,O</td>
<td>21.27 ± 0.6</td>
<td>23.032</td>
<td>22.67</td>
<td>20.74</td>
</tr>
<tr>
<td>π</td>
<td>A,D,L,O</td>
<td>9.59 ± 0.33</td>
<td>10.27</td>
<td>10.08</td>
<td>9.88</td>
</tr>
<tr>
<td>ρ(770)</td>
<td>A,D</td>
<td>1.295 ± 0.125</td>
<td>1.235</td>
<td>1.316</td>
<td>1.07</td>
</tr>
<tr>
<td>π±</td>
<td>A,O</td>
<td>17.04 ± 0.25</td>
<td>16.30</td>
<td>16.95</td>
<td>16.74</td>
</tr>
<tr>
<td>ρ(770)±</td>
<td>O</td>
<td>2.4 ± 0.43</td>
<td>1.99</td>
<td>2.14</td>
<td>2.06</td>
</tr>
<tr>
<td>η</td>
<td>A,L,O</td>
<td>0.956 ± 0.049</td>
<td>0.886</td>
<td>0.893</td>
<td>0.669*</td>
</tr>
<tr>
<td>ω(782)</td>
<td>A,L,O</td>
<td>1.083 ± 0.088</td>
<td>0.859</td>
<td>0.916</td>
<td>1.044</td>
</tr>
<tr>
<td>η'(958)</td>
<td>A,L,O</td>
<td>0.152 ± 0.03</td>
<td>0.13</td>
<td>0.136</td>
<td>0.106</td>
</tr>
<tr>
<td>K</td>
<td>S,A,D,L,O</td>
<td>2.027 ± 0.025</td>
<td>2.121*</td>
<td>2.062</td>
<td>2.026</td>
</tr>
<tr>
<td>K*(892)</td>
<td>A,D,O</td>
<td>0.761 ± 0.032</td>
<td>0.667</td>
<td>0.681</td>
<td>0.583*</td>
</tr>
<tr>
<td>K*(1430)</td>
<td>D,O</td>
<td>0.106 ± 0.06</td>
<td>0.065</td>
<td>0.079</td>
<td>0.072</td>
</tr>
<tr>
<td>K±</td>
<td>A,D,O</td>
<td>2.319 ± 0.079</td>
<td>2.335</td>
<td>2.286</td>
<td>2.250</td>
</tr>
<tr>
<td>K*(892)±</td>
<td>A,D,O</td>
<td>0.731 ± 0.058</td>
<td>0.637</td>
<td>0.657</td>
<td>0.578</td>
</tr>
<tr>
<td>φ(1020)</td>
<td>A,D,O</td>
<td>0.097 ± 0.007</td>
<td>0.107</td>
<td>0.114</td>
<td>0.134*</td>
</tr>
<tr>
<td>p</td>
<td>A,D,O</td>
<td>0.991 ± 0.054</td>
<td>0.981</td>
<td>0.947</td>
<td>1.027</td>
</tr>
<tr>
<td>Δ++</td>
<td>D,O</td>
<td>0.088 ± 0.034</td>
<td>0.185</td>
<td>0.092</td>
<td>0.209*</td>
</tr>
<tr>
<td>Σ</td>
<td>O</td>
<td>0.083 ± 0.011</td>
<td>0.063</td>
<td>0.071</td>
<td>0.071</td>
</tr>
<tr>
<td>Λ</td>
<td>A,D,L,O</td>
<td>0.373 ± 0.008</td>
<td>0.325*</td>
<td>0.384</td>
<td>0.347*</td>
</tr>
<tr>
<td>Σ⁰</td>
<td>A,D,O</td>
<td>0.074 ± 0.009</td>
<td>0.078</td>
<td>0.091</td>
<td>0.063</td>
</tr>
<tr>
<td>Σ⁺</td>
<td>O</td>
<td>0.099 ± 0.015</td>
<td>0.067</td>
<td>0.077</td>
<td>0.088</td>
</tr>
<tr>
<td>Σ(1385)⁺</td>
<td>A,D,O</td>
<td>0.0471 ± 0.0046</td>
<td>0.057</td>
<td>0.0312*</td>
<td>0.061*</td>
</tr>
<tr>
<td>Ξ⁻</td>
<td>A,D,O</td>
<td>0.0262 ± 0.001</td>
<td>0.024</td>
<td>0.0286</td>
<td>0.029</td>
</tr>
<tr>
<td>Ξ(1530)⁻</td>
<td>A,D,O</td>
<td>0.0058 ± 0.001</td>
<td>0.026*</td>
<td>0.0288*</td>
<td>0.009*</td>
</tr>
<tr>
<td>Ω⁻</td>
<td>A,D,O</td>
<td>0.00125 ± 0.00024</td>
<td>0.001</td>
<td>0.00144</td>
<td>0.0009</td>
</tr>
</tbody>
</table>
Hadron Multiplicities (ctd’)

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</tr>
</thead>
<tbody>
<tr>
<td>$f_2(1270)$</td>
<td>D,L,O</td>
<td>0.168 ± 0.021</td>
<td>0.113</td>
<td>0.150</td>
<td>0.173</td>
</tr>
<tr>
<td>$f_2(1525)$</td>
<td>D</td>
<td>0.02 ± 0.008</td>
<td>0.003</td>
<td>0.012</td>
<td>0.012</td>
</tr>
<tr>
<td>$D^\pm$</td>
<td>A,D,O</td>
<td>0.184 ± 0.018</td>
<td>0.322*</td>
<td>0.319*</td>
<td>0.283*</td>
</tr>
<tr>
<td>$D^*(2010)^\pm$</td>
<td>A,D,O</td>
<td>0.182 ± 0.009</td>
<td>0.168</td>
<td>0.180</td>
<td>0.151*</td>
</tr>
<tr>
<td>$D^0$</td>
<td>A,D,O</td>
<td>0.473 ± 0.026</td>
<td>0.625*</td>
<td>0.570*</td>
<td>0.501</td>
</tr>
<tr>
<td>$D_s^\pm$</td>
<td>A,O</td>
<td>0.129 ± 0.013</td>
<td>0.218*</td>
<td>0.195*</td>
<td>0.127</td>
</tr>
<tr>
<td>$D_s^{*\pm}$</td>
<td>O</td>
<td>0.096 ± 0.046</td>
<td>0.082</td>
<td>0.066</td>
<td>0.043</td>
</tr>
<tr>
<td>$J/\Psi$</td>
<td>A,D,L,O</td>
<td>0.00544 ± 0.00029</td>
<td>0.006</td>
<td>0.00361*</td>
<td>0.002*</td>
</tr>
<tr>
<td>$\Lambda_c^+$</td>
<td>D,O</td>
<td>0.077 ± 0.016</td>
<td>0.006*</td>
<td>0.023*</td>
<td>0.001*</td>
</tr>
<tr>
<td>$\Psi'(3685)$</td>
<td>D,L,O</td>
<td>0.00229 ± 0.00041</td>
<td>0.001*</td>
<td>0.00178</td>
<td>0.0008*</td>
</tr>
</tbody>
</table>

# of *’s = observables with more than 3σ deviation:

OldModel : Herwig++ : Fortran = 9 : 7 : 13
Event Shape Variables, Definition

**Thrust**

\[ F(n) = \frac{\sum_{\alpha} |p_{\alpha} \cdot n|}{\sum_{\alpha} |p_{\alpha}|} \]

Find \( n \), such that thrust

\[ T = \max_n F(n) \]
\[ = F(n_T) \]

thrust major

\[ M = \max_{n \perp n_T} F(n) \]
\[ = F(n_M) \]

thrust minor

\[ n_m = n_T \times n_M \]
\[ m = F(n_m) \]

**Sphericity**

\[ Q_{ij} = \frac{\sum_{\alpha} (p_{\alpha})_i (p_{\alpha})_j}{\sum_{\alpha} p_{\alpha}^2} \]

Diagonalize, eigenvalues

\[ \lambda_1 > \lambda_2 > \lambda_3 \]
\[ \lambda_1 + \lambda_2 + \lambda_3 = 1 \]

Then

\[ S = \frac{3}{2} (\lambda_2 + \lambda_3) \]
\[ P = \lambda_2 - \lambda_3 \]
\[ A = \frac{3}{2} \lambda_3 \]

Eigenvector \( n_S \) sphericity axis etc.

**C, D parameter**

\[ L_{ij} = \frac{\sum_{\alpha} (p_{\alpha})_i (p_{\alpha})_j / |p_{\alpha}|}{\sum_{\alpha} |p_{\alpha}|} \]

Diagonalize, eigenvalues

\[ \lambda_1 + \lambda_2 + \lambda_3 = 1 \]

and define

\[ C = 3 (\lambda_1 \lambda_2 + \lambda_2 \lambda_3 + \lambda_3 \lambda_1) \]
\[ D = 27 \lambda_1 \lambda_2 \lambda_3 \]
Thrust — ME Corrections off/on

no ME correction
\( \delta = 1.7 \text{ GeV} \)
\( \delta = 2.3 \text{ GeV} \)
\( \delta = 3.2 \text{ GeV} \)
DELPHI 96

Herwig++ 1.0
\( \delta = 1.7 \text{ GeV} \)
\( \delta = 2.3 \text{ GeV} \)
\( \delta = 3.2 \text{ GeV} \)
DELPHI 96

$p_{\perp,\text{in}}$ — ME corrections off/on

![Graph showing $p_{\perp,\text{in}}$ distributions with and without ME corrections, comparing different models and parameter values.](image-url)
All Thrust–related distributions slightly wide, ie too many 2-jet like on one side and too many spherical events on the other side.
Four Jet Angles

$|\cos \chi_{BZ}|$

$\cos \Phi_{KSW}$

Herwig++ 1.0
- $\delta = 1.7 \text{ GeV}$
- $\delta = 2.3 \text{ GeV}$
- $\delta = 3.2 \text{ GeV}$
- DELPHI
**B-fragmentation function**

**Weakly decaying b hadrons**

Data: SLD

**HERWIG:**
- default
- PSPLT(2)=0.5
- PSPLT(2)=0.2, CLSMR(2)=1.0
- B1LIM=1.0

**HERWIG 6.4, very sensitive on hadronization!**
**B-fragmentation function**

![Graph showing B-fragmentation function with data points and curves representing different values of \( \delta \).]

*Only parton shower parameters varied!*

Jet Multiplicity

Durham algorithm. Smooth interplay between shower and hadronization.

**Partons**

**Herwig++ 1.0**
- $\Lambda_{QCD} = 125\,\text{MeV}$
- $\Lambda_{QCD} = 500\,\text{MeV}$
- $\delta = 1.7\,\text{GeV}$
- $\delta = 2.3\,\text{GeV}$
- $\delta = 3.2\,\text{GeV}$

**Hadrons**

**Herwig++ 1.0**
- $\Lambda_{QCD} = 125\,\text{MeV}$
- $\Lambda_{QCD} = 500\,\text{MeV}$
- $\delta = 1.7\,\text{GeV}$
- $\delta = 2.3\,\text{GeV}$
- $\delta = 3.2\,\text{GeV}$
- OPAL 99
Jet Multiplicity (PETRA, LEP, LEPII)

\( \sqrt{s} = \{35, 91.2, 189\} \text{ GeV} \)

\[
\langle n_{\text{jet}} \rangle = f_{35, 91.2, 189}\text{ GeV}
\]

Jet Multiplicity @ Next Linear Collider

Herwig++ and NLLA pQCD (Catani, Fiorani, Dokshitzer, Webber, 1992); jet events with $n_f = 5$. 

$\sqrt{s} = 0.5\,\text{TeV}$

$\sqrt{s} = 1.0\,\text{TeV}$

Hadrons

$\langle \eta_{\text{jet}} \rangle$ vs $y_{\text{cut}}$ for $\sqrt{s} = 0.5\,\text{TeV}$ and $\sqrt{s} = 1.0\,\text{TeV}$.

$\langle \eta_{\text{jet}} \rangle$ vs $\sqrt{s}$ for different energies.

$\langle \eta_{\text{jet}} \rangle$ vs $k_{\perp \text{cut}}$ for $\sqrt{s} = 91.2\,\text{GeV}$, $189\,\text{GeV}$, $500\,\text{GeV}$, $1000\,\text{GeV}$.
Additional Complications in $pp$

- backward parton evolution
- soft underlying event
What’s next?

Near Future. . .

★ Initial state shower:
  ● Complete implementation and tests.

★ Refine $e^+e^-$:
  ● Full CKKW ME+PS matching.
  ● Precision tune to LEP data should be possible.

★ with IS and FS showers running:
  ● we can start to test Drell–Yan and jets in pp collisions.
  ● cross check with Tevatron data and finally make predictions for the LHC.
  ● Study of DIS possible.

★ Underlying Event.

★ Hadronic Decays: NEW! $\tau$–decays, Spin correlations (P Richardson).

★ New Ideas: soft gluons, improved shower algorithm, NLO, . . .

Schedule?

● Ready for Future Colliders!
Conclusion

We have completed a new event generator for $e^+e^-$ Annihilation:

**Herwig++ 1.0**

http://www.hep.phy.cam.ac.uk/theory/Herwig++